Critical points in holography

How general is holography?

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Appetizer

Quoted from the workshop webpage:

1. How general is holography?

To what extent do (previous) lessons rely on the particular constructions used to date? Are they tied to stringy effects and to string theory in particular, or are they general lessons for quantum gravity?

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Specific question addressed in this talk:

> Does holography apply only to unitary theories?

Focus on very special but generic phenomenon

Leitmotif: critical points

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Basic idea (generic):

$$(i\frac{\mathrm{d}}{\mathrm{d}t} - \omega_1)(i\frac{\mathrm{d}}{\mathrm{d}t} - \omega_2)\psi = 0$$

$\omega_{1,2}$ determined by parameters/coupling constants of theory

Two branches of solutions: $\psi_{1,2} \propto e^{-i\omega_{1,2}\,t}$

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$$\psi^{\log} \propto \lim_{\omega_2\to\omega_1} \frac{\psi_2 - \psi_1}{\omega_2 - \omega_1} \propto te^{-i\omega_1\, t} + \alpha\,\psi_1$$

Hamiltonian $H = i\frac{\mathrm{d}}{\mathrm{d}t}$ acquires Jordan-block structure

$$H\begin{pmatrix} \psi^{\log} \\ \psi_1 \end{pmatrix} = \begin{pmatrix} \omega_1 & 1 \\ 0 & \omega_1 \end{pmatrix}\begin{pmatrix} \psi^{\log} \\ \psi_1 \end{pmatrix}$$

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Holographic version of critical points?

Outline

Gravity in three dimensions

Logarithmic CFTs

AdS$_3$/LCFT$_2$ correspondence

Generalizations

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Gravity in three dimensions

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$AdS_3/LCFT_2$ correspondence

Generalizations

Motivations for studying gravity in 3 dimensions

- Quantum gravity
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ...
  - Integrable models: powerful tools in physics (Coulomb problem, Hydrogen atom, harmonic oscillator, ...)
  - Models should be as simple as possible, but not simpler
- Gauge/gravity duality
  - Deeper understanding of black hole holography
  - $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence best understood
  - Quantum gravity via AdS/CFT? (Witten '07, Li, Song, Strominger '08)
  - Applications to 2D condensed matter systems?
  - Gauge gravity duality beyond standard AdS/CFT: warped AdS, asymptotic Schrödinger/Lifshitz, non-relativistic CFTs, flat space holography, higher spin holography, logarithmic CFTs, ...
- Physics
  - Cosmic strings (Deser, Jackiw, 't Hooft '84, '92)
  - Black hole analog systems in condensed matter physics (graphene, BEC, fluids, ...)

Topologically massive gravity

Deser, Jackiw & Templeton '82

$$I_{\mathrm{TMG}} = \frac{1}{16\pi\, G} \int \mathrm{d}^3x\sqrt{-g}\,\Big[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\,\Gamma^\rho{}_{\lambda\sigma}\big(\partial_\mu\Gamma^\sigma{}_{\nu\rho} + \frac{2}{3}\,\Gamma^\sigma{}_{\mu\tau}\Gamma^\tau{}_{\nu\rho}\big)\Big]$$

Properties:

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- Interesting scaling limits ($\mu\to\infty$, $\mu\to 0$, $\mu\ell\to 1$, $\ell\to\infty$, ...)
- Black hole solutions, massive gravitons

Equations of motion:

$$R_{\mu\nu}-\frac{1}{2}\,g_{\mu\nu}R-\frac{1}{\ell^2}\,g_{\mu\nu}+\frac{1}{\mu}\,C_{\mu\nu}=0$$

Cotton tensor

$$C_{\mu\nu}=\frac{1}{2}\,\varepsilon_\mu{}^{\alpha\beta}\nabla_\alpha R_{\beta\nu}+(\mu\leftrightarrow\nu)$$

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Conformal field theories in two dimensions

- Any CFT has conserved traceless energy momentum tensor (EMT)

$$T_{z\bar{z}} = 0 \qquad T_{zz} = \mathcal{O}^L(z) \qquad T_{\bar{z}\bar{z}} = \mathcal{O}^R(\bar{z})$$

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$$\langle \mathcal{O}^L(z)\, \mathcal{O}^L(0) \rangle = \frac{c_L}{2z^4} \qquad \langle \mathcal{O}^R(\bar{z})\, \mathcal{O}^R(0) \rangle = \frac{c_R}{2\bar{z}^4}$$

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- Minimal models:

$$c_{(p,q)} = 1 - 6\frac{(p-q)^2}{pq} \qquad \Rightarrow c_{(3,2)} = 0$$

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- Concrete $c = 0$ model exist where partition function is not trivial: percolation, self-avoiding polymers, $O(n)$ model in $n \to 0$ limit, systems with quenched disorder, ...
- Cannot be described by minimal (unitary) models

The $c = 0$ catastrophe

Primary field $\mathcal{O}^M$ with conformal weights $(h, \bar{h})$:

$$\langle \mathcal{O}^M(z\,,\bar{z})\mathcal{O}^M(0\,,0)\rangle = \frac{A}{2z^{2h}\,\bar{z}^{2\bar{h}}}$$

OPE:

$$\mathcal{O}^M(z\,,\bar{z})\mathcal{O}^M(0\,,0) \sim \frac{A}{2z^{2h}\,\bar{z}^{2\bar{h}}}\Big(1 + \frac{2h}{c_L}\,z^2\mathcal{O}^L(0) + \dots\Big)$$

Problem: divergence for $c_L \to 0$

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Possible resolutions in limit $c_L \to 0$:

- weight vanishes $h \to 0$
- normalization vanishes $A \to 0$
- other operator(s) arise with $h \to 2$, which cancel divergence

Focus on last possibility

Correlators in logarithmic conformal field theories (LCFTs)

Aghamohammadi, Khorrami & Rahimi Tabar '97; Kogan & Nichols '01; Rasmussen '04

Suppose now that primary has conformal weights $(2+\varepsilon\,,\varepsilon)$:

$$\langle \mathcal{O}^M(z,\bar z)\,\mathcal{O}^M(0,0)\rangle = \frac{A}{2z^{4+2\varepsilon}\,\bar z^{2\varepsilon}}$$

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Obtain 2-point correlators:

$$\langle\mathcal{O}^L(z)\mathcal{O}^L(0,0)\rangle = 0$$

$$\langle\mathcal{O}^L(z)\mathcal{O}^{\log}(0,0)\rangle = \frac{b_L}{2z^4}$$

$$\langle\mathcal{O}^{\log}(z,\bar z)\mathcal{O}^{\log}(0,0)\rangle = -\frac{b_L\,\ln(m^2|z|^2)}{z^4}$$

Critical points and Jordan cells

In terms of leitmotif example:

$$\mathcal{O}^L \sim \psi_1 \qquad \mathcal{O}^M \sim \psi_2 \qquad \mathcal{O}^{\log} \sim \psi^{\log}$$

Log partner $\mathcal{O}^{\log}$ of EMT has same conformal weights as EMT

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> If EMT acquires log partner Hamiltonian cannot be diagonalized

$$H \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O}^L \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \mathcal{O}^{\log} \\ \mathcal{O}^L \end{pmatrix}$$

> Appearance of Jordan cell = defining feature of LCFTs

Note: Jordan cell can be higher rank than 2, but consider only rank 2 case here

LCFTs: Gurarie '93 Reviews on LCFTs: Flohr '01; Gaberdiel '01

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Is there a gravity side of the LCFT story?

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2008-2010:

All items above work for TMG at critical point $\mu\ell = 1$!

Critical point in TMG

Central charges in TMG (Kraus & Larsen '05):

$$c_L = \frac{3\ell}{2G}\Big(1 - \frac{1}{\mu\ell}\Big) \qquad c_R = \frac{3\ell}{2G}\Big(1 + \frac{1}{\mu\ell}\Big)$$

TMG at the critical point is TMG with the tuning

$$\mu\,\ell = 1$$

between the cosmological constant and the Chern–Simons coupling.
Why special? (Li, Song & Strominger '08)

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Interesting possibilities:

- Dual CFT could be chiral (Li, Song & Strominger '08)
- Dual CFT could be logarithmic (DG & Johansson '08)

Checks of LCFT conjecture
Jordan cell structure

Linearization around AdS background, $g = g^{\text{AdS}} + h$ leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\,C^{(1)}_{\mu\nu} = (\mathcal{D}^R\mathcal{D}^L\mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta^\nu_\mu \pm \ell\,\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta^\nu_\mu + \frac{1}{\mu}\varepsilon_\mu{}^{\alpha\nu}\bar{\nabla}_\alpha$$

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Three linearly independent solutions to (1):

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At critical point left (L) and massive (M) branches coincide!

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Jordan cell structure

Linearization around AdS background, $g = g^{\mathrm{AdS}} + h$ leads to linearized EOM that are third order PDE

$$G^{(1)}_{\mu\nu} + \frac{1}{\mu}\, C^{(1)}_{\mu\nu} = (\mathcal{D}^R \mathcal{D}^L \mathcal{D}^M h)_{\mu\nu} = 0 \tag{1}$$

with three mutually commuting first order operators

$$(\mathcal{D}^{L/R})_\mu{}^\nu = \delta^\nu_\mu \pm \ell\, \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha \qquad (\mathcal{D}^M)_\mu{}^\nu = \delta^\nu_\mu + \frac{1}{\mu} \varepsilon_\mu{}^{\alpha\nu} \bar{\nabla}_\alpha$$

At critical point: get log solution (DG & Johansson '08)

$$h^{\log}_{\mu\nu} = \lim_{\mu\ell \to 1} \frac{h^M_{\mu\nu}(\mu\ell) - h^L_{\mu\nu}}{\mu\ell - 1}$$

with property

$$\big(\mathcal{D}^L h^{\log}\big)_{\mu\nu} = \big(\mathcal{D}^M h^{\log}\big)_{\mu\nu} \neq 0\,, \qquad \big((\mathcal{D}^L)^2 h^{\log}\big)_{\mu\nu} = 0$$

Checks of LCFT conjecture

Jordan cell structure

Log mode exhibits interesting property (DG & Johansson '08):

$$H \begin{pmatrix} h^{\log} \\ h^L \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} h^{\log} \\ h^L \end{pmatrix}$$

Here $H = L_0 + \bar{L}_0 \sim \partial_t$ is the Hamilton operator.

Such a Jordan form of H is defining property of a logarithmic CFT!

Jordan-block structure was main motivation for LCFT conjecture

Checks of LCFT conjecture
Finiteness

Properties of logarithmic mode:

- Perturbative solution of linearized EOM, but not pure gauge
- Energy of logarithmic mode is finite

$$E^{\log} = -47/1152G\,\ell^3$$

 and negative $\to$ instability (DG & Johansson '08)
- Logarithmic mode is asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(\gamma_{ij}^{(0)} e^{2\rho/\ell} + \gamma_{ij}^{(1)} \rho + \gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} e^{-2\rho/\ell} + \dots\right) \mathrm{d}x^i \,\mathrm{d}x^j$$

 but violates Brown–Henneaux boundary conditions! ($\gamma_{ij}^{(1)}\big|_{\mathrm{BH}} = 0$)
- Consistent log boundary conditions replacing Brown–Henneaux (DG & Johansson '08, Martinez, Henneaux & Troncoso '09)
- Brown–York stress tensor is finite, conserved and traceless, but not chiral (Martinez, Henneaux & Troncoso '09, Maloney, Song & Strominger '09, Ertl, DG & Johansson '09)
- Log mode persists non-perturbatively, as shown by Hamilton analysis (DG, Jackiw & Johansson '08, Carlip '08)

Checks of LCFT conjecture

Correlators

If LCFT conjecture is correct then following procedure must work:

- Calculate non-normalizable modes for left, right and logarithmic branches by solving linearized EOM on gravity side
- According to $\mathrm{AdS}_3/\mathrm{LCFT}_2$ dictionary these non-normalizable modes are sources for corresponding operators in the dual CFT
- Calculate 2- and 3-point correlators on the gravity side, e.g. by plugging non-normalizable modes into second and third variation of the on-shell action
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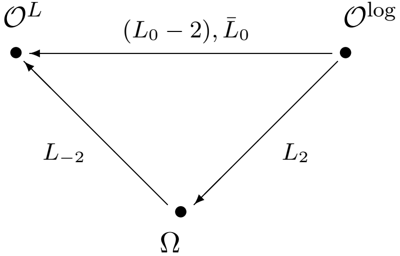
- Works at level of 2-point correlators (Skenderis, Taylor & van Rees '09, DG & Sachs '09)
- Works at level of 3-point correlators (DG & Sachs '09)
- Value of new anomaly: $b_L = -c_R = -3\ell/G$

Checks of LCFT conjecture

1-loop partition function (Gaberdiel, DG & Vassilevich '10)

Structure of low-lying states in LCFT:



Total partition function of Virasoro descendants

$$Z^0_{\mathrm{LCFT}} = Z_\Omega + Z_{\log} = \prod_{n=2}^{\infty} \frac{1}{|1-q^n|^2} \Big(1 + \frac{q^2}{|1-q|^2}\Big)$$

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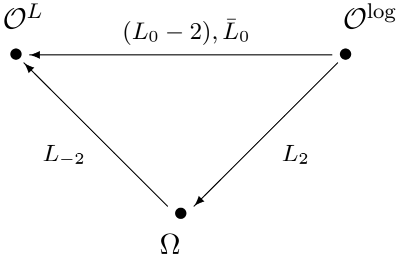

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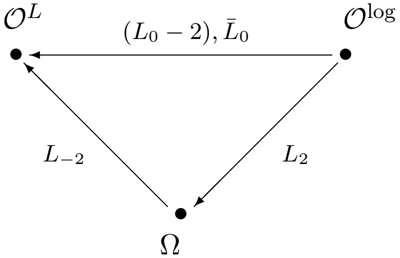

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$$Z_{\text{TMG}} = Z^0_{\text{LCFT}} + \sum_{h,\bar{h}} N_{h,\bar{h}}\, q^h \bar{q}^{\bar{h}} \prod_{n=1}^{\infty} \frac{1}{|1-q^n|^2}$$

All multiplicity coefficients $N_{h,\bar{h}}$ can be shown to be non-negative.
Fairly non-trivial test of the LCFT conjecture!

Checks of LCFT conjecture

Quasi-normal modes (Solodukhin & Sachs '08, Sachs '08)

- Birmingham, Sachs, Solodukhin '01:
  One-to-one correspondence between poles of retarded propagator in CFT and quasi-normal frequencies of linear perturbations of BTZ
- LCFT should have double pole instead of single pole due to degeneration of operators at critical point
- Sachs '08:
  TMG at critical point has standard right-moving QNM, but no left-moving QNM (pure gauge)
- Additional QNM from log modes
- Linear dependence in time of log modes produces the predicted double pole

> QNM spectrum compatible with LCFT conjecture:
> Double poles in retarded correlators

Outline

Gravity in three dimensions

Logarithmic CFTs

$AdS_3/LCFT_2$ correspondence

Generalizations

Summary and generalizations

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- 3D: generic massive gravity theories (DG, Johansson, Zojer '10)
  - New massive gravity, generalized massive gravity, ...
  - higher rank Jordan cells possible in some of these models
  - qualitatively new LCFTs may arise: no log partner of EMT

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- String construction leading to critical gravity?
  Perhaps possible in ABJM context in higgsed phase (Chu & Nilsson '09, Chu, Nastase, Nilsson & Papageorgakis '10)

Thanks to my collaborators ...
... and thanks for your attention!

Vienna group, March 2012

KITP collaboration, May 2012

LCFT: N. Johansson, R. Jackiw, I. Sachs, O. Hohm, M. Gaberdiel, D. Vassilevich, T. Zojer, S. Ertl, M. Bertin
conformal CS holography: H. Afshar, B. Cvetkovic, N. Johansson, S. Ertl
higher spin holography: M. Gary, R. Rashkov
flat space holography: A. Bagchi, S.Detournay
other holographic aspects: J. Aparicio, E. Lopez, I. Papadimitriou, S. Stricker

Critical points and Jordan cells in quantum mechanics

See "Non-Hermitian quantum mechanics" by Nimrod Moiseyev

Consider the Hamiltonian

$$H = \begin{pmatrix} 1 & \lambda \\ \lambda & -1 \end{pmatrix}$$

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Similarity trafo $J = A^{-1} H A$:

$$J = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Simplest example of Jordan cell in non-hermitian critical quantum mechanics!

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- Idea: Apply $\mathrm{AdS}_3/\mathrm{LCFT}_2$ to describe strongly coupled LCFTs!

Some literature on condensed matter applications of LCFTs

- Cardy '99 Logarithmic correlations in Quenched Random Magnets and Polymers
- Gurarie & Ludwig '99 Conformal algebras of 2D disordered systems
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More applications awaiting to be discovered!