Flat space higher spin gravity

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Dutch String meeting, Groningen, February 2015

based on work w. Afshar, Bagchi, Basu, Detournay, Fareghbal, Gary, Riegler, Rosseel, Salzer, Sarkar, Schöller, Simon, ...

Outline

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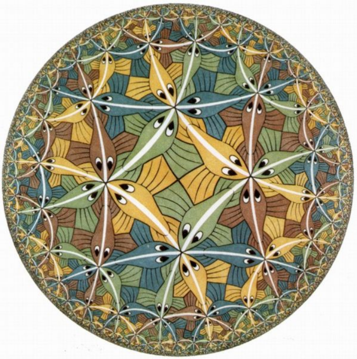
  - Holographic principle realized in Nature? (yes/no)



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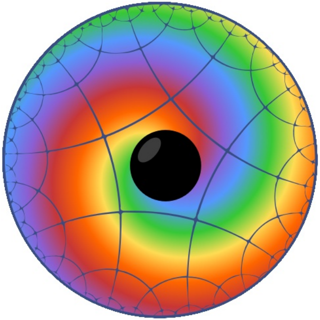

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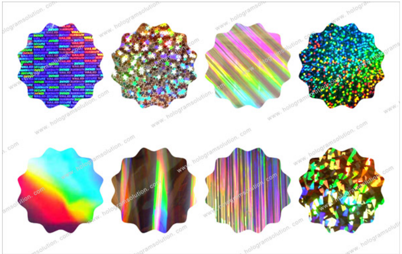

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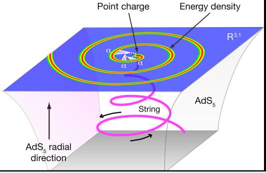
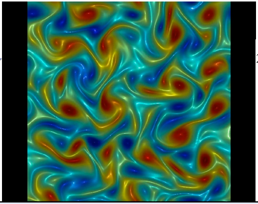
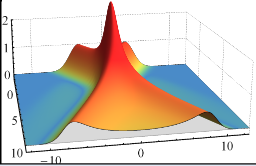
- Applications
  - Gauge gravity correspondence (plasmas, condensed matter, ...)



Specific motivation for 3D

Gravity in 3D is simpler than in higher dimensions

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Specific motivation for flat space higher spin gravity

Massless higher spin theories constrained by no-gos!

- Coleman–Mandula '67
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- Weinberg–Witten '80
- recent summary: Bekaert, Boulanger, Sundell '12

Conclusion: there are no consistent interacting massless higher spin theories in 4- (or higher-) dimensional flat space

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- Vasiliev '87-'90: higher spin theories in (A)dS

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- Vasiliev '87-'90: higher spin theories in (A)dS
- Afshar, Bagchi, Fareghbal, DG, Rosseel '13; Gonzalez, Matulich, Pino, Troncoso '13: flat space higher spin theories in 3d

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Address these issues in 3D!

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Holography basics

Flat space gravity

Flat space higher spin gravity

Gravity in 3D
AdS_3 gravity

- Lowest dimension with black holes and (off-shell) gravitons

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Interesting generic constraints from CFT_2!
e.g. Hellerman '09, Hartman, Keller, Stoica '14

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Caveat: while there are many string compactifications with AdS_3 factor, applying holography just to AdS_3 factor does not capture everything!

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
   Example: Einstein gravity with Dirichlet boundary conditions

$$I = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{|g|}\,\big(R+\frac{2}{\ell^2}\big)$$

   with $\delta g = \text{fixed}$ at the boundary

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
   Example: asymptotically AdS

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \big(e^{2\rho/\ell}\,\gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} + \dots\big)\,\mathrm{d}x^i\,\mathrm{d}x^j$$

   with $\delta\gamma^{(0)} = 0$ for $\rho \to \infty$

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
   - Find and classify all constraints
   - Construct canonical gauge generators
   - Add boundary terms and get (variation of) canonical charges
   - Check integrability of canonical charges
   - Check finiteness of canonical charges
   - Check conservation (in time) of canonical charges
   - Calculate Dirac bracket algebra of canonical charges

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   Example: Brown–Henneaux analysis for 3D Einstein gravity

$$\{Q[\varepsilon],\, Q[\eta]\} = \delta_\varepsilon Q[\eta]$$

$$Q[\varepsilon] \sim \oint \mathrm{d}\varphi\, \mathcal{L}(\varphi)\varepsilon(\varphi)$$

$$\delta_\varepsilon \mathcal{L} = \mathcal{L}\,\varepsilon + 2\mathcal{L}\,\varepsilon' + \frac{\ell}{16\pi G_N}\,\varepsilon'''$$

Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc's
4. Derive (classical) asymptotic symmetry algebra and central charges
   Example: Two copies of Virasoro algebra

$$[\mathcal{L}_n,\, \mathcal{L}_m] = (n-m)\,\mathcal{L}_{n+m} + \frac{c}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

   with Brown–Henneaux central charge

$$c = \frac{3\ell}{2G_N}$$

   Reminder: ASA = quotient algebra of asymptotic symmetries by 'trivial' asymptotic symmetries with zero canonical charges

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5. Improve to quantum ASA
   Example: semi-classical ASA in spin-3 gravity (Henneaux, Rey '10; Campoleoni, Pfenninger, Fredenhagen, Theisen '10)

$$[W_n, W_m] = \frac{16}{5c} \sum_p L_p L_{n+m-p} + \dots$$

   quantum ASA

$$[W_n, W_m] = \frac{16}{5c+22} \sum_p : L_p L_{n+m-p} : + \dots$$

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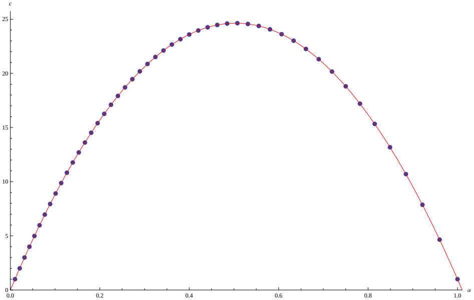

Afshar et al '12

Discrete set of Newton constant values compatible with unitarity
(3D spin-N gravity in next-to-principal embedding)

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6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
   Example: Monster CFT in (flat space) chiral gravity
   Witten '07
   Li, Song & Strominger '08
   Bagchi, Detournay & DG '12

$$Z(q) = J(q) = \frac{1}{q} + (1 + 196883)\, q + \mathcal{O}(q^2)$$

Note: $\ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections}$

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   Examples: too many!

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Goal of this talk:

Apply algorithm above to flat space holography in 3D higher spin theories

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- Example where it does not work at all: highest weight conditions!

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Interesting example:

- unitarity of flat space quantum gravity

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Many open issues in flat space holography!

(Higher spin) gravity as Chern–Simons gauge theory...

...with weird boundary conditions (Achucarro & Townsend '86; Witten '88; Bañados '96)

CS action (for AdS: $\mathrm{sl}(2) \oplus \mathrm{sl}(2)$):

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int \mathrm{CS}(A) - \frac{k}{4\pi} \int \mathrm{CS}(\bar{A})$$

Variational principle:

$$\delta S_{\mathrm{CS}}|_{\mathrm{EOM}} = \frac{k}{4\pi} \int \mathrm{Tr}\, \big(A \wedge \delta A - \bar{A} \wedge \delta \bar{A}\big)$$

Well-defined for boundary conditions (similarly for $\bar{A}$)

$$A_+ = 0 \qquad \text{or} \qquad A_- = 0 \qquad \text{boundary coordinates } x^\pm$$

Example: asymptotically AdS_3 (Cartan-version of Brown–Henneaux)

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Example: asymptotically AdS$_3$ (Cartan-version of Brown–Henneaux)

$$A_\rho = L_0 \qquad \bar{A}_\rho = -L_0$$

$$A_+ = e^\rho\, L_1 + e^{-\rho}\, L(x^+)\, L_{-1} \qquad \bar{A}_+ = 0$$

$$A_- = 0 \qquad \bar{A}_- = -e^\rho\, L_{-1} - e^{-\rho}\, \bar{L}(x^-)\, L_1$$

Dreibein: $e/\ell \sim A - \bar{A}$, spin-connection: $\omega \sim A + \bar{A}$

İnönü–Wigner contraction of Virasoro (Barnich & Compère '06)

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- Take two copies of Virasoro, generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$, central charges c, $\bar{c}$

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BMS_3 and GCA_2 (or rather, URCA_2)

- Take two copies of Virasoro, generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$, central charges c, $\bar{c}$
- Define superrotations L_n and supertranslations M_n

$$L_n := \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n := \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

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BMS_3 and GCA_2 (or rather, URCA_2)

- Take two copies of Virasoro, generators $\mathcal{L}_n$, $\bar{\mathcal{L}}_n$, central charges c, $\bar{c}$
- Define superrotations L_n and supertranslations M_n

$$L_n := \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n := \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

- Make ultrarelativistic boost, $\ell \to \infty$

$$\begin{aligned}
[L_n,\, L_m] &= (n-m)\, L_{n+m} + c_L\, \tfrac{1}{12}\, \delta_{n+m,\,0} \\
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[M_n,\, M_m] &= 0
\end{aligned}$$

Global part: contracted to isl(2) (generators: $L_{\pm 1}, L_0$, $M_{\pm 1}, M_0$)

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Example TMG (with gravitational CS coupling μ and Newton constant G):

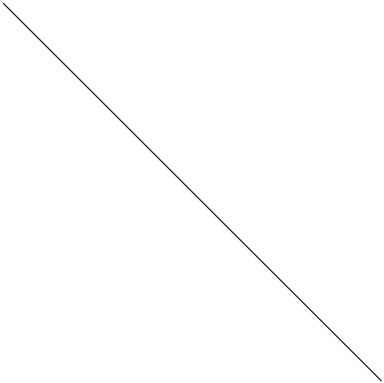
$$c_L = \frac{3}{\mu G} \qquad c_M = \frac{3}{G}$$

Consequence of ultrarelativistic boost for AdS boundary

AdS-boundary:

Limit $\ell \to \infty$

Flat space boundary:



Null infinity holography!

Contraction on gravity side

AdS metric ($\varphi \sim \varphi + 2\pi$):

$$\mathrm{d}s^2_{\text{AdS}} = \mathrm{d}(\ell\rho)^2 - \cosh^2\big(\tfrac{\ell\rho}{\ell}\big)\ \mathrm{d}t^2 + \ell^2\ \sinh^2\big(\tfrac{\ell\rho}{\ell}\big)\ \mathrm{d}\varphi^2$$

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Limit $\ell \to \infty$ ($r = \ell\rho$):

$$\mathrm{d}s^2_{\mathrm{Flat}} = \mathrm{d}r^2 - \mathrm{d}t^2 + r^2\ \mathrm{d}\varphi^2 = -\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + r^2\ \mathrm{d}\varphi^2$$

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BTZ metric:

$$\mathrm{d}s^2_{\mathrm{BTZ}} = -\frac{(\frac{r^2}{\ell^2} - \frac{r_+^2}{\ell^2})(r^2 - r_-^2)}{r^2}\ \mathrm{d}t^2 + \frac{r^2\ \mathrm{d}r^2}{(\frac{r^2}{\ell^2} - \frac{r_+^2}{\ell^2})(r^2 - r_-^2)} + r^2\,\big(\mathrm{d}\varphi - \frac{\frac{r_+}{\ell}\,r_-}{r^2}\ \mathrm{d}t\big)^2$$

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Limit $\ell \to \infty$ ($\hat{r}_+ = \frac{r_+}{\ell} = \text{finite}$):

$$\mathrm{d}s^2_{\text{FSC}} = \hat{r}_+^2\ \big(1 - \tfrac{r_-^2}{r^2}\big)\ \mathrm{d}t^2 - \frac{1}{1 - \frac{r_-^2}{r^2}}\ \frac{\mathrm{d}r^2}{\hat{r}_+^2} + r^2\ \Big(\mathrm{d}\varphi - \frac{\hat{r}_+\,r_-}{r^2}\ \mathrm{d}t\Big)^2$$

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Shifted-boost orbifold studied by Cornalba & Costa more than decade ago

Describes expanding (contracting) Universe in flat space

Cosmological horizon at $r = r_-$, screening CTCs at $r < 0$

Outline

Flat space higher spin gravity

Afshar, Bagchi, Fareghbal, DG, Rosseel '13, Gonzalez, Matulich, Pino, Troncoso '13

- AdS gravity in CS formulation: spin 2 $\to$ spin 3 $\sim$ sl$(2) \to$ sl(3)

Flat space higher spin gravity

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- Flat space: similar!

$$S_{\text{CS}}^{\text{flat}} = \frac{k}{4\pi} \int \text{CS}(\mathcal{A})$$

with isl(3) connection ($e^a =$ "zuvielbein")

$$\mathcal{A} = e^a T_a + \omega^a J_a \qquad T_a = (M_n,\, V_m) \quad J_a = (L_n,\, U_m)$$

isl(3) algebra (spin 3 extension of global part of BMS/GCA algebra)

$$\begin{aligned}
[L_n,\, L_m] &= (n-m) L_{n+m} \\
[L_n,\, M_m] &= (n-m) M_{n+m} \\
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[M_n,\, U_m] = [L_n,\, V_m] &= (2n-m) V_{n+m} \\
[U_n,\, U_m] &= (n-m)(2n^2+2m^2-nm-8) L_{n+m} \\
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- Spin 3 charges:

$$Q[\varepsilon_M,\, \varepsilon_L,\, \varepsilon_V,\, \varepsilon_U] \sim \oint \left(\varepsilon_M(\varphi) M(\varphi) + \varepsilon_L(\varphi) L(\varphi) + \varepsilon_V(\varphi) V(\varphi) + \varepsilon_U(\varphi) U(\varphi) \right)$$

Flat space higher spin gravity

Asymptotic symmetry algebra at finite level k Afshar, Bagchi, Fareghbal, DG, Rosseel '13

- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W_3-algebra

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- Do either Brown–Henneaux type of analysis or İnönü–Wigner contraction of two copies of quantum W_3-algebra
- Obtain new type of W-algebra as extension of BMS ("BMW")

$$[L_n,\, L_m] = (n-m)L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[L_n,\, M_m] = (n-m)M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[U_n,\, U_m] = (n-m)(2n^2+2m^2-nm-8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m} - \frac{96\big(c_L+\frac{44}{5}\big)}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

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$$\Lambda_n = \sum_p :L_p\,M_{n-p}: \, - \tfrac{3}{10}\,(n+2)(n+3)M_n \qquad \Theta_n = \sum_p M_p\,M_{n-p}$$

other commutators as in isl(3) with $n \in \mathbb{Z}$

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- Note quantum shift and poles in central terms!

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- Note quantum shift and poles in central terms!
- Analysis generalizes to flat space contractions of other W-algebras

Unitarity in flat space

Unitarity leads to further contraction DG, Riegler, Rosseel '14

Facts:

- Unitarity in GCA requires $c_M = 0$ (see paper for caveats!)

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- Generalization to contracted higher spin algebras straightforward

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- $c_M = 0$ is necessary for unitarity

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Limit $c_M \to 0$ requires further contraction: $U_n \to c_M\, U_n$

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Doubly contracted algebra has unitary representations:

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[U_n,\, U_m] \propto [U_n, V_m] &= 96(n-m)\sum_p M_p\, M_{n-p}
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Higher spin states decouple and become null states!

Unitarity in flat space

Generic flat space W-algebras DG, Riegler, Rosseel '14

1. NO–GO:

Generically (see paper) you can have only two out of three:

- Unitarity
- Flat space
- Non-trivial higher spin states

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Example:

Flat space chiral gravity

Bagchi, Detournay, DG, 1208.1658

Unitarity in flat space

Generic flat space W-algebras DG, Riegler, Rosseel '14

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Example:

Minimal model holography

Gaberdiel, Gopakumar, 1011.2986, 1207.6697

Unitarity in flat space

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Example:

Flat space higher spin gravity (Galilean W_3 algebra)

Afshar, Bagchi, Fareghbal, DG and Rosseel, 1307.4768

Gonzalez, Matulich, Pino and Troncoso, 1307.5651

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> Compatible with "spirit" of various no-go results in higher dimensions!

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2. YES–GO:

There is (at least) one counter-example, namely a Vasiliev-type of theory, where you can have all three properties!

> Unitary, non-trivial flat space higher spin algebra exists!
> Vasiliev-type flat space chiral higher spin gravity?

Unitarity in flat space

Flat space W_∞-algebra compatible with unitarity DG, Riegler, Rosseel '14

- We do not know if flat space chiral higher spin gravity exists...

Unitarity in flat space

Flat space W_∞-algebra compatible with unitarity DG, Riegler, Rosseel '14

- We do not know if flat space chiral higher spin gravity exists...
- ...but its existence is at least not ruled out by the no-go result!

Unitarity in flat space

Flat space W_∞-algebra compatible with unitarity DG, Riegler, Rosseel '14

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- AdS parent theory: no trace anomaly, but gravitational anomaly (Like in conformal Chern–Simons gravity $\rightarrow$ Vasiliev type analogue?)

Adding chemical potentials Gary, DG, Riegler, Rosseel ’14

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Line-element with spin-2 and spin-3 chemical potentials:

$$g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \Big(r^2\left(\mu_{\mathrm{L}}^2 - 4\mu_{\mathrm{U}}''\mu_{\mathrm{U}} + 3\mu_{\mathrm{U}}'^2 + 4\mathcal{M}\mu_{\mathrm{U}}^2\right) + r\,g_{uu}^{(r)} + g_{uu}^{(0)} + g_{uu}^{(0')}\Big)\,\mathrm{d}u^2 + \Big(r^2\mu_{\mathrm{L}} - r\mu_{\mathrm{M}}' + \mathcal{N}(1+\mu_{\mathrm{M}}) + 8\mathcal{Z}\mu_{\mathrm{V}}\Big)\,2\,\mathrm{d}u\,\mathrm{d}\varphi - (1+\mu_{\mathrm{M}})\,2\,\mathrm{d}r\,\mathrm{d}u + r^2\,\mathrm{d}\varphi^2$$

$$g_{uu}^{(0)} = \mathcal{M}(1+\mu_{\mathrm{M}})^2 + 2(1+\mu_{\mathrm{M}})(\mathcal{N}\mu_{\mathrm{L}} + 12\mathcal{V}\mu_{\mathrm{V}} + 16\mathcal{Z}\mu_{\mathrm{U}}) + 16\mathcal{Z}\mu_{\mathrm{L}}\mu_{\mathrm{V}} + \tfrac{4}{3}(\mathcal{M}^2\mu_{\mathrm{V}}^2 + 4\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}\mu_{\mathrm{V}} + \mathcal{N}^2\mu_{\mathrm{U}}^2)$$

Spin-3 field with same chemical potentials:

$$\Phi_{\mu\nu\lambda}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu\,\mathrm{d}x^\lambda = \Phi_{uuu}\,\mathrm{d}u^3 + \Phi_{ruu}\,\mathrm{d}r\,\mathrm{d}u^2 + \Phi_{uu\varphi}\,\mathrm{d}u^2\,\mathrm{d}\varphi - \left(2\mu_{\mathrm{U}}r^2 - r\mu_{\mathrm{V}}' + 2\mathcal{N}\mu_{\mathrm{V}}\right)\mathrm{d}r\,\mathrm{d}u\,\mathrm{d}\varphi + \mu_{\mathrm{V}}\,\mathrm{d}r^2\,\mathrm{d}u - \left(\mu_{\mathrm{U}}'r^3 - \tfrac{1}{3}r^2(\mu_{\mathrm{V}}'' - \mathcal{M}\mu_{\mathrm{V}} + 4\mathcal{N}\mu_{\mathrm{U}}) + r\mathcal{N}\mu_{\mathrm{V}}' - \mathcal{N}^2\mu_{\mathrm{V}}\right)\mathrm{d}u\,\mathrm{d}\varphi^2$$

$$\Phi_{uuu} = r^2\left[2(1+\mu_{\mathrm{M}})\mu_{\mathrm{U}}(\mathcal{M}\mu_{\mathrm{L}} - 4\mathcal{V}\mu_{\mathrm{U}}) - \tfrac{1}{3}\mu_{\mathrm{L}}^2(\mathcal{M}\mu_{\mathrm{V}} - 4\mathcal{N}\mu_{\mathrm{U}}) + 16\mu_{\mathrm{L}}\mu_{\mathrm{U}}(\mathcal{V}\mu_{\mathrm{V}} + \mathcal{Z}\mu_{\mathrm{U}}) - \tfrac{4}{3}\mathcal{M}\mu_{\mathrm{U}}^2(\mathcal{M}\mu_{\mathrm{V}} + 2\mathcal{N}\mu_{\mathrm{U}})\right] + 2\mathcal{V}(1+\mu_{\mathrm{M}})^3 + \tfrac{2}{3}(1+\mu_{\mathrm{M}})^2(6\mathcal{Z}\mu_{\mathrm{L}} + \mathcal{M}^2\mu_{\mathrm{V}} + 2\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}) + 16\mu_{\mathrm{L}}\mu_{\mathrm{V}}^2(\mathcal{N}\mathcal{V} - \tfrac{1}{3}\mathcal{M}\mathcal{Z}) + \tfrac{2}{3}(1+\mu_{\mathrm{M}})\big((\mathcal{N}\mu_{\mathrm{L}} + 16\mathcal{Z}\mu_{\mathrm{U}})(2\mathcal{M}\mu_{\mathrm{V}} + \mathcal{N}\mu_{\mathrm{U}}) + 12\mathcal{M}\mathcal{V}\mu_{\mathrm{V}}^2\big) + \tfrac{64}{3}\mathcal{Z}\mu_{\mathrm{U}}\mu_{\mathrm{V}}(\mathcal{N}\mu_{\mathrm{L}} + 12\mathcal{V}\mu_{\mathrm{V}} + 12\mathcal{Z}\mu_{\mathrm{U}}) + \mathcal{N}^2\mu_{\mathrm{L}}^2\mu_{\mathrm{V}} + 64\mathcal{V}^2\mu_{\mathrm{V}}^3 - \tfrac{8}{27}(\mathcal{M}^3\mu_{\mathrm{V}}^3 - \mathcal{N}^3\mu_{\mathrm{U}}^3) - \tfrac{4}{9}\mathcal{M}\mathcal{N}\mu_{\mathrm{U}}\mu_{\mathrm{V}}(4\mathcal{M}\mu_{\mathrm{V}} + 5\mathcal{N}\mu_{\mathrm{U}}) + \sum_{n=0}^{3} r^n\,\Phi_{uuu}^{(r^n)}$$

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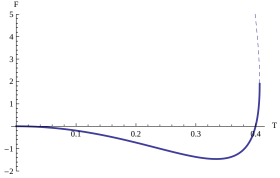
Interesting novel phase transitions of zeroth/first order:



Free energy of four branches of regular solutions as function of temperature for different values of higher spin chemical potential ratio (in AdS: see David, Ferlaino, Kumar '12)

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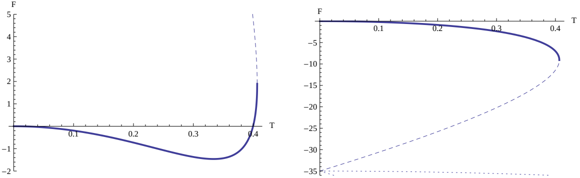

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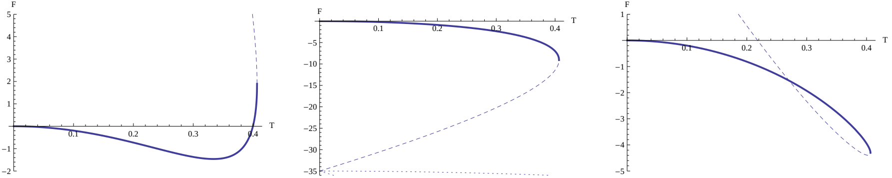

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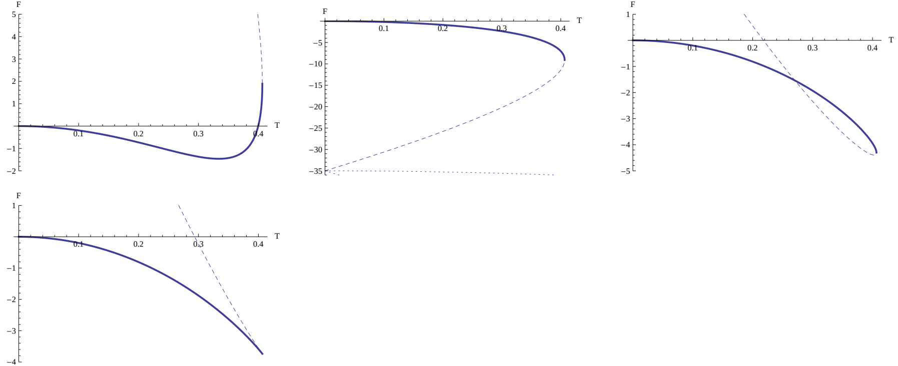

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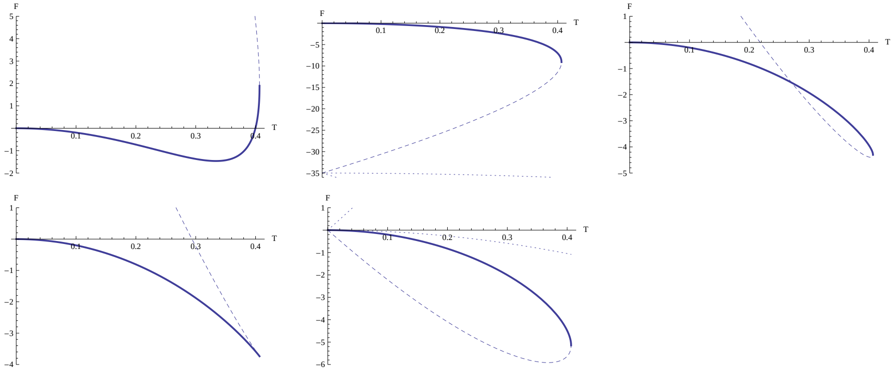

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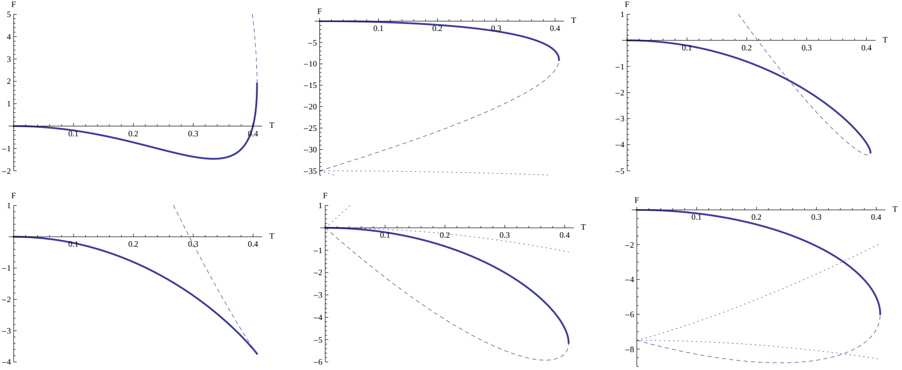

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Flat space higher spin holography provides a new playground
Contributes to long-term goal: find how general is holography

Thanks for your attention!