Black Holes I (136.028)

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Outlook on black hole research, January 2020

FWF

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Appetizer, Part I

Physics of the 20^{th} century: harmonic oscillator

Simple idea:

> Harmonic oscillator: take a physical system and shake it

Amazingly successful:

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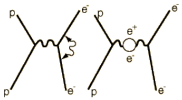

Feynman diagrams contributing to Lamb shift

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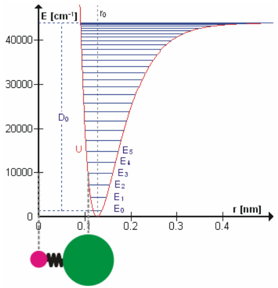

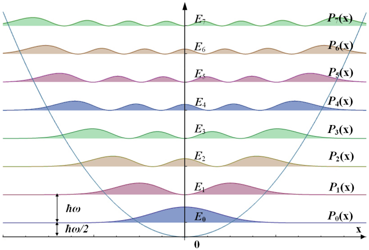

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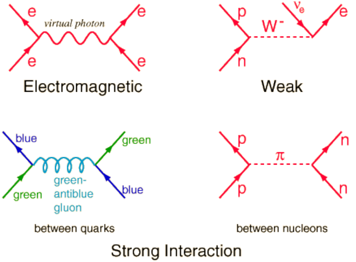

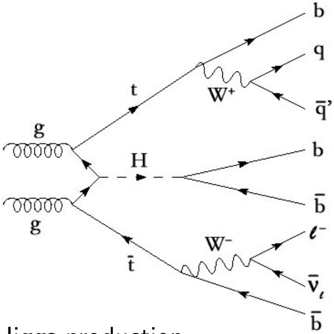

Higgs production

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- see also the TU Wien curriculum

Lectures at TU Wien containing harmonic oscillator (PP = particle physics)

- Intro to PP
- Atomic, nuclear & PP
- Intro to QFT
- Theor. methods in PP
- Astro-PP
- Black holes
- Intro to QED
- Field theory and pheno
- Path integrals
- Thermal field theory
- Cosmology
- various basic lectures

Appetizer, Part II

Physics of the 21$^{\mathrm{st}}$ century: black holes?

Application of harmonic oscillator limited to perturbative phenomena

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Physics of the 21$^{\rm st}$ century: black holes?

Application of harmonic oscillator limited to perturbative phenomena

Many physical systems require non-perturbative physics:

- QCD at low energies
- High T_c superconductors
- Graphene
- Cold atoms
- Gravity at high curvature

Generally speaking:

Strongly coupled systems require new techniques

Application of harmonic oscillator limited to perturbative phenomena

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Generally speaking:

Strongly coupled systems require new techniques

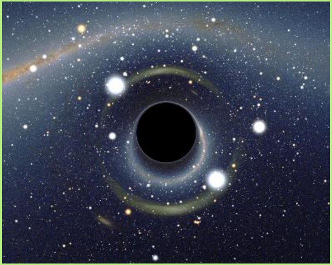
Punch-line of this outlook:

Black hole holography can provide such a technique

Appetizer, Part III

Black holes have apparently paradoxical properties

Black holes: The simplest macroscopic objects in the Universe



Properties determined by:

- Mass M
- Angular momentum J
- Charge(s) Q

Black hole $\sim$ elementary particle!

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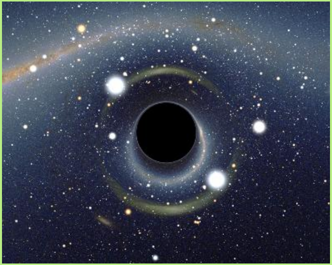

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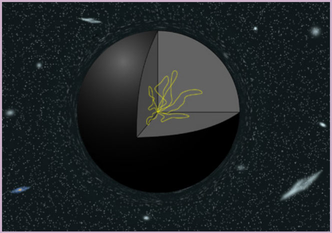
Black holes: The most complicated objects conceivable



Quantum mechanics:

- Black holes radiate
- Black holes have entropy
- Black holes are holographic

Bekenstein–Hawking entropy:

$$S_{\rm BH} \sim A/4$$

Outline

Black hole experiments

Black hole theory

Black hole holography

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Do black holes exist in our Universe? (image source: random webpage)

Ham's scho ans g'sehn? (Ernst Mach to Ludwig Boltzmann conerning the existence of atoms)

Artistic black hole binary impression

Do black holes exist in our Universe? (image source: 1502.03808)

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Numerical black hole simulation (interstellar)

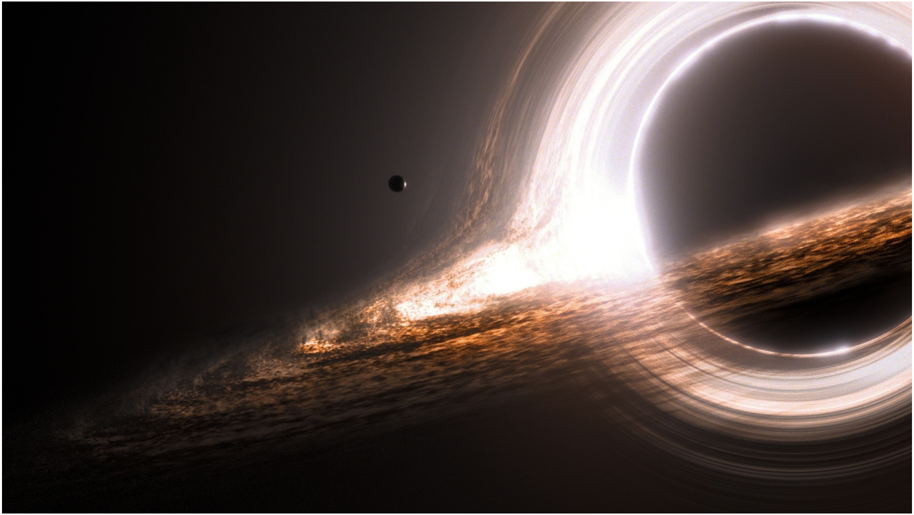

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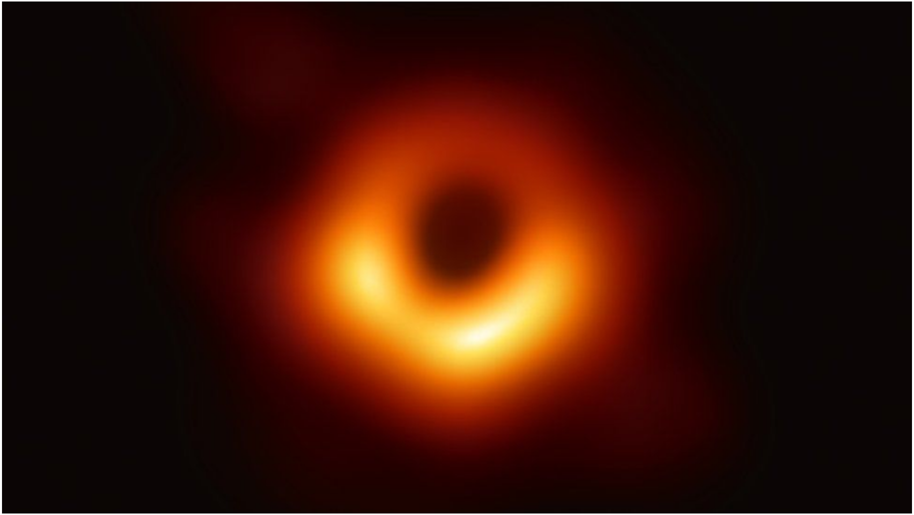
Photo of black hole M87 (EHT collaboration)



Pre-photographic evidence for black holes

"Seeing is believing" is infantile; why did people believe in black holes before the photo?

Experimental evidence for/against various black hole candidates:

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- stellar black holes: gravitational collapse Chandrasekhar 1930

  after fusion in star stops:
  Fermi pressure of electrons/neutrons prevents gravitational collapse

  critical mass from Fermi pressure = gravitational pressure
  rough estimate yields

  $$M_{\text{critical}} \sim \frac{1}{m_N^2} \sim 10^{38} \sim 10^{30} kg \sim M_\odot$$

  more refined calculation yields

  $$M_{\text{critical}} \approx 3 M_\odot$$

  stellar objects with mass $> 3M_\odot$ thus are black holes

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  Objects whose mass is clearly beyond critical $M > 3M_\odot$ (from '06):

System	$P_{\rm orb}$ [days]	$f(M)$ [$M_\odot$]	Donor Spect. Type	Classification	$M_{\rm x}$ [†] [$M_\odot$]
GRS 1915+105[a]	33.5	9.5 ± 3.0	K/M III	LMXB/Transient	14 ± 4
V404 Cyg	6.471	6.09 ± 0.04	K0 IV	,,	12 ± 2
Cyg X-1	5.600	0.244 ± 0.005	09.7 Iab	HMXB/Persistent	10 ± 3
LMC X-1	4.229	0.14 ± 0.05	07 III	,,	>4
XTE J1819-254	2.816	3.13 ± 0.13	B9 III	IMXB/Transient	7.1 ± 0.3
GRO J1655-40	2.620	2.73 ± 0.09	F3/5 IV	,,	6.3 ± 0.3
BW Cir[b]	2.545	5.74 ± 0.29	G5 IV	LMXB/Transient	>7.8
GX 339-4	1.754	5.8 ± 0.5	–	,,	
LMC X-3	1.704	2.3 ± 0.3	B3 V	HMXB/Persistent	7.6 ± 1.3
XTE J1550-564	1.542	6.86 ± 0.71	G8/K8 IV	LMXB/Transient	9.6 ± 1.2
4U 1543-475	1.125	0.25 ± 0.01	A2 V	IMXB/Transient	9.4 ± 1.0
H1705-250	0.520	4.86 ± 0.13	K3/7 V	LMXB/Transient	6 ± 2
GS 1124-684	0.433	3.01 ± 0.15	K3/5 V	,,	7.0 ± 0.6
XTE J1859+226[c]	0.382	7.4 ± 1.1	–	,,	
GS2000+250	0.345	5.01 ± 0.12	K3/7 V	,,	7.5 ± 0.3
A0620-003	0.325	2.72 ± 0.06	K4 V	,,	11 ± 2
XTE J1650-500	0.321	2.73 ± 0.56	K4 V	,,	
GRS 1009-45	0.283	3.17 ± 0.12	K7/M0 V	,,	5.2 ± 0.6
GRO J0422+32	0.212	1.19 ± 0.02	M2 V	,,	4 ± 1
XTE J1118+480	0.171	6.3 ± 0.2	K5/M0 V	,,	6.8 ± 0.4

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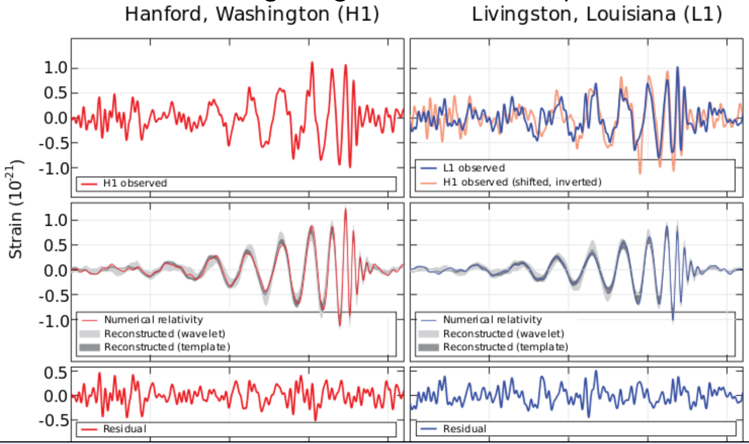

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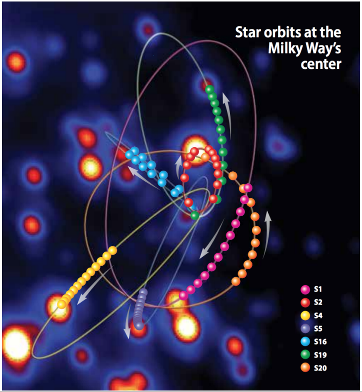

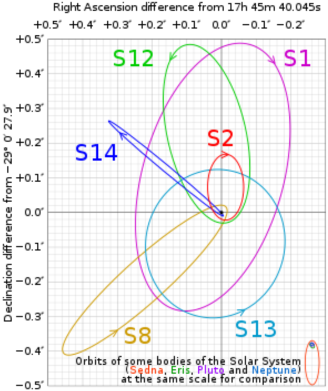

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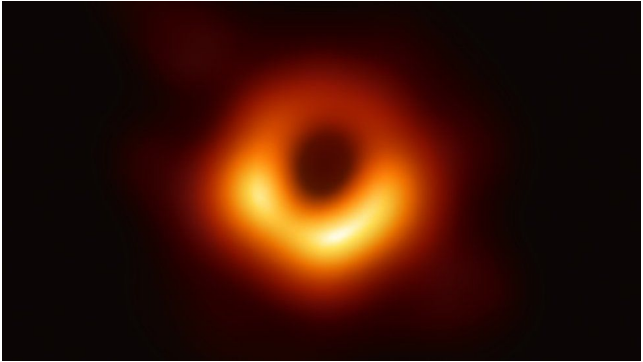

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> - overwhelming evidence for stellar and supermassive black holes
> - confirmed mass ranges so far: $3 - 100 M_\odot$ and $10^6 - 10^{10} M_\odot$
> - black holes could in principle exist for any mass $> M_{\text{Planck}}$

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Definition and observables of black holes

- defining property: black hole horizon

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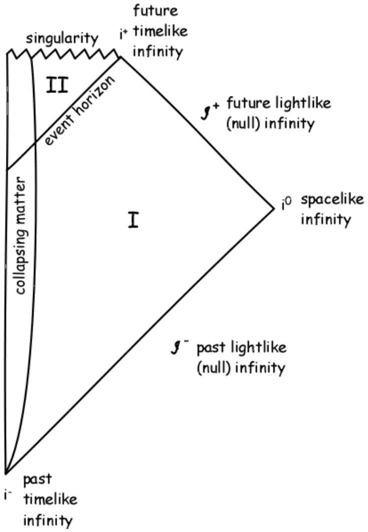

I outside region

II black hole

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  - electric monopole charge = charge Q (irrelevant so far in observations)
  - magnetic monopole charge = probably irrelevant
  - electric monopole mass = mass M
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> Essentially same observables as for elementary particles:
> charge Q, mass M, spin $J \Rightarrow$ amazingly simple!

Thermodynamics and black holes — black hole thermodynamics?

Thermodynamics

Zeroth law:

$T =$ const. in equilibrium

T: temperature

Black hole mechanics

Zeroth law:

$\kappa =$ const. f. stationary black holes

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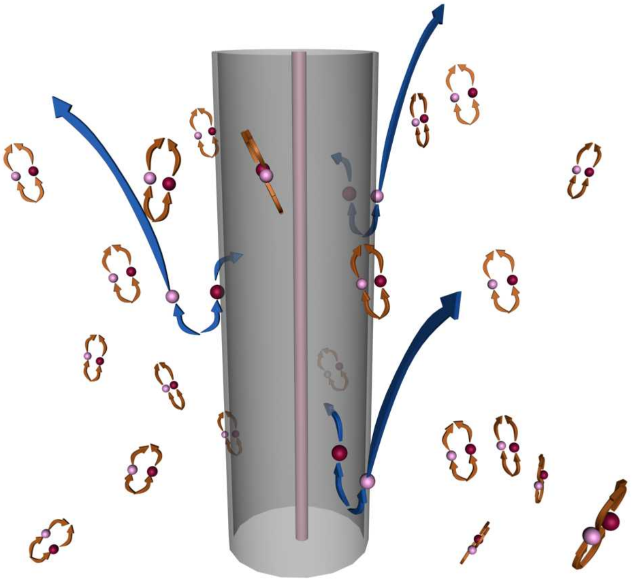
Formal analogy or actual physics?

Hawking effect (QFT under external conditions)

Black holes evaporate due to semi-classical effects!



General black holes:

$$T_{\rm H}=\frac{\kappa}{2\pi}$$

$$\boldsymbol{S_{\rm BH}=\frac{A}{4}}$$

Schwarzschild (SI units):

$$T_{\rm H}=\frac{\hbar c^3}{8\pi G_N k_B M}$$

$$S_{\rm BH}=\frac{c^3 A}{4G_N\hbar}$$

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Currently template for experimental data in quantum gravity

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Holographic principle 't Hooft '93; Susskind '95

Gravity in d spatial dimensions equivalent to QFT in $d-1$ spatial dimensions

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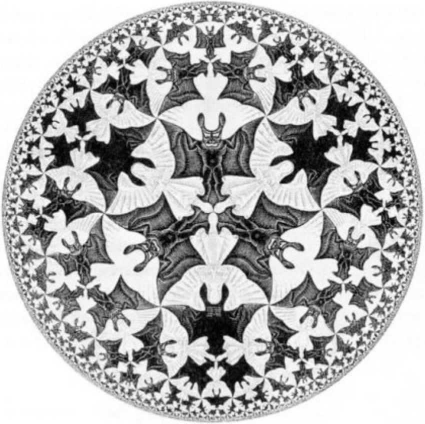
WTF? How can this be true?

Anti-de Sitter/conformal field theory (AdS/CFT) heuristics

Maldacena's `hep-th/9711200` currently has about 15-thousand citations

Best studied realization of holography is AdS/CFT correspondence:

- AdS is a negatively curved spacetime (maximally symmetric)



Open Universe Looking from inside, boundary at infinity
Limit Circle IV, by M. C. Escher

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$$\mathrm{d}s^2 = (E/L)^2 \eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + (L/E)^2\,\mathrm{d}E^2$$

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L sets physical scales and is called "AdS-radius"

This is precisely the line element of AdS in 1 dimension higher!

AdS$_3$/CFT$_2$
Focus on symmetries

2d CFTs allow to apply powerful methods; let us focus on CFT$_2$

- symmetries of CFT$_d$: $SO(d,2)$ if $d > 2$
- symmetries of AdS$_{d+1}$: $SO(d,2)$ if $d > 1$

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$$[L_n,\, L_m] = (n-m)\, L_{n+m} + \frac{c}{12}\,\big(n^3-n\big)\,\delta_{n+m,\,0} \qquad n,m\in\mathbb{Z}$$

symmetry algebra is called 'Virasoro algebra with central charge c'

fineprint: there are actually two copies of this algebra; to reduce clutter we focus on one of them

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- how can this possibly be true in AdS_3?

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symmetry algebra is called 'Virasoro algebra with central charge c'

fineprint: there are actually two copies of this algebra; to reduce clutter we focus on one of them

- how can this possibly be true in AdS_3?
- how can any gravity theory have an infinite amount of symmetries?!

at most $D(D+1)/2$ Killing vectors ξ in D spacetime dimensions

$$\mathcal{L}_\xi g_{\mu\nu} = \xi^\alpha\partial_\alpha g_{\mu\nu} + g_{\mu\alpha}\partial_\nu\xi^\alpha + g_{\nu\alpha}\partial_\mu\xi^\alpha = 0$$

e.g. in $D=4$ for $g_{\mu\nu}=\eta_{\mu\nu}$ get $\xi^\alpha = \xi^\alpha_{(0)} + \Lambda^\alpha{}_\beta x^\beta =$ Poincaré trafos

2d CFTs allow to apply powerful methods; let us focus on CFT_2

- symmetries of CFT_d: $SO(d,2)$ if $d>2$
- symmetries of AdS_{d+1}: $SO(d,2)$ if $d>1$
- matches for $d>2$, but what about $d=2$?
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Last question was resolved already in 1960ies by Bondi, van der Burgh, Metzner and Sachs (BMS)

Asymptotic symmetries

The limit of general relativity at small curvature is special relativity. Right?

- consider metrics $g_{\mu\nu}$ that asymptote to Minkowski metric $\eta_{\mu\nu}$

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Two important lessons

1. BMS symmetries of relevance for QFTs (soft theorems)
2. more generally, gravity theories can have infinitely many symmetries

Back to $\text{AdS}_3/\text{CFT}_2$

- consider asymptotically AdS_3 metrics

$$g_{\mu\nu} = \bar{g}_{\mu\nu}^{\text{AdS}} + \delta g_{\mu\nu}$$

with suitable* choice of $\delta g_{\mu\nu}$

* if you care about details read Max Riegler's DKPI PhD thesis 1609.02733

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recall CFT_2 symmetries

$$[L_n,\, L_m] = (n-m)\,L_{n+m} + \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,\,0}$$

Almost Virasoro, so almost there — but central charge c missing

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- matches perfectly with symmetries of CFT_2!

Consequences of symmetry matching

What we have learned so far about AdS_3 is already impressive:

3d Einstein gravity with negative cosmological constant and Brown–Henneaux boundary conditions, if it exists, is dual to a CFT_2 since the physical Hilbert space falls into representations of two Virasoro algebras

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- CFT_2 correlation functions calculable on gravity side
  e.g. 5-point function of stress-tensor flux components

$$\mathrm{CFT}_2: \qquad \langle T_{++}(z_1)T_{++}(z_2)T_{++}(z_3)T_{++}(z_4)T_{++}(z_5)\rangle = \frac{4c\, g_5(\gamma,\,\zeta)}{\prod_{1\leq i\leq 5} z_{ij}}$$

$\gamma = z_{12}z_{34}/(z_{13}z_{24})$, $\zeta = z_{25}z_{34}/(z_{35}z_{24})$, $z_{ij} = z_i - z_j$ and

$$g_5(\gamma,\,\zeta) = \frac{\gamma+\zeta}{2(\gamma-\zeta)} - \frac{\gamma^2-\gamma\zeta+\zeta^2}{\gamma(\gamma-1)\zeta(\zeta-1)(\gamma-\zeta)}\big([\gamma(\gamma-1)+1][\zeta(\zeta-1)+1]-\gamma\zeta\big)$$

matches precisely with gravity side

$$\frac{\delta^5 I_{\mathrm{EH}}[g_{\mu\nu}]}{\delta g^{++}(z_1)\delta g^{++}(z_2)\delta g^{++}(z_3)\delta g^{++}(z_4)\delta g^{++}(z_5)} = \frac{4c\, g_5(\gamma,\,\zeta)}{\prod_{1\leq i\leq 5} z_{ij}}$$

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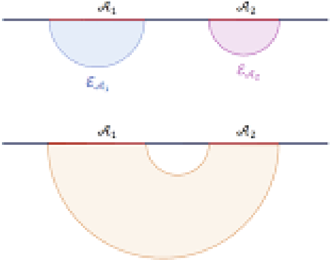

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Holographic dictionary between AdS_3 and CFT_2 observables!

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  example: information loss problem in quantum gravity mapped to information gain problem on CFT side

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  example: calculation of shear viscosity over entropy density at strong coupling mapped to gravitational wave absorption of black hole

$$\frac{\eta}{s} = \frac{1}{4\pi} + \text{finite coupling corrections}$$

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Black hole holography currently seems like a big hammer!
Black hole holography harmonic oscillator of 21st century?

Outlook on selected topics

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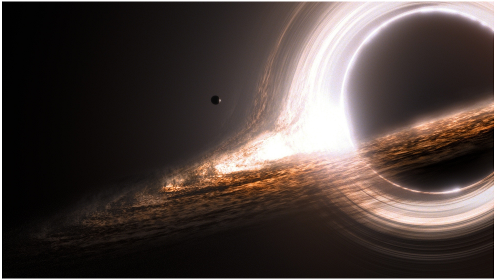
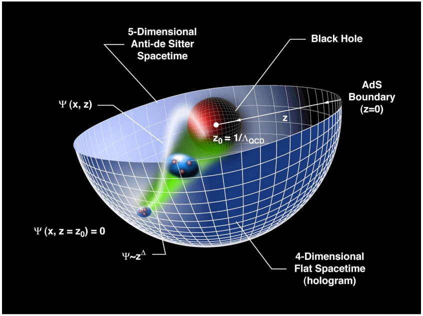
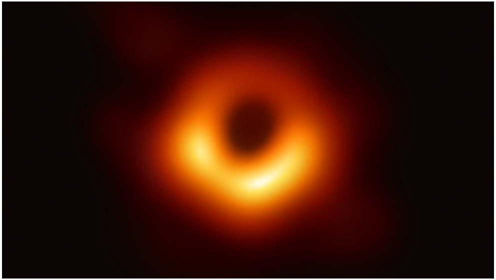
- Will need more research on black holes to resolve all issues!
- Black holes II prepares you for some of the research directions

Thanks for your attention... ...see (some of) you in March!







EHT collaboration, April 2019