Black Holes, AdS/CFT correspondence and Holography

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FWF Der Wissenschaftsfonds.

Introduction

The purpose of the talk is:

- review the properties of black holes and their connection to thermodynamics
- get an idea about AdS/CFT correspondence
- have insight into some real cool stuff

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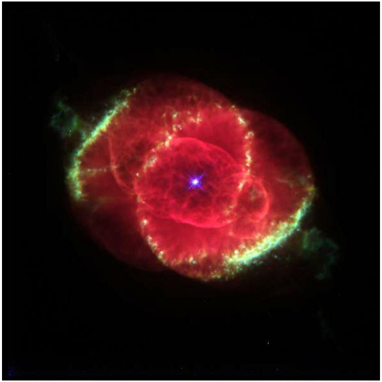


CYGNUS–X1 Black hole

The fishy side of black holes

- point of no return $\leftrightarrow$ black hole horizon
- waterfall $\leftrightarrow$ singularity

Black hole horizon and thermodynamics

Facts:

- area of horizon $\propto$ mass
- The horizon can't decrease! $\delta A \geq 0$
  $\rightarrow$ analogy: second law of thermodynamics: $\delta S \geq 0$

$\rightarrow$ entropy of a black hole: $S \propto \frac{A}{4}$ (Hawking, 1971)

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Maximum entropy

consider: spherical region of space Γ, boundary $\delta\Gamma$ and area of boundary A
$+$ thermodynamic system with entropy S contained in Γ

the total mass of this system can not exceed the mass of a black hole of area A

$\longrightarrow$ the maximum entropy of a region of space is proportional to its area $\longrightarrow$ such bounds are called holographic

Summary

- black hole = horizon + singularity
- horizon can be related to entropy
  $\longrightarrow$ black hole entropy
- maximum entropy
  $\longrightarrow$ holographic bounds

The holographic principle

The scaling of the gravitational entropy has been taken as an indication of a new fundamental principle, the holographic principle

Any quantum gravitational system in (d+1) dimensions should have a dual description in terms of a QFT without gravity in one dimension less.

$\rightarrow$ if gravity is holographic this would explain the scaling behaviour of the black hole entropy, since the entropy of a QFT scales like the volume, which is the same as the area in one dimension higher
(idea by t'Hooft/Susskind (in the 1990ies), discovery by Maldacena (1997))

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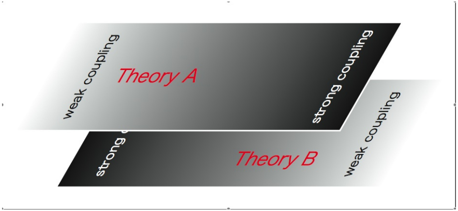
The duality

The AdS/CFT correspondence relates string on AdS to specific gauge theories at the boundary of AdS

Therefore it is a useful tool in string theory for the last 10 years

- Equivalence between two theories which look different
- Complementarity: Cover one theory's hard part (strong coupling physics) by the other's easy part (weak coupling physics)



The duality relates bulk fields with gauge-invariant operators of the boundary theory

Correlation functions of such gauge invariant operators can then be extracted from the asymptotics of the bulk solutions and conversely, given correlation functions of the dual operators, one can reconstruct asymptotic solutions

e.g. $\langle T_{\mu\nu} \rangle_{gauge} = T^{BY}_{\mu\nu}$

AdS/CFT basics

Recall that the AdS_{d+1} metric is

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2}\eta_{ij}dx^i dx^j$$

conformal boundary at $r \to 0$

$\to$ need boundary conditions for all bulk fields at conformal infinity

In the AdS/CFT corresondence

- the fields parameterizing these boundary conditions at conformal infinity are identified with sources that couple to operators of the dual CFT
- the on-shell action is the generating functional of CFT correlation function

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What can we expect/require?

Holography leads to new intuition about the boundary conditions:

- in QFT the sources coupling to operators are unconstrained since one functionally differentiates with respect to them
- one should be able to specify arbitrary functions/tensors as boundary conditions for bulk field

Asymptotically AdS spacetimes

Consider the case where the bulk field is the metric

$\rightarrow$ the metric approaches that of AdS as conformal infinity is approached!

For AdS/CFT however one needs more general boundary conditions!

The boundary conditions must be parameterized by an unconstrained metric since the metric acts as a source for every momentum tensor T_{ij} of the dual CFT

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Asymptotically locally AdS spacetimes

general set up: Fefferman and Graham (1985)
→ the corresponding spacetimes are called asymptotically locally AdS

An alAdS spacetime admits the following metric, located at r=0

$$ds^2 = \frac{dr^2}{r^2} + \frac{1}{r^2} g_{ij}(x,r) dx^i dx^j$$

where $\lim_{r \to 0} g_{ij}(x,r) = g_{(0)ij}(x)$ is a non-degenerate metric

→ requirement on $g_{ij}(x,r)$ a priori is that it has a non-degenerate limit as $r \to 0$
the precise form of the expansion $g_{ij}(x,r)$ is determined by solving the bulk field equations asymptotically

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Fefferman-Graham-Expansion

For Einstein-Gravity in (d+1) dimensions

$$g_{ij}(x,r) = g_{(0)ij} + r^2 g_{(2)ij} + \cdots + r^d (g_{(d)ij} + \log r^2 h_{(d)ij}) + \cdots$$

For the einstein gravity in 3 dimensions:

$$g_{ij}(x,r) = g_{(0)ij} + r^2 g_{(2)ij} + \ldots$$

boundary conditions as above $g_{(0)ij}(x) = \delta_{ij}$

precise form is specific to einstein gravity
$\rightarrow$ coupling to matter changes the coefficients

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The AdS/CFT duality implies a new approach to conserved charges

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Black Holes

Holography or the AdS/CFT correspondence

Example on Holographic Renormalization

To infinity ... and beyond!

- In QFT the energy is computed using the energy momentum tensor

$$E = \langle H \rangle = \int_C d^{d-1}x \langle T_{00} \rangle$$

generically, this expression needs renormalization to UV infinites

- in the AdS/CFT correspondence

$$\langle T_{ij} \rangle = \frac{\delta S_{onshell}[\rho_0]}{\delta g_0^{ij}}$$

this expression is finite, due to infinite volume of spacetime (IR divergence) and needs renormalization

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One can holographically renormalize the theory by adding local boundary covariant counterterms

one then obtains a finite one point function for T_{ij} for general AlAdS and one can proof that the holographic charges are the correct gravitational charges

Summary

- holographic principle
- The AdS/CFT correspondence relates strings on AdS to specific gauge theories at the boundary of AdS
- Asymptotically AdS
- Asymptotically locally AdS

Quantum mechanical toy model

Hamiltonian bulk action

$$I_B = \int_{t_i}^{t_f} dt \, [-\dot{p}q - H(q,p)]$$

want: Dirichlet boundary value problem q=fixed at t_i, t_f
Problem: $\delta I_B = 0$ requires $q\delta p = 0$ at the boundary
$\rightarrow$ add the GHY- boundary term

$$I_E = I_B + I_{GHY}$$

$$I_{GHY} = pq|_{t_i}^{t_f}$$

as expected

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AdS_2

Bulk action

$$I_B = -\int_{\mathcal{M}} d^2x\sqrt{g}\left[X(R+\frac{2}{\ell^2})\right]$$

Variation with respect to this field yields:

$$R = -\frac{2}{\ell^2}$$

Variation with respect to the metric g yields the equations of motion

$$\nabla_\mu\nabla_\nu X - g_{\mu\nu}\Box X + g_{\mu\nu}\frac{X}{\ell^2} = 0$$

those solved by $X = r$ and

$$g_{\mu\nu}dx^\mu dx^\nu = \left(\frac{r^2}{\ell^2} - M\right)dt^2 + \frac{dr^2}{\frac{r^2}{\ell^2} - M}$$

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GHY-term

$$I_{GHY} = -\int_{\partial\mathcal{M}} dx\sqrt{\gamma}XK$$

Action with correct boundary value problem:

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the boundary lies at $r = r_0$ with $r_0 \to \infty$
question is: are we done? NO!

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The variation of I_E yields

$$\delta I_E \sim EOM + \delta X(boundary - term) - \lim_{r_0 \to \infty} \int_{\partial \mathcal{M}} dt \delta\gamma$$

γ is the asymptotic metric; $\delta\gamma$ may be finite!

$\rightarrow \delta I_E \neq 0$ for some variations that preserve boundary conditions!

observations: the dirichlet boundary problem is not changed under the transformation of the bulk action by adding the GHY and the counter term.

$$I_E \rightarrow \Gamma = I_B + I_{GHY} - I_{CT}$$

with $I_{CT} = S(q,t)|_f^t$

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Hamilton's Principal function

- Solves the Hamilton-Jacobi equation
- Does not change boundary value problem when added to action
- is capable to render $\delta\Gamma = 0$ even when $\delta I_E \neq 0$
- reasonable ansatz:
  holographic counterterm = solution of Hamilton-Jacobi equation!

This Ansatz yields

$$I_{CT} = -\int_{\mathcal{M}} dx \sqrt{\gamma} \frac{X}{\ell}$$

The action is consistent with boundary value problem and variational principle:

$$\Gamma = -\frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{g}\left[X(R+\frac{2}{\ell^2})\right] - \int_{\partial\mathcal{M}} dx\sqrt{\gamma}XK + \int_{\partial\mathcal{M}} dx\sqrt{\gamma}\frac{X}{\ell}$$

Now $\delta\Gamma = 0$ for all variations that preserve the boundary conditions!

Summary and algorithm of holographic renormalization

- start with a bulk action I_B
- check consistency of boundary value problem
- if necessary, add boundary term I_{GHY}
- check consistency of variational principle
- if necessary, subtract holographic counterterm I_{CT}
- use improved action

$$\Gamma = I_B + I_{GHY} - I_{CT}$$

3D vs. 2D gravity

2D

- testing area of new ideas about quantum gravity
- exact results
- might help to address some conceptual problems

BUT: no gravitons and lacks good analog for horizon area

3D

$\rightarrow$ black holes and gravitons

'keep it simple, stupid - but never oversimplify'

(Albert Einstein, KISS principle)

3D gravity

Several gravity theories:

- pure gravity
  $\rightarrow$ no propagating degrees of freedom
- massive gravity
- cosmological topologically massive gravity
- cosmological topologically massive gravity at the chiral point
- new gravity

$$I_{CTMG} = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\varepsilon^{\alpha\beta\gamma}\Gamma^{\rho}_{\alpha\sigma}\left(\partial_\beta\Gamma^{\sigma}_{\gamma\rho} + \frac{2}{3}\Gamma^{\sigma}_{\beta\tau}\Gamma^{\tau}_{\gamma\rho}\right)\right]$$

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Warped AdS

Anti-de Sitter squashed and stretched

warped AdS is to AdS what the squashed/stretched sphere is to the round sphere

Why is this of interest?

warped AdS was proposed as holographic dual of the nonrelativistic CFT describing cold atoms

arise as exact solutions in CTMG

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Why is this of interest?

warped AdS was proposed as holographic dual of the nonrelativistic CFT describing cold atoms

arise as exact solutions in CTMG

Prepare for warp

review: asymptotically AdS metric in d+1 dimensions

$$ds^2 = \frac{\ell^2 d\hat{\rho}^2}{4\hat{\rho}^2} + \frac{1}{\hat{\rho}}\tilde{\gamma}_{ij} dx^i dx^j$$

asymptotic expansion around $\hat{\rho} = 0$

$$\tilde{\gamma}_{ij} = \tilde{\gamma}_{ij}^{(0)} + \hat{\rho}^{1/2}\tilde{\gamma}_{ij}^{(1)} + \cdots + \hat{\rho}^{1/2}\tilde{\gamma}_{ij}^{(d)} + \hat{\rho}^{d/2}\ln(\hat{\rho})\tilde{h}_{ij}^{(d)}$$

Gaussion normal coordinates: using $\hat{\rho} = e^{-2\rho/\ell}$

$$ds^2 = d\rho + \gamma_{ij} dx^i dx^j$$

with

$$\gamma_{ij} = e^{2\rho/\ell}\gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} + e^{-2\rho/\ell}\gamma_{ij}^{(4)} + \ldots$$

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Engage!

define: asymptotically warped AdS by deforming aAdS

$$k_i = e^{2\rho/\ell} k_i^{(0)} + k_i^{(2)} + e^{-2\rho/\ell} k_i^{(4)} + \dots$$

can construct a new asymptotically AdS spacetime

$$ds^2 = d\rho^2 + \left(\gamma_{ij} + k_i k_j\right) dx^i dx^j = d\rho^2 + \tilde{\gamma}_{ij}\, dx^i dx^j$$

asymptotic expansion of the metric:

$$\tilde{\gamma}_{ij} = \tilde{\gamma}_{ij}^{(-2)} e^{4\rho/\ell} + \tilde{\gamma}_{ij}^{(0)} e^{2\rho/\ell} + \tilde{\gamma}_{ij}^{(2)} + \tilde{\gamma}_{ij}^{(4)} e^{-2\rho/\ell} + \tilde{\gamma}_{ij}^{(6)} e^{-4\rho/\ell} + \dots$$

$\rightarrow$ not asymptotically AdS

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...to boldly go where no man has gone before ...

Remember

$$I_{CTMG} = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left[R + \frac{2}{\ell^2} + \frac{1}{2\mu}\varepsilon^{\alpha\beta\gamma}\Gamma^{\rho}_{\alpha\sigma}\left(\partial_\beta\Gamma^{\sigma}_{\gamma\rho} + \frac{2}{3}\Gamma^{\sigma}_{\beta\tau}\Gamma^{\tau}_{\gamma\rho}\right)\right]$$

$$\delta I_{\mathrm{CTMG}}\big|_{\mathrm{EOM}} = -\int_{\partial\mathcal{M}} d^2x\sqrt{-\gamma}\left(\tilde{K}^{ij} - (\tilde{K} - \frac{1}{\ell_c})\tilde{\gamma}^{ij}\right)\delta\tilde{\gamma}_{ij}$$
$$+\frac{1}{\mu}\int_{\partial\mathcal{M}} d^2x\,\epsilon^{ij}\left(-\tilde{R}^{k\rho}{}_{j\rho}\,\delta\tilde{\gamma}_{ik} + \tilde{K}_i{}^k\,\delta\tilde{K}_{kj} - \frac{1}{2}\tilde{\Gamma}^k{}_{li}\,\delta\tilde{\Gamma}^l{}_{kj}\right)$$

generic result

$$\delta I\big|_{\mathrm{EOM}} \sim \int_{\partial\mathcal{M}} d^2x\sqrt{-\gamma^{(0)}}\left(J^{ij}\,\delta\tilde{\gamma}^{(-2)} + T^{ij}\,\delta\tilde{\gamma}^{(0)}_{ij} + C^{ij}\,\delta\tilde{\gamma}^{(2)}_{ij} + D^{ij}\,\delta\tilde{\gamma}^{(4)}_{ij}\right)$$

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Thank you for your attention!