Quantum Null Energy Condition

A remarkable inequality in physics

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  Example: $g_{\text{ex}}/2 = 1.00115965218(073)$, $g_{\text{th}}/2 = 1.00115965218(178)$

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take square root and then divide by 2

$$\frac{a+b}{2} \geq \sqrt{ab}$$

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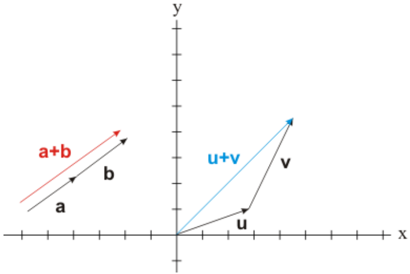

e.g. triangle inequality

$$|u| + |v| \geq |u + v|$$

graphic proof evident

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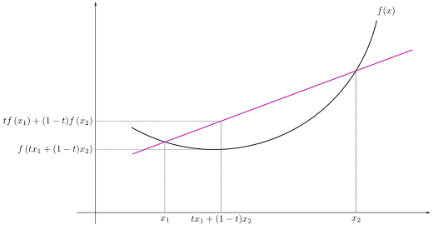

special case of Jensen's inequality: secant always above convex curve between intersection points x_1, x_2

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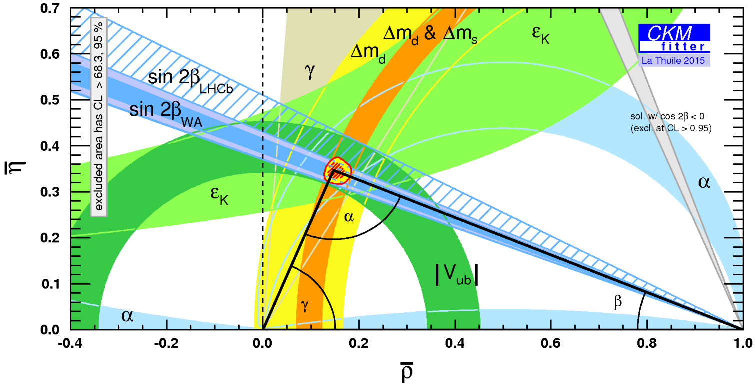

Example: unitarity constraints on physical parameters in quark mixing matrix

if Standard Model correct then measurements must reproduce unitarity triangle

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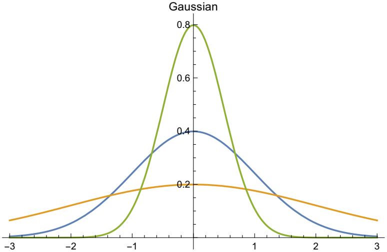

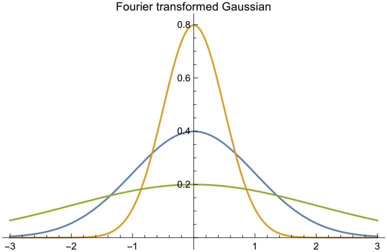

green: localized in coordinate space (x), delocalized in momentum space (p)
blue: mildly (de-)localized in coordinate and momentum space
orange: delocalized in coordinate space (x), localized in momentum space (p)

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  - Definition: (local) inequalities on the stress tensor $T_{\mu\nu}$
    e.g. Null Energy Condition (NEC)

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Are there quantum energy conditions?

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  Faulkner, Leigh, Parrikar and Wang 1605.08072
  Hartman, Kundu and Tajdini 1610.05308

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Quantum null energy condition (QNEC)

Proposed by Bousso, Fisher, Leichenauer and Wall in 1506.02669

QNEC (in $D > 2$) is the following inequality

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Obvious observations:

- if r.h.s. vanishes: semi-classical version of NEC
- if r.h.s. negative: weaker condition than NEC (NEC can be violated while QNEC holds)
- if r.h.s. positive: stronger condition than NEC (if QNEC holds also NEC holds)

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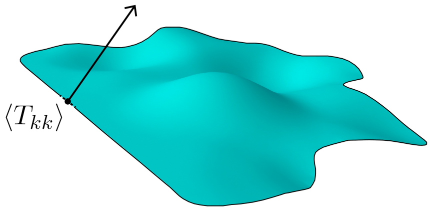

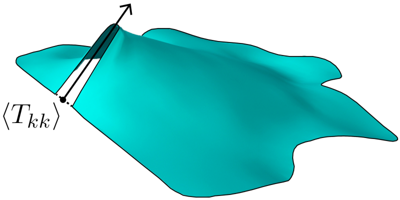

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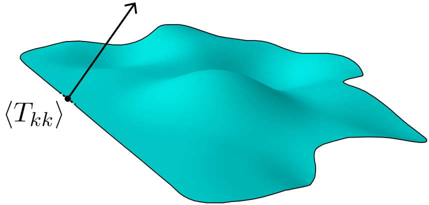

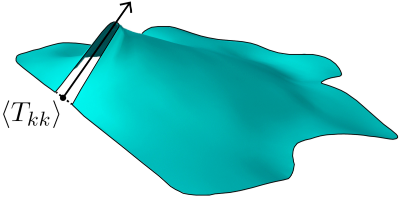

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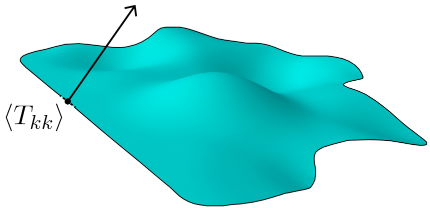

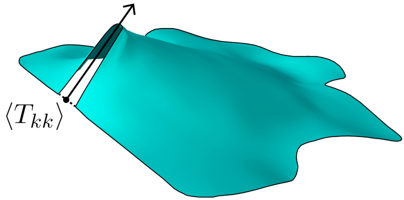

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- $\sqrt{\gamma}$: induced volume form of entangling region (black boundary curve)

Proofs ($D > 2$)

- For free QFTs: Bousso, Fisher, Koeller, Leichenauer and Wall, 1509.02542
- For holographic CFTs: Koeller and Leichenauer, 1512.06109
- For general CFTs: Balakrishnan, Faulkner, Khandker and Wang, 1706.09432
- Saturation of QNEC for contact terms ("Energy is Entanglement"): Leichenauer, Levine and Shahbazi-Moghaddam, 1802.02584

Proofs and counter examples ($D = 2$)

Ongoing work with Ecker, Stanzer and van der Schee

QNEC (in CFT$_2$) is the following inequality

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- QNEC can be violated in hol. CFT_2 with negative bulk energy fluxes

Calculating QNEC holographically

calculating CFT observable holographically = some gravity calculation

AdS/CFT:

Maldacena hep-th/9711200 (> 13700 citations; > 50 in May 2018)
Gubser, Klebanov and Polyakov hep-th/9802109
Witten hep-th/9802150

holographic stress tensor:

Henningson and Skenderis hep-th/9806087
Balasubramanian and Kraus hep-th/9902121
Emparan, Johnson and Myers hep-th/9903238
de Haro, Solodukhin and Skenderis hep-th/0002230

holographic entanglement entropy (HEE):

Ryu and Takayanagi hep-th/0603001
Hubeny, Rangamani and Takayanagi 0705.0016
Swingle 0905.1317 (possible relation between MERA and holography)

Calculating QNEC holographically

calculating CFT observable holographically = some gravity calculation

- need holographic computation of $\langle T_{kk} \rangle$

  well-known AdS/CFT prescription: extract boundary stress tensor from bulk metric expanded near AdS boundary

  Example: $\text{AdS}_3/\text{CFT}_2$

$$\mathrm{d}s^2 = \frac{\ell^2}{z^2}\left(\mathrm{d}z^2 + 2\,\mathrm{d}x^+\,\mathrm{d}x^-\right) + \langle T_{++}\rangle\left(\mathrm{d}x^+\right)^2 + \langle T_{--}\rangle\left(\mathrm{d}x^-\right)^2 + \mathcal{O}(z^2)$$

  AdS_3 boundary: $z \to 0$
  $\mathcal{O}(1)$ terms in metric: flux components of stress tensor $\langle T_{\pm\pm}\rangle$
  (trace vanishes, $\langle T_{+-}\rangle = 0$)
  ℓ: so-called AdS-radius (cosmological constant $\Lambda = -1/\ell^2$)

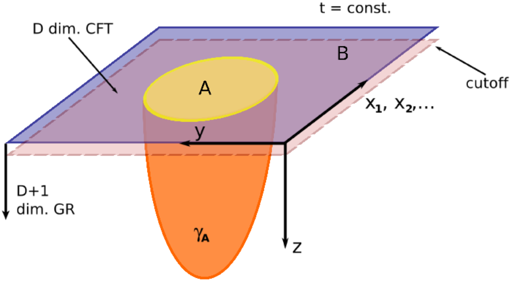
Calculating QNEC holographically

calculating CFT observable holographically = some gravity calculation

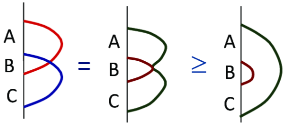
- need holographic computation of $\langle T_{kk} \rangle$
- need holographic computation of (deformations of) EE

HEE = area of extremal surface

simple to calculate!



also: simple proof of strong subadditivity inequalities



$$\Rightarrow \; S_{A\cup B} + S_{B\cup C} \geq S_{A\cup B\cup C} + S_B$$

Thermal case

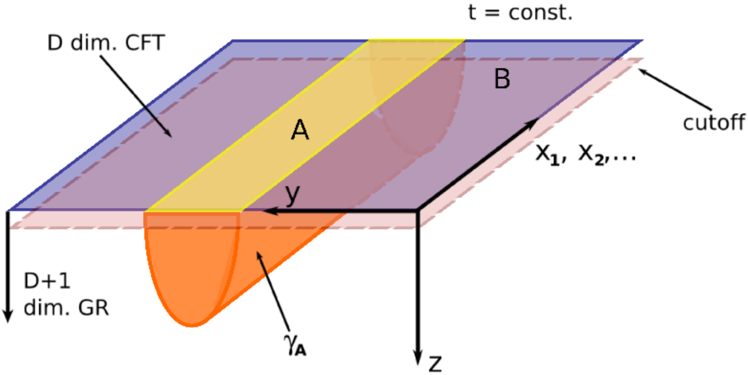
see work with Ecker, Stanzer and van der Schee 1710.09837

thermal states in CFT_4 = black holes in AdS_5

- paper-and-pencil calculation starts with Schwarzschild black brane

$$\mathrm{d}s^2 = \frac{1}{z^2}\Big(-f(z)\,\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + \mathrm{d}y^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2\Big)$$

with $f(z) = 1 - \pi^4 T^4 z^4$



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- determine area of minimal surfaces for small temperature, $T\ell \ll 1$, and extract HEE (ℓ = width of strip)

$$\frac{1}{2\pi}\,S'' \approx -\frac{0.065}{\ell^4} + 0.019\,\pi^4 T^4 - 0.083\,\ell^4 \pi^8 T^8 + \mathcal{O}\big(\ell^8 T^{12}\big)$$

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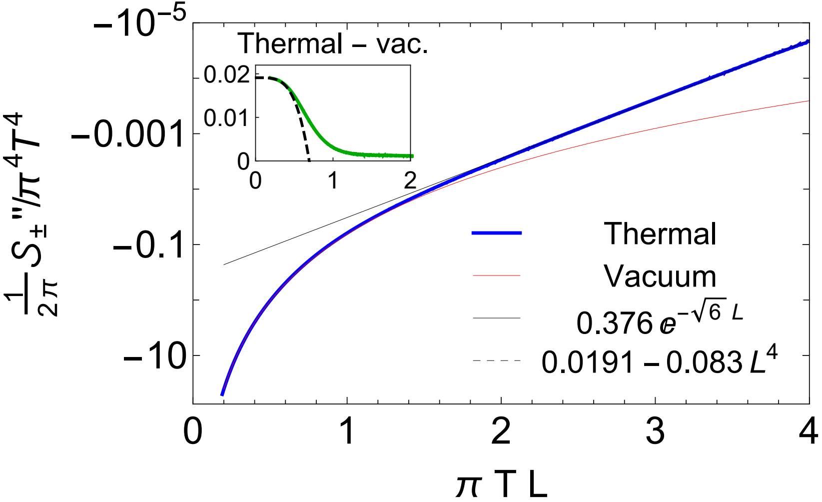

notational alert: L in the plot corresponds to width ℓ

Colliding gravitational shockwaves and QNEC saturation

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plasma formation in CFT_4 = colliding gravitational shock waves in AdS_5

toy model for quark-gluon plasma formation in heavy ion collisions

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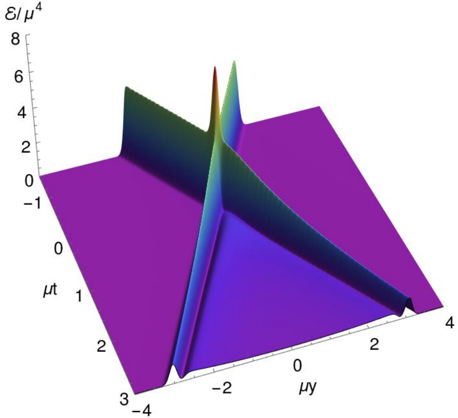

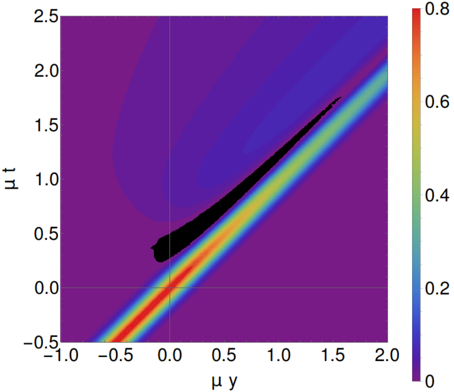

Left: energy density plot

Right: black region violates NEC

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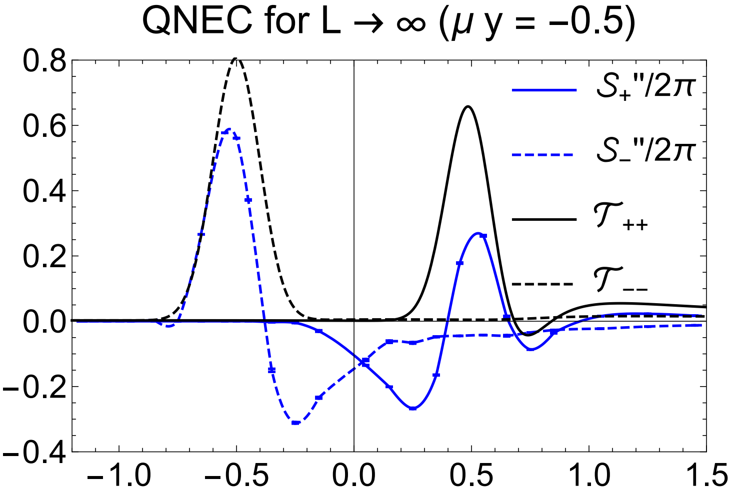

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Thanks for your attention!