The D3-D7 Model of AdS/CFT with Flavour
(with Special Feature)

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Lot’s of known stuff (Review: hep-th/0711.4467 ) +
JHEP 0712:091,2007 (with J. Erdmenger & J. P. Shock)
Outline

1. Introduction to AdS/CFT
2. The D3-D7 model: Adding Flavour to AdS/CFT
3. Electric and Magnetic Field Backgrounds
   - Magnetic Field
   - Electric Field
4. Summary
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Motivation

New tools for strongly coupled gauge theories?

- QCD in the infrared is strongly coupled (Conformal window?)
  
  [Deur, Korsch et. al: hep-ph/0509113]

- QGP produced at strong coupling \( (T > \Lambda_{QCD}) \)
Our new tool: AdS/CFT Holography
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The Original Correspondence (weakest form)

IIB Supergravity on $AdS_5 \times S^5$ with $(R/\ell_s)^4 = 2\lambda \gg 1$

$$dS^2_{AdS_5 \times S^5} = R^2 \left( \frac{dx^\mu dx^\mu + du^2}{u^2} + d\Omega_5^2 \right)$$

$\Leftrightarrow$

large $N_c$ limit of $\mathcal{N} = 4$ $SU(N_c)$ Super Yang-Mills with $\lambda = g_{YM}^2 N_c$

1. Strong-Weak Coupling Duality: $G_N \propto g_s^2 = g_{YM}^4$
2. Gubser-Klebanov-Polyakov-Witten relation:
   $$\langle e^{i \int d^4 x J_O O} \rangle = e^{i S_{IIB, onshell}[J_O]}$$
3. Operator-Field Dictionary:
   $$\phi_{m^2} = \Delta(\Delta - 4) \approx u^{4-\Delta} J_O + u^\Delta \langle O \rangle$$
A short look at string theory

Basic Objects:
- **Open** and **Closed** Strings, D-Branes

Strings have length: $\ell_s$
Strings have tension: $T_s = \frac{1}{2\pi\alpha'} = \frac{1}{2\pi\ell_s^2}$

“Minimal Surface” action principle: Nambu-Goto Action

$$S_{NG} = \frac{1}{2\pi\ell_s^2} \int d\tau d\sigma \sqrt{-\det P[G]_{ab}}$$

$$P[G]_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b}$$

[Becker, Becker, Schwarz]
**D-Branes:** Dirichlet boundary conditions for the string (e.g. $X^1(\sigma, \tau) = 0$ for $\sigma = 0, \pi$)

$1^{st}$ Quantized Strings:

- **Open String:** “Gauge Theory”
  
  $A_\mu, \Phi^I, \text{Fermions}$

- **Closed String:** “Gravity”
  
  $G_{\mu\nu}, B_{\mu\nu}, \Phi, \text{Fermions}$

[hep-ph/0701201]
**A short look at string theory**

**D-Branes**: Dirichlet boundary conditions for the string (e.g. $X^1(\sigma, \tau) = 0$ for $\sigma = 0, \pi$)

**D-Branes Curve Spacetime:**

Open-Closed String Duality
**Introduction to AdS/CFT**

**A short look at string theory**

**D-Branes:** Dirichlet boundary conditions for the string (e.g. \( X^1(\sigma, \tau) = 0 \) for \( \sigma = 0, \pi \))

**D-Branes Curve Spacetime:**

“Graviton” Absorption/Emission
**D-Branes:** Dirichlet boundary conditions for the string (e.g. \(X^1(\sigma, \tau) = 0\) for \(\sigma = 0, \pi\))

- **Dp-Branes Have Tension:**
  \[
  T_p = \frac{1}{(2\pi)^p \ell_s^{(p+1)} g_s}
  
  g_s \ldots \text{string coupling}
  
  \]

“Graviton” Absorption/Emission
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\[
T_p = \frac{1}{(2\pi)^p \ell_s^{(p+1)} g_s}
\]

\( g_s \) ... string coupling

---

String Splitting/Merging

[“D-Branes”, C. V. Johnson]
**D-Branes:** Dirichlet boundary conditions for the string (e.g. $X^1(\sigma, \tau) = 0$ for $\sigma = 0, \pi$)

- Dp-Branes are “Minimal Hypersurfaces”: $(\alpha' = \ell_s^2)$

\[
S_{Dp} = S_{DBI} + S_{CS}
\]

\[
S_{DBI} = -T_p \int d^{(p+1)}\xi \sqrt{-\det (P[G + B]_{ab} + 2\pi \alpha' F_{ab})}
\]

\[
S_{CS} = +T_p g_s \sum_p \int_{\text{topform}} P[C^{(p+1)} \wedge e^B] \wedge e^{2\pi \alpha' F}
\]
**D-Branes:** Dirichlet boundary conditions for the string (e.g. $X^1(\sigma, \tau) = 0$ for $\sigma = 0, \pi$)

- **Dp-Branes carry Gauge Theories:** e.g. flat D3-Brane in flat space in the limit $\alpha' \ll 1$

$$S_{DBI,p=3} = -\frac{1}{(2\pi)^3 \alpha'^2 g_s} \int d^4\xi \sqrt{-\text{det} (P[G]_{ab} + 2\pi \alpha' F_{ab})}$$

$$\approx -T_3 \int d^4\xi \sqrt{-\text{det} P[\eta]} - \frac{1}{8\pi g_s} \int d^4\xi \sqrt{-\text{det} P[\eta]} F_{ab} F^{ab}$$

$$= -T_3 \text{Vol}(\mathbb{R}^4) - \frac{1}{8\pi g_s} \int d^4\xi F_{ab} F^{ab}$$
\( \mathcal{N} = 4 \ d = 4 \) Super-Yang-Mills-Theory

- More precisely: The gauge theory on a stack of \( N_c \) D3-branes is the unique maximally supersymmetric gauge theory in 4D, \( \mathcal{N} = 4 \) Super-Yang-Mills Theory (SYM)

- SUSYs: 16 Poincaré + 16 Conformal = \( \frac{64}{2} \) real supercharges = \( \frac{1}{2} \) BPS

- Fields: \( \mathcal{N} = 4 \) Vector Multiplet in 4D = \( (A_\mu, \lambda_\alpha A, \phi^i) = (V, \Phi_I) \)

\[
\mathcal{L} = \text{tr} \left\{ -\frac{1}{2g_{YM}^2} F_{\mu\nu}^2 + \frac{\Theta_I}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} - i \sum_{A=1}^{4} \bar{\lambda}^A \tilde{\sigma}_{\mu} D_{\mu} \lambda_A - \sum_{i=1}^{6} D_\mu \phi^i D^{\mu} \phi^i \right. \\
\left. + \frac{g_{YM}^2}{2} \sum_{i,j} [\phi^i, \phi^j]^2 + g_{YM} \sum_{A, B, i} (C_i^{AB} \lambda_A [\phi^i, \lambda_B] + h.c.) \right\}
\]

- Global Symmetries: \( SO(4, 2) \times SU(4)_R \) (\( \beta(g_{YM}) = 0 \) !)
D3-Branes in IIB Supergravity

- Have seen:
  1. D3-Branes carry $SU(N_c) \mathcal{N} = 4$ SYM theory
  2. D3-Branes curve space in a supersymmetric way
- D3-Branes are solitonic $\frac{1}{2}$ BPS solutions to IIB Supergravity [Horowitz, Strominger Nucl.Phys.B360(1991)]

$$d s^2 = H(r)^{-\frac{1}{2}} d x^\mu^2 + H(r)^{\frac{1}{2}} (d r^2 + r^2 d \Omega_5^2)$$

$$C_4 = H(r)^{-1} d t \wedge d x \wedge d y \wedge d z, \quad H(r) = 1 + \frac{R^4}{r^4}, \quad \frac{R^4}{\alpha'^2} = 4\pi g_s N_c$$

- Asymptotically flat
- Killing Horizon: $g_{tt} = 0 = H(r)^{-\frac{1}{2}}$ at $r = 0$
- Near horizon geometry: $AdS_5 \times S^5$ (w. $N_c$ units of 5-form flux on $S^5$)

$$d s^2_{AdS_5 \times S^5} = \frac{r^2}{R^2} d x^\mu^2 + \frac{R^2}{r^2} d r^2 + R^2 d \Omega_5^2$$

Q: Connection between these two descriptions of D3-Branes, i.e. between open and closed string physics?
Field Theory Perspective: D3-Branes in flat space,

\[ G_N = g_s^2 \ell_s^8 = \kappa^2 \ll 1 \] (backreaction \( \sim G_N \))

\[ S_{\text{eff}} = S_{\text{IIB}} + S_{D3} + S_{\text{int}} \]

\[ \frac{1}{2\kappa^2} \int \sqrt{|g|} R + \ldots \sim \frac{1}{\kappa} \sim \kappa^# > 0 \]

→ Low Energy Limit \( \ell_s \to 0, g_s, N_c, \ldots \) fixed:

Free IIB SUGRA & \( \mathcal{N} = 4 \) SYM Theory

Soliton Perspective: Observer at Infinity sees (\( \omega \ll 1/\ell_s \))

1. Low-Energy Bulk Modes \( \sigma_{D3} \sim \omega^3 R^8 \approx 0 \)

→ Decoupling of Near-Horizon Region → Free IIB SUGRA

2. Redshifted modes of arbitrary (!) energy from near the horizon

→ IIB STRING THEORY on \( AdS_5 \times S^5 \)
\[ SU(N_c) \mathcal{N} = 4 \text{ SYM} \quad \Leftrightarrow \quad \text{IIB String Theory on } AdS_5 \times S^5 (N_c \text{ units of five-form flux,}) \\
\quad R^4_{AdS_5/S^5} = 4\pi\alpha'^2 g_s N_c = 2\alpha'^2 g_{YM}^2 N_c \]

\'t Hooft Limit: \[ N_c \rightarrow \infty, \quad \lambda = g_{YM}^2 N_c \text{ fix} \]

2. Planar \[ SU(N_c = \infty) \mathcal{N} = 4 \text{ SYM} \quad \Leftrightarrow \quad \text{Semiclassical Strings on } AdS_5 \times S^5 \]
\[ 2\pi g_s = g_{YM}^2 \rightarrow 0 \]

Strong Coupling Limit: \[ \frac{R^4}{\ell_s^4} = 2\lambda >> 1 \]

3. Planar \[ SU(N_c = \infty) \mathcal{N} = 4 \text{ SYM at strong 't Hooft coupling} \]
\[ \Leftrightarrow \quad \text{IIB Supergravity on } AdS_5 \times S^5 \]
The AdS/CFT Correspondence

The Original Correspondence (weakest form)

IIB Supergravity on $AdS_5 \times S^5$ with $(R/\ell_s)^4 = 2\lambda \gg 1$

$$ds^2_{AdS_5 \times S^5} = R^2 \left( \frac{dx^\mu dx^\mu + du^2}{u^2} + d\Omega_5^2 \right)$$

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$$\phi m^2 = \Delta(\Delta-4) \sim u^{4-\Delta} J_\mathcal{O} + u^\Delta \langle \mathcal{O} \rangle$$
The AdS/CFT Correspondence

- Closed string sector
  - AdS$_5 \times S^5$ geometry

- Open string sector
  - Large N D3 stack at bottom of throat
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\( \mathcal{N} = 4 \) Vector Multiplet \((V, \Phi_i), \ i = 1, 2, 3: \) All Adjoint

How to introduce “Quarks” = fundamental degrees of freedom?

→ Open String Sector in Gravity Dual → Probe Branes

**\( N_c \) D3-\( N_f \) D7 Intersection:**

- \( \text{D3:} (x^0, x^1, x^2, x^3) \)
  - \( \text{D7:} (x^0, x^1, x^2, x^3, y^4, y^5, y^6, y^7) \)

- \( \mathcal{N} = 2 \) Supersymmetric

- Field Content:
  - 3-3: \( \mathcal{N} = 4 \) vector multiplet
  - 7-3: \( N_c - N_f \) chiral multiplet \( Q^I \)
  - 3-7: \( \tilde{N}_c - \tilde{N}_f \) chiral multiplet \( \tilde{Q}^I \)
  - 7-7: \( U(N_f) \) DBI theory

- “Quark” masses:
  \[ m_q = LT_s = \frac{L}{2\pi\alpha'} \]
How to A) Not spoil Maldacenas argument?
B) Decouple 7-7 $U(N_f)$ theory from the field theory?

\[ \text{'t Hooft Limit = Probe Limit} \quad N_c \to \infty, \quad N_f = \text{fix} \]

1. Field Theory
   - Decoupling limit still holds: $T_7 \sim \frac{1}{(g_s \ell_s^8)} = \frac{g_s}{\kappa^2}$
   - $U(N_f)$ becomes global: $\lambda_8 = g_{YM,8}^2 N_f \sim g_s \ell_s^4 N_c \frac{N_f}{N_c} \sim \lambda \ell_s^4 \frac{N_f}{N_c} \to 0$
     \[ \to \text{Quenched Approximation!} \]

2. Gravity Side:
   \[ V_{\text{Newton}} = G_N N_f T_7 \sim N_f g_s \lambda \frac{N_f}{N_c} \to 0 \]
   \[ \to \text{Neglect D7 backreaction on D3 soliton background} \]
Extended Correspondence: $N_f$ D7 Branes in $AdS_5 \times S^5$

4d $\mathcal{N} = 4$ SU(N) Super Yang-Mills theory coupled to 4d $N_f \mathcal{N} = 2$ hypermultiplets in the Quenched Approximation

$$\lambda \gg 1 \quad \leftrightarrow \quad \text{type IIB SUGRA on } AdS_5 \times S^5 + \quad \text{Dirac-Born-Infeld & Chern-Simons theory on D7 with Neglected Backreaction}$$
D3/D7-Model: Field Theory

\[ \mathcal{N} = 4 \text{ Glue} & \mathcal{N} = 2 \text{ Quarks} \]

\[ \mathcal{L} = \Im \left[ \tau \int d^2 \theta d^2 \bar{\theta} \left( \text{tr}(\Phi_i e^V \Phi_i e^{-V}) + Q_i^\dagger e^V Q' + \bar{Q}_I^\dagger e^{-V} \bar{Q}_I \right) + \tau \int d^2 \theta \left( \text{tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) + \text{tr}(\varepsilon_{ijk} \Phi_i \Phi_j \Phi_k) + \bar{Q}_I (m_q + \Phi_3) Q' \right) \right], \]

\[ \Phi_1 = \phi_1 + i \phi_2 + \ldots, \Phi_2 = \phi_3 + i \phi_4 + \ldots, \Phi_3 = \phi_5 + i \phi_6 + \ldots; \]

\[ \tau = \frac{\theta}{2\pi} + i \frac{4\pi}{g_{ym}^2} \]

1. **Field Content:**
   - \( \mathcal{N} = 4 \) “Glue” Vector Multiplet: \( V, \Phi_i, i = 1, 2, 3 \)
   - \( \mathcal{N} = 2 \) “Quark” Hypermultiplet: \( Q', \bar{Q}_I \)

2. **Conformal in the** \( N_c \rightarrow \infty \) **limit:** \( \beta(\lambda) = N_c \beta(g_{ym}^2) \propto \lambda^2 \frac{N_f}{N_c} \rightarrow 0 \)

3. **Global Symmetries** (\( m_q = 0, \) VEVs=0):

\[ SO(4,2) \times SU(2)_\Phi \times SU(2)_R \times U(1)_R \times U(N_f) \]
The D3-D7 model: Adding Flavour to AdS/CFT

**D3/D7-Model: Field Theory**

\( \mathcal{N} = 4 \) Glue & \( N_f \) \( \mathcal{N} = 2 \) Quarks

\[
\mathcal{L} = \mathcal{S} \left[ \tau \int d^2 \theta d^2 \bar{\theta} \left( \text{tr}(\Phi_i e^V \Phi_i e^{-V}) + Q_i^\dagger e^V Q^i + \tilde{Q}^i \dagger e^{-V} \tilde{Q}_i \right) + \right. \\
+ \tau \int d^2 \theta \left( \text{tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) + \text{tr}(\epsilon_{ijk} \Phi_i \Phi_j \Phi_k) + \tilde{Q}_i (m_q + \Phi_3) Q^i \right) \right],
\]

\[
\Phi_1 = \phi_1 + i \phi_2 + \ldots, \Phi_2 = \phi_3 + i \phi_4 + \ldots, \Phi_3 = \phi_5 + i \phi_6 + \ldots;
\]

\[
\tau = \frac{\theta}{2\pi} + i \frac{4\pi}{g_{ym}^2}
\]

<table>
<thead>
<tr>
<th>components</th>
<th>spin</th>
<th>( SU(2)<em>\Phi \times SU(2)</em>\mathcal{R} )</th>
<th>( U(1)_\mathcal{R} )</th>
<th>( \Delta )</th>
<th>( U(N_f) )</th>
<th>( U(1)_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_1, \Phi_2 )</td>
<td>( \phi_1, \phi_2, \phi_3, \phi_4 )</td>
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<td>0, 1/2</td>
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</tr>
<tr>
<td>( \psi_i = (\psi, \bar{\psi})^\dagger )</td>
<td>( q^m = (q, \bar{q}) )</td>
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\( \rightarrow U(1)_\mathcal{R} \) acts like a chiral symmetry: \( \ldots m_q (i \bar{\psi} \psi + h.c.) \ldots \)
Supersymmetric Embeddings ($N_f = 1$)

- **Rewrite $AdS_5 \times S^5$:**

  \[ r^2 = (y^4)^2 + (y^5)^2 + (y^6)^2 + (y^7)^2 + (z^8)^2 + (z^9)^2 = \rho^2 + L^2 \]

  \[
  ds^2_{AdS_5 \times S^5} = \frac{r^2}{R^2} dx^\mu dx^\mu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega^2_5
  \]

  \[
  = \frac{\rho^2 + L^2}{R^2} dx^\mu dx^\mu + \frac{R^2}{\rho^2 + L^2} \left( d\rho^2 + \rho^2 d\Omega^2_3 + \frac{dL^2 + L^2 d\Phi^2}{d\vec{y}^2} + \frac{dL^2 + L^2 d\Phi^2}{d\vec{z}^2} \right)
  \]

- **Match Geometric Symmetries:**

  \[
  \begin{array}{cccccc}
  SO(4, 2) & SU(2)_\Phi & SU(2)_R & U(1)_R & U(N_f) \\
  Iso(AdS_5) & SU(2)_{L,\vec{y}} & SU(2)_{R,\vec{y}} & U(1)_{\vec{z}} & U(N_f) \\
  \end{array}
  \]

- **Embedding:**

  \[ \xi^\alpha = (x^\mu, \rho, S^3), \quad L = L(\rho), \quad \Phi = 0 \]

- **Constant $L$:**

  \[ \mathcal{L}_{DBI} \propto \rho^3 \sqrt{1 + L'^2} \Rightarrow \left( \frac{\rho^3 L'}{\sqrt{1 + L'^2}} \right)' = 0 \Rightarrow L = 2\pi \alpha' m_q \]

- **$\mathcal{N} = 2$ SUSY?:**

  \[ \frac{\partial S_{onshell}}{\partial m_q} = 0 \text{ (or check } \kappa \text{-symmetry)} \]
Supersymmetric Embeddings ($N_f = 1$)

- **Dual Operator:** $\Phi_3 = \phi + \ldots$

\[ O_{mq} = i(\bar{\psi}\psi + \psi^\dagger\bar{\psi}^\dagger) + m_q(\bar{q}\bar{q}^\dagger + q^\dagger q) + \sqrt{2}(\bar{q}\phi\bar{q}^\dagger + q^\dagger \phi q + \text{h.c.}) \]

- **Why no VEV in $\text{AdS}_5 \times S^5$? A Priori:**

\[
\left( \frac{\rho^3 L'}{\sqrt{1 + L'^2}} \right)' = 0 \Rightarrow L(\rho) \xrightarrow{\rho \to \infty} 2\pi \alpha' m_q + \frac{(2\pi \alpha')^3 \langle O_{mq} \rangle}{\rho^2}
\]

- **Reason 1:** $O_m$ is an F-Term $\int d^2 \theta \tilde{Q}(m_q + \Phi_3)Q$
- **Reason 2:** Embeddings not well-behaved

\[
\frac{\rho^3 L'}{\sqrt{1 + L'^2}} = c_1 \Rightarrow L' = \frac{c_1}{\sqrt{\rho^6 - c_1^2}} \xrightarrow{\rho \to c_1^+} \infty, \text{ unless } c_1 = 0!
\]
Flavour at Finite Temperature

Flavour Physics at Finite Temperature

\[ AdS\text{-}\text{Schwarzschild} \times S^5 \text{ (Black brane), } T = T_{\text{Hawking}} \propto r_s \]

1. **Embedding:** \( L(\rho) \overset{\rho \to \infty}{\sim} 2\pi\alpha' m_q + \frac{(2\pi\alpha')^3}{\rho^2} \langle O_{m_q} \rangle \)
2. No “Spontaneous CSB”: \( \langle O_{m_q} \rangle (m_q = 0) = 0 \)
3. Fluctuations: Mesons \(\rightarrow\) **Meson Melting Transition**

Picture: [hep-th/0611099]
D7 in AdS-Schwarzschild
First order Phase Transition in AdS-Schwarzschild

\[ F(m_q, T) = -S_{D3, \text{onshell}}(m_q, T) \Rightarrow -\langle O_{m_q} \rangle = \frac{\partial F(m_q, T)}{\partial m_q} \]
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**Electric/Magnetic \( B \)-Field** [see also 0709.1547, 0709.1554 (hep-th)]

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### Ansatz for the Kalb-Ramond Field

\[ B_{el} = B dt \wedge dx \quad \text{\&} \quad B_{mag} = B dy \wedge dz \]

- \( dB = 0 \Rightarrow \) No deformation of AdS-Schwarzschild

\[ -\frac{T_7}{g_2} \sqrt{-\det (P[G + B] + 2\pi \alpha' F)} + \sum_p P[C_p \wedge e^B] \wedge e^{2\pi \alpha' F} \]

- Affects Brane (Flavour) physics: D7 embeddings, thermodynamics, phase transitions, meson spectra ...

- Effect: Background for \( U(1)_F \) gauge field, mimics constant \( U(1) \in U(N_c) \) field strength
Magnetic Field

\[ B_{\text{mag}} = B \, dy \wedge dz \]

- **Zero Temperature:** [Filev et al. hep-th/0701001]
  CSB, Goldstone Boson with \( M \propto \sqrt{m_q} \) for small \( m_q \), Zeeman splitting

- **Finite Temperature:**
  - Small B: Meson Melting Transition
  - No molten phase & CSB above a critical magnetic field strength (GMOR)
  - Phase diagram
  - Spectrum of Pseudoscalar Mesons
**Magnetic Finite Temperature Embeddings**

\[
\chi_{SB} : \quad \tilde{B}_{\text{crit}} \approx 16, \quad \tilde{B} \propto \frac{B}{T^2}
\]
Meson Melting Transition below $\tilde{B}_{\text{crit}}$

No molten phase and spontaneous CSB above $\tilde{B}_{\text{crit}}$

Magnetic KR-Field acts repells the D7s from the origin
\( \Phi \) Meson Spectrum (Upper Branch, \( \tilde{M} = \frac{M\sqrt{\lambda}}{\sqrt{\pi}m_q} \))

\( \tilde{B}=0 \)

\( \tilde{B}=5 \)

\( \tilde{B}=10 \)

\( \tilde{B}=17 \)
Goldstone Boson ($\tilde{B} = 16$)

\[ \tilde{M} = 5\sqrt{\tilde{m}} \]
Electric $B_{\mu\nu}$

$$B_{el} = B dt \wedge dx$$

- Problem: **Zero Locus** of DBI action

$$\sqrt{- \det P [G + B]} = 0 \quad \text{for} \quad \rho_{IR}^2 + L(\rho_{IR})^2 = \frac{BR^2}{2} + \frac{1}{2} \sqrt{4r_s^4 + B^2 R^4}$$

Zero Locus
Electric $B_{\mu\nu}$

$$B_{el} = B dt \wedge dx$$

- Solution: $U(1)_f$ gauge field $A_x(\rho), A_t(\rho)$ [hep-th:0705.3890]

$$\partial_a \left( \frac{\delta L_{D7}}{\delta \partial_a A_b} \right) = 0 \Rightarrow \text{Two Conserved Quantities}$$

$$A_t(\rho) \simeq \mu - \frac{D}{\rho^2} \leftrightarrow \text{finite baryon number density} \quad \langle J_t \rangle = D$$

$$A_x(\rho) \simeq \frac{B}{\rho^2} \leftrightarrow \text{baryon number current in } x\text{-direction} \quad \langle J_x \rangle = B$$

$$\Rightarrow A'_x(\rho) = f'(D, B, \rho), \quad A'_t(\rho) = g'(D, B, \rho) \Rightarrow \text{Legendre transform}$$

Require $\tilde{S}_{D7}[L(\rho), D, B]$ to be well-defined:

$$B = B(\rho_{IR}, T, B, D)$$

$$\Rightarrow \text{Well-defined EOMs for } L(\rho)!$$
Electric Embeddings at $T = 0$: No CSB, Phase Transition

→ Dissociation of Mesons?
→ Conical singularities?
Condensate vs. Mass
Condensate vs. Mass: Phase Transition

\[ \tilde{c} \]

\[ \tilde{m} \]

Area \( \propto F \)

René Meyer (MPI Munich)
Electric and Magnetic Field Backgrounds

Electric Field

\[ \Phi (l=0) \text{ Meson Spectrum at } T = 0: \Delta M < 0 \]

Incoming Wave Boundary Conditions $\rightarrow$ Dissociation of Mesons!
What to expect at Finite Temperature?

- Meson melting enhanced by dissociation
- Any nonzero electric field will decrease the melting temperature
- No SCSB
- Finite T: One or two transitions?

Open Questions

- Fate of the conically singular solutions? They are either
  - physical → What creates the singularity?
  - unphysical → What happens in that mass range?
- Energy of the system is time-dependent →
  - Is this still equilibrium physics?
  - Thermodynamics in the presence of external currents?
Outline

1. Introduction to AdS/CFT
2. The D3-D7 model: Adding Flavour to AdS/CFT
3. Electric and Magnetic Field Backgrounds
   - Magnetic Field
   - Electric Field
4. Summary
Summary: Electric/Magnetic Background Fields

**Magnetic Background**
- Induced SCSB
- Magnetic Field Stabilizes Mesons
- Mesons don’t melt for large enough $B$ (SCSB)
- Zeeman splitting

**Electric Background**
- No SCSB
- Dissociation (& Meson Melting)
- Stark shift
Backup Slides
Condensate vs. Mass

\[ \tilde{B} = 0 \]
\[ \tilde{B} = 0.5 \]
\[ \tilde{B} = 1 \]
\[ \tilde{B} = 5 \]
\[ \tilde{B} = 10 \]
\[ \tilde{B} = 12 \]
\[ \tilde{B} = 15 \]
\[ \tilde{B} = 17 \]
\[ \tilde{B} = 20 \]
\[ \tilde{B} = 30 \]
\[ \tilde{B} = 60 \]
\[ \tilde{B} = 1000 \]