Unitarity in flat space holography

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based on work with
Rosseel and Riegler, 1403.5297
and work with Afshar, Bagchi, Detournay, Fareghbal, Simon

Motivation

- Holographic principle, if correct, must work beyond AdS/CFT
  holographic principle: 't Hooft '93; Susskind '94

  AdS/CFT precursor: Brown, Henneaux '86
  AdS/CFT: Maldacena '97; Gubser, Klebanov, Polyakov '98; Witten '98

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- Does it work in flat space?

  Polchinski '99

  Susskind '99

  Giddings '00

  Gary, Giddings, Penedones '09; Gary, Giddings '09; ...

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- Can we find models realizing flat space/field theory correspondences?

  Barnich, Compere '06

  Barnich et al. '10-'14

  Bagchi et al. '10-'14

  Strominger et al. '13-'14

  ...

  flat space chiral gravity: Bagchi, Detournay, DG '12

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- Are there higher-spin versions of such models?
  Afshar, Bagchi, Fareghbal, DG, Rosseel '13
  Gonzalez, Matulich, Pino, Troncoso '13

  part of larger program: non-AdS holography in higher spin gravity
  Gary, DG, Rashkov '12
  Afshar, Gary, DG, Rashkov, Riegler '12
  Gutperle, Hijano, Samani '13
  Gary, DG, Prohazka, Rey (in prep.) '14

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- Does this correspondence emerge as limit of (A)dS/CFT?

  some aspects: yes; other aspects: no

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- (When) are these models unitary?

  this talk!

  see also Barnich, Oblak '14 (induced representations of BMS_3)

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- (When) are these models unitary?
- Is there an analog of Hawking–Page phase transition?
  yes: Bagchi, Detournay, DG, Simon '13
  Detournay, DG, Schöller, Simon '14

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Address unitarity question in two boundary dimensions!
Particular interest: unitarity in flat space higher spin gravity?

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Work mostly on CFT side and study “landscape” of possible flat space asymptotic symmetry algebras:

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In this talk I will address 1.-5., but not 6.!

Main results

1. NO–GO:

Generically (see later) you can have only two out of three:

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Example:

Flat space chiral gravity

Bagchi, Detournay, DG, 1208.1658

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Example:

Minimal model holography

Gaberdiel, Gopakumar, 1011.2986, 1207.6697

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Example:

Flat space higher spin gravity (Galilean W_3 algebra)

Afshar, Bagchi, Fareghbal, DG and Rosseel, 1307.4768

Gonzalez, Matulich, Pino and Troncoso, 1307.5651

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2. YES–GO:
There is (at least) one counter-example, namely a Vasiliev-type of theory, where you can have all three properties!

> Unitary, non-trivial flat space higher spin algebra exists!
> Vasiliev-type flat space chiral higher spin gravity?

İnönü–Wigner contraction of Virasoro (Barnich & Compère '06)
BMS_3 and GCA_2/$URCA_2$

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- Define superrotations L_n and supertranslations M_n

$$\begin{aligned} &\text{GCA:} \qquad && L_n := \mathcal{L}_n + \bar{\mathcal{L}}_n \qquad && M_n := -\epsilon\left(\mathcal{L}_n - \bar{\mathcal{L}}_n\right) \\ &\text{URCA:} \qquad && L_n := \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad && M_n := \epsilon\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right) \end{aligned}$$

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- Galilean limit/ultrarelativistic boost: $\epsilon \sim 1/\ell \to 0$

$$\begin{aligned} [L_n,\, L_m] &= (n-m)\, L_{n+m} + c_L\, \tfrac{1}{12}\, \delta_{n+m,\,0} \\ [L_n,\, M_m] &= (n-m)\, M_{n+m} + c_M\, \tfrac{1}{12}\, \delta_{n+m,\,0} \\ [M_n,\, M_m] &= 0 \end{aligned}$$

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- Central charges:

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- Example: URCA TMG: $c_L = 3/\mu G$ and $c_M = 3/G$ [dimensionful!]

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Assumptions:

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Conclusions:

- Gram matrix of level 2 descendants:

$$\begin{pmatrix} \langle 0|L_2 L_{-2}|0\rangle = \frac{c_L}{2} & \langle 0|L_2 M_{-2}|0\rangle = \frac{c_M}{2} \\ \langle 0|M_2 L_{-2}|0\rangle = \frac{c_M}{2} & \langle 0|M_2 M_{-2}|0\rangle = 0 \end{pmatrix}$$

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- $c_M = 0$, $c_L < 0$: one negative norm and one null state

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Generalization to higher spin asymptotic symmetry algebras

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Example: Galilean W_3 algebra: $[L_n, L_m]$ and $[L_n, M_m]$ as before,

$$[L_n, U_m] = (2n-m)U_{n+m} \qquad [L_n, V_m] = [M_n, U_m] = (2n-m)V_{n+m}$$

$$[U_n, U_m] = (n-m)(2n^2+2m^2-nm-8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m} - \frac{96\left(c_L + \frac{44}{5}\right)}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

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Unitarity leads to further contraction! (Afshar et al. 1307.4768)

İnönü–Wigner contraction of two W-algebras (generators $\mathcal{L}, \mathcal{W}$):

$$L_n := \mathcal{L}_n + \bar{\mathcal{L}}_n \qquad M_n := -\epsilon\left(\mathcal{L}_n - \bar{\mathcal{L}}_n\right)$$

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Doubly contracted algebra has unitary representations:

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Higher spin states decouple and become null states!

How general is this decoupling mechanism? (DG, Riegler, Rosseel '14)

Same conclusions — all higher spin states become null states — for

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  Unitary for $1 \leq c_L \leq 32$
- Contracted Feigin–Semikhatov (Galilean $W_N^{(2)}$)
  Unitary for $1 \leq c_L \leq 2(N-1)^2(N+1) \sim N^3$ (for large N)

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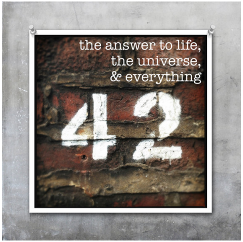

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  Upper bound from current algebra part:

$$\hat{\mathfrak{u}}(1): \qquad [O_n,\, O_m] = \frac{2(54-c_L)}{3}\, n\, \delta_{n+m,\,0}$$

$$\hat{\mathfrak{su}}(2): \qquad [Q_n^a,\, Q_m^b] = (a-b)Q_{n+m}^{a+b} + \frac{42-c_L}{6}\, n\, \delta_{a+b,\,0}\, \delta_{n+m,\,0}$$

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  Lower bound: non-negativity of c_{bare} in

$$c_L = c_{\hat{\mathfrak{u}}(1)} + c_{\hat{\mathfrak{su}}(2)} + c_{\text{bare}}$$

with $c_{\dots} = k \dim g/(k + h^\vee)$ with level $k = (42 - c_L)/6$, thus:

$$c_{\hat{\mathfrak{u}}(1)} = 1 \qquad c_{\hat{\mathfrak{su}}(2)} = 3(42 - c_L)/(54 - c_L)$$

$\Rightarrow c_{\text{bare}} \geq 0$ implies $c_L \geq 29 - \sqrt{661} \approx 3.29$

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In all cases above direct consequence of non-linearity in W-algebra!

General NO–GO result

See `1403.5297` for details!

Idea of no-go proof:

- Assume non-linearity in W-algebra, e.g. $(\lim_{c\to\infty} f(c) \to 0)$

$$[W_n,\, W_m] = \ldots + f(c)\ :\!AB\!:_{n+m} + \ldots + \omega(c) \prod_{j=-(s-1)}^{s-1} (n+j)\, \delta_{n+m,\,0}$$

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- Define Galilean contraction in usual way, e.g.

$$\begin{aligned} U_n &:= W_n + \bar{W}_n & \qquad V_n &:= -\epsilon\left(W_n - \bar{W}_n\right) \\ C_n &:= A_n + \bar{A}_n & D_n &:= -\epsilon\left(A_n - \bar{A}_n\right) \\ E_n &:= B_n + \bar{B}_n & F_n &:= -\epsilon\left(B_n - \bar{B}_n\right) \end{aligned}$$

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  No central terms in higher spin generators:

$$[V_n,\, V_m] = 0 \quad \text{(trivial)}$$
$$[U_n,\, V_m] = \cdots + \#\, c_M\, \delta_{n+m,\,0} \quad \text{(dimensions!)}$$

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$$[U_n,\, U_m] = \ldots + \mathcal{O}(1/c_M)\,(:CF:_{n+m} + \ :DE:_{n+m})$$
$$+\ \underbrace{\mathcal{O}(\frac{1}{c_M^2})}_{\neq 0}\ :DF:_{n+m} + \tilde{\omega}(c_L) \prod_{j=-(s-1)}^{s-1} (n+j)\, \delta_{n+m,\,0},$$

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$$\begin{aligned}[][c_M U_n,\, c_M U_m] &= \ldots + \mathcal{O}(c_M)\, (:CF:_{n+m} + \ :DE:_{n+m}) \\ &\quad + \mathcal{O}(1)\ :DF:_{n+m} + c_M^2 \tilde{\omega}(c_L) \prod_{j=-(s-1)}^{s-1} (n+j)\, \delta_{n+m,\,0},\end{aligned}$$

- Need rescaling $U_n \to c_M U_n = \tilde{U}_n$ before taking limit $c_M \to 0$

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- This is why higher spin states become null states and decouple!

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... every no-go result is only as good as its premises!

Drop assumption of non-linearity!

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- Linear higher spin algebra: Pope–Romans–Shen W_∞ algebra!

$$[V_m^i, V_n^j] = \sum_{r=0}^{\lfloor \frac{i+j}{2} \rfloor} g_{2r}^{ij}(m,n)\, V_{m+n}^{i+j-2r} + c^i(m)\, \delta^{ij}\, \delta_{m+n,0}$$

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- Ultra-relativistic contraction of generators

$$\mathcal{V}^i_m = V^i_m - \bar{V}^i_{-m}\,, \qquad \mathcal{W}^i_m = \epsilon\left(V^i_m + \bar{V}^i_{-m}\right)$$

  and central charges

$$c_{\mathcal{V}} = c - \bar{c}\,, \qquad c_{\mathcal{W}} = \epsilon\,(c + \bar{c})$$

The Treachery of Algebras — Ceci n'est pas une théorie.

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- AdS parent theory: no trace anomaly, but gravitational anomaly (Like in conformal Chern–Simons gravity $\rightarrow$ Vasiliev type analogue?)

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Main results:

- NO–GO: Generically, only two out of three are possible: unitarity, flat space, non-trivial higher spin states
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Interpretation: Perhaps flat space $+$ unitarity allows only (specific) Vasiliev-type of theories and no truncation to finite spin?

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- Construction of unitary flat space chiral higher spin gravity ($\mathrm{FS}\chi\mathrm{HSG}_3$)?

Evidence so far for unitary $\mathrm{FS}\chi\mathrm{HSG}_3$:
constructed its asymptotic symmetry algebra!

… hopefully shed Empire of Light on existence of FSχHSG$_3$ in the future!

References

Thanks for your attention!

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