Black hole thermodynamics

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FWF

with R. McNees and J. Salzer: 1402.5127

Main statements and overview

Main statement 1

Black holes are thermal states

- four laws of black hole mechanics/thermodynamics
- phase transitions between black holes and vacuum

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Main statement 1

Black holes are thermal states

Main statement 2

Black hole thermodynamics useful for quantum gravity checks

- quantum gravity entropy matching with semi-classical prediction

$$S_{BH} = \underbrace{\frac{k_B c^3}{\hbar G_N}}_{=1 \text{ in this talk}} \frac{A_h}{4} + \mathcal{O}(\ln A_h)$$

- semi-classical log corrections of entropy

Main statements and overview

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Black holes are thermal states

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Black hole thermodynamics useful for quantum gravity concepts

- information loss, fuzzballs, firewalls, ...
- black hole holography, AdS/CFT, gauge/gravity correspondence, ...

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Everything phrased in terms of geometry! Classical gravity = geometry!

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Even before Bekenstein–Hawking:

> Non-extensive entropy expected from/predicted by GR!

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$T =$ const. in equilibrium

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Formal analogy or actual physics?

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- Hawking: indeed!

$$S_{BH} = \frac{1}{4}\, A_h$$

  using semi-classical gravity

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- Result: Hawking temperature!

$$T_H = \frac{1}{8\pi M}$$

Consider Euclidean path integral (Gibbons, Hawking, 1977)

$$\mathcal{Z} = \int \mathscr{D}g\mathscr{D}X \, \exp\left(-\frac{1}{\hbar}\, I_E[g, X]\right)$$

Free energy from Euclidean path integral
Main idea

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Requires periodicity in Euclidean time and accessibility of semi-classical approximation

Consider small perturbation around classical solution

$$I_E[g_{cl} + \delta g, X_{cl} + \delta X] = I_E[g_{cl}, X_{cl}] + \delta I_E[g_{cl}, X_{cl}; \delta g, \delta X] + \frac{1}{2}\,\delta^2 I_E[g_{cl}, X_{cl}; \delta g, \delta X] + \dots$$

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If nothing goes wrong:

$$\mathcal{Z} \sim \exp\left(-\frac{1}{\hbar}\, I_E[g_{cl}, X_{cl}]\right) \int \mathscr{D}\delta g\, \mathscr{D}\delta X \exp\left(-\frac{1}{2\hbar}\,\delta^2 I_E\right) \times \dots$$

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What could go Wrong?

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Typical gravitational actions evaluated on black hole solutions:

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Holographic renormalization resolves first and second problem!

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Holographic renormalization

Subtract suitable boundary terms from the action

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such that second problem resolved; typically also resolves first problem

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Leading order (set $\hbar = 1$):

$$\mathcal{Z}(T,\, X) = e^{-\Gamma(T,\, X)} = e^{-\beta F(T,\, X)}$$

Here F is the Helmholtz free energy

Example: Hawking–Page phase transition of AdS_3/BTZ

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- Critical Hawking–Page temperature: $T_{\text{crit.}} = 1/(2\pi)$

Works also for flat space and expanding universe (in 2+1)

Bagchi, Detournay, Grumiller & Simon PRL '13

Hot flat space $(\varphi \sim \varphi + 2\pi)$

$$\mathrm{d}s^2 = \mathrm{d}t^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\varphi^2$$

Works also for flat space and expanding universe (in 2+1)

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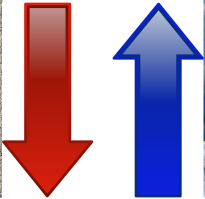

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + \frac{(E\tau)^2 \, \mathrm{d}x^2}{1 + (E\tau)^2} + \left(1 + (E\tau)^2\right) \left(\mathrm{d}y + \frac{(E\tau)^2}{1 + (E\tau)^2} \, \mathrm{d}x\right)^2$$

Flat space cosmology $(y \sim y + 2\pi r_0)$

Summary, outlook and rest of the talk (if time permits)

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- Black hole are thermal states
- Calculation of free energy requires holographic renormalization
- Interesting phase transitions possible
- Generalizable to (flat space) cosmologies

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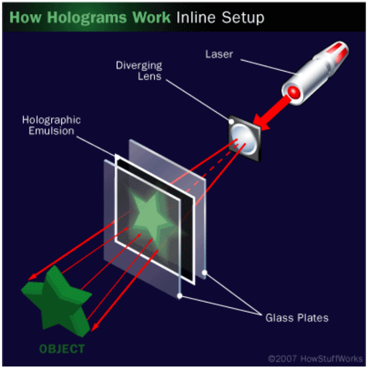
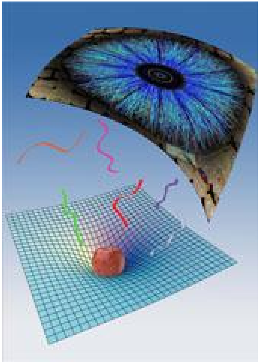
Holography — Main idea
aka gauge/gravity duality, aka AdS/CFT correspondence





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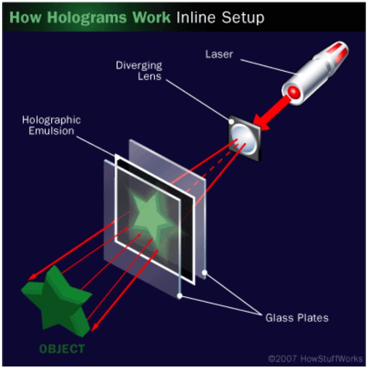

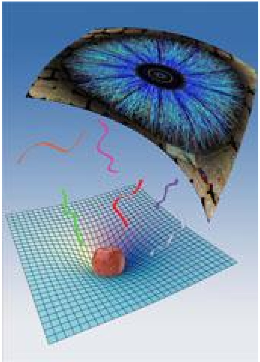

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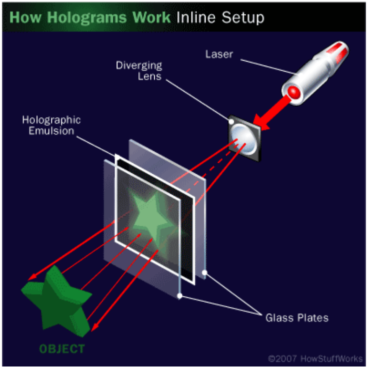

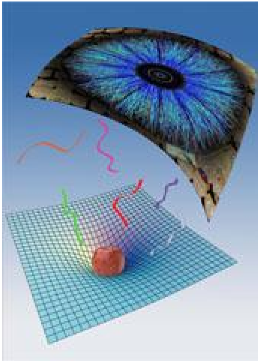

One of the most fruitful ideas in contemporary theoretical physics:

- The number of dimensions is a matter of perspective
- We can choose to describe the same physical situation using two different formulations in two different dimensions
- The formulation in higher dimensions is a theory with gravity
- The formulation in lower dimensions is a theory without gravity

Why gravity?

The holographic principle in black hole physics

Boltzmann/Planck: entropy of photon gas in d spatial dimensions

$$S_{\text{gauge}} \propto \text{volume} \propto L^d$$

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$$\text{e.g.} \quad \langle T_{\mu\nu}\rangle_{\text{gauge}} = T^{BY}_{\mu\nu} \qquad \delta(\text{gravity action}) = \int \mathrm{d}^d x \sqrt{|h|}\, T^{BY}_{\mu\nu}\, \delta h^{\mu\nu}$$

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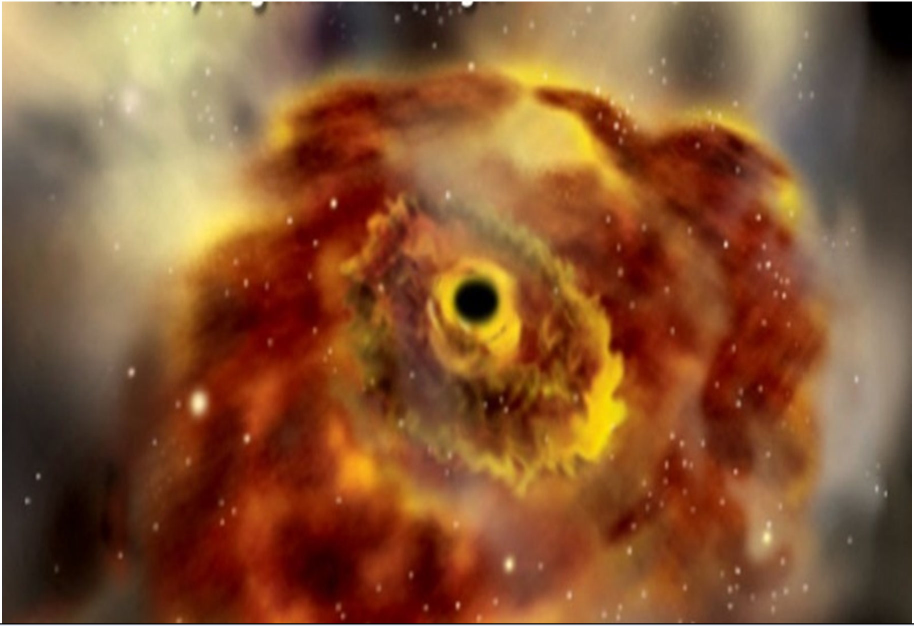
We can expect many new applications in the next decade!

Thanks for your attention!



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