Conformal blocks from Wilson lines with loop corrections

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Based on: [arXiv:1708.08657], [arXiv:1801.08549], [arXiv:1806.05836]

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Introduction

- Holography offers a way to learn quantum aspects

Quantum gravity ⟷ Conformal field theory

- Quantum effects (Expansion in Newton's constant)
- $1/c$ $(1/N)$ corrections

Large c limit

- 3d sl(N) Chern-Simons theory w/ matters→Wilson lines
- 2d conformal model w/ W_N symmetry

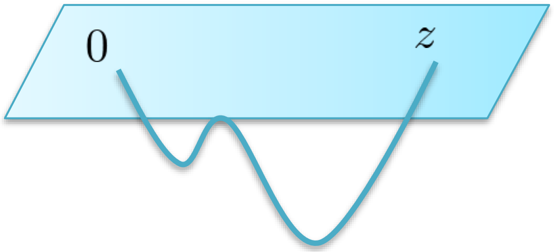
[Castro-Gopakumar-Gutperle-Raeymaekers '11, Gaberdiel-Gopakumar, Perlmutter-Prochazka-Raeymaekers '12]

How can the boundary data tell us the quantum effects in bulk gravity?

Introduction

- Approach

Wilson line



- sl(2) Chern-Simons gravity + Wilson line
  $\rightarrow$ Liouville conformal blocks [Verlinde '90]

- Expectation value of Wilson line (1/*c* leading order)

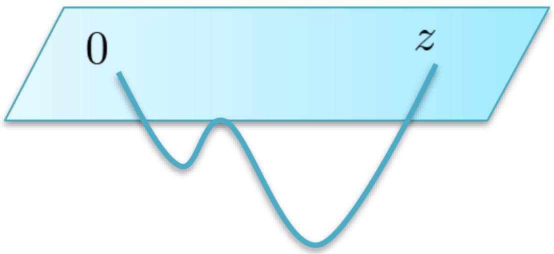
$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle = \langle W_{h_0}(z)\rangle$$

- Quantum aspects of these Wilson lines?

Introduction

- Previous works

Wilson line



- Wilson line + CFT $\longrightarrow$ $1/c$ expansion of conformal blocks

[Fitzpatric-Kaplan-Li-Wang '16]

$\longrightarrow$ purely bulk picture?

- Wilson line $\longrightarrow$ Conformal weight at $1/c$ order

[Besken-Hedge-Kraus '17]

$\longrightarrow$ How to deal divergences?

Introduction

How can the boundary data tell us the quantum effects in bulk gravity?

An answer (my talk)

The boundary symmetry fix the renormalization prescription

- What we did
  - Proposed a new renormalization prescription
  - Applied this prescription to conformal blocks and calculate new results at 1/c order.

Plan of talk

1. Introduction
2. Wilson line methods
3. Renormalization prescription & CB
4. Summary

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Wilson line methods

- Correlation functions in CFT

$$h \equiv h_0 + \frac{h_1}{c} + \frac{h_2}{c^2} + \mathcal{O}(c^{-3})$$

  - 2 point function

From bulk computation…?

$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle = \frac{1}{z^{2h}}$$

Expand

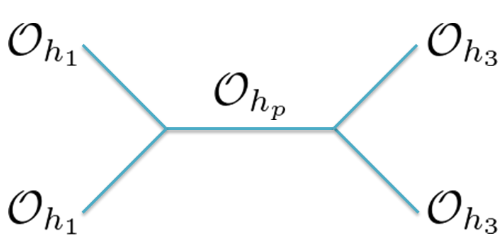
$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle = \frac{1}{z^{2h_0}}\left[1 - \frac{1}{c}2h_1\log(z) + \frac{1}{c^2}(2h_1^2\log^2(z) - 2h_2\log(z))\right] + \cdots$$

  - 3 point function (Ward-Takahashi identity)

$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)T(\infty)\rangle = hz^2\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle$$

  - 4 point function & Conformal blocks

$$\langle \mathcal{O}_{h_1}(z)\mathcal{O}_{h_1}(0)\mathcal{O}_{h_3}(\infty)\mathcal{O}_{h_3}(1)\rangle = \sum_p C_{11}{}^p C_{33}{}^p \mathcal{F}(h_1, h_2; h_p; z)\bar{\mathcal{F}}(h_1, h_2; h_p; \bar{z})$$



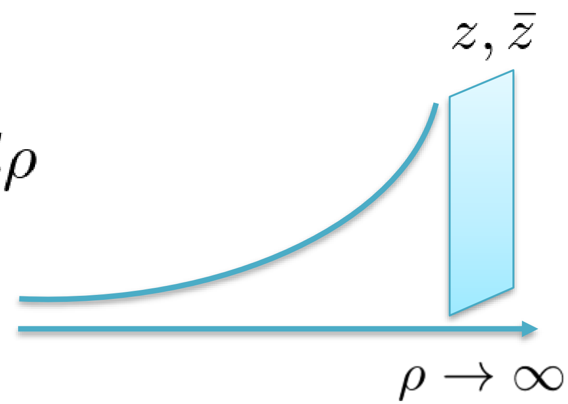
Conformal blocks

Wilson line methods

- sl(2) Chern-Simons gravity

  - Gauge field (Solution of EOM)

$$A = e^{-\rho L_0} a(z) e^{\rho L_0} dz + L_0 d\rho$$



  - Boundary DOF

    → sl(2) WZW model

- Asymptotic AdS condition

$$(A - A_{AdS})\,|_{\rho \to \infty} = \mathcal{O}(1)$$

$$\longrightarrow \quad a(z) = L_1 + \frac{6}{c} T(z) L_{-1}$$

  - Virasoro symmetry in boundary

Wilson line methods

- Wilson line operator

A gauge transformation

$$W(z_f, z_i) = P\exp\left[\int_{z_i}^{z_f} A_\mu dx^\mu\right] \to P\exp\left[\int_{z_i}^{z_f} a(z)dz\right]$$

$$= P\exp\left[\int_{z_i}^{z_f} (L_1 + \frac{6}{c}T(z)L_{-1})dz\right]$$

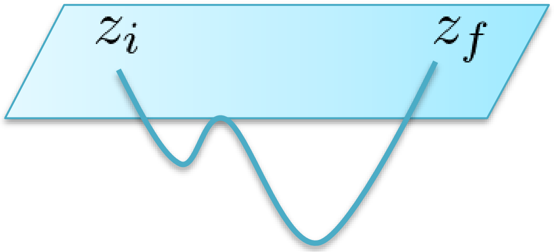
  - Expectation value of Wilson line (leading order)

$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle = \langle W_{h_0}(z)\rangle$$

$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)T(\infty)\rangle = \langle W_{h_0}(z)T(\infty)\rangle$$

  - $1/c$, $1/c^2$,… order?

- Divergences in computation



$$\langle T(z_1)T(z_2)\rangle = \frac{c/2}{(z_1 - z_2)^4}$$

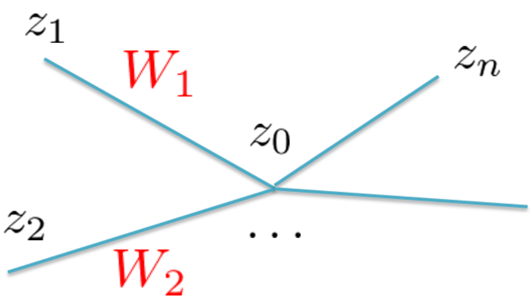
  - Divergences arise at the coincident points in the integral

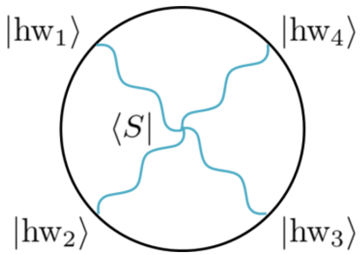
Wilson line methods

- Conformal blocks

  - n-point function

$$\langle \mathcal{O}_{h_1}(z_1)\cdots\mathcal{O}_{h_n}(z_n)\rangle = \langle S|\prod_{i=1}^{n} W_{j_i}(z_0;z_j)\,|\mathrm{hw}_i\rangle$$





Representation in sl(2)

  - $\langle S|$ belongs to a single representation in $\bigotimes_{j=1}^{n} \Lambda_j$

  - Choose a singlet ⟶ Leads to conformal blocks

[Bhatta-Raman-Suryanarayana, Besken-Hedge-Hijano-Kraus '16, '17]

  - Divergences arise in integral…

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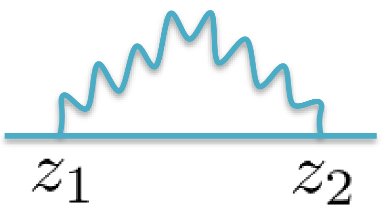
Renormalization prescription & CB

- Prescription for regularization

  - Divergences

$$\langle T(z_1)T(z_2)\rangle = \frac{c/2}{(z_1 - z_2)^4}$$



  - Removing divergences

1. Regulator

$$\langle T(z_1)T(z_2)\rangle = \frac{c/2}{(z_1 - z_2)^{4-2\epsilon}}$$

2. Overall factor & Shift parameter

$$W(z_f, z_i) = \mathcal{N}_2 P \exp\left[\int_{z_i}^{z_f} (L_1 + \frac{6}{c} c_2 T(z) L_{-1}) dz\right]$$

3. Ward-Takahashi identity for c_2

$$\langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)T(\infty)\rangle = h z^2 \langle \mathcal{O}_h(z)\bar{\mathcal{O}}_h(0)\rangle$$

Renormalization prescription & CB

- Reproduce h_1 & Fix c_2
  - 1-loop corrections of $\langle \mathcal{O}(z)\mathcal{O}(0)\rangle$

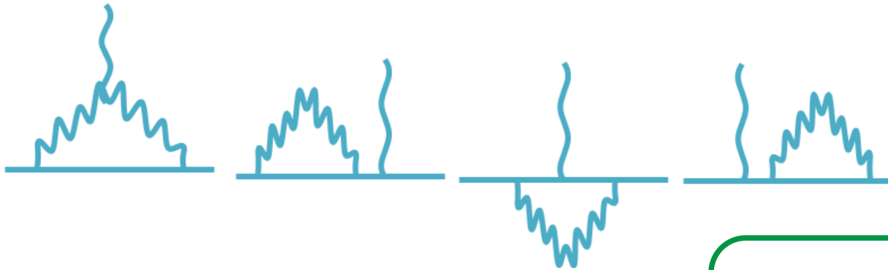
Absorbed by redefining N_2

$$\frac{1}{cz^{2h_0}}\left[\frac{6(h_0-1)h_0}{\epsilon}+12h_0(h_0-1)\log z+2h_0(5h_0-2)\right]$$

$$c_2 = 1+\mathcal{O}(c^{-1})$$

Reproduce h_1!!

  - 1-loop corrections of $\langle \mathcal{O}(z)\mathcal{O}(0)T(\infty)\rangle$

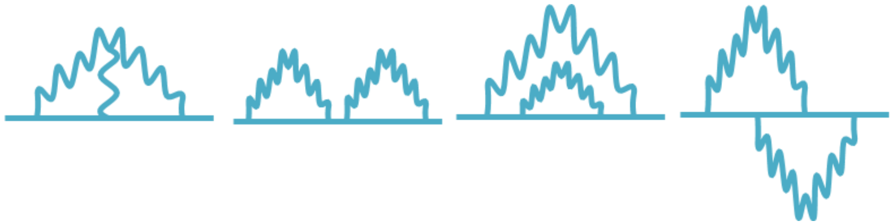


$$\left(h_0+\frac{h_1}{c}\right)z^2\langle \mathcal{O}(z)\mathcal{O}(0)\rangle$$

$$c_2 = 1+\frac{1}{c}\left(\frac{6}{\epsilon}+3\right)+\mathcal{O}(c^{-2})$$

Renormalization prescription & CB

- Redefine N_2, c_2 & Reproduce h_2
  - 2-loop corrections of $\langle \mathcal{O}(z)\mathcal{O}(0)\rangle$



$$72h_0^2(h_0-1)^2\log^2 z + 156h_0(h_0-1)\log z$$

$$c_2 = 1 + \frac{1}{c}\left(\frac{6}{\epsilon} + 3\right) + \mathcal{O}(c^{-2})$$

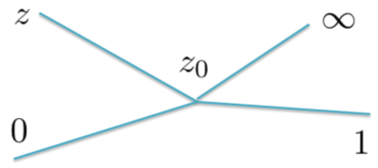
Reproduce h_2!!

- Apply to more higher order
  - Is our method true for higher orders in $1/c$?

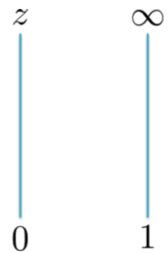
→ $1/c^3$ result [Besken-D'Hoker-Hedge-Kraus '18]

Renormalization prescription & CB

- Identity blocks up to $1/c^2$ order
  - Intermediate state is identity operator or its descendants

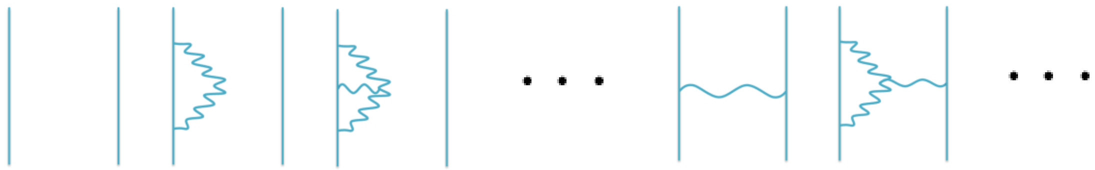


→



$$= \langle W_{j_1}(z,0) W_{j_3}(\infty,1)\rangle$$

  - Self-energy type & T exchange



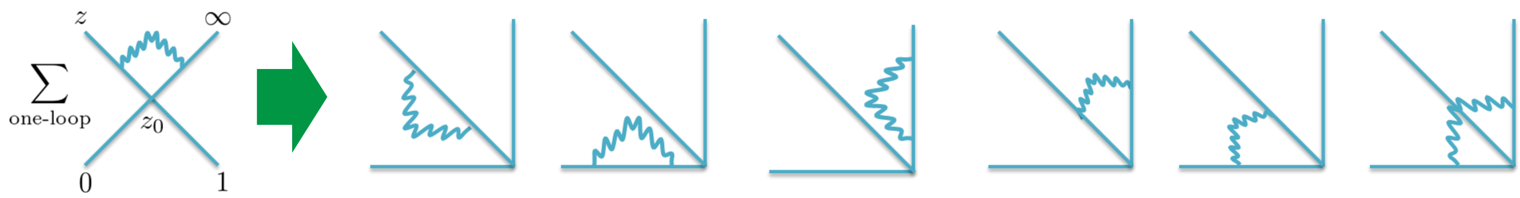
→ Only shift the conformal weight

  - TT exchange

→ Reduce to the analysis of Fitzpatrick et al [Fitzpatrick-Kaplan-Li-Wang '16]

Renormalization prescription & CB

- General blocks up to 1/c order
  - Choose $z_0 = z_4 = 1$



  - Reproduce known results [Fitzpatrick-Kaplan-Li-Wang '16]

$$\mathcal{F}(h_1, h_3; h_p; z) = z^{-2h_3+h_p}\,{}_2F_1(h_p, h_p; 2p; z) + \frac{1}{c}[h_1 h_3 f_a(h_p, z) + (h_1 + h_3) f_b(h_p, z) + f_c(h_p, z)] + \cdots$$

Known in all order of *z*

Known in first few orders of *z*

  - And obtain the all order expression of f_c in z

We obtained new CFT results from bulk computation

Recent work in CFT [Bombini-Giusto-Russo '18]

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→

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- What we did
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Summary

- Future directions

  - Higher order in $1/c$

    Our method is true for all order?

  - Supersymmetric case $\mathcal{N}=1$: [arXiv:1806.05836]

  - Treatment of heavy operator

  - Higher spin case spin 3: [arXiv:1708.08657], [arXiv:1801.08549]

    Is our method true for sl(N) CS theory and W_N symmetry?

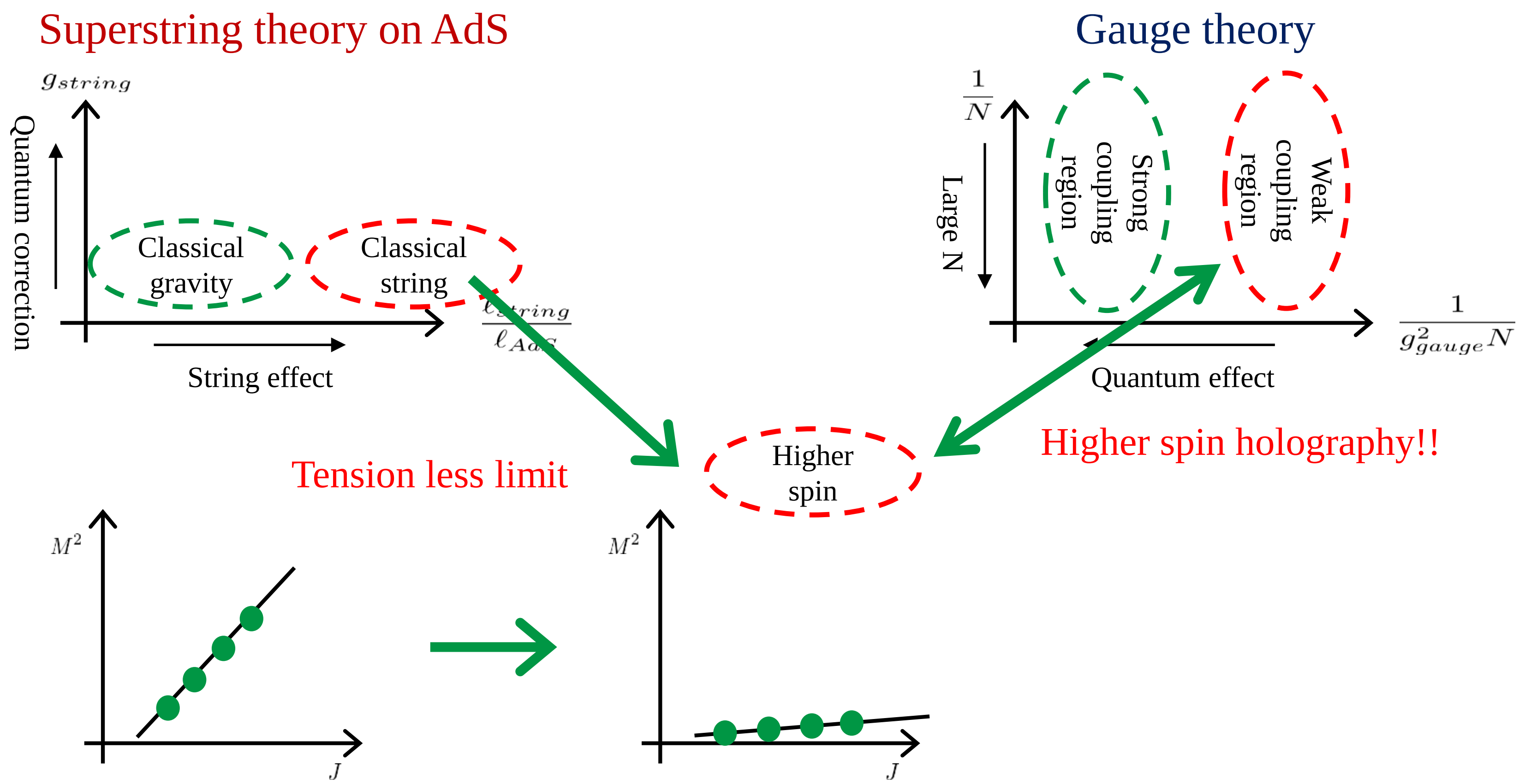
That's all for my presentation

Thank you very much

Back up slides

Higher spin holography

- Higher spin holography



- Weak/weak duality

Higher spin holography

- Gaberdiel-Gopakumar conjecture [Gaberdiel-Gopakumar '10]

  - Large N limit but finite λ

3d Vasiliev theory ⟷ 2d W_N minimal model

$$\frac{\mathrm{su}(N)_k \oplus \mathrm{su}(N)_1}{\mathrm{su}(N)_{k+1}} \qquad c = (N-1)\left(1 - \frac{N(N+1)}{(k+N)(k+N+1)}\right)$$

  - Large c limit but finite N

[Castro-Gopakumar-Gutperle-Raeymaekers '11, Gaberdiel-Gopakumar, Perlmutter-Prochazka-Raeymaekers '12]

3d sl(N) Chern-Simons theory w/ matters ⟷ 2d W_N minimal model

$$k = -1 - N + \frac{N(N^2-1)}{c} + \frac{N(1-N^2)(1-N^3)}{c^2} + \mathcal{O}(c^{-3})$$

Dual CFT data

- Dual conformal model
  - coset description and central charge

$$\frac{\mathrm{su}(2)_k \oplus \mathrm{su}(2)_1}{\mathrm{su}(2)_{k+1}} \qquad c = 1 - \frac{6}{(k+2)(k+3)}$$

  - Large *c* limit

$$k = -3 + \frac{6}{c} + \frac{42}{c^2} + \mathcal{O}(c^{-1})$$

  - Conformal weight of primary state (spin *j*)

$$\begin{aligned} h_j &= -j - \frac{6j(j+1)}{c} - \frac{78j(j+1)}{c^2} + \mathcal{O}(c^{-3}) \\ &\equiv h_0 + \frac{h_1}{c} + \frac{h_2}{c^2} + \mathcal{O}(c^{-3}) \end{aligned}$$

SUSY extension

- Supersymmetric case ($\mathcal{N}$ = 1 SUSY)

sl(2) → osp(1|2), $\langle T(z_1)T(z_2)\rangle$ + $\langle G(z_1)G(z_2)\rangle$

  - Only consider the NS-sector for light operator

    → R-sector is heavy operator $\mathcal{O}(c^1)$

  - Obtained the closed form expressions of super conformal blocks including 1/c corrections

    → Our new result

- Extended SUSY
  - $\mathcal{N}$ = 2 SUSY (in progress)
  - More interesting case: $\mathcal{N}$=3, $\mathcal{N}$=4

    → Relation to superstring?