Quantum space-time, higher spin and gravity from matrix models

Harold Steinacker

Department of Physics, University of Vienna

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Quantum Structure of Spacetime

expect quantum structure of space-time; how?

guidelines:

- simple
- gauge theory (Minkowski signature)
- finite dof per volume (Planck scale)
  $\rightarrow$ underlying d.o.f. non-geometric
- GR established only in IR regime
  space-time & gravity may emerge from other d.o.f.

Matrix Models (of Yang-Mills type)

$S = Tr([X^\mu, X^\nu][X_\mu, X_\nu] + ...)$ provide suitable framework!

- simple
- describe dynamical NC / fuzzy spaces, gauge theory

$$X^a \to U^{-1} X^a U$$

- well suited for quantization: $\int dX\, e^{-S[X]}$
  - generic models: serious UV/IR mixing problem
  - preferred model: maximal SUSY = IKKT model

    shares features of string theory, cut the “landscape”
- gravity?

outline:

- the IKKT matrix model & NC gauge theory
- fuzzy H_n^4 & twistor space
  tower of higher-spin modes, truncated at n
- cosmological (3+1)-dim. space-time $\mathcal{M}_n^{3,1}$
  (FLRW cosmology, Big Bounce)
- fluctuations $\rightarrow$ higher spin gauge theory
- spin 2 modes & gravitons
- linearized Einstein-Hilbert action

HS, arXiv:1606.00769
HS, arXiv:1710.11495
M. Sperling, HS arXiv:1806.05907
M. Sperling, HS arXiv:1901.03522

The IKKT matrix model

IKKT or IIB model — Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[Y,\Psi] = -Tr\Big([Y^a,Y^b][Y^{a'},Y^{b'}]\eta_{aa'}\eta_{bb'} \; + m^2 Y^a Y_a \; + \bar\Psi\gamma_a[Y^a,\Psi]\Big)$$

$$Y^a = Y^{a\dagger} \in Mat(N,\mathbb{C}), \qquad a = 0,...,9, \qquad N \to \infty$$

gauge invariance $Y^a \to UY^aU^{-1}$, $SO(9,1)$, SUSY

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point, N large
- equations of motion:
  $\Box Y^a + m^2 Y^a = 0, \qquad \Box \equiv \eta_{ab}[Y^a,[Y^b,.]]$
- quantization: $Z = \int dYd\Psi\, e^{iS[Y]}$, SUSY essential

strategy:

- look for solutions $\to$ space(time)
- fluctuations $\to$ gauge theory, dynamical geometry, gravity ?!
- matrix integral = (Feynman) path integral, incl. geometry

---

allows also non-perturbative (numerical) approach !

cf. Kim, Nishimura, Tsuchiya arXiv:1108.1540 ff class of solutions: fuzzy spaces = quantized symplectic manifolds

$$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{9,1}$$

$[X^a, X^b] \sim i\theta^{ab}(x)$ ... (quantized) Poisson tensor

algebra of functions on fuzzy space: $\mathrm{End}(\mathcal{H})$

- Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$:

$$[X^a, X^b] = i\theta^{ab}\,\mathbb{1}$$

quantized symplectic space $(\mathbb{R}^4, \omega)$

admits translations $X^a \to X^a + c^a\mathbb{1}$, no rotation invariance

- fuzzy 2-sphere S^2_N

$$X_1^2 + X_2^2 + X_3^2 = R_N^2, \qquad [X_i, X_j] = i\epsilon_{ijk}X_k$$

fully covariant under $SO(3)$ (Hoppe; Madore)

fluctuations in M.M. on fuzzy space $\to$ NC gauge theory

Let X^μ ... fuzzy space $(\mathcal{M}, \theta^{\mu\nu})$, $\phi \in End(\mathcal{H})$ ... quant. "function"
derivations:

$$[X^\mu, \phi] =: i\theta^{\mu\nu}\partial_\nu \phi$$

consider fluctuations around background X^a

$$Y^a = X^a + \mathcal{A}^a$$

$$\begin{aligned}
[Y^\mu, \phi] &= i\theta^{\mu\nu} D_\nu \phi, \qquad D_\mu = \partial_\mu + i[A_\mu, .] \\
[Y^\mu, Y^\nu] &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'} \underbrace{(\partial_{\mu'} A_{\nu'} - \partial_{\nu'} A_{\mu'} + [A_{\mu'}, A_{\nu'}])}_{F_{\mu'\nu'}} \\
S &= Tr([Y^\mu, Y^\nu][Y_\mu, Y_\nu]) \sim \int F_{\mu\nu} F^{\mu\nu} + c.
\end{aligned}$$

$\to$ YM gauge theory

$$A_\mu \to U^{-1} A_\mu U + U^{-1}\partial_\mu U$$

$\leftrightarrow$ dynamical geometry ("emergent gravity") (emergent) gravity $\leftrightarrow$ deformed $\mathcal{M}_\theta^4$? H.S. arXiv:1003.4134 ff

cf. H.Yang, hep-th/0611174 ff

eff. metric encoded in $\Box = [X_a, [X^a, .]] \sim -e^\sigma(x)\Delta_G$

$$G^{\mu\nu} = e^{-\sigma}\theta^{\mu\mu'}\theta^{\mu\mu'}g_{\mu'\nu'}$$

fluctuations $X^a + \mathcal{A}^a(X)$ $\rightarrow$ dynamical metric

cf. Rivelles hep-th/0212262

bare M.M action: 2 Ricci-flat metric fluctuations, not full Einstein eq.

quantum effects $\rightarrow$ induced gravity (Sakharov)

problems:

- $\theta^{\mu\nu}$ breaks Lorentz invariance $\rightarrow$ other terms possible (e.g. $R_{\mu\nu\alpha\beta}\theta^{\mu\nu}\theta^{\alpha\beta}$ D. Klammer, H.S. arXiv:0909.5298)
- huge cosm. constant

issues seem resolved for covariant quantum spaces:

4D covariant quantum spaces

... reconciling symplectic ω with (Lorentz/Euclid.) invar. in 4D

- example: fuzzy four-sphere S_N^4

  Grosse-Klimcik-Presnajder; Castelino-Lee-Taylor; Medina-o'Connor; Ramgoolam; Kimura; Karabail-Nair; Zhang-Hu 2001 (QHE) ...

  $\to$ higher-spin gauge theory in IKKT HS arXiv:1606.00769
- noncompact H_n^4 $\to$ higher-spin gauge theory

  M. Sperling, HS arXiv:1806.05907
- projection of H_n^4 $\to$ cosmological space-time $\mathcal{M}_n^{3,1}$

  HS, arXiv:1710.11495, arXiv:1709.10480
- spin 2 modes on $\mathcal{M}_n^{3,1}$ $\to$ gravity M. Sperling, HS arXiv:1901.03522

Euclidean fuzzy hyperboloid H_n^4 (=$EAdS_n^4$)

Hasebe arXiv:1207.1968

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) .$$

$$\eta^{ab} = \mathrm{diag}(-1,1,1,1,1,-1)$$

choose “short” discrete unitary irreps $\mathcal{H}_n$ (“minireps”, doubletons)

special properties:

- irreps under $\mathfrak{so}(4,1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\mathrm{spec}(\mathcal{M}^{05}) = \{E_0, E_0+1, ...\}, \qquad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace is $n+1$-dim. irrep of $SU(2)_L$: fuzzy S_n^2

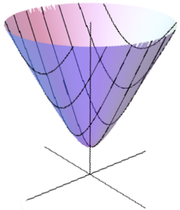
fuzzy hyperboloid H_n^4

def.

$$\begin{aligned} X^a &:= r\mathcal{M}^{a5}, \qquad a = 0, \ldots, 4 \\ [X^a, X^b] &= ir^2\mathcal{M}^{ab} =: i\Theta^{ab} \end{aligned}$$

5 hermitian generators X^a satisfy (cf. Snyder)

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbf{1}, \qquad R^2 = r^2(n^2-4)$$



one-sided hyperboloid in $\mathbb{R}^{1,4}$, covariant under $SO(4,1)$

note: induced metric = Euclidean AdS^4

oscillator construction & twistor space:

classical: $\mathbb{C}P^{2,1} \cong \mathbb{C}^4/_{\mathbb{C}^*}$ ... fund. rep of $\mathfrak{su}(2,2) \cong \mathfrak{so}(4,2)$

Hopf map

$$\begin{array}{ccc} \mathbb{C}P^{2,1} & \ni & \psi \\ \downarrow & & \downarrow \\ S^4 & \ni & x^a = \bar\psi\gamma^a\psi \end{array}$$

quantum (fuzzy) $\mathbb{C}P_n^{2,1}$: 4 bosonic oscillators $\psi_\alpha := \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \\ b_1 \\ b_2 \end{pmatrix}$

$$[\psi_\alpha, \bar\psi^\beta] = \delta_\alpha^\beta, \quad \text{fund. rep. of } \mathfrak{su}(2,2)$$

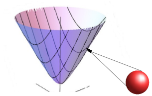
def.

$$\begin{aligned} X^a &:= r\bar\psi\gamma^a\psi, \qquad \gamma^a = \Sigma^{a5} \\ \mathcal{M}^{ab} &:= \bar\psi\Sigma^{ab}\psi \end{aligned}$$

$\{\mathcal{M}^{ab}, X^a = \mathcal{M}^{a5}, \ a = 0, ..., 4\}$ ... $\mathfrak{so}(4,2)$ generators

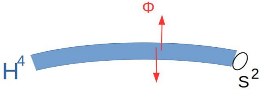
minireps $\mathcal{H}_n$: on $|\Omega\rangle = a_{i_1}^\dagger \dots a_{i_n}^\dagger |0\rangle$ ... lowest weight state therefore:

$H_n^4 \equiv End(\mathcal{H}_n) =$ quantized $\mathbb{C}P^{1,2} = S^2$ bundle over H^4, selfdual $\theta^{\mu\nu}$



symplectic form ω depends on S^2, averaged on H^4 $\Rightarrow$ covariance

functions on $H_n^4 \overset{loc}{\cong} S^2 \times H^4$ = harmonics on S^2 $\times$ functions on H^4



local stabilizer on H^4 acts on S^2

$\rightarrow$ harmonics on H_n^4 = higher spin modes on H^4

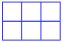
fuzzy "functions" on H_n^4:

$$(End(\mathcal{H}_n) \rightsquigarrow) \qquad HS(\mathcal{H}_n) = \int\limits_{\mathbb{C}P^{1,2}} f(m)\,|m\rangle\langle m| \;\cong\; \bigoplus_{s=0}^{n} \mathcal{C}^s$$

$\mathcal{C}^0$ = scalar functions on H^4: $\qquad \phi(X)$

$\mathcal{C}^1$ = selfdual 2-forms on H^4: $\qquad \phi_{\mu\nu}(X)\theta^{\mu\nu} \;=$ 

$\vdots$

$End(\mathcal{H}_n) \cong$ fields on H^4 taking values in $\mathfrak{hs} = \oplus$  $\ni \theta^{\mu_1\nu_1}\ldots\theta^{\mu_s\nu_s}$

higher spin modes = would-be KK modes on S^2

i.e. higher spin theory, truncated at n

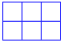
M. Sperling, HS arXiv:1806.05907

semi-classical limit & higher spin

general functions on H_n^4:

$$\phi = \phi_{\underline{\alpha}}(x)\,\Xi^{\underline{\alpha}}$$

where

$$\Xi^{\underline{\alpha}} = \mathcal{M}...\mathcal{M} \qquad \in \ \mathfrak{hs} \ \cong \ \oplus$$ 

as vector space, (Poisson) Lie algebra

$$\{\mathcal{M}^{ab}, \mathcal{M}^{b_1c_1}...\mathcal{M}^{b_sc_s}\} = \eta^{ab_1}\mathcal{M}^{bc_1}...\mathcal{M}^{b_sc_s} \pm \ldots$$

relations

$$\mathcal{M}^{ab}\mathcal{M}^{ac} = \frac{R^2}{\theta}P_T^{ab}, \qquad \varepsilon_{abcde}\mathcal{M}^{ab}\mathcal{M}^{cd} = \frac{R}{\theta}x^e$$

functions on H_n^4 $\approx$ $\mathfrak{hs}$ - valued functions on H^4

cf. Vasiliev theory!

relation with spin s fields: isomorphism

$$
\begin{aligned}
\Gamma^{(s)} H^4 &\to \mathcal{C}^s \\
\phi^{(s)}_{a_1 \ldots a_s}(x) &\mapsto \phi^{(s)} = \{x^{a_1}, \ldots \{x^{a_s}, \phi^{(s)}_{a_1 \ldots a_s}\} \ldots\} \\
\{x_{a_1}, \ldots \{x_{a_s}, \phi^{(s)}\} \ldots\}_0 &\leftarrow \phi^{(s)}
\end{aligned}
$$

$\phi^{(s)}_{a_1 \ldots a_s}(x)$ ... "symbol" of $\phi^{(s)} \in \mathcal{C}^s$ M. Sperling & HS, arXiv:1806.05907

= symm., traceless, tangential, div.-free rank s tensor field on H^4

$$
\begin{aligned}
\phi_{c_1 \ldots c_s}(x) x^{c_i} &= 0 \,, \\
\phi_{c_1 \ldots c_s}(x) \eta^{c_1 c_2} &= 0 \,, \\
\partial^{c_i} \phi_{c_1 \ldots c_s}(x) &= 0 \,.
\end{aligned}
$$

H_n^4 is starting point for cosmological quantum space-times $\mathcal{M}_n^{3,1}$:

- exactly homogeneous & isotropic, Big Bounce
- on-shell higher-spin fluctuations obtained
- spin 2 metric fluctuations $\rightarrow$ gravity (linearized)

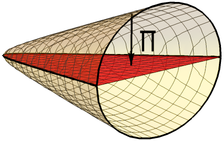
open FRW universe from H_n^4

HS arXiv:1710.11495

$\mathcal{M}_n^{3,1}$ = H_n^4 projected to $\mathbb{R}^{1,3}$ via

$$Y^\mu \sim y^\mu : \ \mathbb{C}P^{1,2} \to H^4 \subset \mathbb{R}^{1,4} \stackrel{\Pi}{\longrightarrow} \mathbb{R}^{1,3} \ .$$

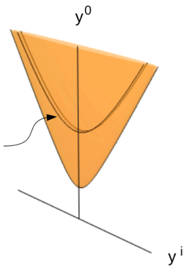
induced metric has Minkowski signature!



algebraically: $\mathcal{M}_n^{3,1}$ generated by

$$\boxed{Y^\mu := X^\mu, \quad \text{for} \ \ \mu = 0,1,2,3 \qquad \text{(drop } X^4 \text{)}}$$

geometric properties:



- $SO(3,1)$ manifest $\Rightarrow$ foliation into $SO(3,1)$-invariant space-like 3-hyperboloids H^3_τ
- double-covered FRW space-time with hyperbolic ($k=-1$) spatial geometries

$$ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2,$$

$d\Sigma^2$ ... $SO(3,1)$-invariant metric on space-like H^3

functions on $\mathcal{M}^{3,1}$:

generated by $X^\mu = r\mathcal{M}^{\mu 5} \sim x^\mu$ and $T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4} \sim t^\mu$, with CR

$$\begin{aligned} \{t^\mu, x^\nu\} &= \sinh(\eta)\eta^{\mu\nu} \\ \{x^\mu, x^\nu\} &= \theta^{\mu\nu} \\ \{t^\mu, t^\nu\} &= -\frac{1}{r^2R^2}\theta^{\mu\nu} \end{aligned}$$

constraints

$$\begin{aligned} x_\mu x^\mu &= -R^2\cosh^2(\eta), \qquad x^4 = R\sinh(\eta) \\ t_\mu t^\mu &= r^{-2}\cosh^2(\eta)\,, \\ t_\mu x^\mu &= 0, \\ t_\mu \theta^{\mu\alpha} &= -\sinh(\eta)x^\alpha, \\ x_\mu \theta^{\mu\alpha} &= -r^2R^2\sinh(\eta)t^\alpha, \\ \eta_{\mu\nu}\theta^{\mu\alpha}\theta^{\nu\beta} &= R^2r^2\eta^{\alpha\beta} - R^2r^4t^\alpha t^\beta - r^2x^\alpha x^\beta \\ \theta^{\mu\nu}\theta_{\mu\nu} &= 2R^2r^2(2-\cosh^2(\eta)) \end{aligned}$$

hence: t^μ ... space-like S^2

self-duality on H^4 $\Rightarrow$ $\theta^{\mu\nu} = c(x^\mu t^\nu - x^\nu t^\mu) + b\epsilon^{\mu\nu\alpha\beta}x_\alpha t_\beta$

functions as higher-spin modes:

$$\phi = \sum_{s=0}^{n} \phi^{(s)} \quad \in \mathrm{End}(\mathcal{H}_n), \qquad \phi^{(s)} \in \mathcal{C}^s$$

2 points of view:

- functions on H_n^4: full $SO(4,1)$ covariance

  represent $\phi^{(s)}$ as

$$\begin{aligned} \phi^H_{a_1 \ldots a_s} &\propto \{x^{a_1}, \ldots \{x^{a_s}, \phi^{(s)}\} \ldots\}_0 \\ \phi^{(s)} &= \{x^{a_1}, \ldots \{x^{a_s}, \phi^H_{a_1 \ldots a_s}\} \ldots\} \end{aligned}$$

- functions on $\mathcal{M}_n^{3,1}$: reduced $SO(3,1)$ covariance

$$\phi^{(s)} = \phi^{(s)}_{\mu_1 \ldots \mu_s}(x) t^{\mu_1} \ldots t^{\mu_s}$$

$t_\mu x^\mu = 0 \;\Rightarrow\;$ "space-like gauge"

$$\boxed{x^{\mu_i} \phi^{(s)}_{\mu_1 \ldots \mu_s} = 0}$$

($\rightarrow$ no ghosts!)

$\mathcal{M}^{3,1}$ realization in IKKT model:

$$S[Y] = Tr(-[Y^\mu, Y^\nu][Y_\mu, Y_\nu] + m^2 Y^\mu Y_\mu)$$

background solution:

$$Y^\mu := T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4}$$

satisfies

$$\Box T^\mu = 3R^{-2} T^\mu, \qquad \Box = [T^\mu, [T_\mu, .]]$$

- $[\Box, \mathcal{S}^2] = 0, \quad \mathcal{S}^2 = [\mathcal{M}^{ab}, [\mathcal{M}_{ab}, \cdot]] + r^{-2}[X_a, [X^a, \cdot]]$

  ... spin Casimir, selects spin s sectors $\mathcal{C}^s$

  $\Rightarrow$ higher-spin expansion $\phi = \phi(X) + \phi_\mu(X) T^\mu + \ldots$ on $\mathcal{M}^{3,1}$
- $\Box \sim \alpha^{-1} \Box_G$ encodes eff. FRW metric $ds^2_G = -dt^2 + a(t)^2 d\Sigma^2$, asymptotically coasting $a(t) \propto t$
- Big Bounce, initial $a(t) \sim t^{1/5}$ singularity

fluctuations & higher spin gauge theory

$$S[Y] = Tr(-[Y^a, Y^b][Y_a, Y_b] + m^2 Y^a Y_a) = S[U^{-1} Y U]$$

background solution: $S_N^4,\ H_n^4,\ \mathcal{M}_n^{3,1}$

add fluctuations $Y^\mu = T^\mu + \mathcal{A}^\mu,$

gauge trafos $\mathcal{A}^\mu \to [\Lambda, \mathcal{A}^\mu] + [\Lambda, T^\mu], \qquad \Lambda \in End(\mathcal{H})$

expand action to second oder in $\mathcal{A}^\mu$

$$S[Y] = S[T] + \frac{2}{g^2}\mathrm{Tr}\mathcal{A}_\mu\Big(\underbrace{(\Box + \frac{1}{2}m^2)\delta^\mu_\nu + 2[[T^\mu, T^\nu], .]}_{\mathcal{D}^2} - \underbrace{[T^\mu, [T^\nu, .]]}_{g.f.}\Big)\mathcal{A}_\nu$$

$\mathcal{A}_\mu$ ... $\mathfrak{hs}$-valued field on $\mathcal{M}$, incl. spin 2 diagonalization & eigenmodes on $\mathcal{M}_n^{3,1}$: background $\overline{Y}^\mu = T^\mu$

M. Sperling, HS: arXiv:1901.03522

symmetry: only space-like $SO(3,1)$
underlying $SO(4,1)$ & $SO(4,2)$ extremely useful

- ansatz:

$$\begin{aligned}\mathcal{A}_\mu^{(+)}[\phi^{(s)}] &:= \{x_\mu, \phi^{(s)}\}_+ \\ \mathcal{A}_\mu^{(-)}[\phi^{(s)}] &:= \{x_\mu, \phi^{(s)}\}_- \ .\end{aligned}$$

can show using $\mathfrak{so}(4,2)$ structure: are eigenmodes

$$\begin{aligned}\mathcal{D}^2 \mathcal{A}_\mu^{(+)}[\phi^{(s)}] &= \mathcal{A}_\mu^{(+)}\left[\left(\Box + \tfrac{2s+5}{R^2}\right)\phi^{(s)}\right] \\ \mathcal{D}^2 \mathcal{A}_\mu^{(-)}[\phi^{(s)}] &= \mathcal{A}_\mu^{(-)}\left[\left(\Box + \tfrac{-2s+3}{R^2}\right)\phi^{(s)}\right] \ .\end{aligned}$$

$\rightarrow$ 2 off-shell eigenmodes,

1 on-shell physical mode $\mathcal{A}_\mu^{(-)}[\phi^{(s)}]$ with $(\Box - \frac{2s}{R^2})\phi^{(s)} = 0$

$\mathcal{H}_{\text{phys}} = \{\mathcal{D}^2\mathcal{A} = 0, \ \ \mathcal{A} \ \text{gauge fixed}\}/\{\text{pure gauge}\}$

- pure gauge mode $\mathcal{A}_\mu^{(g)}[\phi^{(s)}] = \{t_\mu, \phi^{(s)}\}$
- one off-shell mode missing (?), suspect no extra physical mode

- conjecture: no ghosts (cf. YM !)
- one propagating physical on-shell modes found for each spin $s \leq n$
- same propagation for all physical modes, governed by universal metric $G^{\mu\nu}$

  even though $SO(3,1)$ only space-like

  $\exists$ Lorentz-violating structures: $x_\mu\{t^\mu, .\} \sim \frac{\partial}{\partial\tau}$ ... time-like VF (cosmic background!)

vielbein, metric & dynamical geometry

consider scalar field $\phi = \phi(x)$ (e.g. transversal fluctuation)

kinetic term $-Tr[T^\alpha,\phi][T_\alpha,\phi] \sim \int e^\alpha\phi e_\alpha\phi = \int \gamma^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

vielbein

$$
\begin{aligned}
e^\alpha &:= \{T^\alpha,.\} = e^{\alpha\mu}\partial_\mu \\
e^{\alpha\mu} &= \sinh(\eta)\eta^{\alpha\mu}
\end{aligned}
$$

metric $\gamma^{\mu\nu} = \eta_{\alpha\beta}e^{\alpha\mu}\, e^{\beta\nu}$

perturbed vielbein: $Y^\alpha = T^\alpha + \mathcal{A}^\alpha$

$$
\begin{aligned}
e^\alpha &= \{T^\alpha + \mathcal{A}^\alpha,.\} = e^{\alpha\mu}[\mathcal{A}]\partial_\mu \\
\delta_\mathcal{A}\gamma^{\mu\nu} &\sim \{\mathcal{A}^\mu, x^\nu\} + (\mu\leftrightarrow\nu)
\end{aligned}
$$

linearize & average over fiber $\rightarrow$ $h^{\mu\nu} = [\delta_\mathcal{A}\gamma^{\mu\nu}]_0$
coupling to matter:

$$S[\text{matter}] \sim \int_\mathcal{M} d^4x\, h^{\mu\nu}T_{\mu\nu}$$

vielbein, metric & dynamical geometry

consider scalar field $\phi = \phi(x)$ (e.g. transversal fluctuation)

kinetic term $-Tr[T^\alpha,\phi][T_\alpha,\phi] \sim \int e^\alpha\phi e_\alpha\phi = \int \gamma^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

vielbein

$$\begin{aligned} e^\alpha &:= \{T^\alpha,.\} = e^{\alpha\mu}\partial_\mu \\ e^{\alpha\mu} &= \sinh(\eta)\eta^{\alpha\mu} \end{aligned}$$

metric $\gamma^{\mu\nu} = \eta_{\alpha\beta}e^{\alpha\mu}\, e^{\beta\nu}$

perturbed vielbein: $Y^\alpha = T^\alpha + \mathcal{A}^\alpha$

$$\begin{aligned} e^\alpha &= \{T^\alpha + \mathcal{A}^\alpha,.\} = e^{\alpha\mu}[\mathcal{A}]\partial_\mu \\ \delta_{\mathcal{A}}\gamma^{\mu\nu} &\sim \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu) \end{aligned}$$

linearize & average over fiber $\to$ $h^{\mu\nu} = [\delta_{\mathcal{A}}\gamma^{\mu\nu}]_0$

coupling to matter:

$$S[\text{matter}] \sim \int_{\mathcal{M}} d^4x\, h^{\mu\nu}T_{\mu\nu}$$

effective metric $G^{\mu\nu}$ & conformal factor:

encoded in Laplacian $\Box_Y = [Y_\mu,[Y^\mu,.]] \sim \frac{1}{\sqrt{|G|}}\partial_\mu(\sqrt{|G|}G^{\mu\nu}\partial_\nu .)$:

$$
\begin{aligned}
G^{\mu\nu} &= \alpha\,\gamma^{\mu\nu}\,, \qquad \alpha = \sqrt{\frac{|\theta^{\mu\nu}|}{|\gamma^{\mu\nu}|}}\,, \\
\gamma^{\mu\nu} &= g_{\mu'\nu'}[\theta^{\mu'\mu}\theta^{\nu'\nu}]_{S^2}
\end{aligned}
$$

$[.]_{S^2}$ ... averaging over the internal S^2.

$\rightarrow$ scale factor of FLRW background:

- late times:

$$a(t) \approx \frac{3}{2}t, \qquad t\to\infty\,.$$

... coasting universe (no too bad !)

- big bounce:

$$a(t) \;\propto\; (t-t_0)^{\frac{1}{5}}$$

towards gravity on $\mathcal{M}^{3,1}$

linearized metric: $h^{\mu\nu} \propto \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu)$

contains all dof required for gravity

5+1 off-shell dof from $\mathcal{A}^{(-)}[\phi^{(2)}]$ and $\mathcal{A}^{(+)}[\phi^{(0)}]$, 3 pure gauge (!)

lin. Ricci:

$$\mathcal{R}^{\mu\nu}_{(\mathrm{lin})}[h[\mathcal{A}]] \approx \frac{1}{2} \underbrace{\Box h_{\mu\nu}[\mathcal{A}]}_{h_{\mu\nu}[\mathcal{D}^2\mathcal{A}]\approx 0} - \frac{1}{4}\left(\{t_\mu, \{t_\nu, h\}\} + (\mu \leftrightarrow \nu)\right)$$

(up to cosm. scales)

on-shell (vacuum) in M.M.:

$\mathcal{A}^-[\phi^{(2,0)}]$: $h \approx 0 \Rightarrow \mathcal{R}^{\mu\nu}_{(\mathrm{lin})} \approx 0$ ... 2 graviton modes (massless !)

$\mathcal{A}^-[\phi^{(2,1)}]$: $h \approx 0 \Rightarrow \mathcal{R}^{\mu\nu}_{(\mathrm{lin})} \approx 0$ ... trivial (on-shell)

$\mathcal{A}^-[\phi^{(2,2)}]$, $\mathcal{R}^{\mu\nu}_{(\mathrm{lin})} \sim 0$ ... scalar mode (lin. Schwarzschild ?!)

gauge transformations

-of functions: $\phi \mapsto \{\Lambda, \phi\}$

spin 1 trafos: $\Lambda = v^\mu(x) t_\mu \ \in \mathcal{C}^1$:

$$\{v^\mu t_\mu, \phi\}_0 \ = \frac{1}{3}\left(\sinh(\eta)\left(3 v^\mu \partial_\mu + (\mathrm{div} v)\tau - \tau v^\mu \partial_\mu\right) + x_\gamma \varepsilon^{\gamma\mu\alpha\beta}\partial_\alpha v_\mu \partial_\beta\right)\phi$$

3 (rather than 4) diffeomorphisms !

due to invar. symplectic volume on $\mathbb{C}P^{1,2}$

-of gravitons:

$$\boxed{\delta G_{\mu\nu} = \nabla_\mu \mathcal{A}_\nu + \nabla_\mu \mathcal{A}_\nu,} \qquad \mathcal{A}_\mu = \{x_\mu, \Lambda\}_- \ \text{... VF}$$

∇ ... covariant w.r.t. FLRW background

$\nabla_\alpha(a(t)^{\frac{2}{3}}\mathcal{A}^\alpha) = 0$ ...(almost) volume preserving

-of gauge fields: $\mathcal{A}^\mu \mapsto \{\Lambda, T^\mu + \mathcal{A}^\mu\}$

linearized Einstein-Hilbert action

- determined by gauge invariance $\delta_\phi S_{\rm EH} = 0$
  result:

$$S_{\rm EH} := S_1 - \frac{1}{24} S_h - \frac{1}{R^2} S_3 - \frac{1}{2R^2} S_4 + \frac{3}{R^2}\left(\frac{1}{2} S_{M3} - 2 S_{gf2} + \frac{3}{2} S_{gf3}\right)$$

$\approx$ linearized Einstein-Hilbert for traceless modes

where

$$\begin{aligned}
S_1 &= \int \left(h_{\mu\nu} - \tfrac{1}{3} h\, \eta_{\mu\nu}\right) \mathcal{G}^{\mu\nu}_{(\rm lin)} \left[h_{\alpha\beta} - \tfrac{1}{3} h\, \eta_{\alpha\beta}\right] \\
S_h &= \int h \Box h\,, \\
S_3 &= \int \delta g^0_{\mu\nu} \delta g_0^{\mu\nu}\,, \\
S_4 &= \int \delta g^0_{\mu\nu} \{\mathcal{M}^{\mu\rho}, \delta g_0^{\rho\nu}\}\,, \\
S_{M3} &= \int f_{\mu\nu} \{\mathcal{M}^{\nu\rho}, h^{\rho\mu}\}\,, \\
S_{gf2} &= \int \{x_\mu, A^\mu\}_- D^- \{t_\rho, A^\rho\}\,, \\
S_{gf3} &= \int (D^- \{t_\nu, A^\nu\}) \Box^{-1} (D^- \{t_\nu, A^\nu\})\,,
\end{aligned}$$

- expected to be induced by quantum effects (cf. Sakharov 1967)

- Ricci-flat vacuum solutions for both bare M.M. and induced S_{EH}

  S_{EH} may be needed to recover inhomogeneous Einstein eq.

  $$G_{\mu\nu} \propto T_{\mu\nu}$$

  $\Rightarrow$ expect $\approx$ linearized GR at intermediate scale,
  good agreement with solar system tests
- model is fully non-linear (to be understood)
- no cosm. const. $\int d^4x \sqrt{g}$ (?)
  replaced by YM-action, stabilizes $\mathcal{M}^{3,1}$

  $\rightarrow$ no cosm. const. problem ?!
- significant differences at cosmic scales,

  reasonable (coasting) cosmology without any fine-tuning !!

bare YM action & gravitons:

can rewrite

$$S_2[\mathcal{A}^{(-)}[\phi^{(2,0)}]] \propto -\int h^{\mu\nu}[\phi^{(2,0)}](\Box - \frac{2}{R^2})(\Box_H - 2r^2)^{-1} h_{\mu\nu}[\phi^{(2,0)}]$$

leading to

$$(\Box - \frac{2}{R^2})h_{\mu\nu} \sim -(\Box_H - 2r^2)T_{\mu\nu}$$

summary

- matrix models:
  natural framework for quantum theory of space-time & matter
- 4D covariant quantum spaces $\rightarrow$ higher spin theories
- $\exists$ nice cosmological FRW space-time solutions
  reg. BB, finite density of microstates
- fluct. $\rightarrow$ all ingredients for (lin.) gravity
- Yang-Mills structure $\rightarrow$ emergent gravity rather than GR
- good UV behavior (SUSY), well suited for quantization

... seems to work!! needs to be elaborated

breaking $SO(4,1) \to SO(3,1)$ and sub-structure

consider

$$D\phi := -i[X^4, \phi], \qquad \text{respects } SO(3,1)$$

acts on spin s modes as follows

$$D = \underbrace{div^{(3)}\phi}_{D^-\phi} + \underbrace{t^\mu \nabla^{(3)}_\mu \phi}_{D^+\phi} : \quad \mathcal{C}^s \to \mathcal{C}^{s+1} \oplus \mathcal{C}^{s-1}$$

decomposition into $SO(3,1)$ irreps on $H^3 \subset H^4$

$$\mathcal{C}^{(s)} = \mathcal{C}^{(s,0)} \oplus \mathcal{C}^{(s,1)} \oplus \ldots \oplus \mathcal{C}^{(s,s)}$$

D^- resp. D^+ act as

$$D^- : \mathcal{C}^{(s,k)} \to \mathcal{C}^{(s-1,k-1)}, \qquad D^+ : \mathcal{C}^{(s,k)} \to \mathcal{C}^{(s+1,k+1)} .$$