Can Chern-Simons or Rarita-Schwinger be a Volkov-Akulov Goldstone?

Dmitri Sorokin

INFN Padova Section and Uni PD

arXiv:1806.05945 with Sukriti Bansal

ESI Programme “Higher spins and holography”, Vienna, 20 March 2019

Motivation

- How to break higher-spin symmetries?
- Are they broken spontaneously? What is the mechanism?
- Can one construct consistent non-linear models of higher-spin Goldstone fields associated with spontaneous breaking of HS symmetries?
- To start with a simplified set-up based on simple but yet non-trivial finite HS algebras (similar to SUSY) and see….

History

- In 1975 Hietarinta constructed (super)algebras that generalize to higher-spins conventional susy algebras and proposed to use their non-linear realizations for the description of HS Goldstone fields ala Volkov-Akulov
- Independently, in 1977 Baaklini considered the case of spin-3/2 superalgebra of this type in D=4. 89'-10' Shima, Tanii, Tsuda, Pilot, Rajpoot…
- In 1984 Aragon & Deser constructed, in D=3, "Hypergravity" a system of (non-dynamical) gravity and fermionic spin-(2n+3)/2 gauge field invariant under local spin-(2n+1)/2 supersymmetry transformations
- Recently Hypergravity was revisited by Zinoviev '14, Bunster et.al '14, Fuentealba et.al '15, Henneaux et.al '15, …

Our motivation: study effects of spontaneous symmetry breaking and properties of non-linear Lagrangians for higher-spin goldstone fields

Simplest cases to be considered: spin-1 and spin-3/2 Goldstones in D=2+1

Hietarinta algebras

Poincaré SUSY algebra

$$\{Q_\alpha, Q_\beta\} = 2\Gamma^c_{\alpha\beta} P_c$$

$$[Q_\alpha, P_c] = 0$$

$$(a = 0, 1, \ldots, D-1)$$

α - spinor indices

$$\longrightarrow$$

$$\{Q_\alpha^{a_1 \ldots a_n}, Q_\beta^{b_1 \ldots b_m}\} = f_{\alpha\beta}^{a_1 \ldots a_n, b_1 \ldots b_m, c} P_c$$

$$[Q_\alpha^{a_1 \ldots a_n}, P_c] = 0$$

Lorentz-invariant structure constants

Bosonic Hietarinta algebras

$$[S^{a_1 \ldots a_n}, S^{b_1 \ldots b_m}] = f^{a_1 \ldots a_n, b_1 \ldots b_m, c} P_c$$

$$[S^{a_1 \ldots a_n}, P_c] = 0$$

Difference from the conventional HS algebras:

$$[T_{s_1}, T_{s_2}] = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \ldots + T_{|s_1-s_2|+2}$$

(Anti)commutators of the HS generators in the Hietarinta algebras do not produce other yet higher spin generators, i.e. the Hietarinta algebras are finite dimensional.

- Hietarinta algebras can be obtained by contractions of HS algebras

Volkov-Akulov Goldstino models '72

(Describe effects of spontaneous susy breaking in terms of non-linear effective Lagrangians for Goldstone fields on which susy is realized non-linearly)

Poincaré SUSY algebra

$$\{Q_\alpha, Q_\beta\} = 2\Gamma^c_{\alpha\beta} P_c \qquad Q_\alpha \quad \leftrightarrow \quad \psi_\alpha(x)$$

Spin ½ Goldstino

$$[Q_\alpha, P_c] = 0$$

$$(a = 0, 1, \ldots, D-1)$$

VA Construction :

i) Find a susy-invariant Cartan one-form:

$$\Omega = -\mathtt{i} g^{-1} dg = E^a P_a + E^\alpha Q_\alpha, \qquad g = e^{ix^a P_a + i\theta^\alpha Q_\alpha}$$

$$E^a = dx^a + i\bar{\theta}\Gamma^a d\theta, \qquad E^\alpha = d\theta^\alpha$$

SUSY transformations: $\theta'^\alpha = \theta^\alpha + \epsilon^\alpha, \quad x'^a = x^a - \mathtt{i}\bar{\epsilon}\Gamma^a\theta$

Volkov-Akulov Goldstino models

ii) Replace the Grassmann coordinate with the field: $\theta^{\alpha} \quad \rightarrow \quad f^{-1}\psi^{\alpha}(x)$

susy breaking scale

Susy invariant one-form in space-time:

$$E^a = dx^a + if^{-2}\psi\Gamma^a d\psi = dx^b(\delta_b^a + if^{-2}\psi\Gamma^a\partial_b\psi) \equiv dx^b E_b^a(\psi)$$

susy variations: $\delta x^a = -if^{-2}\varepsilon\Gamma^a\psi, \quad \delta\psi^{\alpha} = \varepsilon^{\alpha} + if^{-2}\varepsilon\Gamma^b\psi\partial_b\psi^{\alpha}$

Non-linear transformation

iii) Construct susy invariant action:

$$S = \tfrac{f^2}{D!}\int \varepsilon_{a_1\ldots a_D} E^{a_1} \wedge E^{a_2} \cdots \wedge E^{a_D} = -f^2\int d^D x \det E_b^a$$

In D=3

$$S_{1/2} = \int d^3x(-i\psi\Gamma^a\partial_a\psi + f^{-2}(\psi\psi)\partial_a\psi\Gamma_b\partial_c\psi\varepsilon^{abc} + \cancel{f}^2)$$

$$a = 0,1,2 \quad \alpha = 1,2 \quad \psi\psi \equiv \psi^{\alpha}C_{\alpha\beta}\psi^{\beta}$$

D=3 Chern-Simons Goldstones

$$[S^a, S^b] = 2\mathrm{i}\varepsilon^{abc} P_c$$
$$[S^a, P_b] = 0$$
$$[M^a, S^b] = 2\mathrm{i}\varepsilon^{abc} S_c$$

$$S^a \quad \rightarrow \quad A_a(x)$$

$$\delta A_a(x) = c_a + f^{-2}\varepsilon^{dbc} c_b A_c \partial_d A_a, \qquad \delta x^a = -f^{-2}\varepsilon^{dbc} c_b A_c$$

constant vector parameter

Invariant one-form: $E^a = dx^a + f^{-2}\varepsilon^{abc} A_b dA_c = dx^d(\delta^a_d + f^{-2}\varepsilon^{abc} A_b \partial_d A_c) \equiv dx^d E^a_d$

Invariant action:

Chern-Simons

$$S = -f^2 \int d^3x\,(\det E^a_d - 1) = \int d^3x \left(\varepsilon^{abc} A_a \partial_b A_c - \frac{f^{-2}}{2} \varepsilon^{abc} \varepsilon^{dfg} A_a A_d \partial_b A_f \partial_c A_g \right)$$

Does the action have a gauge symmetry? $\delta A_a = \partial_a \lambda(x) + \ldots$

NO $\longrightarrow$ vector field acquires a propagating degree of freedom

Propagating CS Goldstone is Galileon

Stueckelberg trick - make the action gauge invariant by replacing

$$A_a \rightarrow A'_a = A_a - f^{1/2}\partial_a \varphi(x), \qquad \delta A_a = \partial_a \lambda, \qquad \delta\varphi = f^{-1/2}\lambda$$

Make this substitution in the action and consider a so-called decoupling limit $f \rightarrow \infty$

$$\mathcal{L}|_{f\to\infty} = \varepsilon^{abc}\, A_a\, \partial_b A_c - \frac{1}{2}\, \varepsilon^{abc}\varepsilon^{def}\, \partial_a\varphi\, \partial_d\varphi\, \partial_e\partial_b\varphi\, \partial_f\partial_c\varphi\, .$$

Quartic Galileon Lagrangian (appears in modified gravities):

$$\begin{aligned}\mathcal{L}(\varphi) &= \frac{1}{2}\,\varphi\,\varepsilon^{abc}\varepsilon^{def}\,\partial_a\partial_d\varphi\,\partial_e\partial_b\varphi\,\partial_f\partial_c\varphi \\ &= -3\varphi\,\det(\partial_a\partial^b\varphi) \\ &= -\frac{1}{2}\varphi\left((\Box\varphi)^3 - 3\,\Box\varphi\,\partial_a\partial^b\varphi\,\partial_b\partial^a\hat{\varphi} + 2\,\partial_a\partial^b\varphi\,\partial_b\partial^c\varphi\,\partial_c\partial^a\varphi\right)\end{aligned}$$

Galilean symmetry: $\delta\varphi = c + c_a x^a$

Consequence: Galileon Lagrangians are quadratic in time derivatives

Hamiltonian analysis

$$S = \int d^3x \left(\varepsilon^{abc} A_a \partial_b A_c - \frac{f^{-2}}{2} \varepsilon^{abc} \varepsilon^{dfg} A_a A_d \partial_b A_f \partial_c A_g \right)$$

The action is of the first order in time derivative $\partial_0 A_a$

Conjugate momenta are constrained:

$$p^i = \varepsilon^{ij} A_j + f^{-2} F^i(A_b, \partial_i A_c) \quad i = 1, 2, \qquad \varepsilon^{ij} \partial_i A_j + O(f^{-2}) \approx 0$$

secondary constraint

$$p^0 = f^{-2} F^i(A_b, \partial_i A_c)$$

The 4 constraints are of the second class and reduce the number of dofs to 1

On the constraint surface Hamiltonian is

$$H = 6 f^{-2} A_0^2 \det \partial_i A_j, \qquad H_\varphi^{Galileon} = 6(\partial_0 \varphi)^2 \det \partial_i \partial_j \varphi$$

Hamiltonian is not bounded from below.
Fluctuations around certain zero-energy configurations may have negative energies leading to instabilities

3d Rarita-Schwinger goldstino model

Spin-3/2 Superalgebra

$$\{Q^a_\alpha, Q^b_\beta\} = 2\, C_{\alpha\beta}\, \varepsilon^{abc}\, P_c + \mathrm{b}\, \Gamma^{(a}_{\alpha\beta} P^{b)} + \mathrm{c}\, \eta^{ab}\, \Gamma^c_{\alpha\beta}\, P_c\,, \qquad [Q^a_\alpha, P_b] = 0$$

Special cases:

b=4, c=-2 $\quad Q_\alpha \equiv (\Gamma_a Q^a)_\alpha, \qquad \hat{Q}^a = Q^a - \frac{1}{3}\Gamma^a Q, \qquad \Gamma_a \hat{Q}^a \equiv 0$

$$\{Q_\alpha, Q_\beta\} = 2\Gamma^a_{\alpha\beta} P_a, \quad \{Q_\alpha, \hat{Q}^a_\beta\} = 0 = \{\hat{Q}^a_\alpha, \hat{Q}^b_\beta\}$$

b=-4/5, c=8/5 $\quad \{Q_\alpha, \hat{Q}^a_\beta\} = 0 = \{Q_\alpha, Q_\beta\}, \quad \{\hat{Q}^a_\alpha, \hat{Q}^b_\beta\} \neq 0$

<u>b=c=0</u> $\quad \{Q^a_\alpha, Q^b_\beta\} = 2\, C_{\alpha\beta}\, \varepsilon^{abc}\, P_c\,, \qquad [Q^a_\alpha, P_b] = 0$

The case of our interest

3d Rarita-Schwinger goldstino model

Spin-3/2 Superalgebra $\left\{Q^a_\alpha, Q^b_\beta\right\} = 2C_{\alpha\beta}\varepsilon^{abc}P_c, \quad \left[Q^a_\alpha, P_b\right] = 0 \quad \alpha,\beta=1,2$

$$Q^a_\alpha \leftrightarrow \chi^a_\alpha(x), \qquad \delta\chi^\alpha_a(x) = \zeta^\alpha_a + \mathtt{i}\, f^{-2}\varepsilon^{dbc}\left(\zeta^\beta_b\,\chi_{c\beta}\right)\partial_d\chi^\alpha_a\,,$$

Susy invariant one-form $E^a = dx^a + if^{-2}\epsilon^{abc}\chi^\alpha_b d\chi_{c\alpha}$

Rarita-Schwinger

Action
$$S = \int d^3x\Big(\underbrace{i\varepsilon^{abc}\,\chi_a\partial_b\chi_c}_{\text{Rarita-Schwinger}} + \frac{f^{-2}}{2}(\varepsilon^{abc}\varepsilon^{dfg} - \varepsilon^{afc}\varepsilon^{dbg})\chi_a\partial_b\chi_c\chi_d\partial_f\chi_g$$
$$+\frac{if^{-4}}{6}(\varepsilon^{abc}\varepsilon^{dfg} - \varepsilon^{abg}\varepsilon^{dfc})\varepsilon^{pqr}\chi_c\partial_p\chi_g\chi_a\partial_q\chi_b\chi_d\partial_r\chi_f\Big)$$

Gauge symmetry:

$$\delta\chi^\alpha_a + \frac{\mathtt{i}f^{-2}}{3}\,\varepsilon^{dfg}\,\partial_a\left(\chi^\alpha_d\,(\chi_f\,\delta\chi_g)\right) + \mathtt{i}\,f^{-2}\,\varepsilon^{dfg}\,(\delta\chi_d\,\partial_a\chi_f)\,\chi^\alpha_g = \partial_a\epsilon^\alpha(x)$$

Reduction of the non-linear 3/2-goldstino action to the quadratic RS action

$$S_{\rm RS} = {\rm i} \int d^3x \, \varepsilon^{abc} \, \hat\chi_a \, \partial_b \hat\chi_c , \qquad \delta \hat\chi_a^\alpha = \partial_a \epsilon^\alpha(x),$$ linearized local «susy»

where $$\hat\chi_a^\alpha = \chi_a^\alpha + \frac{{\rm i} f^{-2}}{3} \, \varepsilon^{dfg} \, \chi_d^\alpha \, (\chi_f \, \partial_a \chi_g)$$

Non-linear realization of the rigid spin-3/2 susy:

$$\delta \chi_a^\alpha(x) = \zeta_a^\alpha + {\rm i} \, f^{-2} \varepsilon^{dbc} \left(\zeta_b^\beta \, \chi_{c\beta} \right) \partial_d \chi_a^\alpha \, ,$$

$$\delta \hat\chi_a^\alpha = \zeta_a^\alpha + {\rm i} \, f^{-2} \, \varepsilon^{dbc} \, (\zeta_b \, \hat\chi_c) \, \partial_d \hat\chi_a^\alpha + \frac{{\rm i} f^{-2}}{3} \, \varepsilon^{dbc} \Big((\hat\chi_b \, \partial_a \hat\chi_c) \, \zeta_d^\alpha + (\zeta_b \, \partial_a \hat\chi_c) \hat\chi_d^\alpha \Big) + \mathcal{O}(f^{-4})$$

$$[\delta_2, \delta_1] \, \hat\chi_a^\alpha = \xi^d \, \partial_d \hat\chi_a^\alpha \, , \qquad \xi^d = 2 \, {\rm i} \, f^{-2} \, \varepsilon^{dbc} \, \zeta_b^1 \, \zeta_c^2$$ - closes on translation

spin-3/2 goldstino is a pure gauge: $\hat\chi_a^\alpha = \partial_a \psi^\alpha \quad ({\rm on-shell})$

Summary and things to do

- The simplest examples of spontaneous breaking of Hietarinta symmetries and corresponding Goldstone models turned out to be peculiar non-linear generalizations of the Chern-Simons and Rarita-Schwinger Lagrangians

- The CS Goldstone propagates a scalar mode which is a Galileon field that appears in modified theories of gravity

✓ It would be of interest to consider coupling of the CS Goldstone to a **3d bi-gravity** theory which is invariant under local symmetry associated with the algebra

$$[S^a, S^b] = 2\mathrm{i}\varepsilon^{abc}P_c, \quad [S^a, P^b] = [P^a, P^b] = 0 \qquad \text{2 spin-2 gauge fields}$$

$$[M^a, S^b] = 2\mathrm{i}\varepsilon^{abc}S_c, \quad [M^a, P^b] = 2\mathrm{i}\varepsilon^{abc}P_c \quad \rightarrow \quad \text{spin-connection}$$

$$[M^a, M^b] = 2\mathrm{i}\varepsilon^{abc}M_c$$

"Duality" relation with the 3d Maxwell algebra:

$$S^a \leftrightarrow P^a : \quad [P^a, P^b] = 2\mathrm{i}\varepsilon^{abc}S_c \equiv iS^{ab}, \quad [S^a, S^b] = 0 = [P^a, S^b]$$

3d "Maxwell-Chern-Simons" gravity

P. Salgado, R.J. Szabo and O. Valdivia '14

L. Avilés, E. Frodden, J. Gomis, D. Hidalgo and J. Zanelli '18

$$P^a \;\leftrightarrow\; e^a(x) = dx^m e^a_m(x), \quad M^a \;\leftrightarrow\; \omega^a(x) = dx^m \omega^a_m(x),$$

$$S^a \;\leftrightarrow\; f^a(x) = dx^m f^a_m(x),$$

Local symmetries: $\delta e^a = D\xi^a(x) + \epsilon^{abc} e_b \lambda_c(x) + \varepsilon^{abc} f_b \zeta_c(x)$

$$\delta f^a = D\zeta^a(x) + \epsilon^{abc} f_b \lambda_c(x), \qquad \delta\omega^a = D\lambda^a(x)$$

$$\begin{aligned} S &= \mathsf{a}_1 \int (e^a \wedge R_a + \frac{1}{2} f^a \wedge D f_a) + \mathsf{a}_2 \int f^a \wedge R_a, \\ &+ \mathsf{a}_3 \int (\omega^a d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a\omega^b\omega^c) \end{aligned}$$

$$R^a = d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b \wedge \omega_c, \qquad D f_a = df_a + \varepsilon_{abc}\omega^b \wedge f^c$$

Higher spin generalization s=n+1

$$[S^{a_1 \ldots a_n}, S^{b_1 \ldots b_n}] = 2\mathtt{i}\varepsilon^{a_1 b_1 c} P_c \, \eta^{a_2 b_2} \ldots \eta^{a_n b_n}, \quad [S^{(a)}, P^b] = [P^a, P^b] = 0$$

$$\begin{aligned} S_{n+1} &= \mathtt{a}_1 \int (e^a \wedge R_a + \frac{1}{2} f^{a_1 \ldots a_n} \wedge D f_{a_1 \ldots a_n}), \\ &+ \mathtt{a}_2 \int (\omega^a d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^a \omega^b \omega^c) \end{aligned}$$

Can this action be obtained by a contraction of the conventional CS HS theory based on SL(n,R) x SL(2,R)?

Conclusion and outlook

- In the spin-3/2 case the spontaneous breaking of the rigid spin-3/2 susy retains local symmetry of the Rarita-Schwinger Lagrangian. The non-linear spin-3/2 goldstino Lagrangian reduces to the quadratic RS Lagrangian upon a non-linear field redefinition. This causes the 3d RS goldstino be non-propagating field

- Problems to study:

✓ To couple the RS goldstino to conventional 3d (super)gravity and Hypergravity (with spin-2 and spin-5/2 gauge fields) and to study properties of these models

✓ To consider the RS goldstino model in D=4 associated with the algebra

$$\{Q^a_\alpha, Q^b_\beta\} = 2\varepsilon^{abcd}(\Gamma_5\Gamma_c)_{\alpha\beta}P_d \qquad \alpha, \beta = 1, ..., 4$$

✓ Generalization to higher-spin Goldstone fields