Symmetric orbifolds and tensionless string field theory

Joris Raeymaekers

CEICO, Academy of Sciences, Prague

Higher Spins and Holography, ESI Vienna

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

MINISTRY OF EDUCATION,
YOUTH AND SPORTS

Will review parts of Gaberdiel-Gopakumar's groundbreaking papers

- M. R. Gaberdiel and R. Gopakumar, *Higher Spins & Strings*, JHEP **1411**, 044 (2014) [arXiv:1406.6103 [hep-th]]
- M. R. Gaberdiel and R. Gopakumar, *Stringy Symmetries and the Higher Spin Square*, J. Phys. A **48**, no. 18, 185402 (2015) [arXiv:1501.07236 [hep-th]].
- M. R. Gaberdiel and R. Gopakumar, *String Theory as a Higher Spin Theory*, JHEP **1609**, 085 (2016) [arXiv:1512.07237 [hep-th]].

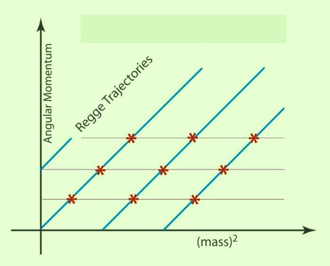
Based on:

- J.R., *On tensionless string field theory in AdS_3*, arXiv:1903.xxxxx [hep-th]
- P. Kessel and J.R., *Simple unfolded equations for massive higher spins in AdS_3*, JHEP **1808**, 076 (2018) [arXiv:1805.07279 [hep-th]].
- J.R., *On matter coupled to the higher spin square*, J. Phys. A **49**, no. 35, 355402 (2016) [arXiv:1603.07845 [hep-th]].

MOTIVATION

What is the **symmetry principle underlying string theory**? Can we find a formulation where it is manifest?

- String theory in Minkowski is a massive higher spin theory



- High energy behaviour suggests huge underlying higher spin gauge symmetry, spontaneously broken in Minkowksi vacuum (Gross 1988)
- Tensionless limits should make (portion of) this symmetry manifest

TENSIONLESS STRINGS AND SYMMETRIC ORBIFOLD

- Vasiliev 1990: interacting massless higher spins natural in (A)dS space ⇒ take tensionless limit in AdS background
- Exciting recent progress! Eberhardt, Gaberdiel, Gopakumar 2018: tensionless limit of worldsheet string theory on $AdS_3 \times S^3 \times T^4$ with NS flux. Direct derivation of dual CFT.
- Arises from decoupling limit of system of k NS5 branes and N F1's with

$$k = \left(\frac{l_{AdS}}{l_s}\right)^2, \qquad g_s \sim \frac{1}{\sqrt{N}}$$

Minimal value $k = 1$ is tensionless point.

- Dual CFT is symmetric orbifold of T^4 CFT with 4 bosons and 4 fermions:

$$Sym^N(T^4) = \frac{(T^4)^N}{S_N}$$

- Argued to be dual CFT long ago from S-dual D1-D5 point of view Strominger, Vafa 1996, though connection was less direct

TENSIONLESS AdS_3 SFT

- Gaberdiel, Gopakumar 2014-15: properties of HS field theory from boundary CFT
- Massless higher spin sector: huge gauge invariance: 'higher spin square' *hss*
- Matter: massive HS fields, come in multiplets of *hss*
- Here: field equations to linear order in matter fields

Massless HS:	*hss* Chern-Simons
Matter:	Vasiliev-like unfolded equations

TENSIONLESS AdS_3 SFT

- Gaberdiel, Gopakumar: properties of HS field theory from boundary CFT
- Massless higher spin sector: huge gauge invariance 'higher spin square' *hss*
- Matter: massive HS fields, come in multiplets of *hss*
- Here: field equations to linear order in matter fields

$$\begin{array}{ll} \text{Massless HS:} & dA + A \wedge A = d\bar{A} + \bar{A} \wedge \bar{A} = 0 \\ \text{Matter:} & \left(d + A^{(n)} + \bar{A}^{(n)}\right) |C^{(n)}\rangle = 0 \end{array}$$

- Capture cubic couplings $AAA, \; CCA$

OUTLINE

- Symmetric orbifolds at large N
- Bulk dual HS theory: free scalar example
- Tensionless AdS_3 strings
- Outlook

I. Symmetric orbifolds at large N

Symmetric orbifolds

- Start from a CFT $\mathcal{C}$. Want to describe **symmetric orbifold CFT**

$$Sym^N(\mathcal{C}) = \frac{(\mathcal{C})^N}{S_N}$$

- Contains **twisted sectors** where

$$\phi_i(\sigma + 2\pi) = \phi_{g(i)}(\sigma), \qquad g \in S^N, i = 1, \ldots, N$$

  One-to-one with conjugacy classes $[g]$ of S_N

- Conjugacy classes labelled by partitions of N giving cycle decomposition

$$(1)^{N_1}(2)^{N_2}\ldots(n)^{N_n}\ldots, \qquad \sum_n nN_n = N$$

  A length-n cycle cyclically permutes n copies, e.g.

$$\phi_1 \to \phi_2 \to \ldots \to \phi_n \to \phi_1$$

Spectrum and DMVV formula

- **Partition function** of seed theory $\mathcal{C}$

$$Z \equiv \mathrm{tr}\, q^{L_0} \bar{q}^{\bar{L}_0} = \sum_{\Delta, \bar{\Delta}} c(\Delta, \bar{\Delta}) q^{\Delta} \bar{q}^{\bar{\Delta}}$$

- **Symmetric orbifold spectrum** determined by seed degeneracies $c(\Delta, \bar{\Delta})$. Dijkgraaf, Moore, Verlinde, Verlinde 1996: generating function for orbifold partition functions

$$\mathcal{Z} \equiv \sum_N p^N Z(Sym^N(\mathcal{C})) = \prod_{n>0} \prod_{\Delta, \bar{\Delta}} \left(1 - p^n q^{\frac{\Delta}{n}} \bar{q}^{\frac{\bar{\Delta}}{n}}\right)^{-c(\Delta, \bar{\Delta}) \delta^{(n)}_{\Delta - \bar{\Delta}}}$$

where

$$\delta^{(n)}_{\Delta - \bar{\Delta}} = \begin{cases} 1 \text{ if } \Delta = \bar{\Delta} \bmod n \\ 0 \text{ otherwise} \end{cases}$$

- If we know seed theory Hilbert space , can describe underlying **states** explicitly

EXAMPLE: FREE SCALAR

- Example: **free compact scalar** X.
  Restrict to **zero momentum and winding** sector for simplicity.
  Fock space $\mathcal{H}$ states of form

$$\alpha_{-\vec{m}}\bar{\alpha}_{-\vec{p}}|0\rangle$$

e.g. $\vec{m} = (5, 4, 4, 1, 1, 1) \to \alpha_{-\vec{m}} = \alpha_{-5}(\alpha_{-4})^2(\alpha_{-1})^3$

- Partition function

$$\begin{aligned} \tilde{Z}_X &= \left| \prod_{n=1}^{\infty} (1-q^n)^{-1} \right|^2 \\ &= \sum_{h,\bar{h}\in\mathbb{N}} P(h)P(\bar{h})q^h q^{\bar{h}} \end{aligned}$$

with $P(h) \equiv$ number of partitions of h

n-CYCLE TWISTED HILBERT SPACE

- **Twist by n-cyclic permutation** Ω: David, Mandal, Wadia 2002

$$X_i(\sigma + 2\pi) = \Omega X_i(\sigma), \qquad \Omega : X_1 \to X_2 \to \ldots \to X_n \to X_1$$

Strands of 'long string' with period $2\pi n$.

- Diagonalize Ω:

$$X^{\left(\frac{k}{n}\right)} = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} e^{-\frac{2\pi i k(j-1)}{n}} X_j, \qquad \Omega X^{\left(\frac{k}{n}\right)} = e^{\frac{2\pi i k}{n}} X^{\left(\frac{k}{n}\right)}, \qquad k = 0, \ldots, n-1$$

$X^{\left(\frac{k}{n}\right)}$ have modes with fractional part $\frac{k}{n}$

$$\partial X^{\left(\frac{k}{n}\right)} = -i \sum_{m \in \mathbb{Z}} \alpha_{m+\frac{k}{n}} z^{-m-\frac{k}{n}-1}$$

- Project on Ω-invariant states: $L_0 - \bar{L}_0 \in \mathbb{Z} \to$ build up **Hilbert space** $\mathcal{H}_{(n)}$

$$\alpha_{-\frac{\vec{m}}{n}} \bar{\alpha}_{-\frac{\vec{p}}{n}} |0\rangle_n, \qquad \sum_a m_a = \sum_a p_a \pmod{n}$$

Large-N limit and single-trace operators

At **large** N, large central charge

$$N \sim \frac{\ell}{G_N}$$

and weakly coupled gravity dual. Expect spectrum to separate into

- **'single-trace states'** $\leftrightarrow$ single-particle excitations of elementary bulk fields
- **'multi-trace states'** $\leftrightarrow$ multi-particle excitations in bulk

How to find 'single-trace' spectrum in symmetric orbifold theory?

Large-N limit of DMVV

Extract p^N coefficient at large N in

$$\mathcal{Z} \equiv \sum_N p^N Z(Sym^N(\mathcal{C})) = \prod_{n>0} \prod_{\Delta,\bar\Delta} \left(1 - p^n q^{\frac{\Delta}{n}} \bar q^{\frac{\bar\Delta}{n}}\right)^{-c(\Delta,\bar\Delta)\delta^{(n)}_{\Delta-\bar\Delta}}$$

Isolate contribution from vacuum, set $\tilde p = p(q\bar q)^{-\frac{1}{24}}$ (Keller 2011)

$$\mathcal{Z} = \frac{1}{1-\tilde p} \underbrace{\prod_{n>1} \prod_{\Delta,\bar\Delta}{}' \left(1 - \tilde p^n q^{\frac{\Delta}{n}+\frac{cn}{24}} \bar q^{\frac{\bar\Delta}{n}+\frac{cn}{24}}\right)^{-c(\Delta,\bar\Delta)\delta^{(n)}_{\Delta-\bar\Delta}}}_{\equiv R(\tilde p)}$$

Write $R(\tilde p) = \sum_k a_k \tilde p^k$. Coefficient of $\tilde p^N$ is

$$\sum_{k=0}^{N} a_k \to R(1)$$

Takes multi-particle form: product of factors $\left(1 - q^{\Delta_1}\bar q^{\bar\Delta_1}\right)^{-1}$
$\{(\Delta_1, \bar\Delta_1)\}$ form 1-particle spectrum.

Single-particle states

$$\Rightarrow \tilde{Z}_{1-\text{part.}} = (\tilde{Z} - 1) + \sum_{n=2}^{\infty} \tilde{Z}\left(\mathcal{H}_{(n)}\right)$$

Single-particle states counted by this formula:

- In **untwisted sector**: one-to-one with states in $\mathcal{H}$ (except vacuum)

$$\alpha_{-\vec{m}}\bar{\alpha}_{-\vec{p}}|0\rangle \leftrightarrow \left(\alpha_{-\vec{m}}\bar{\alpha}_{-\vec{p}}|0\rangle\right) \otimes \underbrace{|0\rangle \otimes \ldots |0\rangle}_{N-1 \text{ times}} + \text{cyclic}$$

- In $(1)^{N-n}(n)$ **twist sector**, e.g. twisted by

$$X_1 \to X_2 \to \ldots X_n \to X_1, \qquad X_{n+1} \to X_{n+1}, \ldots X_N \to X_N$$

  1-particle states one-to-one with states in $\mathcal{H}_{(n)}$:

$$\alpha_{-\frac{\vec{m}}{n}}\bar{\alpha}_{-\frac{\vec{p}}{n}}|0\rangle_n \leftrightarrow \left(\alpha_{-\frac{\vec{m}}{n}}\bar{\alpha}_{-\frac{\vec{p}}{n}}|0\rangle_n\right) \otimes \underbrace{|0\rangle \otimes \ldots |0\rangle}_{N-n \text{ times}} + \ldots$$

II. The bulk higher spin theory

LINEARIZED THEORY AROUND ADS

- AdS_3 has $sl(2) \oplus \overline{sl(2)}$ isometry
- Single-particle states are organized in **primary irreps** $(h, \bar{h})$ **of** $sl(2) \oplus \overline{sl(2)}$ with character $\chi_h \bar{\chi}_{\bar{h}}$,

$$\chi_h = \frac{q^h}{1-q} \qquad \text{for } h > 0, \qquad\qquad \chi_0 = 1$$

- Correspond to particle with **mass** m **and spin** s

$$m^2 = (h + \bar{h} - s)(h + \bar{h} + s - 2), \qquad s = |h - \bar{h}|$$

- Decompose single-particle partition function into $sl(2) \oplus \overline{sl(2)}$ characters

$$Z(\mathcal{H}_{(n)}) = \sum_{h,\bar{h}} \underbrace{N_{(n)}(h, \bar{h})}_{\#\,(h,\bar{h}) \text{ particles in length-}n\text{ twist sector}} \chi_h \bar{\chi}_{\bar{h}}$$

Trick: use $q^h = \chi_h - \chi_{h+1}$

Linearized theory around AdS

Massless higher spin sector:

- From purely (anti-) chiral excitations in untwisted sector, get $N_{(1)}(s,0)$ spin-s Fronsdal fields Φ_s, with

$$N_{(1)}(s,0) = N_{(1)}(0,s) = P(s) - P(s-1), \qquad s \in \mathbb{N}_{>0}$$

$$\Rightarrow \Phi_1 + \Phi_2 + \Phi_3 + 2\Phi_4 + 2\Phi_5 + 4\Phi_6 + 4\Phi_7 + 7\Phi_8 + 8\Phi_9 + 12\Phi_{10} + \dots$$

'stringy' asymptotics $N_{(1)}(s,0) \sim e^{2\pi\sqrt{\frac{s}{6}}}$

Matter sector:

- From nonchiral excitations in untwisted sector and n-cycle twisted sectors, get $N_{(n)}(h,\bar{h})$ Fierz-Pauli fields $\Psi_{(m^2,s)}$, with

$$\begin{aligned} N_{(n)}(h,\bar{h}) \quad = \quad & \delta^{(1)}_{h-\bar{h}} \left(P\left(nh + \frac{1}{24}(n^2-1)\right) - P\left(n(h-1)h + \frac{1}{24}(n^2-1)\right)\right) \times \\ & \left(P\left(n\bar{h} + \frac{1}{24}(n^2-1)\right) - P\left(n(\bar{h}-1)h + \frac{1}{24}(n^2-1)\right)\right) \end{aligned}$$

BULK THEORY AROUND AdS

$$
\begin{aligned}
n = 1: &\quad \Psi_{(0,0)} + \Psi_{(4,1)} + \Psi_{(8,0)} + \Psi_{(8,2)} + 2\Psi_{(12,3)} + \Psi_{(16,1)} + 2\Psi_{(16,4)} + 2\Psi_{(24,2)} + \Psi_{(24,0)} + \ldots \\
n = 2: &\quad \Psi_{(-\frac{63}{64},0)} + \Psi_{(-\frac{15}{64},0)} + \Psi_{(\frac{1}{64},1)} + 3\Psi_{(\frac{17}{64},2)} + \Psi_{(\frac{17}{64},0)} + 2\Psi_{(\frac{81}{64},1)} + 4\Psi_{(\frac{225}{64},2)} + \ldots \\
n = 3: &\quad \Psi_{(-\frac{80}{81},0)} + 4\Psi_{(-\frac{56}{81},0)} + \Psi_{(-\frac{32}{81},0)} + 2\Psi_{(\frac{4}{81},1)} + 4\Psi_{(\frac{40}{81},0)} + 4\Psi_{(\frac{64}{81},1)} + 16\Psi_{(\frac{208}{81},0)} + \ldots
\end{aligned}
$$

- Should combine into multiplets of higher spin gauge algebra
- Gauge-invariant formulation ⇒ couplings determined by gauge invariance

Higher spin gauge algebra

Gaberdiel, Gopakumar 2014, 2015

- $Sym^N(X)$ has huge **algebra of chiral operators**, one-to-one with chiral algebra of seed theory

$$\mathcal{U}(z) \leftrightarrow \sum_{i=1}^{N} \mathcal{U}_i(z)$$

Basis of quasi-primary operators $\mathcal{U}_{\underline{i}}^{(h)}(z),\ \underline{i} = 1, \ldots, N_{(1)}(h, 0)$

- **Bulk higher spin algebra is wedge subalgebra** of chiral algebra: modes which annihilate vacuum from left and right. Spanned by

$$\left(U_{\underline{i}}^{(h)}\right)_a \text{ with } |a| < h$$

- Wedge modes are (infinite) **combinations of normal-ordered monomials**

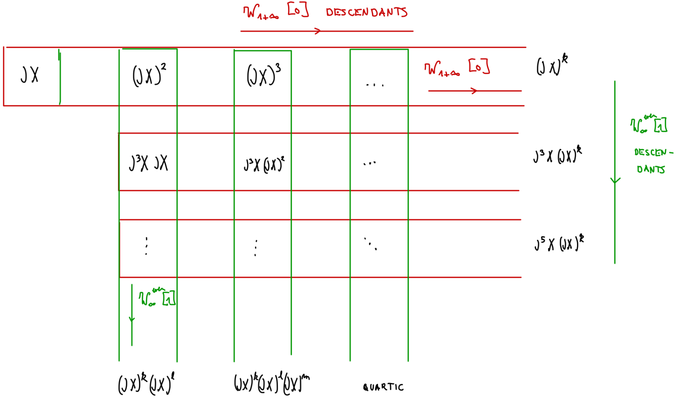
$$\alpha_{-\vec{m}}\alpha_{\vec{p}} \qquad \text{with } \vec{m}, \vec{p} \neq \emptyset$$

The higher spin square

Wedge algebra contains two higher spin subalgebras

- 'vertical' $hs^{even}[1]$: wedge modes of bilinears $\partial^k X \partial^l X$ spanning $\mathcal{W}_\infty^{even}[1]$
- 'horizontal' $hs[0]$: wedge modes of multilinears $(\partial X)^k$ spanning $\mathcal{W}_{1+\infty}[0]$

Form a **'higher spin square'** algebra *hss*



‘More tensionless’ points in moduli space

- Have neglected **states with momentum and winding** which contribute

$$h = \frac{1}{2}\left(\frac{M}{R} + \frac{WR}{2}\right)^2, \qquad \bar{h} = \frac{1}{2}\left(\frac{M}{R} - \frac{WR}{2}\right)^2, \qquad M, W \in \mathbb{Z}$$

- At special radii, chiral algebra is extended. E.g. at **selfdual radius** $R = \sqrt{2}$ all states with $M = \pm W$ are (anti-)chiral of spin M^2. Chiral algebra is enlarged, mixes different momentum-winding sectors.
- How does higher spin square structure extend to selfdual case?

MASSLESS HS SECTOR AS CHERN-SIMONS THEORY

- Massless higher spin fields have no local degrees of freedom → suggest **Chern-Simons formulation** with gauge algebra $hss \oplus \overline{hss}$

$$F = dA + A \wedge A = 0, \qquad \bar{F} = d\bar{A} + \bar{A} \wedge \bar{A} = 0$$

- Expansion around pure AdS solution

$$A_{AdS} = L_0 d\rho + \left(e^{\rho} L_1 + \frac{1}{4} e^{-\rho} L_{-1} \right) dx_+, \qquad \bar{A}_{AdS} = \bar{L}_0 d\rho - \left(e^{\rho} \bar{L}_1 + \frac{1}{4} e^{-\rho} \bar{L}_{-1} \right) dx_-$$

  yields right spectrum of decoupled Fronsdal equations (Campoleoni, Fredenhagen, Pfenninger, Theisen 2010)

- Next step:
  - Reproduce CFT chiral algebra as asymptotic symmetry algebra?
  - Generalization of Drinfeld-Sokolov reduction?
  - Boundary conditions?

Matter sector equations

J.R. 1603.07845, P. Kessel & J.R. to appear

Criteria:

- **Gauge-invariant** under $hss \oplus \overline{hss}$
- Linearized around AdS, describe **correct spectrum of Fierz-Pauli fields**

Our proposal:

- For each n, introduce **0-form** $|C^{(n)}(x)\rangle$ **taking values in internal space** $\mathcal{H}_{(n)}$, i.e. n-cycle twisted Hilbert space
- **Field equations**

$$\left(d + A^{(n)} + \bar{A}^{(n)}\right)|C^{(n)}\rangle = 0$$

Covariant constancy $\leftrightarrow$ unfolded description (more below)

- Superscripts $A^{(n)}, \bar{A}^{(n)}$: $hss \oplus \overline{hss}$ **evaluated in length-n twist representation**

LENGTH-n TWIST SECTOR REPRESENTATION OF hss

How do $hss \oplus \overline{hss}$ **generators act on length-n twist sector** Hilbert space $\mathcal{H}_{(n)}$?

Untwisted sector is trivial:

- $hss \oplus \overline{hss}$ generators

$$\alpha_{-\vec{m}}\alpha_{\vec{p}}, \qquad \bar{\alpha}_{-\vec{m}}\bar{\alpha}_{\vec{p}} \qquad \text{with } \vec{m}, \vec{p} \neq \emptyset$$

- act naturally on massive untwisted states

$$\alpha_{-\vec{m}}\bar{\alpha}_{-\vec{p}}|0\rangle \qquad \text{with } \vec{m}, \vec{p} \neq \emptyset$$

Gives irrep, the **'defining' or 'minimal' representation** of $hss \oplus \overline{hss}$

n-CYCLE TWIST SECTOR REPRESENTATION OF hss

Length-n twist sector: **algorithm to find representation on $\mathcal{H}_{(n)}$** of wedge modes of arbitrary operator $\mathcal{O} =: \mathcal{F}(\partial X, \partial^2 X, \ldots) :$

- Take symmetric combination in $X^{\otimes n}$:

$$\mathcal{O}^{(n)} = \sum_{i=1}^{n} : \mathcal{F}(\partial X_i, \partial^2 X_i, \ldots) :$$

  has well-defined action on $\mathcal{H}_{(n)}$

- Write ito fields $X^{\left(\frac{k}{n}\right)}$ with $\frac{k}{n}$- fractional modes (diagonalize $\Omega : X_i \to X_{i+1(\mathrm{mod}\, n)}$)

$$X_j = \frac{1}{\sqrt{n}} \sum_{k=0}^{n-1} e^{2\pi i k(j-1)} X^{\left(\frac{k}{n}\right)}$$

- Convert conformal normal ordering : : to creation-annihilation normal ordering $\circ\circ$, see Polchinski's book Ch. 2.
  2-point function:

$$: \partial X^{\left(\frac{k}{n}\right)}(z) \partial X^{\left(\frac{l}{n}\right)}(z') : - \circ \partial X^{\left(\frac{k}{n}\right)}(z) \partial X^{\left(\frac{l}{n}\right)}(z') \circ = \frac{1 - \left(\frac{z'}{z}\right)^{\frac{k}{n}} \left(1 + \frac{k}{n}\left(\frac{z}{z'} - 1\right)\right)}{(z - z')^2} \delta_{k+l,n}$$

  \+ Wick contractions for general operators

n-CYCLE TWIST SECTOR REPRESENTATION OF hss

- Obtain

$$\mathcal{O}^{(n)} = \sum_{i=1}^{n} {}^{\circ}_{\circ}\mathcal{F}(\partial X_i, \partial^2 X_i, \ldots){}^{\circ}_{\circ} + \text{corrections}$$

- Extract desired wedge mode

EXAMPLES

hss representation in twisted sectors **involves the fractional moded oscillators**. Examples:

- $sl(2)$ generators

$$L_m^{(n)} = \frac{1}{2} \sum_{p \in \mathbb{Z}} {}^{\circ}_{\circ} \alpha_{\frac{p}{n}} \alpha_{m-\frac{p}{n}} {}^{\circ}_{\circ} + \underbrace{\frac{1}{24}\left(n - \frac{1}{n}\right)\delta_{m,0}}_{\text{0-point energy}}$$

- wedge modes of spin-3 current $\mathcal{U} \equiv (\partial X)^3$

$$U_a^{(n)} = \frac{i}{\sqrt{n}} \sum_{p,q \in \mathbb{Z}} {}^{\circ}_{\circ} \alpha_{\frac{p}{n}} \alpha_{\frac{q}{n}} \alpha_{a-\frac{p+q}{n}} {}^{\circ}_{\circ} - \frac{i(n-1)(2n-1)}{4n^{\frac{3}{2}}} \alpha_a$$

Wave equations around AdS_3 background

P. Kessel, J.R. 1805.07279

- **pure AdS_3 background** $\in sl(2) \oplus \overline{sl(2)}$ in Poincaré coordinates

$$A_{AdS} = L_0 d\rho + e^{\rho} L_1 dx_+ = g^{-1} dg, \qquad \bar{A}_{AdS} = \bar{L}_0 d\rho - e^{\rho} \bar{L}_1 dx_- = \bar{g}^{-1} d\bar{g}$$

- Internal Hilbert spaces $\mathcal{H}_{(n)}$ decompose under $sl(2) \oplus \overline{sl(2)}$ in $N_{(n)}(h, \bar{h})$ irreps of type $(h, \bar{h})$. Gives **decoupled equations** of form

$$\left(d + A_{\rm AdS} + \bar{A}_{\rm AdS}\right) |C(x)\rangle = 0$$

all fields evaluated in $(h, \bar{h})$ irrep.

- Solution space

$$|C(x)\rangle = (g\bar{g})^{-1}(x)|C_0\rangle, \qquad C_0 \in (h, \bar{h}) \text{ irrep}$$

gives representation of $(h, \bar{h})$ on fields

Wave equations around AdS_3 background

- Equations are of unfolded type, contain ∞ **number of auxiliary fields**. Eliminate in terms of physical fields?
- Can construct vectors in $(h, \bar{h})$ irrep

$$|\alpha_1 \ldots \alpha_{2s}\rangle, \qquad s = |h - \bar{h}|, \qquad \alpha_i = +, -$$

transforming as **spin-s under Lorentz subalgebra** (generated by $M_m = L_m - \bar{L}_{-m}$) (See also Iazeolla, Sundell 2008)

- Not normalizeable, but have finite overlap with solutions $|C(x)\rangle$. Construct projections

$$\phi_{\alpha_1 \ldots \alpha_{2s}}(x) = \langle \alpha_1 \ldots \alpha_{2s} | C(x) \rangle$$

- Satisfy **topogically massive wave equation** propagating $(h, \bar{h})$ Deser, Jackiw, Templeton 1982, Tyutin, Vasiliev 1997

$$\nabla_{\alpha_1}{}^{\beta} \phi_{\beta \alpha_2 \ldots \alpha_{2s}} + \left(\text{sgn}\,(h - \bar{h})\right) (h + \bar{h} - 1) \phi_{\alpha_1 \ldots \alpha_{2s}} = 0$$

Pair of parity-related fields combines into single Fierz-Pauli field.

Relation to other unfolded eqs.

Relation to Vasiliev-like unfolded system Vasiliev 1994

- Can construct spin-$s + m$ Lorentz tensors, $m \in \mathbb{N}$ and projections

$$\phi_{\alpha_1 \ldots \alpha_{2(s+m)}} = \langle \alpha_1 \ldots \alpha_{2(s+m)} | C(x) \rangle$$

- Combine in Vasiliev-like master field

$$C_V = \sum_m \phi_{\alpha_1 \ldots \alpha_{2(s+m)}} y^{\alpha_1} \ldots y^{\alpha_(s+m)}$$

Satisfies **unfolded topologically massive equations** of Boulanger, Ponomarev, Sezgin, Sundell 2015

III. Tensionless AdS_3 string

The T^4 CFT

- Seed theory: $\mathcal{N} = (4,4)$ sigma model on T^4. Contains **2 complex bosons and 2 complex fermions**.
- $\mathcal{N} = 4$ SCA: bosonic currents $T(z), J^a(z)$, fermionic $G^{\pm}(z), \tilde{G}_{\pm}(z)$
- Refined partition function in NSNS sector (in zero momentum and winding sector):

$$\begin{aligned}\tilde{Z}_{T^4} &= (q\bar{q})^{\frac{1}{4}} \mathrm{tr}_{NSNS} q^{L_0} \bar{q}^{\bar{L}_0} y^{2J_0^3} \bar{y}^{2\bar{J}_0^3} \\ &= \left| \prod_{n=1}^{\infty} \frac{(1+yq^{n-\frac{1}{2}})^2 (1+y^{-1}q^{n-\frac{1}{2}})^2}{(1-q^n)^4} \right|^2\end{aligned}$$

Extra factors keep track of R-charge.

Single-particle Hilbert space

- Want to find the large-N limit of the **symmetric orbifold in the NSNS sector**, which describes perturbative excitations on AdS
- DMVV formula generalizes straighforwardly to the RR sector. Convert to NSNS using **spectral flow** De Boer 1998, Maldacena, Moore, Strominger 1999

$$\Delta_{NS} = \Delta_R - j_R + \frac{c}{24}, \qquad j_{NS} = j_R - \frac{c}{12}$$

- single-particle states one-to-one with states in n**-cycle twisted Hilbert spaces** $\mathcal{H}_{(n)}$

Single-particle Hilbert space

- **Fermionic modes** in n-cycle twist sector are

$$\begin{cases} \psi^a_{\frac{m}{n}}, & \tilde{\psi}^a_{\frac{m}{n}} & \text{for } n \text{ even}, \\ \psi^a_{\frac{m+\frac{1}{2}}{n}}, & \tilde{\psi}^a_{\frac{m+\frac{1}{2}}{n}} & \text{for } n \text{ odd}, \end{cases} \qquad m \in \mathbb{Z}$$

- Act on spin-$\frac{n-1}{2}$ **multiplet of ground states** with Lunin, Mathur 2001

$$h = \frac{n-1}{2}$$

- $\mathcal{H}_{(n)}$ consists of Fock space states satisfying twist-invariance condition

$$L_0 + J_0^3 - (\bar{L}_0 + \bar{J}_0^3) \in \mathbb{Z}$$

THE BULK HS THEORY

- Flat gauge fields + covariantly constant matter fields

$$
\begin{aligned}
F = dA + A \wedge A &= 0, \qquad \bar{F} = d\bar{A} + \bar{A} \wedge \bar{A} = 0 \\
\left(d + A^{(n)} + \bar{A}^{(n)}\right) |C^{(n)}\rangle &= 0, \qquad n = 1, 2, \ldots
\end{aligned}
$$

- Higher spin algebra $HSS \cong$ wedge subalgebra of chiral algebra. Structure of higher spin square.
- Matter fields $|C^{(n)}\rangle$ valued in n-cycle twisted Hilbert space $\mathcal{H}_{(n)}$
- Oscillator representation of HSS on $\mathcal{H}_{(n)}$ through

$$
\begin{aligned}
:\partial X^{\left(\frac{k}{n}\right)a}(z)\partial\tilde{X}^{\left(\frac{l}{n}\right)b}(z'): - {}^{\circ}_{\circ}\partial X^{\left(\frac{k}{n}\right)a}(z)\partial\tilde{X}^{\left(\frac{l}{n}\right)b}(z'){}^{\circ}_{\circ} &= \delta^{a,b}\delta^{k,l}\frac{1 - \left(\frac{z'}{z}\right)^{\frac{k}{n}}\left(1 + \frac{k}{n}\left(\frac{z}{z'} - 1\right)\right)}{(z - z')^2} \\
:\psi^{\left(\frac{k}{n}\right)a}(z)\tilde{\psi}^{\left(\frac{l}{n}\right)b}(z'): - {}^{\circ}_{\circ}\psi^{\left(\frac{k}{n}\right)a}(z)\tilde{\psi}^{\left(\frac{l}{n}\right)b}(z'){}^{\circ}_{\circ} &= \delta^{a,b}\delta^{k,l}\frac{\left(\frac{z'}{z}\right)^{\frac{k}{n}} - 1}{(z - z')}
\end{aligned}
$$

Spacetime supermultiplets

- Spacetime superalgebra is $psu(1,1|2) \oplus \overline{psu(1,1|2)}$, the wedge algebra of the $\mathcal{N} = (4,4)$ SCA.
- $psu(1,1|2)$ has bosonic subalgebra $sl(2) \oplus su(2)$, representations labelled by (h, j) Two kinds:

$j < h$:	long multiplets,	character $\mathrm{ch}_{h,j}$
$j = h$:	short multiplets,	character ch_h

- $\Rightarrow$ counting formulas for supermultiplet content
- Massless HS sector:

$$\tilde{Z}_{chiral} = 2\,\mathrm{ch}_{\frac{1}{2}} + \mathrm{ch}_1 + 3\,\mathrm{ch}_{1,0} + 2\,\mathrm{ch}_{\frac{3}{2},\frac{1}{2}} + 10\,\mathrm{ch}_{2,0} + 8\,\mathrm{ch}_{\frac{5}{2},\frac{1}{2}} + \mathrm{ch}_{3,1} + 29\,\mathrm{ch}_{3,0} + \ldots$$

Spacetime supermultiplets

Matter content from n-cycle twist sector:

$$\tilde{Z}_{(1)} = 4\,\mathrm{ch}_{\frac{1}{2}}\overline{\mathrm{ch}}_{\frac{1}{2}} + 2\left(\mathrm{ch}_{\frac{1}{2}}\overline{\mathrm{ch}}_{1} + \mathrm{ch}_{1}\overline{\mathrm{ch}}_{\frac{1}{2}}\right) + \mathrm{ch}_{1}\overline{\mathrm{ch}}_{1} + 9\,\mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{1,0} + 6\left(\mathrm{ch}_{\frac{1}{2}}\overline{\mathrm{ch}}_{1,0} + \mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{\frac{1}{2}}\right) + 3\left(\mathrm{ch}_{1}\overline{\mathrm{ch}}_{1,0} + \mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{1}\right) + \ldots$$

$$\tilde{Z}_{(2)} = \mathrm{ch}_{\frac{1}{2}}\overline{\mathrm{ch}}_{\frac{1}{2}} + 2\left(\mathrm{ch}_{\frac{1}{2}}\overline{\mathrm{ch}}_{1} + \mathrm{ch}_{1}\overline{\mathrm{ch}}_{\frac{1}{2}}\right) + 4\,\mathrm{ch}_{1}\overline{\mathrm{ch}}_{1} + 4\,\mathrm{ch}_{\frac{1}{2},0}\overline{\mathrm{ch}}_{\frac{1}{2},0} + 8\left(\mathrm{ch}_{\frac{1}{2},0}\overline{\mathrm{ch}}_{1,\frac{1}{2}} + \mathrm{ch}_{1,\frac{1}{2}}\overline{\mathrm{ch}}_{1,0}\right) + 64\,\mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{1,0} + 8\left(\mathrm{ch}_{\frac{1}{2}}\overline{\mathrm{ch}}_{1,0} + \mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{\frac{1}{2}}\right) + 16\left(\mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{1} + \mathrm{ch}_{1}\overline{\mathrm{ch}}_{1,0}\right) + 16\,\mathrm{ch}_{1,\frac{1}{2}}\overline{\mathrm{ch}}_{1,\frac{1}{2}} + \ldots$$

$$\tilde{Z}_{(3)} = \mathrm{ch}_{1}\overline{\mathrm{ch}}_{1} + \mathrm{ch}_{\frac{2}{3},0}\overline{\mathrm{ch}}_{\frac{2}{3},0} + 4\,\mathrm{ch}_{\frac{5}{6},\frac{1}{2}}\overline{\mathrm{ch}}_{\frac{5}{6},\frac{1}{2}} + 49\,\mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{1,0} + 7\left(\mathrm{ch}_{1}\overline{\mathrm{ch}}_{1,0} + \mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{1}\right) + \ldots$$

$$\tilde{Z}_{(4)} = 4\,\mathrm{ch}_{1,0}\overline{\mathrm{ch}}_{1,0} + \mathrm{ch}_{1,\frac{1}{2}}\overline{\mathrm{ch}}_{1,\frac{1}{2}} + 100\,\mathrm{ch}_{\frac{5}{4},0}\overline{\mathrm{ch}}_{\frac{5}{4},0} + 20\left(\mathrm{ch}_{\frac{5}{4},0}\overline{\mathrm{ch}}_{\frac{5}{4},1} + \mathrm{ch}_{\frac{5}{4},1}\overline{\mathrm{ch}}_{\frac{5}{4},0}\right) + 64\,\mathrm{ch}_{\frac{5}{4},\frac{1}{2}}\overline{\mathrm{ch}}_{\frac{5}{4},\frac{1}{2}} + 4\,\mathrm{ch}_{\frac{5}{4},1}\overline{\mathrm{ch}}_{\frac{5}{4},1} + \ldots$$

Gives consistency check on (some of) our derivations

OUTLOOK

Studied tensionless SFT on AdS_3. Proposed field equations capturing *AAA* and *CCA* couplings

- Included twisted sectors
- Vasiliev-like unfolded formulation, showed how to project on physical fields

Future:

- **Representation theory of *HSS***
- Holographic *CCA* **three-point functions** a la Ammon, Kraus, Perlmutter 2011 (In progress with K. Farnsworth, O. Hulik, O.Vasilakis, J. Vosmera)
- **Further couplings**?
  Field redefinitions — 'local frame' reproducing CFT
  Formal deformations — absent? (Sharapov, Skvortsov 2019)

Thank you!