TWO APPLICATIONS OF THE FACTORIZATION OF INFRARED DYNAMICS

BASED ON

U. Kol, MP, R. Javadinezhad JHEP 1901(2019) 89
arxiv:1808.02987 [hep-th]

AND PREVIOUS WORK

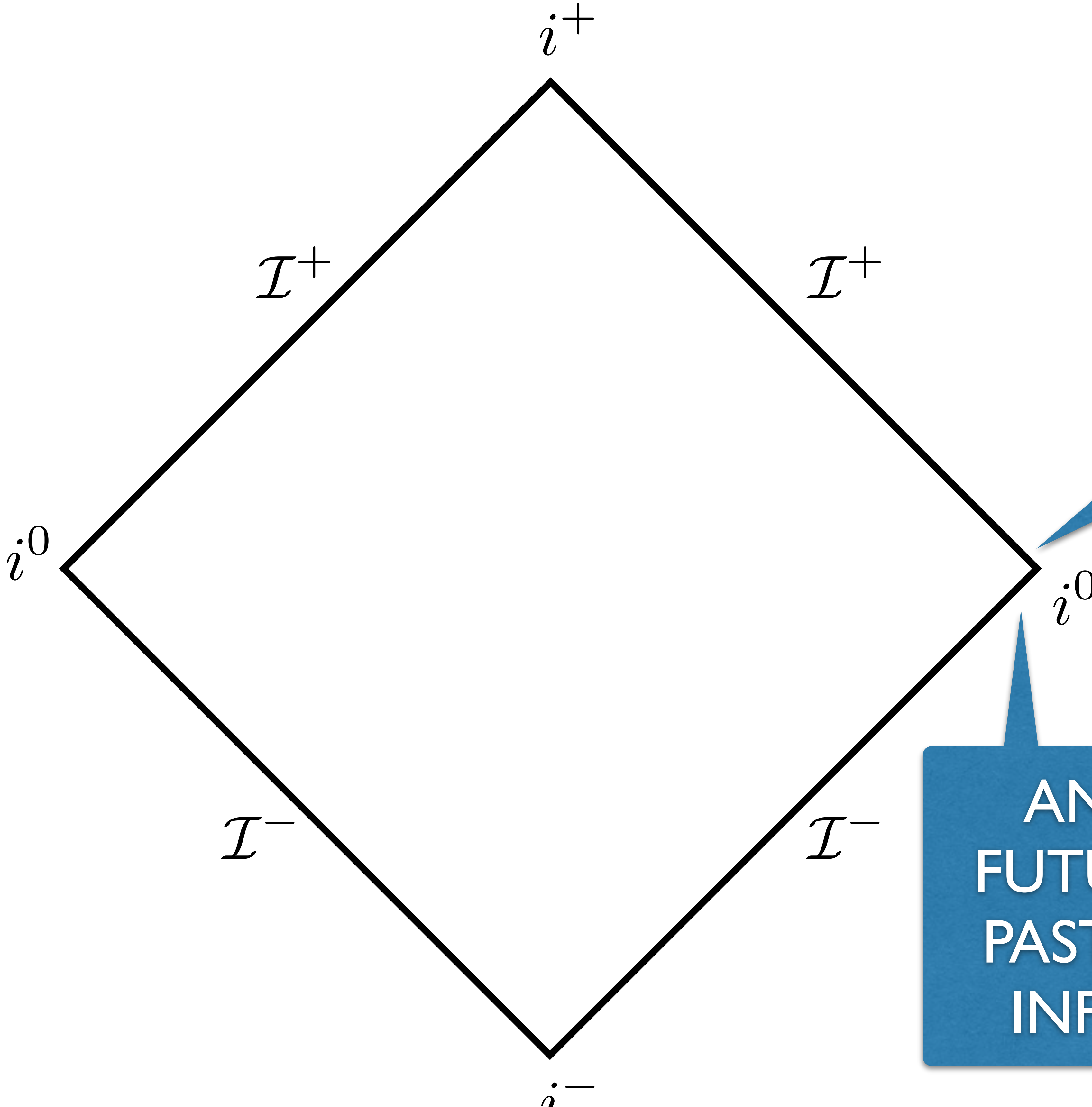
M. Mirbabayi MP, PRL 117 (2016) n. 21, 211301
arxiv:1607.03120 [hep-th]

R. Bousso, MP, CQG 34 (2017) n. 20, 204001
arxiv:1706.00436 [hep-th]
R. Bousso, MP, PRD 96 (2017) n. 8, 086016
arxiv:1706.09280 [hep-th]

- BMS CHARGES AND SOFT BMS CHARGES
- BMS CHARGE CONSERVATION LAWS
- CANONICALLY CONJUGATE IR DEGREES OF FREEDOM
- THE OTHER CONSERVATION LAW
- CANONICAL TRANSFORMATIONS ON IR VARIABLES
- FACTORIZATION OF IR DYNAMICS ON OPERATORS
- A “BETTER LORENTZ TRANSFORMATION”?
- CONSISTENCY OF THE NEW LORENTZ ALGEBRA
- HAWKING RADIATION IN THE PRESENCE OF IR DEGREES OF FREEDOM (SOFT HAIR)

- FACTORIZATION OF IR DYNAMICS ON FIXED BLACK-HOLE BACKGROUND GEOMETRY
- ENTANGLEMENT OF SOFT HAIR WITH THE HAWKING RADIATION: AN EXACT CALCULATION IN THE ZERO-FREQUENCY LIMIT
- ABSENCE OF OBSERVABLE EFFECTS

BMS CHARGES



CHARGES ARE DEFINED AT PAST OF FUTURE NULL INFINITY

AND AT FUTURE OF PAST NULL INFINITY

BMS CHARGES AT FUTURE INFINITY

$$ds^2 = -du^2 + \frac{2Gm}{r}du^2 - 2dudr + r^2\gamma_{AB}d\theta^A d\theta^B + rC^+_{AB}d\theta^A d\theta^B$$

$$N^+_{AB} = \partial_u C^+_{AB}$$

$$Q^+_{h\ lm} = \frac{1}{4\pi}\int_{I^+_-} d^2\theta\sqrt{\gamma}Y_{lm}(\theta)m$$

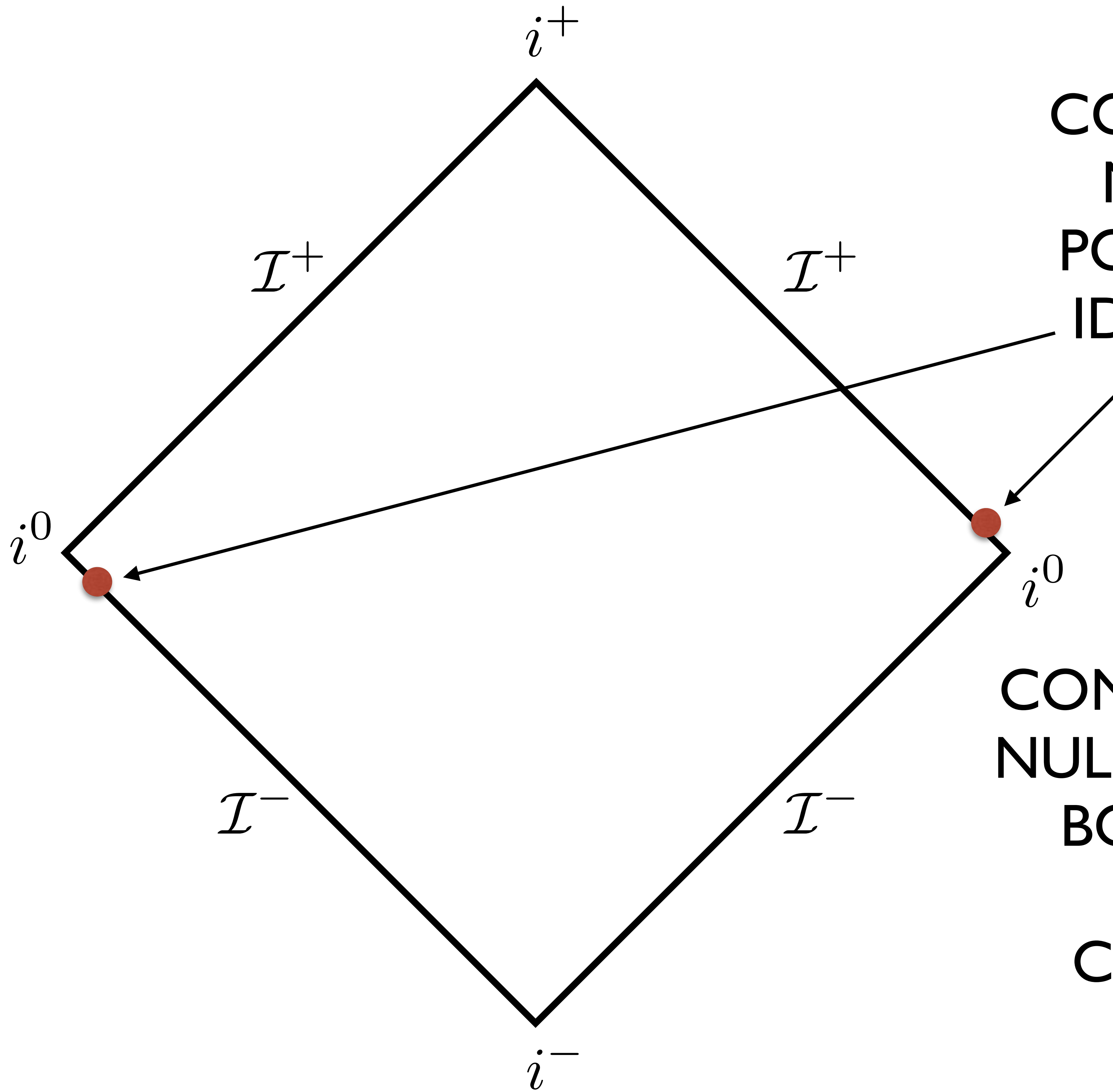
BONDI MASS ASPECT

$$Q^+_{lm} = Q^+_{s\ lm} + Q^+_{h\ lm}$$

$$Q^+_{h\ lm} = \frac{1}{4\pi}\int_{I^+} dud^2\theta\sqrt{\gamma}Y_{lm}(\theta)\Big[\lim_{r\to\infty} r^2 T_{uu} - \frac{1}{8G}N^+_{AB}N^{+\ AB}\Big]$$

$$Q^+_{s\ lm} = -\frac{1}{16\pi G}\int_{I^+} dud^2\theta\sqrt{\gamma}Y_{lm}(\theta)D^A D^B N^+_{AB}, \qquad l \geq 2$$

BMS CHARGES CONSERVATION



CONFORMAL COMPACTIFICATION MAKES THE TWO POINTS CLOSE AND IDENTIFIES ANGLES ANTIPODALLY

$$\phi^- = \phi^+ + 2\pi$$

$$\theta^- = \pi - \theta^+$$

CONTINUITY ALONG NULL GENERATOR OF BOUNDARY=BMS CHARGE CONSERVATION

$$Q^+_{lm} = Q^-_{lm}$$

BMS CHARGE CONSERVATION = SOFT THEOREMS, BUT WE CAN DO BETTER!

$$[Q^+_{s\,lm}, N^+_{AB}(u,\theta)] = 0$$

SOFT CHARGE NEEDS CONJUGATE VARIABLE

(OTHERWISE IT IS A CONSTANT ON IRREPS OF LOCAL OPERATOR ALGEBRA)

ADDITIONAL DEGREE OF FREEDOM: BOUNDARY GRAVITON

$$\lim_{u\to-\infty} C^+_{AB}(u,\theta) = -2D_A D_B C^+(\theta) + \gamma_{AB} D^2 C^+(\theta)$$

$$[Q^+_{s\,lm}, C^+_{kn}] = -i\delta_{lk}\delta_{mn} \qquad l,k \geq 2$$

CANONICAL COMMUTATION RELATIONS

THE BOUNDARY GRAVITON ALSO OBEYS A SIMPLE MATCHING CONDITION WHEN ANGLES AT PAST AND FUTURE NULL INFINITY ARE IDENTIFIED ANTIPODALLY

$$C^{+}_{lm} = C^{-}_{lm}$$

SEVERAL ARGUMENTS SUPPORT THIS MATCHING CONDITION

IN PARTICULAR, IT IS THE ONLY LORENTZ AND CPT INVARIANT BOUNDARY CONDITION, IT IS USED IN MOST GENERAL RELATIVITY COMPUTATIONS IN ASYMPTOTICALLY FLAT SPACETIME AND IT IS VALID TO ALL-ORDER PERTURBATIVE GRAVITY COMPUTATIONS

A CANONICAL TRANSFORMATION

DEFINE IT AS FOLLOWS

$$UQ^{+}_{s\,lm}U^{-1} = Q^{+}_{lm} \qquad UC^{+}_{lm}U^{-1} = C^{+}_{lm}$$

EXPLICITLY

$$U = \exp[-i\sum_{l=2}^{\infty}\sum_{m=-l}^{l} Q^{+}_{h\,lm}C^{+}_{lm}]$$

DEFINE DRESSED VARIABLES BY THE SAME CANONICAL TRANSFORMATION

$$UN^{+}_{AB}U^{-1} \equiv \hat{N}^{+}_{AB}$$

KEY PROPERTY: THEY COMMUTE WITH THE BMS CHARGES

$$[\hat{N}^{+}_{AB}, Q^{+}_{lm}] = U[N^{+}_{AB}, Q^{+}_{s\,lm}]U^{-1} = 0$$

SO $\hat{N}^{+}_{AB},\ Q^{+}_{lm},\ C^{+}_{lm}$ FORM A COMPLETE SET OF CANONICAL VARIABLES WITH CCR

FACTORIZATION OF IR DYNAMICS ON OPERATORS

USE THE HEISENBERG PICTURE: OPERATORS EVOLVE IN TIME, STATES DO NOT

$$O^+ = \Omega^{-1} O^- \Omega$$

BMS CHARGES AND BOUNDARY GRAVITONS COMMUTE WITH TIME EVOLUTION OPERATOR BECAUSE OF MATCHING CONDITIONS

$$Q^+_{lm} = \Omega^{-1} Q^-_{lm} \Omega = Q^-_{lm} \qquad C^+_{lm} = \Omega^{-1} C^-_{lm} \Omega = C^-_{lm}$$

CCR: $Q^\pm_{lm} = -i \dfrac{\partial}{\partial C^\pm_{lm}}$

CCR+MATCHING CONDITIONS:

$$[Q^\pm_{lm}, \Omega] = -i \frac{\partial \Omega}{\partial C^\pm_{lm}} = 0 \qquad [C^\pm_{lm}, \Omega] = 0 \rightarrow \Omega = \Omega(\hat{N})$$

HEISENBERG TIME EVOLUTION INDEPENDENT OF ALL IR DEGREES OF FREEDOM!

$$[Q^{\pm}_{lm}, \Omega] = -i\frac{\partial \Omega}{\partial C^{\pm}_{lm}} = 0 \qquad [C^{\pm}_{lm}, \Omega] = 0 \rightarrow \Omega = \Omega(\hat{N})$$

APPLICATION #1: USE DRESSING TO DEFINE A NEW "BETTER" LORENTZ

LORENTZ DOES NOT COMMUTE WITH BMS SO VACUA RELATED BY GENERAL SUPERTRANSLATIONS ARE NOT SIMULTANEOUSLY LORENTZ INVARIANT.

SOLUTION: DEFINE A NEW LORENTZ THAT DOES COMMUTE WITH BMS

$$\delta_L \hat{N}^{+}_{AB} = [\hat{L}^{+}, \hat{N}^{+}_{AB}] = U[L, N^{+}_{AB}]U^{-1}$$

$$\delta_L C^{+}_{lm} = \delta_L Q^{+}_{lm} = 0 \qquad \hat{L}^{+} = ULU^{-1}$$

CONSISTENCY CHECKS

1) NEW LORENTZ COMMUTES WITH TIME EVOLUTION

NEW LORENTZ AT PAST NULL INFINITY

$$\delta_L \hat{N}^-_{AB} = [\hat{L}^-, \hat{N}^-_{AB}] = V[L, N^-_{AB}]V^{-1}$$

$$\delta_L C^-_{lm} = \delta_L Q^-_{lm} = 0 \qquad \hat{L}^- = VLV^{-1} \qquad V = \Omega U \Omega^{-1}$$

LORENTZ TRANSFORM THEN EVOLVE

=

EVOLVE THEN LORENTZ TRANSFORM

$$\Omega^{-1}\delta_L \hat{N}^-_{AB}\Omega = \Omega^{-1}V[L, N^-_{AB}]V^{-1}\Omega = U[L, \Omega^{-1}N^-_{AB}\Omega]U = U[L, N^+_{AB}]U^{-1} = \delta_L N^+_{AB}$$

FROM DEFINITION OF U AND L=SYMMETRY OF TIME EVOLUTION

CONSISTENCY CHECKS

2) JACOBI IDENTITIES

$$\delta_\xi \hat{N}^+_{AB} = U[L, N^+_{AB}]U^{-1} = U\mathcal{L}_\xi N^+_{AB} U^{-1} = \mathcal{L}_\xi \hat{N}^+_{AB}$$

LIE DERIVATIVE

$$\delta_\xi(\delta_\eta X) - \delta_\eta(\delta_\xi X) = \delta_{[\xi,\eta]} X, \qquad X = C^+_{lm}, Q^+_{lm}, N^+_{AB}$$

SATISFIED

$$[Y, \delta_\xi X] + \delta_\xi [X, Y] + [\delta_\xi Y, X] = 0, \qquad X, Y = Q^+_{lm}, C^+_{lm} \; or \; X, Y = N^+_{AB}$$

SATISFIED

$$[\delta_\xi \hat{N}^+_{AB}, X] + [\hat{N}^+_{AB}, \delta_\xi X] + \delta_\xi [X, \hat{N}^+_{AB}] = [\mathcal{L}_\xi \hat{N}^+_{AB}, X] = 0, \qquad X = Q^+_{lm}, C^+_{lm}$$

SATISFIED

THE NEW LORENTZ IS CONSISTENT AS ISOMORPHISM OF THE ASYMPTOTIC OPERATOR ALGEBRA.
A CHARGE IS A HERMITIAN OPERATOR DEFINED ON AN IRREDUCIBLE REPRESENTATION OF THE CCR.
TO PROMOTE THE ISOMORPHISM TO AN OPERATOR WE HAVE TO DEFINE ITS ACTION ON THE VACUUM, CHECK THAT THE DOMAIN OF DEFINITION IS DENSE AND THAT IT COINCIDES WITH THE DOMAIN OF THE ADJOINT.

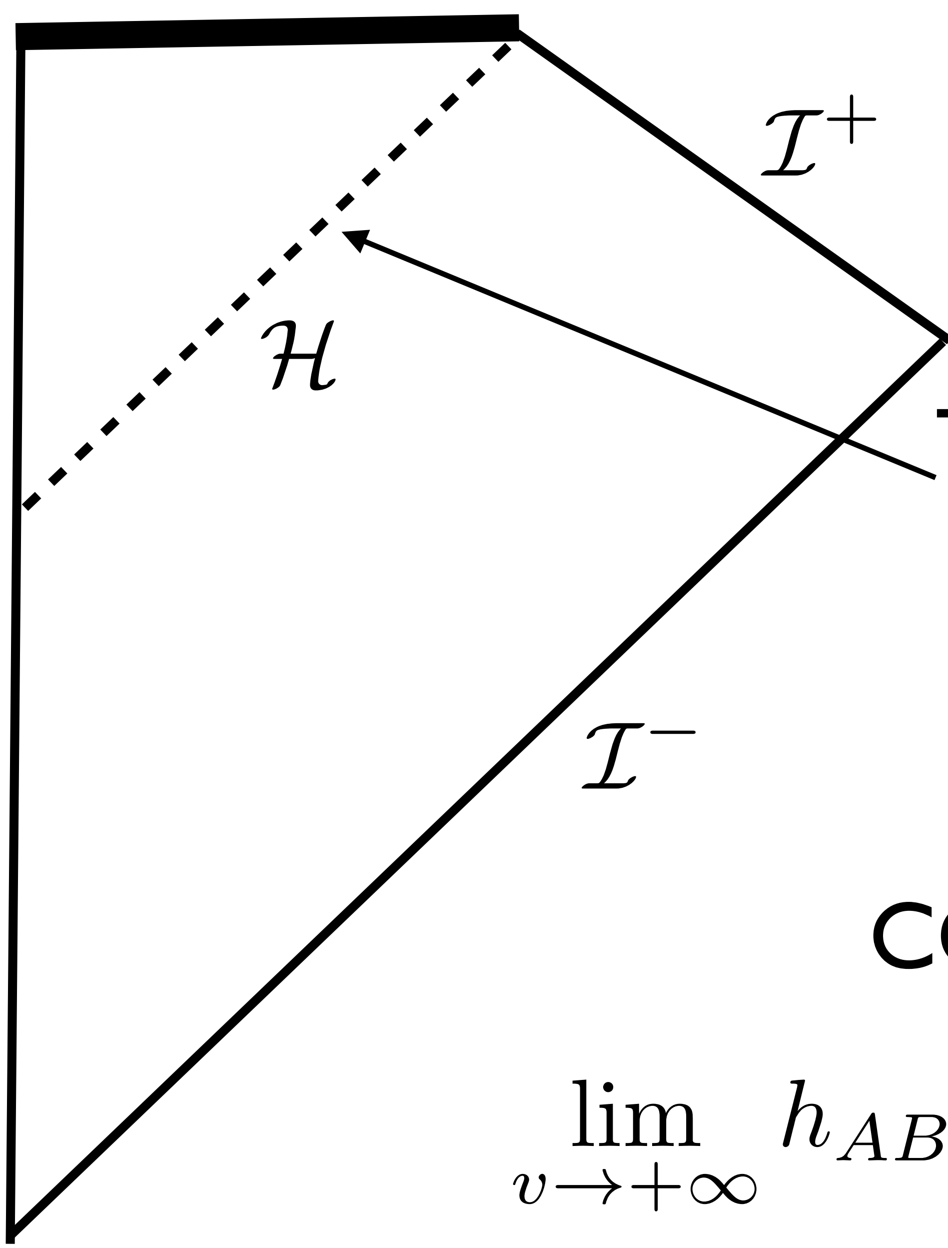
SO THE CONSTRUCTION IS NOT COMPLETE YET, BUT IT IS A FIRST STEP TOWARDS DEFINING NEW LORENTZ CHARGES WHICH DO NOT BELONG TO THE BMS ALGEBRA AND THAT COMMUTE WITH SUPERTRANSLATIONS

APPLICATION #2: HAWKING RADIATION IN THE PRESENCE OF SOFT HAIR

WE WORK IN THE LIMIT

$$G \to 0, \qquad GM = \text{ constant}$$

(NOTHING TO SAY ABOUT INFORMATION PUZZLE)



METRIC: VAIDYA

TO FIRST ORDER IN FLUCTUATIONS CHARGES CAN BE DEFINED HERE

$$Q^H_{lm} = Q^H_{s\ lm} + Q^H_{h\ lm}$$

CONJUGATE VARIABLE TO $Q^H_{s\ lm}$

$$\lim_{v \to +\infty} h_{AB}\Big|_{\mathcal{H}} = (4MD_A D_B - 2M\gamma_{AB}D^2)F$$

CHARGES AND BOUNDARY GRAVITONS AT NULL INFINITY CAN BE DEFINED EXACTLY AS BEFORE.
WE WILL NEED ONLY THE MATCHING CONDITIONS

$$Q^H_{s\ lm} + Q^H_{h\ lm} + Q^+_{s\ lm} + Q^+_{h\ lm} = Q^-_{s\ lm} + Q^-_{h\ lm}$$

$$C^+ = C^-$$

DEFINE TWO CANONICAL TRANSFORMATIONS

$$U_- Q^-_{s\ lm} U_-^{-1} = Q^-_{lm} \qquad U_+ (Q^H_{s\ lm} + Q^+_{s\ lm}) U_+^{-1} = Q^H_{lm} + Q^+_{lm}$$

$$U_- = \exp[-i \sum_{lm} Q^-_{h\ lm} C^-_{lm}]$$

$$U_+ = \exp[-i \sum_{lm} Q^+_{h\ lm} C^+_{lm}] \exp[-i \sum_{lm} Q^H_{h\ lm} F_{lm}]$$

DRESSED OPERATORS AT PAST AND FUTURE NULL INFINITY

$$\hat{\phi}^+ = U_+ \phi^+ U_+^{-1} \qquad \hat{\phi}^- = U_- \phi^- U_-^{-1}$$

DRESSED OPERATORS AT PAST NULL INFINITY COMMUTE WITH

$$C^-_{lm},\ Q^-_{lm}$$

SINCE VARIABLES AT THE HORIZON COMMUTE WITH VARIABLES AT FUTURE NULL INFINITY BY LOCALITY, DRESSED OPERATORS AT FUTURE NULL INFINITY COMMUTE WITH

$$C^+_{lm},\ F_{lm},\ Q^H_{lm},\ Q^+_{lm}$$

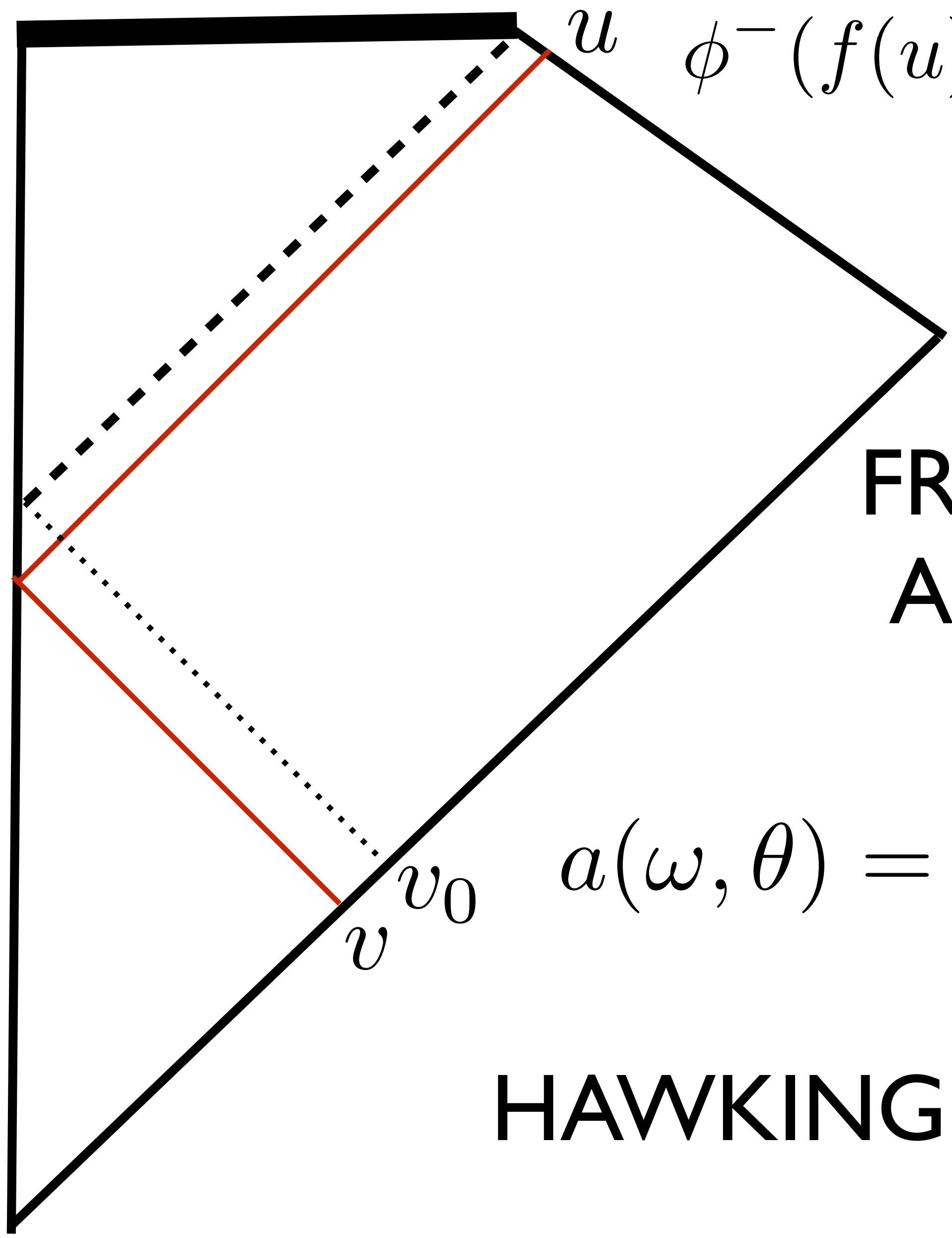
DRESSING CANCELS THE EFFECT OF SUPERTRANSLATIONS. SUPERTRANSLATIONS ARE ANGLE-DEPENDENT SHIFTS OF NULL COORDINATES, HENCE:

$$\hat{\phi}^+(u,\theta) = \phi^+(u + C^+(\theta), \theta) \qquad \hat{\phi}^- = \phi^-(v + C^-(\theta), \theta)$$

SCATTERING IN FIXED BACKGROUND (HAWKING 1974)

$$\phi^+(u,\theta) = \phi^-(v,\theta)\Big|_{v=f(u)}, \qquad f(u) = v_0 - C\exp(-\kappa u) + \dots$$

$$a(\omega,\theta) = \int du e^{i\omega u}\phi^+(u,\theta) = \int du e^{i\omega u}\phi^-(f(u),\theta)$$



$$\phi^-(f(u),\theta) = \int_0^\infty \frac{đ\,\nu}{2\nu}\Big[e^{-i\nu f(u)}b(\nu,\theta) + e^{i\nu f(u)}b^\dagger(\nu,\theta)\Big]$$

THE FOURIER TRANSFORM IN u NOW CONTAINS BOTH POSITIVE FREQUENCY ANNIHILATION OPERATORS AND NEGATIVE FREQUENCY CREATION OPERATORS

$$a(\omega,\theta) = \int_0^\infty \frac{đ\,\nu}{2\nu}\Big[\alpha(\omega,\nu)b(\nu,\theta) + \beta(\omega,\nu)b^\dagger(\nu,\theta)\Big]$$

HAWKING FLUX $\propto \int_0^\infty \frac{đ\,\nu}{2\nu}\Big|\beta(\omega,\nu)\Big|^2$

INTRODUCING SOFT HAIR

$$\phi^{\pm} \to \hat{\phi}^{\pm}$$

$$\hat{a}(\omega,\theta) = \int du e^{i\omega u}\phi^{+}(u + C^{+},\theta) = \int du e^{i\omega u}\phi^{-}(f(u) + C^{-},\theta)$$

$$\phi^{-}(f(u) + C^{-},\theta) = \int_0^\infty \frac{đ\,\nu}{2\nu}\left[e^{-i\nu f(u)}e^{-i\nu C^{-}}b(\nu,\theta) + e^{i\nu f(u)}e^{i\nu C^{-}}b^{\dagger}(\nu,\theta)\right]$$

$$\hat{a}(\omega,\theta) = e^{-i\omega C^{+}}a(\omega,\theta)$$

SO THE BOGOLIOUBOV COEFFICIENTS CHANGE BY A FREQUENCY-DEPENDENT PHASE

$$\beta(\omega,\nu) \to \beta'(\omega,\nu) = e^{i\omega C^{+} + i\nu C^{-}}\beta(\omega,\nu)$$

THE HAWKING FLUX IS UNCHANGED

$$\int_0^\infty \frac{đ\,\nu}{2\nu}\left|\beta(\omega,\nu)\right|^2 \to \int_0^\infty \frac{đ\,\nu}{2\nu}\left|\beta'(\omega,\nu)\right|^2 = \int_0^\infty \frac{đ\,\nu}{2\nu}\left|\beta(\omega,\nu)\right|^2$$

CONCLUSIONS

- THE ZERO-FREQUENCY SECTOR OF GRAVITY IN ASYMPTOTICALLY FLAT SPACE FACTORIZES UNDER VERY GENERAL ASSUMPTIONS, INDEPENDENTLY OF THE DETAILED DYNAMICS IN THE INTERIOR OF THE SPACETIME

- FACTORIZATION IS PARTICULARLY CLEAR WHEN THE CANONICAL FORMALISM IS USED AND FACTORIZATION IS APPLIED TO OPERATORS INSTEAD OF STATES

- FACTORIZATION OF IR DYNAMICS ON OPERATORS IS ACHIEVED BY A CANONICAL TRANSFORMATION, WHICH WE CALLED "DRESSING."

- DRESSING MAY BE USED TO DEFINE A "BETTER LORENTZ" SYMMETRY THAT COMMUTES WITH SUPERTRANSLATIONS

- DRESSING ALLOWS TO COMPUTE IN A SIMPLE MANNER THE EFFECT OF INFRARED HAIR ON HAWKING RADIATION IN THE NO-BACKREACTION APPROXIMATION

- THERE IS AN EFFECT ON BOGOLIOUBOV COEFFICIENTS BUT SUCH EFFECT DISAPPEARS FROM THE HAWKING FLUX (AND FROM CORRELATOR OF LOCAL OBSERVABLES COMPUTED AT FUTURE NULL INFINITY)

LAUNDRY LIST

- PROMOTE NEW LORENTZ TO UNITARY OPERATOR

- GO BEYOND THE STRICT ZERO-FREQUENCY LIMIT

- INCORPORATE THE EFFECT OF BACKREACTION ON HAWKING RADIATION COMPUTATION