Supersymmetric Higher Spin Current Multiplets

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- Based on: Universe **4** (2018) no.1, 6
  JHEP **1803** (2018) 119
  JHEP **1805** (2018) 204
  JHEP **1808** (2018) 055
  arXiv:**1811.12858** [hep-th]

- A collaboration with I. L. Buchbinder, S. J. Gates and R. von Unge

Why Study (supersymmetric) Higher Spins ?

- Fact: $0 \leq$ spin of elementary particles ≤ 2 $\leftarrow$ No-Go theorems

  $0\ ,\ 1/2\ ,\ 1\ ,\ \boxed{3/2}\ ,\ 2$ $\rightarrow$ Supersymmetry

- First Motivation: General structures in (Supersymmetric) Field Theory

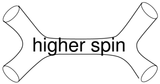
  Particles
  Unitary, irreps of (super) Poincaré Group $\longrightarrow$ Classify them (m,j) $\longrightarrow$ Physical requirements $(m^2 \geq 0, \boxed{j < \infty})$

  No reason to stop at $j \leq 2$

- Second Motivation: String Theory [SUSY + higher spin states]

$m = 0,\ j = 1$ no tachyons $\longleftrightarrow$ Supersymmetry

$m = 0,\ j = 2$ $+$ tower of massive spins



UV success $\longleftrightarrow$ Higher Spin propagation

$E \gg m \rightarrow$ {ignore massess} $\rightarrow$ {symmetries emerge} $\longleftarrow$ Higher Spin gauge theories

Why Study (supersymmetric) Higher Spins ?

- Third Motivation: Holography (AdS/CFT)
  [QFT in fixed spacetime (D) = Gravity in different spacetime (AdS) (D+1)]

Succesful Models for
higher spin interactions
(Vasiliev's theory, 3D CS hs-gravity)

Two features
$m = 0, j = 2$
Not Flat Spacetime (AdS)

Higher spins a playground
for Holography

[supersymmetric h.s. consistent with AdS]

Interacting (supersymmetric) Theories

$[T^{m}, T^{HS}] = T^{HS}$
$[T^{mn}, T^{HS}] = T^{HS}$
$\delta_{HS} h_{mn} = h_{HS}$

- Fact: Consistent Interacting Theories = Difficult Problem
  (without guiding principle)
- spin 1 (YM): Principle Bundles
- spin 2 (GR): Riemannian Manifolds

  } Supergeometry
- higher spins: No Geometrical Input $\rightarrow$ Alternative methods
- Physical requirement: d.o.f. interacting theory = d.o.f. free theory

  Gauge Invariance $\leftarrow$
  (manifest susy)

(supersymmetric) h.s. interactions respect (susy covariant) gauge symmetries

Noether's Method

Noether's method: A systematic, perturbative, analysis of invariance

Coupling Matter to Higher Spins

- Two types of fields: matter fields (ϕ), gauge fields (h)
- Expand the interacting action $S[\phi, h]$ in terms of coupling constant g
  the field transformations $\delta\phi$, δh

$$S[\phi,h] = S_0[\phi] + g\, S_1[\phi,h] + g^2\, S_2[\phi,h] + \dots\ ,$$
$$\delta\phi = 0 + g\, \delta_1\phi + g^2\, \delta_2\phi + \dots\ ,$$
$$\delta h = \delta_0 h + g\, \delta_1 h + g^2\, \delta_2 h + \dots$$

- Invariance of order g: $\int \left\{ \frac{\delta S_0}{\delta\phi}\, \delta_1\phi + \frac{\delta S_1}{\delta h}\, \delta_0 h \right\} = 0$
- Current [$\mathcal{J}$] generated cubic terms: $S_1[\phi,h] \sim \int \mathcal{J}[\phi]\, h$

$$\int \left\{ \frac{\delta \boxed{S_0}}{\delta\phi}\, \boxed{\delta_1\phi} + \mathcal{J}[\phi]\, \boxed{\delta_0 h} \right\} = 0 \xrightarrow{\text{on-shell}} \text{conservation of } \mathcal{J}$$

Supersymmetric matter and gauge superfields

- Apply Noether's method to supersymmetric theories
- matter fields $\rightarrow$ matter supermiltiplets (Φ, Σ)

$\bar{\mathrm{D}}^2\Sigma = 0$

Σ → complex linear superfield

Φ → chiral superfield

$\bar{\mathrm{D}}_{\dot{\alpha}}\Phi = 0$

- higher spin fields $\rightarrow$ higher spin supermultiplets

1. Half-integer superspin $Y = s + 1/2$: $(j = s+1 \,,\, j = s+1/2)$

$$H_{\alpha(s)\dot{\alpha}(s)} : \; \delta_0 H_{\alpha(s)\dot{\alpha}(s)} = \tfrac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot{\alpha}(s)} - \tfrac{1}{s!}\bar{\mathrm{D}}_{(\dot{\alpha}_s}L_{\alpha(s)\dot{\alpha}(s-1))}$$

$$\chi_{\alpha(s)\dot{\alpha}(s-1)} : \; \delta_0 \chi_{\alpha(s)\dot{\alpha}(s-1)} = \bar{\mathrm{D}}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \mathrm{D}^{\alpha_{s+1}}\Lambda_{\alpha(s+1)\dot{\alpha}(s-1)}$$

2. Integer superspin $Y = s$: $(j = s+1/2 \,,\, j = s)$

$$\Psi_{\alpha(s)\dot{\alpha}(s-1)} : \; \delta_0 \Psi_{\alpha(s)\dot{\alpha}(s-1)} = -\mathrm{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \tfrac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}}\Lambda_{\alpha(s)\dot{\alpha}(s-2))}$$

$$V_{\alpha(s-1)\dot{\alpha}(s-1)} : \; \delta_0 V_{\alpha(s-1)\dot{\alpha}(s-1)} = \mathrm{D}^{\alpha_s}L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^{\dot{\alpha}_s}\bar{L}_{\alpha(s-1)\dot{\alpha}(s)}$$

Super-Poincaré Algebra and Superspace

- Super-Poincaré algebra

$$[P_A, P_B\} = f_{AB}{}^{C} P_C \ , \ P_A = \{Q_\alpha, \bar{Q}_{\dot{\alpha}}, P_m\} \ , \ A = \{\alpha, \ \dot{\alpha}, \ m\}$$
$$[J, P] \sim P \ , \ [J, J] \sim J$$

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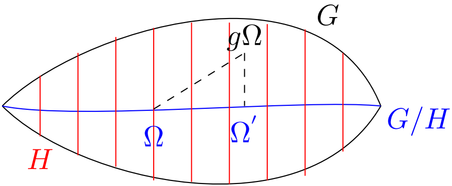
- Construct Left Coset $\Omega \ (G/H)$



$$g(\omega, x, \theta, \bar{\theta}) = \Omega(x, \theta, \bar{\theta}) h(\omega)$$
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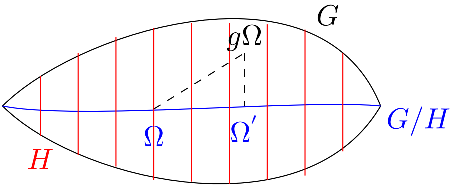
- Construct Left Coset Ω (G/H)



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- Superspace $(\mathbb{R}^{4|4})$: $(x_m, \ \theta^\alpha, \ \bar{\theta}^{\dot{\alpha}})$

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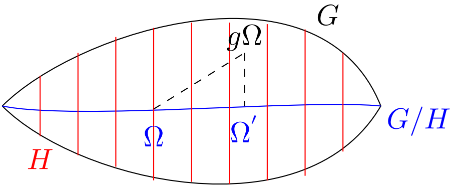

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- Superspace ($\mathbb{R}^{4|4}$) : $(x_m, \ \theta^\alpha, \ \bar\theta^{\dot\alpha})$
- Superfields : 1. $\Phi \ : \ \mathbb{R}^{4|4} \to \mathbb{R}^{4|4}$

  2. $\delta_S \Phi(x, \theta, \bar\theta) = i\epsilon^\alpha Q_\alpha \Phi + i\bar\epsilon^{\dot\alpha} \bar{Q}_{\dot\alpha} \Phi$

$$Q_\alpha = i\partial_\alpha + \frac{1}{2}\bar\theta^{\dot\alpha}\sigma^m_{\alpha\dot\alpha}\partial_m, \qquad \bar{Q}_{\dot\alpha} = i\bar\partial_{\dot\alpha} + \frac{1}{2}\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m$$

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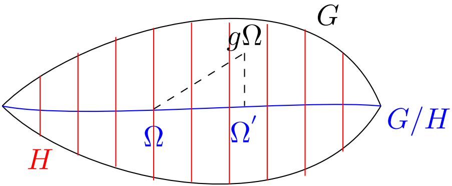

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- Covariant derivatives: $\mathrm{D}_\alpha = \partial_\alpha + \frac{i}{2}\bar{\theta}^{\dot{\alpha}} \sigma^m_{\alpha\dot{\alpha}} \partial_m, \qquad \bar{\mathrm{D}}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}} + \frac{i}{2}\theta^\alpha \sigma^m_{\alpha\dot{\alpha}} \partial_m$

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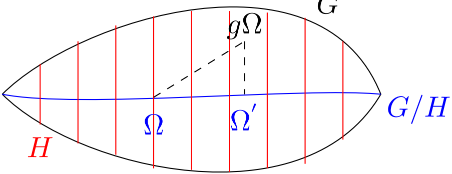

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Superfields & Components

- Superfields as functions of θ and $\bar{\theta}$

$$\Phi(x,\theta,\bar{\theta}) = A(x) + \theta^{\alpha}\psi_{\alpha}(x) + \bar{\theta}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}}(x) + \theta^2 F(x) + \bar{\theta}^2 G(x) + \theta^{\alpha}\bar{\theta}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} + \theta^2\bar{\theta}^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}}(x) + \bar{\theta}^2\theta^{\alpha}\rho_{\alpha}(x) + \theta^2\bar{\theta}^2 D(x)$$

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- Component fields are the coefficients in the expansion.

$$\Phi \sim \theta^{n}\bar{\theta}^{m}\Phi_{(n,m)} \rightarrow \Phi_{(n,m)} \sim \partial_{\alpha}^{n}\bar{\partial}_{\dot{\alpha}}^{m}\Phi|_{\theta=0,\ \bar{\theta}=0}$$

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- Component fields are the coefficients in the expansion.

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- Components in terms of the covariant derivatives

$$\Phi^{(0,0)}_{\alpha(n)\dot{\alpha}(m)} = \Phi_{\alpha(n)\dot{\alpha}(m)}|$$

$$\Phi^{(1,0)}_{\beta\alpha(n)\dot{\alpha}(m)} = \mathrm{D}_{\beta}\Phi_{\alpha(n)\dot{\alpha}(m)}|$$

$$\Phi^{(0,1)}_{\alpha(n)\dot{\beta}\dot{\alpha}(m)} = \bar{\mathrm{D}}_{\dot{\beta}}\Phi_{\alpha(n)\dot{\alpha}(m)}|$$

$$\Phi^{(1,1)}_{\beta\alpha(n)\dot{\beta}\dot{\alpha}(m)} = -\frac{1}{2}\left[\mathrm{D}_{\beta},\bar{\mathrm{D}}_{\dot{\beta}}\right]\Phi_{\alpha(n)\dot{\alpha}(m)}|$$

$$\Phi^{(2,0)}_{\alpha(n)\dot{\alpha}(m)} = -\mathrm{D}^2\Phi_{\alpha(n)\dot{\alpha}(m)}|$$

$$\Phi^{(0,2)}_{\alpha(n)\dot{\alpha}(m)}| = -\bar{\mathrm{D}}^2\Phi_{\alpha(n)\dot{\alpha}(m)}|$$

$$\Phi^{(2,1)}_{\alpha(n)\dot{\beta}\dot{\alpha}(m)} = -\frac{1}{2}\left\{\mathrm{D}^2,\bar{\mathrm{D}}_{\dot{\beta}}\right\}\Phi_{\alpha(n)\dot{\alpha}m}|$$

$$\Phi^{(1,2)}_{\beta\alpha(n)\dot{\alpha}(m)} = -\frac{1}{2}\left\{\bar{\mathrm{D}}^2,\mathrm{D}_{\beta}\right\}\Phi_{\alpha(n)\dot{\alpha}(m)}|$$

$$\Phi^{(2,2)}_{\alpha(n)\dot{\alpha}(m)} = \frac{1}{4}\Box\Phi_{\alpha(n)\dot{\alpha}(m)}| + \frac{1}{2}\mathrm{D}^{\gamma}\bar{\mathrm{D}}^2\mathrm{D}_{\gamma}\Phi_{\alpha(n)\dot{\alpha}(m)}|$$

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- Superfields as functions of θ and $\bar{\theta}$

$$\Phi(x,\theta,\bar{\theta}) = A(x) + \theta^{\alpha}\psi_{\alpha}(x) + \bar{\theta}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}}(x) + \theta^2 F(x) + \bar{\theta}^2 G(x)$$
$$+ \theta^{\alpha}\bar{\theta}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} + \theta^2\bar{\theta}^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}}(x) + \bar{\theta}^2\theta^{\alpha}\rho_{\alpha}(x) + \theta^2\bar{\theta}^2 D(x)$$

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$$\Phi \quad \Phi_{(n,m)} \sim \partial^n_{\alpha}\bar{\partial}^m_{\dot{\alpha}}\Phi|_{\theta=0,\ \bar{\theta}=0}$$

$$\alpha(n) : \alpha_1\alpha_2\alpha_3\ldots\alpha_n$$

- Componen … derivatives

$$\Phi^{(0,0)}_{\alpha(n)\dot{\alpha}(m)} = \Phi_{\alpha(n}$$

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Supersymmetric matter and gauge superfields

- Apply Noether's method to supersymmetric theories
- matter fields → matter supermiltiplets (Φ, Σ)

  - Σ → complex linear superfield: $\bar{\mathrm{D}}^2\Sigma = 0$
  - Φ → chiral superfield: $\bar{\mathrm{D}}_{\dot\alpha}\Phi = 0$

- higher spin fields → higher spin supermultiplets

1. Half-integer superspin $Y = s + 1/2$: $(j = s+1\ ,\ j = s+1/2)$

$$H_{\alpha(s)\dot\alpha(s)} :\ \delta_0 H_{\alpha(s)\dot\alpha(s)} = \tfrac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot\alpha(s)} - \tfrac{1}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s}L_{\alpha(s)\dot\alpha(s-1))}$$

$$\chi_{\alpha(s)\dot\alpha(s-1)} :\ \delta_0\chi_{\alpha(s)\dot\alpha(s-1)} = \bar{\mathrm{D}}^2 L_{\alpha(s)\dot\alpha(s-1)} + \mathrm{D}^{\alpha_{s+1}}\Lambda_{\alpha(s+1)\dot\alpha(s-1)}$$

2. Integer superspin $Y = s$: $(j = s+1/2\ ,\ j = s)$

$$\Psi_{\alpha(s)\dot\alpha(s-1)} :\ \delta_0\Psi_{\alpha(s)\dot\alpha(s-1)} = -\mathrm{D}^2 L_{\alpha(s)\dot\alpha(s-1)} + \tfrac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot\alpha_{s-1}}\Lambda_{\alpha(s)\dot\alpha(s-2))}$$

$$V_{\alpha(s-1)\dot\alpha(s-1)} :\ \delta_0 V_{\alpha(s-1)\dot\alpha(s-1)} = \mathrm{D}^{\alpha_s}L_{\alpha(s)\dot\alpha(s-1)} + \bar{\mathrm{D}}^{\dot\alpha_s}\bar{L}_{\alpha(s-1)\dot\alpha(s)}$$

Chiral supermultiplet

Goal: Find a non-trivial $\delta_1\Phi$

can not be absorbed by redefinitions
doesn't depend on e.o.m
(doesn't vanish on-shell)

Properties of $\delta_1\Phi$:
1. First order in Φ
2. Consistent with chiral constraint: $\bar{\mathrm{D}}_{\dot\alpha}\delta_1\Phi = 0$

Inspiration: Cubic interactions between (super)matter and (super)gravity

linearized (super)diffeomorhism

$$\delta_1\Phi = A^{\alpha}\,\mathrm{D}_{\alpha}\Phi + \Delta^{\alpha\dot\alpha}\,\partial_{\alpha\dot\alpha}\Phi$$

Ansatz:

$$\delta_1\Phi = \sum_{k=0}^{\infty}\Big\{\; A^{\alpha(k+1)\dot\alpha(k)}\,\mathrm{D}_{\alpha_{k+1}}\bar{\mathrm{D}}_{\dot\alpha_k}\mathrm{D}_{\alpha_k}\dots\bar{\mathrm{D}}_{\dot\alpha_1}\mathrm{D}_{\alpha_1}\Phi + \Gamma^{\alpha(k)\dot\alpha(k+1)}\,\bar{\mathrm{D}}_{\dot\alpha_{k+1}}\mathrm{D}^2\bar{\mathrm{D}}_{\dot\alpha_k}\mathrm{D}_{\alpha_k}\dots\bar{\mathrm{D}}_{\dot\alpha_1}\mathrm{D}_{\alpha_1}\Phi + \Delta^{\alpha(k)\dot\alpha(k)}\,\bar{\mathrm{D}}_{\dot\alpha_k}\mathrm{D}_{\alpha_k}\dots\bar{\mathrm{D}}_{\dot\alpha_1}\mathrm{D}_{\alpha_1}\Phi + E^{\alpha(k)\dot\alpha(k)}\,\mathrm{D}^2\bar{\mathrm{D}}_{\dot\alpha_k}\mathrm{D}_{\alpha_k}\dots\bar{\mathrm{D}}_{\dot\alpha_1}\mathrm{D}_{\alpha_1}\Phi\Big\}$$

"Higher Spin transformation" of Chiral superfield

$$\delta_0 \chi_{\alpha(s)\dot\alpha(s-1)} = \bar{\mathrm{D}}^2 L_{\alpha(s)\dot\alpha(s-1)} + \mathrm{D}^{\alpha_{s+1}} \Lambda_{\alpha(s+1)\dot\alpha(s-1)}$$

- Chirality + Non-triviality:

$$\delta_1 \Phi = -\sum_{k=0}^{\infty} \Big\{ \bar{\mathrm{D}}^2 \ell^{\alpha(k+1)\dot\alpha(k)} \, \mathrm{D}_{\alpha_{k+1}} \bar{\mathrm{D}}_{\dot\alpha_k} \mathrm{D}_{\alpha_k} \dots \bar{\mathrm{D}}_{\dot\alpha_1} \mathrm{D}_{\alpha_1} \Phi - \frac{1}{(k+1)!} \bar{\mathrm{D}}^{(\dot\alpha_{k+1}} \ell^{\alpha(k+1)\dot\alpha(k))} \, \bar{\mathrm{D}}_{\dot\alpha_{k+1}} \mathrm{D}_{\alpha_{k+1}} \dots \bar{\mathrm{D}}_{\dot\alpha_1} \mathrm{D}_{\alpha_1} \Phi \Big\} + \bar{\mathrm{D}}^2 \ell \, \Phi$$

$$\delta_0 H_{\alpha(s)\dot\alpha(s)} = -\frac{1}{s!} \bar{\mathrm{D}}_{(\dot\alpha_s} L_{\alpha(s)\dot\alpha(s-1))} + \frac{1}{s!} \mathrm{D}_{(\alpha_s} \bar{L}_{\alpha(s-1))\dot\alpha(s)}$$

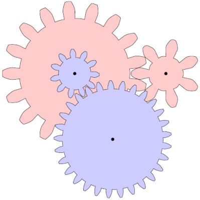
$$\delta_0 H = \bar{\mathrm{D}}^2 L + \mathrm{D}^2 \bar{L}$$

Matter - Higher Spin cubic interactions

Starting point: Single, free, massless chiral $S_0 = \int d^8z \, \bar{\Phi}\Phi$

Noether's method:

$$\xrightarrow{(S_0\ ,\ \delta_1\Phi)}$$



$$\xrightarrow{(\ S_{\Phi-\text{HS}}\ )}$$

Cubic interactions:

higher spin *supertrace*

$$S_{\Phi-\text{HS}} = g \int \sum_{s=0}^{\infty} \left\{ H^{\alpha(s)\dot{\alpha}(s)} \, \mathcal{J}_{\alpha(s)\dot{\alpha}(s)} + \left[\chi^{\alpha(s)\dot{\alpha}(s-1)} \, \mathrm{D}_{\alpha_s} \mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} + c.c. \right] \right\}$$

higher spin *supercurrent*

Higher Spin Supercurrent multiplet of chiral

Canonical supercurrent multiplet: $\left\{\mathcal{J}_{\alpha(s)\dot{\alpha}(s)}\ ,\ \mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)}\right\}$

$\bar{\mathrm{D}}^{\dot{\alpha}_s} A_{\alpha(s)\dot{\alpha}(s)} = \bar{\mathrm{D}}^2 B_{\alpha(s)\dot{\alpha}(s-1)}$

partially used this freedom during Noether's procedure

Properties:

- Not uniquely defined (Improvement terms)
- Include higher derivatives (Metsaev bound : ✓)
- Interactions only with half-integer superspin supermultiplets ($s+1, s+1/2$)
- Conservation equations:

$$\bar{\mathrm{D}}^{\dot{\alpha}_{k+1}} \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)} = \tfrac{1}{(k+1)!} \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_{k+1}} \mathcal{T}_{\alpha(k))\dot{\alpha}(k)}\ , \quad k = 0, 1, 2, \ldots$$

$$\bar{\mathrm{D}}^2 \mathcal{J} = 0$$

$s_{max} \leq \#\partial \leq s_1 + s_2 + \ldots$

Minimal supercurrent multiplet

Explore improvement terms freedom:

$$\delta\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)} = \frac{1}{(k+1)!}\,\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^2\bar{U}_{\alpha(k))\dot\alpha(k+1)} - \frac{1}{(k+1)!}\,\bar{\mathrm{D}}_{(\dot\alpha_{k+1}}\mathrm{D}^2U_{\alpha(k+1)\dot\alpha(k))}$$

$$\delta\mathcal{T}_{\alpha(k)\dot\alpha(k)} = \frac{k+2}{k+1}\mathrm{D}^{\alpha_{k+1}}U_{\alpha(k+1)\dot\alpha(k)} + \bar{\mathrm{D}}^{\dot\alpha_{k+1}}\bar{U}_{\alpha(k)\dot\alpha(k+1)}$$

Minimal supercurrent multiplet: $\left\{\mathcal{J}^{min}_{\alpha(s)\dot\alpha(s)}\ ,\ 0\right\}$

- There is unique choice of $U_{a(k+1)\dot\alpha(k)}$ such that $\mathcal{T}_{\alpha(k)\dot\alpha(k)} = 0$
- Not true in general (property of S_0)
- Conservation equation : $\bar{\mathrm{D}}^{\dot\alpha_s}\mathcal{J}^{min}_{\alpha(s)\dot\alpha(s)} = 0$
- Cubic interactions:

$$S_{\Phi-\mathbf{HS}} = g\sum_{s=0}\int H^{\alpha(s)\dot\alpha(s)}\ \mathcal{J}^{min}_{\alpha(s)\dot\alpha(s)}$$

$$s=0\ :\ \ \mathcal{J}^{min} = -\Phi\ \bar\Phi$$

$$s=1\ :\ \mathcal{J}^{min}_{\alpha\dot\alpha} = \frac{1}{3}\,\mathrm{D}_\alpha\Phi\ \bar{\mathrm{D}}_{\dot\alpha}\bar\Phi + \frac{i}{3}\,\partial_{\alpha\dot\alpha}\Phi\ \bar\Phi - \frac{i}{3}\,\Phi\ \partial_{\alpha\dot\alpha}\bar\Phi$$

$$s=2\ :\ \mathcal{J}^{min}_{\alpha\beta\dot\alpha\dot\beta} = -\frac{1}{10}\,\partial^{(2)}\Phi\ \bar\Phi - \frac{1}{10}\,\Phi\ \partial^{(2)}\bar\Phi + \frac{2}{5}\,\partial\Phi\ \partial\bar\Phi - \frac{i}{5}\,\mathrm{D}\Phi\ \partial\bar{\mathrm{D}}\bar\Phi + \frac{i}{5}\,\partial\mathrm{D}\Phi\ \bar{\mathrm{D}}\bar\Phi$$

Conformal higher spin supercurrents

Poincaré v.s. Conformal $(s+1\ ,\ s+1/2)$ supermultiplets :

- Poincaré : $H_{\alpha(s)\dot{\alpha}(s)}\ ,\ \chi_{\alpha(s)\dot{\alpha}(s-1)}$

$$\longrightarrow \quad \delta_0 H_{\alpha(s)\dot{\alpha}(s)} = \frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot{\alpha}(s)} - \frac{1}{s!}\bar{\mathrm{D}}_{(\dot{\alpha}_s}L_{\alpha(s)\dot{\alpha}(s-1))}$$
$$\delta_0 \chi_{\alpha(s)\dot{\alpha}(s-1)} = \bar{\mathrm{D}}^2 L_{\alpha(s)\dot{\alpha}(s-1)} + \mathrm{D}^{\alpha_{s+1}}\Lambda_{\alpha(s+1)\dot{\alpha}(s-1)}$$

- Conformal : $H_{\alpha(s)\dot{\alpha}(s)}$

$$\longrightarrow \quad \delta_0 H_{\alpha(s)\dot{\alpha}(s)} = \frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot{\alpha}(s)} - \frac{1}{s!}\bar{\mathrm{D}}_{(\dot{\alpha}_s}L_{\alpha(s)\dot{\alpha}(s-1))}$$

- Superfield strength : $\mathcal{W}_{\alpha(2s+1)}$

$$\longrightarrow \quad \mathcal{W}_{\alpha(2s+1)} \sim \bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_{2s+1}}\partial_{\alpha_{2s}}{}^{\dot{\alpha}_s}\dots\partial_{\alpha_(s+1)}{}^{\dot{\alpha}_1}H_{\alpha(s))\dot{\alpha}(s)}$$

Minimal supercurrent = conformal supercurrent

$$S = \int \bar{\Phi}\Phi + g\sum_{s=0}\int H^{\alpha(s)\dot{\alpha}(s)}\ \mathcal{J}_{\alpha(s)\dot{\alpha}(s)}$$

$$\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} = 0$$

conformal invariant

[Sezgin-Sundell 2002] [Kuzenko-Manvelyan-Theisen 2017]

Massive chiral supermultiplet

- Consider a mass term correction:

$$S_0 = \int d^8z \; \bar{\Phi}\Phi \; + \frac{m}{2} \int d^6z \; \Phi^2 \; + \frac{m}{2} \int d^6\bar{z} \; \bar{\Phi}^2$$

- Starting action is no longer conformal invariant
- Minimal supercurrent multiplet can not be reached : $\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} \neq 0$
- Supercurrent and supertrace acquire mass corrections

$$\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} = \mathcal{J}^{min}_{\alpha(s)\dot{\alpha}(s)} + m \ldots$$

$$\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} = \; 0 + m \ldots$$

Massive chiral sup...

- Conside...
-

$$(\Phi = \bar{D}^2 \Lambda)$$

$$\begin{aligned}\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} =& \mathcal{J}^{min}_{\alpha(s)\dot{\alpha}(s)} \\ &+m \sum_{p=0}^{s-1} \gamma_p \, \partial^{(p)} D\bar{D}\Lambda \, \partial^{(s-1-p)} \Phi + m \sum_{p=0}^{s-1} \delta_p \, \partial^{(p)} \bar{D}\Lambda \, \partial^{(s-1-p)} D\Phi \\ &-m \sum_{p=0}^{s-1} \gamma^*_p \, \partial^{(p)} \bar{D}D\bar{\Lambda} \, \partial^{(s-1-p)} \bar{\Phi} - m \sum_{p=0}^{s-1} \delta^*_p \, \partial^{(p)} D\bar{\Lambda} \, \partial^{(s-1-p)} \bar{D}\bar{\Phi}\end{aligned}$$

$$\begin{aligned}\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} =& \; 0 + m \sum_{p=0}^{s-1} \zeta_p \, \partial^{(p)} \Lambda \, \partial^{(s-1-p)} \Phi + m \sum_{p=0}^{s-1} \xi_p \, \partial^{(p)} \bar{\Lambda} \, \partial^{(s-1-p)} \bar{\Phi} \\ &+ m \sum_{p=0}^{s-2} \sigma_p \, \partial^{(p)} \bar{D}D\Lambda \, \partial^{(s-2-p)} \Phi\end{aligned}$$

Massive chiral supermultiplet

- Consider a mass term correction:

$$S_0 = \int d^8z\, \bar{\Phi}\Phi + \frac{m}{2}\int d^6z\, \Phi^2 + \frac{m}{2}\int d^6\bar{z}\, \bar{\Phi}^2$$

- Starting action is no longer conformal invariant
- Minimal supercurrent multiplet can not be reached : $\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} \neq 0$
- Supercurrent and supertrace acquire mass corrections

$$\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} = \mathcal{J}^{min}_{\alpha(s)\dot{\alpha}(s)} + m \dots$$

$$\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} = 0 + m \dots$$

- Check conservation equation: $\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} = \frac{1}{s!}\bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_s}\mathcal{T}_{\alpha(s-1))\dot{\alpha}(s-1)}$

  Answer: YES BUT ONLY FOR ODD s $[\, s = 2l+1\ ,\ l = 0, 1, 2, \dots\,]$

  consistent coupling only with $(2l+2\ ,\ 2l+3/2)$ supermultiplets

- Includes supergravity ($l = 0$) but NOT the vector supermultiplet

Nonlinear sigma model

- A more general starting point:

$$S_0 = \int d^8z\, \mathcal{K}(\Phi,\bar{\Phi}) + \int d^6z\, \mathcal{W}(\Phi) + \int d^6\bar{z}\, \bar{\mathcal{W}}(\bar{\Phi})$$

$$\mathcal{K}(\Phi,\bar{\Phi}) \sim \mathcal{K}(\Phi,\bar{\Phi}) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi})\ ,$$

$$\mathcal{W}(\Phi) \sim \mathcal{W}(\Phi) + \textit{constant}$$

- Search for a supercurrent multiplet $\{\mathcal{J}_{\alpha(s)\dot{\alpha}(s)}\ ,\ \mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)}\}$ such that

$$\bar{\mathrm{D}}^{\dot{\alpha}_s}\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} = \tfrac{1}{s!}\bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_s}\mathcal{T}_{\alpha(s-1))\dot{\alpha}(s-1)}$$

$$\bar{\mathrm{D}}^2\mathcal{J} = 0$$

$$S_I \sim \int d^8z \left\{ H^{\alpha(s)\dot{\alpha}(s)}\mathcal{J}_{\alpha(s)\dot{\alpha}(s)} + [\chi^{\alpha(s)\dot{\alpha}(s-1)}\mathrm{D}_{\alpha_s}\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} + c.c.] \right\}$$

Nonlinear sigma model: Results

- $s = 0$ (vector supermultiplet):

  Global $U(1)$ $\xrightarrow{\text{gauging}}$ vector supermultiplet

  ONLY IF $\mathcal{K} = \mathcal{K}(\Phi\bar{\Phi}),\ \mathcal{W}(\Phi) = 0$

$$\mathcal{J} = \Phi \mathcal{K}_{\Phi}$$

- $s = 1$ (supergravity supermultiplet):

  YES for any $\mathcal{K},\ \mathcal{W}$ $\quad (\Phi = \bar{\mathrm{D}}^2\Lambda,\ \ \mathcal{W} = \Phi\mathcal{F})$

$$\mathcal{J}_{\alpha\dot{\alpha}} = \mathrm{D}_{\alpha}\Phi\, \bar{\mathrm{D}}_{\dot{\alpha}}\mathcal{K}_{\Phi} - \mathrm{D}_{\alpha}\bar{\mathrm{D}}_{\dot{\alpha}}(\Lambda\mathcal{F}) + \bar{\mathrm{D}}_{\dot{\alpha}}\mathrm{D}_{\alpha}(\bar{\Lambda}\bar{\mathcal{F}})$$

$$\mathcal{T} = -\mathcal{K} + \Lambda\mathcal{F} + 2\bar{\Lambda}\bar{\mathcal{F}}$$

Expected!
Anything can be coupled to supergravity

Nonlinear sigma model: Results

- $s \geq 2$ (higher spin supermultiplets):

ONLY IF

1.) $\mathcal{K} = \bar{\Phi}\Phi$, $\mathcal{W} = 0$: Free, massless, chiral

2.) $\mathcal{K} = \bar{\Phi}\Phi$, $\mathcal{W} = f\Phi$: Free, chiral with linear superpotential

3.) $\mathcal{K} = \bar{\Phi}\Phi$, $\mathcal{W} = m\Phi^2$: Free, massive, chiral (s-odd selection rule)

- Results are consistent with expectations from:

1. Coleman-Mandula (assumes non-trivial $\mathcal{S}$-matrix)
2. Maldacena-Zhiboedov (assumes non-trivial h.s. current)

Non-minimal matter supermultiplet:

- Consider a free, massless, complex linear

$$S_0 = -\int d^8z \, \bar{\Sigma}\Sigma$$

- Repeat: Find $\delta_1 \Sigma \; \rightarrow \; \ldots$

- OR use duality:

$$S = -\int d^8z \, \bar{\sigma}\, \sigma \; + \; \int d^8z \, \Phi \, \sigma \; + \; \int d^8z \, \bar{\Phi} \, \bar{\sigma} \; + \; g_\sigma \int d^8z \sum_{s=0}^{\infty} H^{\alpha(s)\dot{\alpha}(s)} \mathcal{J}_{\alpha(s)\dot{\alpha}(s)}$$

Results:

$$\mathcal{J}^{(\Sigma)}_{\alpha(s)\dot{\alpha}(s)} = \mathcal{J}^{(\Phi)}_{\alpha(s)\dot{\alpha}(s)}\Big|_{\Phi\rightarrow\Sigma} \quad , \quad g_\Sigma = (-1)^{s+1} g_\Phi$$

1. For even spin $j = s + 1$ (supergravity, ...) same charge (attractive forces)
2. For odd spin $j = s + 1$ (vector, ...) opposite charge (repulsive forces)

What about integer superspins $(s+1/2\ ,\ s)$?

Thus far: $\delta_1\Phi$ linear in Φ gives interactions with $(s+1\ ,\ s+1/2)$

like superdiffeo and supergravity

What about integer superspin supermultiplets $(s+1/2\ ,\ s)$?

- Poincaré : $\Psi_{\alpha(s)\dot{\alpha}(s-1)}\ ,\ V_{\alpha(s-1)\dot{\alpha}(s-1)}$

$$\delta_0\Psi_{\alpha(s)\dot{\alpha}(s-1)} = -\boxed{\mathrm{D}^2 L_{\alpha(s)\dot{\alpha}(s-1)}} + \frac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}}\Lambda_{\alpha(s)\dot{\alpha}(s-2))}$$

$$\delta_0 V_{\alpha(s-1)\dot{\alpha}(s-1)} = \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot{\alpha}(s-1)} + \bar{\mathrm{D}}^{\dot{\alpha}_s}\bar{L}_{\alpha(s-1)\dot{\alpha}(s)}$$

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- Conformal : $\Psi_{\alpha(s)\dot{\alpha}(s-1)}$

$$\delta_0\Psi_{\alpha(s)\dot{\alpha}(s-1)} = \frac{1}{s!}\boxed{\mathrm{D}_{(\alpha_s}\Xi_{\alpha(s-1))\dot{\alpha}(s-1)}} + \frac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot{\alpha}_{s-1}}\Lambda_{\alpha(s)\dot{\alpha}(s-2))}$$

What about integer superspins $(s + 1/2\ ,\ s)$?

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- Superfield strength : $\mathcal{W}_{\alpha(2s)}$

$$\mathcal{W}_{\alpha(2s)} \sim \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_{2s}}\partial_{\alpha_{2s-1}}{}^{\dot{\alpha}_s}\dots\partial_{\alpha_(s+1)}{}^{\dot{\alpha}_1}\Psi_{\alpha(s))\dot{\alpha}(s-1)}$$

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$$\delta_0V_{\alpha(s-1)\dot\alpha(s-1)} = \mathrm{D}^{\alpha_s}L_{\alpha(s)\dot\alpha(s-1)} + \bar{\mathrm{D}}^{\dot\alpha_s}\bar{L}_{\alpha(s-1)\dot\alpha(s)}$$

- Conformal : $\Psi_{\alpha(s)\dot\alpha(s-1)}$

$$\delta_0\Psi_{\alpha(s)\dot\alpha(s-1)} = \frac{1}{s!}\boxed{\mathrm{D}_{(\alpha_s}\Xi_{\alpha(s-1))\dot\alpha(s-1)}} + \frac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot\alpha_{s-1}}\Lambda_{\alpha(s)\dot\alpha(s-2))}$$

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$$\mathcal{W}_{\alpha(2s)} \sim \bar{\mathrm{D}}^2\mathrm{D}_{(\alpha_{2s}}\partial_{\alpha_{2s-1}}{}^{\dot\alpha_s}\ldots\partial_{\alpha_(s+1)}{}^{\dot\alpha_1}\Psi_{\alpha(s))\dot\alpha(s-1)}$$

- Conservation equations:

Poincaré: $\mathrm{D}^2\mathcal{J}_{\alpha(s)\dot\alpha(s-1)} = \frac{1}{s!}\,\mathrm{D}_{(\alpha_s}\mathcal{T}_{\alpha(s-1))\dot\alpha(s-1)}\ ,\ \bar{\mathrm{D}}^{\dot\alpha_{s-1}}\mathcal{J}_{\alpha(s)\dot\alpha(s-1)} = 0$

Conformal: $\mathrm{D}^{\alpha_s}\mathcal{J}_{\alpha(s)\dot\alpha(s-1)} = 0\ ,\ \bar{\mathrm{D}}^{\alpha_{s-1}}\mathcal{J}_{\alpha(s)\dot\alpha(s-1)} = 0$

Integer superspin supercurrents

Consider $\delta_1\Phi$ linear in $\bar{\Phi}$

$$\delta_1\Phi = \sum_{k=0}^{\infty}\Big\{ A^{\alpha(k)\dot{\alpha}(k+1)}\,\bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot{\alpha}_1}\bar{\Phi} + \Delta^{\alpha(k)\dot{\alpha}(k)}\,\mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot{\alpha}_1}\bar{\Phi} + \Gamma^{\alpha(k+1)\alpha(k)}\,\mathrm{D}_{\alpha_{k+1}}\bar{\mathrm{D}}^2\mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot{\alpha}_1}\bar{\Phi} + E^{\alpha(k)\dot{\alpha}(k)}\,\bar{\mathrm{D}}^2\mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot{\alpha}_1}\bar{\Phi}\Big\}$$

Chirality + Non-triviality :

integer conformal superspin

1. $$\delta_1\Phi = \sum_{k=0}^{\infty}\Big\{\tfrac{1}{(k+1)!}\,\bar{\mathrm{D}}^{(\dot{\alpha}_{k+1}}\bar{\xi}^{\alpha(k)\dot{\alpha}(k))}\,\bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot{\alpha}_1}\bar{\Phi} + \bar{\mathrm{D}}^2\bar{\xi}^{\alpha(k)\dot{\alpha}(k)}\,\mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot{\alpha}_1}\bar{\Phi}\Big\}$$

2. $$\delta_1\Phi = \sum_{k=0}^{\infty}\Big\{\bar{\mathrm{D}}^2\bar{\ell}^{\alpha(k)\dot{\alpha}(k+1)}\,\bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot{\alpha}_1}\bar{\Phi}\Big\}$$

integer Poincaré superspin

Matter - Integer Superspin cubic interactions

Starting point: Single, free, massless chiral $S_0 = \int d^8z\, \bar{\Phi}\Phi$

Cubic interactions: $S_I \sim \int d^8z\, \Psi^{\alpha(s)\dot{\alpha}(s-1)}\, \mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} + c.c.$

- Conformal: Only for $\boxed{s = 2l+2}$ [For Ads: Buchbinder-Hutomo-Kuzenko -2018]

$$\mathcal{J}_{\alpha(2l+2)\dot{\alpha}(2l+1)} = \frac{i(-1)^l}{2l+3} \sum_{p=0}^{2l+1} (-1)^p \binom{2l+1}{p} \binom{2l+2}{p+1} \partial^{(p)}\mathrm{D}\Phi\, \partial^{(2l+1-p)}\Phi$$

- Poincaré: For all $\boxed{s \geq 1}$ $\mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} = (-i)^{(s-1)}\Phi\, \partial^{(s-1)}\mathrm{D}\Phi$

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- Poincaré: For all $\boxed{s \geq 1}$ $\mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} = (-i)^{(s-1)}\Phi\, \partial^{(s-1)}\mathrm{D}\Phi$

Starting point: $S_0 = \int d^8z\, \bar{\Phi}\Phi + f \int d^6z\, \Phi + f^* \int d^6\bar{z}\, \bar{\Phi}$

For all $\boxed{s \geq 1}$

$$\mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} = (-i)^{(s-1)}\Phi\, \partial^{(s-1)}\mathrm{D}\Phi \ ,$$
$$\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} = (-i)^{(s-1)}\, f^*\, \partial^{(s-1)}\Phi + (i)^{(s-1)}\, f\, \partial^{(s-1)}\bar{\Phi}$$

Matter - Integer Superspin cubic interactions

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Cubic interactions: $S_I \sim \int d^8z \, \Psi^{\alpha(s)\dot{\alpha}(s-1)} \, \mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} \, + \, c.c.$

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$$\mathcal{J}_{\alpha(2l+2)\dot{\alpha}(2l+1)} = \frac{i(-1)^l}{2l+3} \sum_{p=0}^{2l+1} (-1)^p \binom{2l+1}{p} \binom{2l+2}{p+1} \partial^{(p)} \mathrm{D}\Phi \, \partial^{(2l+1-p)} \Phi$$

- Poincaré: For all $\boxed{s \geq 1}$ $\mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} = (-i)^{(s-1)} \Phi \, \partial^{(s-1)} \mathrm{D}\Phi$

Starting point: $S_0 = \int d^8z \, \bar{\Phi}\Phi \, + \, f \, \int d^6z \, \Phi \, + \, f^* \, \int d^6\bar{z} \, \bar{\Phi}$

For all $\boxed{s \geq 1}$

$$\mathcal{J}_{\alpha(s)\dot{\alpha}(s-1)} = (-i)^{(s-1)} \Phi \, \partial^{(s-1)} \mathrm{D}\Phi \ ,$$

$$\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)} = (-i)^{(s-1)} \, f^* \, \partial^{(s-1)} \Phi \, + \, (i)^{(s-1)} \, f \, \partial^{(s-1)} \bar{\Phi}$$

Starting point: $S_0 = \int d^8z \, \bar{\Phi}\Phi \, + \, m \, \int d^6z \Phi^2 \, + \, m \, \int d^6\bar{z} \bar{\Phi}^2$

Only for $\boxed{s = 2l+2}$

$$\mathcal{J}_{\alpha(2l+2)\dot{\alpha}(2l+1)} = (-1)^{(l+1)} \, i \, \Phi \, \partial^{(2l+1)} \mathrm{D}\Phi$$

$$\mathcal{T}_{\alpha(2l+1)\dot{\alpha}(2l+1)} = (-1)^{(l+1)} \, i \, m \, \bar{\Phi} \, \partial^{(2l+1)} \Phi \, - \, (-1)^{(l+1)} \, i \, m \, \Phi \, \partial^{(2l+1)} \bar{\Phi}$$

Part II

Supersymmetric Higher Spin Currents

of higher spin multiplets

Beyond matter theories

- Consider cubic interactions of higher spin supermultiplets
- Focus on non-minimal coupling (higher derivative lagrangians)
- Special type of vertices : hj where j is written in terms of field strengths
- Distinctive feature : Uniqueness (up to trivial field redefinitions)
  [Berends, Burgers, van Dame 1986] [Gelfond, Skvortsov, Vasiliev 2008]
- Superfield strengths of higher spin supermultiplets:

$$Y = s + 1/2: \quad \mathcal{W}_{\alpha(2s+1)} \sim \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_{2s+1}} \partial_{\alpha_{2s}}{}^{\dot{\alpha}_s} \partial_{\alpha_{2s-1}}{}^{\dot{\alpha}_{s-1}} \ldots \partial_{\alpha_{s+1}}{}^{\dot{\alpha}_1} H_{\alpha(s))\dot{\alpha}(s)}$$

$$Y = s: \quad \mathcal{W}_{\alpha(2s)} \sim \bar{\mathrm{D}}^2 \mathrm{D}_{(\alpha_{2s}} \partial_{\alpha_{2s-1}}{}^{\dot{\alpha}_{s-1}} \partial_{\alpha_{2s-2}}{}^{\dot{\alpha}_{s-2}} \ldots \partial_{\alpha_{s+1}}{}^{\dot{\alpha}_1} \Psi_{\alpha(s))\dot{\alpha}(s-1)}$$

$$\bar{\mathrm{D}}_{\dot{\beta}} \mathcal{W}_{\alpha(k)} = 0 \ \text{(chiral)} \quad , \quad \mathrm{D}^{\alpha_k} \mathcal{W}_{\alpha(k)} = 0 \ \text{(e.o.m)}$$

$Y_1 - Y_2 - Y_2$

- Consider the cubic vertex $Y_1 - Y_2 - Y_2$ where

  $\underline{Y_1 = s_1 + 1/2}$ and $\underline{Y_2 \text{ is arbitrary}}$

- Interactions generated by a supercurrent $\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$

$$S_I = g \int d^8z \; H^{\alpha(s_1)\dot{\alpha}(s_1)} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)}$$

$$\bar{\mathrm{D}}^{\dot{\alpha}_{s_1}} \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = 0 \quad , \quad \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = \bar{\mathcal{J}}_{\alpha(s_1)\dot{\alpha}(s_1)}$$

- General ansatz:

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = \sum_{p=0}^{s_1-2Y_2} \alpha_p \, \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)} \, \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}$$
$$+ \sum_{p=0}^{s_1-2Y_2-1} \beta_p \, \partial^{(p)} \mathrm{D}\mathcal{W}_{\alpha(2Y_2)} \, \partial^{(s_1-2Y_2-1-p)} \bar{\mathrm{D}} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}$$

$Y_1 - Y_2 - Y_2$

Reality + conservation = Unique answer

$$\mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = c(i)^{s_1-2Y_2} \sum_{p=0}^{s_1-2Y_2} (-1)^p \frac{\binom{s_1-2Y_2}{p}\binom{s}{p}}{\binom{2Y_2+p}{2Y_2}} \Big\{ \partial^{(p)} \mathcal{W}_{\alpha(2Y_2)}\, \partial^{(s_1-2Y_2-p)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}$$

$$+ i(-1)^{2Y_2} \frac{s_1-2Y_2-p}{2Y_2+1+p}\, \partial^{(p)} \mathrm{D}\mathcal{W}_{\alpha(2Y_2)}\, \partial^{(s_1-2Y_2-1-p)} \bar{\mathrm{D}} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)} \Big\}$$

Observations :

- Constrained superspin values: $s_1 \geq 2Y_2$
  Supersymmetric extention of Weinberg-Witten theorem
- Special case (I): $s_1 = 2Y_2, \quad \mathcal{J}_{\alpha(s_1)\dot{\alpha}(s_1)} = c\, \mathcal{W}_{\alpha(2Y_2)} \bar{\mathcal{W}}_{\dot{\alpha}(2Y_2)}$
  Supersymmetric and higher spin extention of the Bel-Robinson tensor
  [Howe, Stelle, Townsend 1981]
- Special case (II): $Y_2 = 0$ (matter theory)
  Recover matter higher spin supercurrents

THANK YOU!