Conformal Higher Spin Gravity

a review with a few news

Euihun Joung

Kyung Hee Univ (Korea)

with Thomas Basile & Xavier Bekaert [1808.07728]

Conformal Higher Spin (CHS) Gravity:

A. Higher spin analog of Conformal Gravity

B. Theory of interacting conformal higher spin fields

1. What is CHS Gravity, more precisely?

2. Why CHS Gravity is interesting for me (and for you)?

3. Some news on CHS Gravity

What is CHS Gravity?

A. Higher spin analog of Conformal Gravity

- Conformal Gravity in even d dim

(1) Made by Weyl tensor: ex. (Weyl tensor)^2 in d=4

(2) Gauge Symmetry: Diffeo + Weyl

(3) Global Symmetry: Conformal Group SO(2,d)

(4) (Holographic) Weyl Anomaly (even d)

A. Higher spin analog of Conformal Gravity

Fradkin, Linetsky, Tseytlin, …

(1) Spin s Weyl tensor:

- s derivative of rank s field
- traceless (s,s) Young diagram $\begin{array}{|c|}\hline \quad\quad\quad s \quad\quad\quad \\ \hline \quad\quad\quad s \quad\quad\quad \\ \hline\end{array}$
- Form a multiplet under HS gauge symmetry
- HS gauge symmetry?

A. Higher spin analog of Conformal Gravity

(2) Gauge Symmetry: HS gauge (diffeo) + HS Weyl

- HS gauge symmetry
  - Gauge symmetry of HS Gravity?
  - Spin s symmetry generated by rank s-1 tensor
- HS Weyl symmetry
  - Spin s symmetry generated by rank s-2 tensor

A. Higher spin analog of Conformal Gravity

(2) Gauge Symmetry: HS gauge (diffeo) + HS Weyl

- Linearization

$$h_s \sim h_s + \partial\, \xi_{s-1} + \eta\, \sigma_{s-2}$$

A. Higher spin analog of Conformal Gravity

(3) Global Symmetry

- HS conformal Killing $\partial\,\xi_{s-1} + \eta\,\sigma_{s-2} = 0$
- HS analog of conformal symmetry algebra so(2,d)

➡ HSA(2,d)

❖ HSA(2,d-1): HS analog of isometry algebra so(2,d-1)

Fradkin, Vasiliev, …

A. Higher spin analog of Conformal Gravity

(4) (Holographic) Weyl Anomaly

- From Bulk Segal; Bekaert, EJ, Mourad, …
  - HS Gravity in D=d+1 dimensional bulk (AdS)
  - Anomaly of radial direction diffeomorphism
  - Match of Global Symmetries

✤ Anomaly of radial direction HS gauge symmetry?

A. Higher spin analog of Conformal Gravity

(4) (Holographic) Weyl Anomaly

- From Boundary

  Segal; Bekaert, EJ, Mourad; Ponomarev

  Bonora, Cvitan, Dominis Prester, Giaccari, Lima de Souza, Stemberga

  - d-dim CFT dual of D-dim HS Gravity
    - For type A, B, C: it’s free scalar, spinor, vector!
  - Anomaly of Weyl symmetry

Why CHS Gravity is interesting?

- Action principle as Weyl anomaly of free CFT

- Metric formulation

  (and also unfolded and twistor formulation)

  Vasiliev, Shaynkman   Adamo, Hahnel, McLoughlin

- All interaction vertices are local

B. Theory of interacting conformal higher spin fields

- Free conformal spin s field (Fradkin-Tseytlin field)

$$h_s \sim h_s + \partial\,\xi_{s-1} + \eta\,\sigma_{s-2}$$

- Conformal Killing tensor

$$\partial\,\xi_{s-1} + \eta\,\sigma_{s-2} = 0$$

- Equation invariant under linearized gauge symmetry

$$\mathbb{P}^{s}_{\mathrm{TT}}\,\Box^{s+\frac{d-4}{2}}\,h_s \approx 0$$

Fradkin, Tseytlin, Vasiliev, Metsaev, …

B. Theory of interacting conformal higher spin fields

- Free conformal spin s field (Fradkin-Tseytlin field)

$$\chi_{\mathcal{S}(2-s;(s))} = \chi_{\mathcal{V}(2-s;(s))} - \chi_{\mathcal{V}(1-s;(s-1))} + \chi_{\mathcal{D}(1-s;(s-1))}$$

- Conformal Killing tensor

$$\chi_{\mathcal{D}(1-s;(s-1))}$$

- Equation invariant under linearized gauge symmetry

$$\chi_{\mathcal{D}(s+d-2;(s))} = \chi_{\mathcal{V}(s+d-2;(s))} - \chi_{\mathcal{V}(s+d-1;(s-1))}$$

Shaynkman, Tipunin, Vasiliev; Bekaert, Beccaria, Tseytlin

B. Theory of interacting conformal higher spin fields

- On-shell free conformal spin s field

$$\chi_{\mathcal{D}(2;(s,s))} = \chi_{\mathcal{S}(2-s;(s))} - \chi_{\mathcal{D}(s+d-2;(s))}$$

$$= \chi_{\mathcal{D}(1-s;(s-1))} + \chi_{\mathcal{V}(2-s;(s))} - \chi_{\mathcal{V}(1-s;(s-1))}$$

$$- \chi_{\mathcal{V}(s+d-2;(s))} + \chi_{\mathcal{V}(s+d-1;(s-1))}$$

B. Theory of interacting conformal higher spin fields

- On-shell free conformal spin s field

$$\chi_{\mathcal{D}(2;(s,s))} = \chi_{\mathcal{D}(1-s;(s-1))} + \chi_{\mathcal{V}(2-s;(s))} - \chi_{\mathcal{V}(1-s;(s-1))}$$
$$\boxed{-\chi_{\mathcal{V}(s+d-2;(s))} + \chi_{\mathcal{V}(s+d-1;(s-1))}}$$

$$\chi_{\mathcal{D}(s+d-2;(s))} = \chi_{\mathcal{V}(s+d-2;(s))} - \chi_{\mathcal{V}(s+d-1;(s-1))}$$

B. Theory of interacting conformal higher spin fields

- On-shell free conformal spin s field

$$\chi_{\mathcal{D}(2;(s,s))} = \chi_{\mathcal{D}(1-s;(s-1))} \boxed{+\chi_{\mathcal{V}(2-s;(s))} - \chi_{\mathcal{V}(1-s;(s-1))}}$$

$$-\chi_{\mathcal{V}(s+d-2;(s))} + \chi_{\mathcal{V}(s+d-1;(s-1))}$$

$$\chi_{\mathcal{V}(\Delta,\mathbb{Y})}(q^{-1},\boldsymbol{x}) = (-1)^d \chi_{\mathcal{V}(d-\Delta,\mathbb{Y})}(q,\boldsymbol{x})$$

$$\chi_{\mathcal{V}(2-s;(s))}(q,\boldsymbol{x}) - \chi_{\mathcal{V}(1-s;(s-1))}(q,\boldsymbol{x}) = (-)^d \chi_{\mathcal{D}(s+d-2;(s))}(q^{-1},\boldsymbol{x})$$

B. Theory of interacting conformal higher spin fields

- On-shell free conformal spin s field

In even dimension d

$$\chi_{\mathcal{D}(2;(s,s))}(q,\boldsymbol{x}) = \chi_{\mathcal{D}(1-s;(s-1))}(q,\boldsymbol{x}) + \chi_{\mathcal{D}(s+d-2;(s))}(q^{-1},\boldsymbol{x})$$
$$- \chi_{\mathcal{D}(s+d-2;(s))}(q,\boldsymbol{x})$$

Bekaert, Beccaria, Tseytlin

✤ PF of CHS = PF of HS $_{\text{Neumann}}$ - PF of HS $_{\text{Dirichelt}}$

Giombi, Klebanov, Pufu, Safidi, Tarnopolsky

Partition Function of CHS Gravity

$$\sum_{s=0}^{\infty} \chi_{\mathcal{D}(2;(s,s))}(q,\boldsymbol{x}) = \sum_{s=1}^{\infty} \chi_{\mathcal{D}(1-s;(s-1))}(q,\boldsymbol{x}) + \\ + \boxed{\sum_{s=0}^{\infty} \chi_{\mathcal{D}(s+d-2;(s))}(q^{-1},\boldsymbol{x})} - \boxed{\sum_{s=0}^{\infty} \chi_{\mathcal{D}(s+d-2;(s))}(q,\boldsymbol{x})}$$

- Flato-Fronsdal

$$\Big(\chi_{\mathrm{Rac}}(q,\boldsymbol{x})\Big)^2 = \sum_{s=0}^{\infty} \chi_{\mathcal{D}(s+d-2;(s))}(q,\boldsymbol{x})$$

$$\chi_{\mathrm{Rac}}(q^{-1},\boldsymbol{x}) = (-1)^{d+1}\,\chi_{\mathrm{Rac}}(q,\boldsymbol{x})$$

Partition Function of CHS Gravity

$$\sum_{s=0}^{\infty} \chi_{\mathcal{D}(2;(s,s))} = \sum_{s=1}^{\infty} \chi_{\mathcal{D}(1-s;(s-1))}$$

- Partition Function of Linearized Fields (1-Loop PF)

||

- Partition Function of Symmetry Generators

Partition Function of CHS Gravity

$$\sum_{s=0}^{\infty} \chi_{\mathcal{D}(2;(s,s))} = \sum_{s=1}^{\infty} \chi_{\mathcal{D}(1-s;(s-1))}$$

- Remarks
  - Divergent series
  - Converge only as a distribution
  - PF: chemical potential as a natural regulator

Why CHS Gravity is interesting?

- Very special scattering amplitudes
  - Zero scattering of external conformal scalars

    Joung, Nakach, Tseytlin
  - Zero scattering of conformal spin 1 and 2

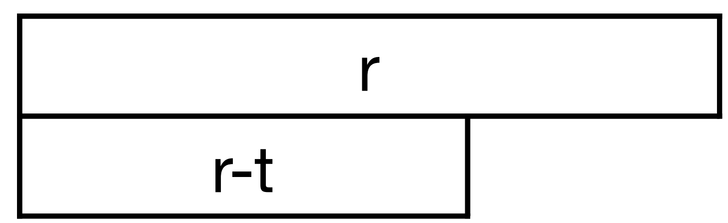
    Beccaria, Nakach, Tseytlin
- Very special partition function
  - Zero Casimir energy and a-anomaly

    Giombi, Klebanov, Pufu, Safidi, Tarnopolsky, Tseytlin
  - Linearized spectrum = Symmetry Algebra

Higher Order Extension

Type-Aℓ HS Gravity

Bekaert, Grigoriev; Brust, Hinterbichler

- AdS dual of $\bar{\phi}\,\Box^{\ell}\,\phi$
- PM fields of spin s and depth t=1,3,…,2ℓ-1
- Type-Aℓ HSA: generated by 

    - Any d HSA with sp2 projector to 2ℓ-1 dim rep
- Any d Vasiliev equation with sp2 projector to 2ℓ-1 dim rep

Type-Aℓ HS Algebra

Alkalaev, Grigoriev; EJ, Mkrtchyan

- Howe duality
  - SO(2,d): $M_{ab} = y_{\alpha a}\, y^{\alpha}{}_{b}$
  - Sp(2): $K_{\alpha\beta} = y_{\alpha} \cdot y_{\beta}$
- Type-Aλ
  - Quotient: $\frac{1}{2}\, K_{\alpha\beta} \star K^{\alpha\beta} \sim (1-\lambda)(1+\lambda)$
  - Type-A$_{1/2}$ HSA(2,d): Subalgebra of Type-A HSA(2,d+1)

Type-Aℓ HS Algebra

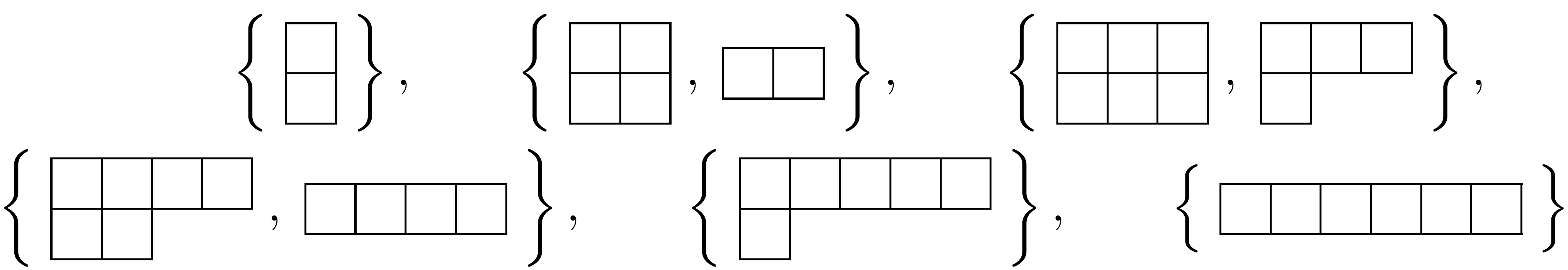
- Type-Aℓ
  - Type-Aλ has an ideal when λ=ℓ
  - Quotient: $K_{(\alpha_1\alpha_2} \star K_{\alpha_3\alpha_4} \star \cdots \star K_{\alpha_{2\ell-1}\alpha_{2\ell})} \sim 0$
  - Projector:

$$D_\lambda = N_\lambda \int_0^1 dx\, x^{\frac{1}{2}}\,(1-x)^{\frac{d-4}{2}}\,{}_2F_1\left(1+\lambda\,,\,1-\lambda\,;\,\frac{3}{2}\,;\,\frac{1}{1-x}\right) e^{-2\sqrt{x}\,y_+\cdot y_-}$$

$$N_\lambda = \frac{(-1)^{\lambda-1}\,\Gamma(d+1)}{2^{d-1}\,\boxed{\Gamma(\frac{d}{2}-\lambda)}\,\Gamma(\frac{d}{2}+\lambda)}$$

Type-Aℓ HS Algebra

- Type-Aℓ with $\ell \geq d/2$
  - has an ideal { (r,2n) | r>n+ℓ-d/2 }
  - Finite dim algebra as coset (ex. ℓ-d/2=3)



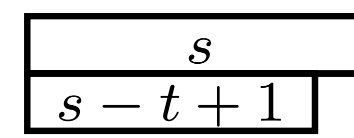
  - Endomorphism of $\boxed{\quad \ell \text{ - } d/2 \quad}$

Type-Aℓ CHS Gravity

- Conformal fields of spin s and depth t=1, 3, … , 2ℓ-1

- Gauge symmetry: $\delta_{\xi,\sigma} h_s^{(t)} = \partial^t \, \xi_{s-t} + \eta \, \sigma_{s-2}$

- Weyl tensor: $C^{(t)}_{s,s-t+1} = \mathbb{P}_{\mathrm{T}}^{s,s-t+1} \, \partial^{s-t+1} \, h_s^{(t)} \underset{\mathfrak{so}(1,d-1)}{\sim}$



- Action:

$$S_{\mathrm{FT}^{(t)}}[h_s^{(t)}] = (-1)^{s-t+1} \int_{M_d} \mathrm{d}^d x \; C^{(t)}_{s,s-t+1} \, \Box^{\frac{d-4}{2}} \, C^{(t)}_{s,s-t+1}$$

$$= \int_{M_d} \mathrm{d}^d x \; h_s^{(t)} \, \mathbb{P}^s_{\mathrm{T}^t\mathrm{T}} \, \Box^{s-t+\frac{d-2}{2}} h_s^{(t)}$$

- Special conformal fields: 1+s ≤ t ≤ s+(d-2)/2

Metsaev

Type-Aℓ CHS Gravity

- Partially conserved currents (k=1,…,ℓ) Brust, Hinterbichler

$$J_s^{(2k-1)} = \bar{\phi}\,\partial^s\,\Box^{\ell-k}\,\phi + \cdots$$

  - Not (partially-)conserved for t=2k-1≥s+1
  - For ℓ ≤d/4
    - Basis for single trace operator with Δ=s+d-2k

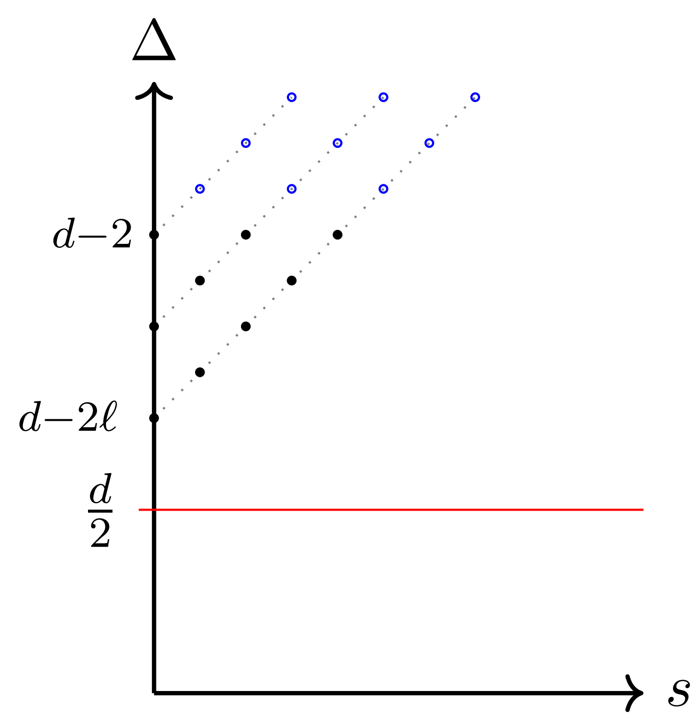
$$\{J_s^{(2k-1)}, \Box\, J_s^{(2k+1)}, \ldots, \Box^{\ell}\, J_s^{(2k+2\ell-1)}\}$$

Type-Aℓ CHS Gravity

- For ℓ >d/4, "Extension" Brust, Hinterbichler
  - Degeneracy (both primary and descendent)

$$J_s^{(2k-1)} \propto \Box^{k'-k}\, J_s^{(2k'-1)} \qquad k+k' = s+\frac{d}{2}\,, \qquad k \leqslant k'$$

  - New generator (neither primary nor descendent) $\tilde{J}_s^{(2k-1)}$



Type-Aℓ CHS Gravity

- For ℓ >d/4, "Extension"
  - Two point fn of current operators

$$\langle J_s^{(2k'-1)}(x)\,\tilde{J}_s^{(2k-1)}(0)\rangle \propto \frac{\eta^s}{|x|^{d+\epsilon}}+\cdots \xrightarrow[\epsilon\sim 0]{} \frac{\rho_{\frac{d}{2}}}{\epsilon}\,\eta^s\,\delta^{(d)}(x)$$

  - Quadratic Lagrangian of CHS Gravity

$$\tilde{h}_s^{(2k-1)}\left(\Box^{s+\frac{d}{2}-2k}+\cdots\right)\tilde{h}_s^{(2k-1)}+\tilde{h}_s^{(2k-1)}\,h_s^{(2k'-1)}$$

    - Both fields are absent in the on-shell spectrum

Type-Aℓ CHS Gravity

- PF of linearized fields = PF of symmetry generators
  - For ℓ <d/4

$$\sum_{t=1,3,\ldots}^{2\ell-1}\left[\sum_{s=0}^{t-1}\chi_{\mathcal{D}(1+t-s;(s))}+\sum_{s=t}^{\infty}\chi_{\mathcal{D}(2;(s,s-t+1))}\right]=\sum_{t=1,3,\ldots}^{2\ell-1}\sum_{s=t}^{\infty}\chi_{\mathcal{D}(1-s;(s-t))}$$

  - For d/4≤ ℓ <d/2

$$\sum_{t=1,3,\ldots}^{2\ell-1}\left[\sum_{s=\max\{0,\frac{t-d+3}{2}+\ell\}}^{t-1}\chi_{\mathcal{D}(1+t-s;(s))}+\sum_{s=t}^{\infty}\chi_{\mathcal{D}(2;(s,s-t+1))}\right]=\sum_{t=1,3,\ldots}^{2\ell-1}\sum_{s=t}^{\infty}\chi_{\mathcal{D}(1-s;(s-t))}$$

Type-Aℓ CHS Gravity

- A few more remarks
  - $\ell \geq d/2$
  - Type-Bℓ
  - Double trace deformation

$$\int_{M_d} \mathrm{d}^d x \left(\bar{\phi}^a \, \Box^\ell \, \phi_a + \sum_{s,k'} g_{s,k} \, J_s^{(2k'-1)} \, J_s^{(2k'-1)} \right)$$

Branching Rule

Branching Rule

- Decomposition of CHS field into PM fields
  - Around AdS background Metsaev; Nutma, Taronna; EJ, Mkrtchyan
  - Bach flat background Kuzenko, Ponds
- Higher depth CHS fields Grigoriev, Hancharuk

$$\mathcal{D}\big(2;(s,s-t+1)\big) \quad \underset{\mathfrak{so}(2,d-1)}{\overset{\mathfrak{so}(2,d)}{\downarrow}} \quad \bigoplus_{\sigma=s-t+1}^{s} \; \bigoplus_{\tau=\sigma-s+t}^{\sigma+\frac{d-4}{2}} \mathcal{S}\big(1+\tau-\sigma;(\sigma)\big) \oplus \mathcal{D}\big(\sigma+d-\tau-2;(\sigma)\big)$$

Why CHS Gravity is interesting?

Why CHSG is interesting?

- Relation to HS Gravity
  - Conformal spin s field around AdS: partially massless spin s and depth 1, 2, … ,s+(d-4)/2
  - Reduction to d-dim (not (d+1)-dim) HS Gravity?
    - Maybe, HSG with Type-$A_{1/2}$ HSA(2,d)

Why CHS Gravity is interesting?

- Action principle as Weyl anomaly of free CFT
- Metric formulation (and also unfolded and twistor formulation)
- All interaction vertices are local

Why CHS Gravity is interesting?

- Very special scattering amplitudes
  - Zero scattering of external conformal scalars
  - Zero scattering of conformal spin 1 and 2
- Very special partition function
  - Zero Casimir energy and a-anomaly
  - Linearized spectrum = Symmetry Algebra