SOLVING THE VASILIEV EQUATIONS:
PERTURBATIVE SCHEMES,
EXACT SOLUTIONS,
LOCAL VS. GLOBAL ASPECTS

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Motivations

- Constructed a wide exact solution space that contains many potentially interesting solutions (e.g., 4D HS black-hole-like solutions) and an effective scheme for superposing fluctuations.

- However, their interpretation requires a better understanding of many problematic issues related with the huge gauge symmetry of the nonlinear HS theory, leading to a *global* formulation:

  - enlarging the set of relevant classical observables and assess their physical meaning: proper HS/stringy generalization of geometry?
  - understanding how to impose boundary conditions on master fields on NC space ;
  - distinguishing small/large gauge transformations.

- Part of the difficulties arise because the tools that are most convenient in solving full eqs. (gauge functions that locally trivialize spacetime, classical moduli encoded in twistor space elements, ...) blur the immediate identifications of Fronsdal fields carrying the dof.

Motivations

- The above problems are all tightly connected to formal, foundational aspects of the non-linear theory:

→ What are criteria that select admissible class of functions/gauge transformations/field redefinitions?

This is at the core of the problem of comparing the perturbative expansion of the Vasiliev equations and the "non-linear Fronsdal" approach.

→ As the theory is formulated with functions of NC variables, changes of orderings are a delicate issue. To what extent can one consider changes of ordering admissible? Is there a preferred basis?

Recent progress achieved by building a map bridging the most convenient "gauge" for solution-building (in which black-hole-like solutions have been found) to the "gauge" in which the extraction of Fronsdal fields makes sense.

Motivations

- Assessing the status of HS gravity wrt GR and stringy completion:

  Find and study the analogues of problematic solutions of GR, such as black holes and cosmologies, and see if the coupling with HS fields solves singularities already at the classical level.

- The embedding of spin 2 inside an infinite-dimensional multiplet, due to HS symmetry, leads to a HS-covariant formulation where all spin-s fields are packed in generating functions (master fields) valued in an infinite-dimensional algebra.

- The resulting pairing between spacetime-coord. and fibre-coord. dependence maps the curvature singularities of an infinite multiplet of spacetime fields into an irregular behaviour of the master fields wrt NC fibre coordinate → a more treatable problem.

Kinematics

- Master-fields living on *correspondence space*, locally $\mathcal{X}$ x $\mathcal{Z}$ x $\mathcal{Y}$:

$$U = dx^\mu U_\mu(Y,Z|x)$$ → gauge fields of all spins + auxiliary

$$\Phi = \Phi(Y,Z|x)$$ → Weyl tensors and their derivatives → local dof

$$S = dz^\alpha S_\alpha(Y,Z|x) + d\bar{z}^{\dot\alpha}\bar{S}_{\dot\alpha}(Y,Z|x)$$ → Z-space connection, no extra local dof

- Commuting oscillators $Y_{\underline{\alpha}} = (y_\alpha, \bar{y}_{\dot\alpha})\ ,\quad Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot\alpha})$ → $\mathfrak{sp}$(4,R) quartets

$$[Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha\beta}} = 2i\begin{pmatrix} \varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot\alpha\dot\beta}\end{pmatrix}\ ,\qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha\beta}}\ ,\quad [Y_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = 0$$

- Star-product:

$$F(Y,Z)\star G(Y,Z) = \int_{\mathcal{R}} \frac{d^4U d^4V}{(2\pi)^4}\, e^{iV^{\underline{\alpha}}U_{\underline{\alpha}}} F(Y+U, Z+U)\, G(Y+V, Z-V)$$

- Inner kleinian operator κ:

$$\kappa = e^{iy^\alpha z_\alpha}\ ,\qquad \kappa\star f(z,y) = f(-z,-y)\star\kappa\ ,\qquad \kappa\star\kappa = 1$$

$$\kappa = \kappa_y\star\kappa_z\ ,\quad \kappa_y\star f(z,y) = f(z,-y)\star\kappa_y\ ,\quad \kappa_y\star\kappa_y = 1\ ,$$

$$\kappa_y = 2\pi\delta^2(y) = 2\pi\delta(y_1)\delta(y_2)$$

4D bosonic Vasiliev equations

- Full equations:

$$\begin{aligned} dU + U \star U &= 0 \\ d\Phi + U \star \Phi - \Phi \star \pi(U) &= 0 \\ dS_\alpha + [U, S_\alpha]_\star &= 0 \\ S_\alpha \star \Phi + \Phi \star \pi(S_\alpha) &= 0 \\ [S_\alpha, S_\beta]_\star &= -2i\epsilon_{\alpha\beta}(1 - b\Phi \star \kappa) \\ [S_\alpha, \bar{S}_{\dot{\beta}}]_\star &= 0 \ , \end{aligned}$$

(Vasiliev, '92)

$$S_\alpha = z_\alpha - 2iV_\alpha$$

$$[S_\alpha, f(Z,Y)] = [z_\alpha, f] + ... = -2i \frac{\partial}{\partial z^\alpha} f + ...$$

- *Z*-oscillators → auxiliary, non-commutative coordinates. Equations fix the evolution along *Z* in such a way that it gives rise to consistent interactions to all orders among physical fields, contained in the (*Z*-independent) initial conditions

$$U_{|Z=0} \ , \qquad \Phi_{|Z=0} \ .$$

- 1st order differential eqs impose a relation between spacetime and twistor space behaviour of their solutions → the physical information can be encoded to a great extent in the twistor-space dependence.

AdS vacuum solution

$$\Phi = \Phi^{(0)} = 0 \,,$$
$$S_\alpha = S_\alpha^{(0)} = z_\alpha \,, \qquad S_{\dot\alpha} = S_{\dot\alpha}^{(0)} = \bar z_{\dot\alpha} \,,$$
$$U = U^{(0)} = \Omega = \tfrac{1}{4i}\left(\omega^{(0)\alpha\beta} y_\alpha y_\beta + \bar\omega^{(0)\dot\alpha\dot\beta}\bar y_{\dot\alpha}\bar y_{\dot\beta} + 2e^{(0)\alpha\dot\beta} y_\alpha \bar y_{\dot\beta}\right)$$

$$e_{(0)}^{\alpha\dot\beta} = -\frac{dx^a(\sigma_a)^{\alpha\dot\beta}}{1-x^2} \,, \qquad \omega_{(0)}^{\alpha\beta} = \frac{x^a dx^b(\sigma_{ab})^{\alpha\beta}}{1-x^2}$$

$$\longrightarrow \quad ds^2_{(0)} = \frac{4dx^2}{(1-x^2)^2}$$

- $U^{(0)}$ is a flat connection, can be represented via a gauge function $L(x|Y)$ = AdS_4 coset element

$$U^{(0)} = \Omega = L^{-1} \star dL$$

$$L(x;y,\bar y) = e_\star^{i\tilde x^\mu(x)\delta_\mu^a P_a} \quad : \quad \mathcal{R}^{3,1} \longrightarrow \frac{SO(3,2)}{SO(3,1)}$$

- The associated adjoint and twisted-adjoint covariant derivatives are defined as

$$D_{\rm ad}^{(0)} f = df + [U^{(0)}, f]_\star \,, \qquad D_{\rm tw}^{(0)} f := df + [U^{(0)}, f]_\pi$$
$$D_{\rm ad}^{(0)} f = df + \Omega^{\underline{\alpha\beta}} Y_{\underline\alpha}\partial^{(Y)}_{\underline\beta} f - i\Omega^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline\alpha}\partial^{(Z)}_{\underline\beta} f$$

Perturbative analysis

- Starting from the initial conditions $U_{|Z=0}$, $\Phi_{|Z=0}$.
  (supplemented with Z-space gauge choices) the eqs can be analyzed in an expansion in $\Phi_{|Z=0}$:

$$\Phi = \sum_{n\geq 1} \Phi^{(n)} , \quad \Phi^{(1)} = \Phi|_{Z=0} , \qquad V_\alpha = \sum_{n\geq 1} V_\alpha^{(n)} \qquad U = \sum_{n\geq 0} U^{(n)} , \quad U^{(0)} = \Omega$$

- The eqs. with at least one component on Z can be integrated to give the Z-dependent fields in terms of non-linear couplings involving the original dof in $\Phi_{|Z=0}$:

$$\Phi^{(n)} = C^{(n)}(Y) - z^\alpha \sum_{k=1}^{n-1} \int_0^1 dt [V_\alpha^{(n-k)}, \Phi^{(k)}]_{\pi, z\to tz} + \text{h.c.}$$

$$V_\alpha^{(n)} = \partial_\alpha \, \xi^{(n)} + z_\alpha \int_0^1 dt\, t \left(\frac{i}{2} \Phi^{(n)} \star \kappa + \sum_{k=1}^{n-1} V^{(k)\alpha} \star V_\alpha^{(n-k)} \right)_{z\to tz} + \bar{z}^{\dot\beta} \sum_{k=1}^{n-1} \int_0^1 tdt [V_\alpha^{(k)}, \bar{V}_{\dot\beta}^{(n-k)}]_{z\to tz}$$

$$U^{(n)} = W^{(n)}(Y) + \frac{i}{2} z^\alpha \sum_{k=0}^{n-1} \int_0^1 dt [W_\mu^{(k)}, V_\alpha^{(n-k)}]_{\star, t\to tz} - \text{h.c.}$$

Perturbative analysis

- The eqs. with at least one component on Z can be integrated in terms of the original dof in $\Phi_{|Z=0}$(x,Y) (with no cohomology for Z-space 1-forms)

$$q := dZ^{\underline{\alpha}} \frac{\partial}{\partial Z^{\underline{\alpha}}} \ , \quad J \ := \ -\frac{i}{4} dz^2 \kappa - \text{h.c.}$$

$$\begin{aligned}
q\Phi^{(1)} &= 0 & \longrightarrow \quad & \Phi^{(1)} = C(Y) \\
D^{(0)}_{\rm tw}\Phi^{(1)} &= 0 & & \\
qV^{(1)} + \Phi^{(1)} \star J &= 0 & \longrightarrow \quad & V^{(1,A)} = V^{(1,A)} = -q^*_A(\Phi^{(1)} \star J) \\
qU^{(1)} + D^{(0)}_{\rm ad} V^{(1)} &= 0 & \longrightarrow \quad & U^{(1,A)} = q^*_B D^{(0)}_{\rm ad} q^*_A(\Phi^{(1)} \star J) + W^{(1,A,B)} \\
D^{(0)}_{\rm ad} U^{(1)} &= 0 & &
\end{aligned}$$

- One is then left with the field equations on $\mathcal{X}$ x Υ :

$$D^{(0)}_{\rm ad} W^{(1,A,B)} = -(D^{(0)}_{\rm ad} q^*_B)(D^{(0)}_{\rm ad} q^*_A)(\Phi^{(1)} \star J) \ , \qquad D^{(0)}_{\rm tw}\Phi^{(1)}|_{Z=0} = 0$$

- The simplest choice corresponds to solving the Z-space eqs. with $q_A{}^* = q_B{}^* = q_0{}^*$, resolving the gauge ambiguity by imposing the Vasiliev gauge

$$\boxed{i_Z V^{(1)} \ \equiv \ Z^{\underline{\alpha}} V^{(1)}_{\underline{\alpha}} \ = \ 0}$$

Perturbative analysis

- Substitution in the remaining equations at first order gives Klein-Gordon, Maxwell, linearized Einstein and Fronsdal eqs. in unfolded form → *Central On-Mass-Shell Theorem* (COMST) :

$$dW + \{\Omega, W\}_\star \;=\; -\frac{i\bar{b}}{4}\, H^{\alpha\beta}\,\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\Phi^{(1)}|_{\bar{y}=0} - \frac{ib}{4}\,\bar{H}^{\dot{\alpha}\dot{\beta}}\,\frac{\partial^2}{\partial \bar{y}^{\dot{\alpha}} \partial \bar{y}^{\dot{\beta}}}\Phi^{(1)}|_{y=0}$$

$$d\Phi^{(1)} + \Omega^{(0)} \star \Phi^{(1)} - \Phi^{(1)} \star \pi(\Omega) \;=\; 0$$

$$H^{\alpha\beta} := e^{(0)\alpha\dot{\gamma}} e^{(0)\beta}{}_{\dot{\gamma}}$$

- The twisted adjoint equation already contains the information on the free propagation of all spin-s fields via the Bargmann-Wigner eqs. on the curvatures. The first equation is a gluing of the Weyl module to the gauge-field module (via the Chevalley-Eilenberg cocycle).

- Going to higher orders gets increasingly difficult (and there is no clear rationale, in the standard perturbation theory, that selects a specific gauge choice).

Perturbative analysis

- Inserting in the pure spacetime equations and setting Z=0, one gets

$$
\begin{aligned}
dW &= \mathcal{V}_2(W,\Phi) := W\star W + \mathcal{V}_2^{(1)}(W,W,\Phi) + \mathcal{V}_2^{(2)}(W,W,\Phi,\Phi) + \ldots \\
d\Phi &= \mathcal{V}_1(W,\Phi) := [W,\Phi]_\pi + \mathcal{V}_1^{(2)}(W,\Phi,\Phi) + \ldots
\end{aligned}
$$

Moreover, on-shell the infinitely many Z-contractions turn into an infinite expansion in derivatives of arbitrarily high order → in a generic frame, one has a non-local, Born-Infeld-like tail at every fixed order in weak fields.

[This depends on the solution scheme for the Z-space eqs.: a different scheme, connected to this one by a non-local field redefinition, cuts the infinite tail to the quasi-local expansion expected via Noether procedure. *(Didenko, Gelfond, Korybut, Vasiliev)*
Not clear how to extend such scheme beyond cubic level.]

- On the other hand, almost all exact solutions have been obtained in different gauges and following the opposite route:
working in the full (*x,Y,Z*)-space in order to take advantage of the simplicity of the eqs. and of the huge gauge freedom of the theory.

Exact solutions

- Surprisingly, constructing exact solutions is simpler than one would think!

- Working in the full (x,Y,Z)-space enables one to keep into account all non-linearities in a manageable, algebraic form, and use to one's advantage the formal Simpicity of the equations
  (The difficulty one encounters, however, is then at the level of interpretation: what is an admissible gauge? Proper class of functions of NC variables? Physical Interpretation of the solution? Meaning of invariants?...)

- In general, one can use all the traditional methods employed for solving complicated differential equations: using some convenient gauge, imposing symmetries, using an algebraically special Ansatz, separating variables…

- Then one usually selects a physical subspace of the possible solutions encoded by the initial choices via physical requirements/global conditions: finiteness of inner product, finiteness and conservation of asymptotic charges...

Exact solutions: gauge function method

- Takes maximum advantage from the fact that the physics is to a large extent encoded in twistor space.

- $\mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$-space eqns:

$$U = g^{-1} \star dg$$

$$\Phi = g^{-1} \star \Phi' \star \pi(g) \; , \qquad d\Phi' = 0$$

$$S_\alpha = g^{-1} \star S'_\alpha \star g \; , \qquad dS'_\alpha = 0$$

- $\mathcal{Y} \times \mathcal{Z}$-space eqns:

$$S'_\alpha \star \Phi' + \Phi' \star \pi(S'_\alpha) = 0$$

$$[S'_\alpha, S'_\beta]_\star = -2i\epsilon_{\alpha\beta}(1 - b\,\Phi' \star \kappa)$$

$$[S'_\alpha, \bar{S}'_{\dot{\beta}}]_\star = 0$$

(Vasiliev, Sezgin-Sundell, C.I.-Sundell, Giombi-Yin…)

- Solve locally all equations with at least one spacetime component via some gauge function $g = g(x,Y,Z)$.
- Then solve the Z-space constraints to determine the spacetime constants $\Phi' = \Phi'(Y,Z)$ and $S'_\alpha = S'_\alpha(Y,Z)$.
- Working in terms of primed fields → *U=0 gauge*.
  Easier to build solutions in $U=0$ gauge, the equations are algebraic. In order to read any spacetime feature (correlation functions, asymptotic charges,…) change gauge and reinstate x-dep. by performing the star-products with g.
- Classical moduli in Φ' , S', g.

Exact solutions: gauge function method

- Choosing a simple, field-independent gauge function, such as $g = L$, the gauge fields seem to remain trivial.
  However, there exists a family of gauge transformations that activate a non-trivial Weyl curvature for $U^{(1)}{}_{|Z=0}$ and keep $U^{(1)}{}_{|Z=0}$ analytic in Y
  → $U^{(1)}{}_{|Z=0}$ can be identified as a generating function for Fronsdal fields.

  [gauge transformation built explicitly at the first order, proposal to extend to all orders via imposing *ALAdS boundary conditions*] *(De Filippi, C.I., Sundell)*

- The behaviour of the fields in (x,Y,Z) depends on a subtle interplay of gauge function and primed fields.
- At fixed gauge function, it is the twistor-space behaviour of the initial Φ' and S'_α that determines the spacetime behaviour of the fields.
  In general, both the local data (primed fields, fibre elements) and the choice of gauge function (through their boundary values) are moduli of the solutions.

  → Restrictions on physically admissible solutions as well as determining the superselection sectors of the theory (to a good extent) coincides with selecting classes of functions in twistor space.

Imposing symmetries in U=0 gauge

- Solving the remaining twistor-space problem can be simplified by imposing symmetries. In the *U*=0 gauge, this can be done directly with rigid generators ε'(Y,Z):

$$\epsilon(x,Y,Z) \;=\; g^{-1}(x,Y,Z)\star\epsilon'(Y,Z)\star g(x,Y,Z)$$

$$\delta_\epsilon\Phi \;=\; [\epsilon,\Phi]_\pi \qquad \longrightarrow \qquad \delta_{\epsilon'}\Phi' \;=\; [\epsilon',\Phi']_\pi$$

- In particular, the AdS Killing vectors

$$J_{AB}(X) \;=\; X_A\frac{\partial}{\partial X^B} - X_B\frac{\partial}{\partial X^A}$$

can be written in terms of the rigid $\mathfrak{so}(3,2)$ isometry generators

$$J_{AB} \;=\; L^{-1}\star M^{(0)\prime}_{AB}(Y)\star L$$

$$M^{(0)\prime}_{AB}(Y) \;=\; M^{(0)\prime}|_{Z=0} \;=\; \frac{1}{8}Y^{\underline{\alpha}}(\Gamma_{AB})_{\underline{\alpha\beta}}Y^{\underline{\beta}}$$

- The Lorentz subalgebra is special, since the form of the nonlinear equations tells us what is the fully non-linear form of the Lorentz generators.

A Lorentz-invariant solution

- In the *U*=0 gauge, imposing Lorentz sym in twistor space:

$$[M'_{\alpha\beta,}\Phi']_\pi = 0\,, \quad [M'_{\alpha\beta},S'_\gamma]_\star = 0 \quad \Rightarrow \quad \Phi' = \Phi'(y^\alpha z_\alpha, \bar{y}^{\dot\alpha}\bar{z}_{\dot\alpha})\,, \quad S'_\alpha = z_\alpha S(y^\alpha z_\alpha, \bar{y}^{\dot\alpha}\bar{z}_{\dot\alpha})$$

- Solving two of the Z-space constraints further fixes the form of the *SO(3,1)-invariant solution* to be the simplest deformation of the AdS vacuum,

$$\Phi' = \nu\,, \qquad S'_\alpha = z_\alpha S(y^\beta z_\beta)\,, \qquad S'_{\dot\alpha} = \bar{z}_{\dot\alpha}\bar{S}(\bar{y}^{\dot\beta}\bar{z}_{\dot\beta})$$

  and the last eqs. become

$$[S'^\alpha, S'_\alpha]_\star = 4i(1-b\nu e^{iy^\alpha z_\alpha})\,, \qquad [\bar{S}'^{\dot\alpha}, \bar{S}'_{\dot\alpha}]_\star = 4i(1-\bar{b}\nu e^{i\bar{y}^{\dot\beta}\bar{z}_{\dot\beta}})$$

  and can be solved via the Laplace-like transform *(Prokushkin-Vasiliev '99)*

$$S(u) = \int_{-1}^{1} ds\; n(s)\; e^{\frac{i}{2}(1+s)u}\,, \quad \bar{S}(\bar{u}) = \int_{-1}^{1} ds\; \bar{n}(s)\; e^{-\frac{i}{2}(1+s)\bar{u}}$$

- Choosing the AdS gauge function L to rotate Φ' and S'_α, the final result is a scalar profile over a conformally rescaled AdS-metric,

$$\boxed{\phi(x) = \nu(1-x^2)\,, \qquad ds^2 = \frac{4\Omega^2 d\tilde{x}^2}{(1-\tilde{x}^2)^2}}$$

(Sezgin-Sundell '05)

(C.I.- J. Raeymaekers '15)

A cosmological "interpretation"

- In coordinates

$$x^0 = \sinh\tau \tanh\frac{\rho}{2} \quad x^i = n^i \cosh\tau \tanh\frac{\rho}{2} \ , \quad n^i n_i = 1 \ , \qquad x^2 > 0 \ ,$$
$$x^0 = \cosh\rho \tan\frac{\tau}{2} \quad x^i = n^i \sinh\rho \tan\frac{\tau}{2} \ , \quad n^i n_i = 1 \ , \qquad x^2 < 0 \ ,$$

  the solution reads

$$\begin{aligned} ds^2 &= d\rho^2 + \eta^2(\rho)\sinh^2\rho(-d\tau^2 + \cosh^2\tau d\Omega_2) \\ \phi &= \nu\, \mathrm{sech}^2\frac{\rho}{2} \ , \qquad x^2 > 0 \ , \\ ds^2 &= -d\tau^2 + \eta^2(\tau)\sin^2\tau(d\rho^2 + \sinh^2\rho d\Omega_2) \\ \phi &= \frac{\nu}{\cos^2\frac{\tau}{2}} \ , \qquad x^2 < 0 \end{aligned}$$

- This has the form of (the continuation to the minkowskian signature of) a Coleman-De Luccia instanton, manifestly O(3,1)-symmetric, describing a bubble of true vacuum inside *AdS*, subject to a big crunch for $\tau = \pi$ (a genuine singularity from the spin-2 point of view).

- However, this description is only valid in terms of ordinary (i.e., spin-2) geometry, NOT HS-invariant. In view of the fast AdS asymptotics we may hope that the corresponding boundary description will not be heavily altered.

Imposing symmetries in U=0 gauge

- Even in the U=0 gauge, imposing symmetries on what is to become a full solution is not easy. The Lorentz-invariant solution is special, since the Lorentz generators are known at full level.
- To build a solution with symmetries including translation generators (such as solutions that may be of cosmological interest) one can only impose the symmetry conditions order by order : i.e., one has to start by imposing them at linear level via the (known) undeformed generators,

$$[M_{ij}, \Phi'^{(1)}]_\pi = 0 \,, \qquad [P_i, \Phi'^{(1)}]_\pi = 0$$

  and then compute the nonlinear correction in weak-field expansion,

$$\epsilon' = \epsilon'^{(0)} + \epsilon'^{(1)} + \epsilon'^{(2)} + \dots \,, \qquad \epsilon'^{(0)} \propto \epsilon^{ij} M_{ij} + \epsilon^i P_i$$

$$(\delta_{\epsilon'} \Phi')^{(2)} = \left([\epsilon'^{(1)}, \Phi'^{(1)}]_\pi + [\epsilon'^{(0)}, \Phi'^{(2)}]_\pi\right)$$

  where the Weyl zero-form to the second order can be computed by the usual perturbation scheme. And so on → increasingly complicated procedure.

- However, a different Ansatz, making use of separation of the non-commutative twistor-space variables, offers a better chance.

Factorized expansion in holomorphic gauge

- A large solution space of interesting solutions (including HS black holes, HSbh + massless scalar, FRW-like solutions,…) take the form :

$$\Phi'(Y,Z) = C'(Y) \ ,$$

$$V'_\alpha(Y,Z) = V'_\alpha(Y,z) = V'_\alpha(\Psi(Y),z) = \sum_{k=1}^{\infty} (\Psi(Y))^{\star k} \star V^{(k)}_\alpha(z) \ ,$$

$$\bar{V}'_{\dot\alpha}(Y,Z) = \bar{V}'_{\dot\alpha}(Y,\bar z) = \bar{V}'_{\dot\alpha}(\bar\Psi(Y),\bar z) = \sum_{k=1}^{\infty} (\bar\Psi(Y))^{\star k} \star \bar{V}^{(k)}_{\dot\alpha}(\bar z)$$

$$\Psi := C' \star \kappa_y \ , \quad \bar\Psi := C' \star \bar\kappa_{\bar y} \ , \qquad [\Psi, \bar\Psi]_\star = 0$$

  → an all-order perturbative expansion in star-powers of the curvatures, absorbing all the *Y*-dependence, with separation of *Y* and *Z* variables and *V* holomorphic in *z*.

- Whereas the ordinary perturbative analysis is organized in powers of $\Phi\star\kappa$ and normal order, this can be considered an expansion in Ψ in Weyl order (no contractions between *Y* and *Z*).
- The different solutions are singled out by the different basis functions (or distributions) of *Y* variables on which one expands C (i.e., Ψ) .
- The expansion in Ψ enables one to solve for the *Z* dependence in a universal way.

Factorized expansion in holomorphic gauge

- This is because $\Phi'=C'(Y)$ → the Z-dependence in the source term is universal and given by κ_z :

$$\partial_{[\alpha}V'_{\beta]} + V'_{[\alpha} \star V'_{\beta]} \quad = \quad -\frac{i}{4}\,\epsilon_{\alpha\beta}\, b\Psi \star \kappa_z$$

First order in Ψ:

$$\partial_{[\alpha}V^{(1)}_{\beta]} \quad = \quad -\frac{i}{4}\,\epsilon_{\alpha\beta}\, b\kappa_z$$

solved by a distributional z-space element

$$\widehat{V}^{(1)\pm} \;\sim\; z^{\pm}\int_{-1}^{1}\frac{d\tau}{(\tau+1)^2}\,e^{i\frac{\tau-1}{\tau+1}z^+z^-} \;\sim\; \frac{1}{z^{\mp}}\lim_{\epsilon\to 0}(1-e^{-\frac{i}{\epsilon}z^+z^-}) \;\sim\; \theta(z^{\pm})\delta(z^{\mp})$$

$$z^{\pm} := u^{\pm\alpha}z_\alpha \;, \quad w_z := z^+z^- \;, \quad [z^-,z^+]_\star = -2i \qquad \rightarrow \qquad \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}e^{-i\frac{1}{\varepsilon}z^+z^-} \;=\; \kappa_z$$

with basis spinors $u^{\pm}{}_{\alpha}$ ($u^{+\alpha}u^{-}{}_{\alpha} = 1$) entering as to achieve an integral realization of a delta function in a Gaussian basis (one could have equally well used a plane wave basis, in which case an auxiliary spinor, the momentum associated to z, would have played that role).

- Higher orders:

$$\partial_{[\alpha}V^{(k)}_{\beta]} + \frac{1}{2}\sum_{p+q=k}\left[V^{(p)}_{\alpha}, V^{(q)}_{\beta}\right]_\star \;=\; 0\;, \quad k\geq 2$$

$$\rightarrow \quad \sum_{k\geq 1} V^{(k)}_{\alpha}\star\Psi^{\star k} \;=\; \int_{-1}^{1}\frac{d\tau}{(\tau+1)^2}\,{}_1F_{1\star}(1/2;2;b\log\tau^2\Psi)\star z_\alpha\, e^{i\frac{\tau-1}{\tau+1}w_z}$$

Comments and observations

- The factorized expansion encodes a (formal) solution space in which Φ' is first-order exact, and the Z-dependence is solved in a universal way
  → gives a systematic procedure to non-linearly deform solutions of the KG and Bargmann-Wigner eqs. into solutions of the full Vasiliev eqs.

  This also facilitates to some extent their physical interpretation as well as the superposition of linearized twisted-adjoint sectors, e.g., $\Psi = \Psi_{bh} + \Psi_{part}$.

- Actual solutions must satisfy:
  1. The star-products $(\Psi)^{*k}$ must be finite → conditions on the fiber algebra $\mathcal{A}(Y)$
  2. The zero-form charges should be finite (e.g., well-defined inner product)
  3. V_α should be at least real-analytic in Z.

  In the case that all Ψ^{*k} can be expanded over a common basis of functions, one can actually write down the full solution in closed form immediately.

- Further constraints placed by requiring the solution to correspond to an asymptotic configuration of Fronsdal fields (over AdS) → analiticity in Y and Z in Vasiliev gauge and finiteness of asymptotic charges.

Particle and HS black-hole modes

- Which solutions of the linearized equations can be dressed into full ones? Which linear sectors can *simultaneously* be dressed into full sectors of the moduli space?

- Factorized Ansatz already used to nonlinearly deform *massless scalar* modes + spherically symmetric *HS black holes*.

- Massless particle modes build up unitary $\mathfrak{so}(3,2)$ LW modules. In D=4 there are 2, unitary scalars, distinguished by Neumann/Dirichlet b.c., with ground states

$$\varphi_{(1,0)} \sim \frac{e^{-it}}{(1+r^2)^{1/2}} \,, \qquad \varphi_{(2,0)} \sim \frac{e^{-2it}}{1+r^2}$$

- Type-D, static scalar consists of the solution singular in the origin $\varphi_{(0,0)} \sim \frac{1}{r}$
  Generalization to arbitrary spin: type-D spin-s Weyl tensors of the form

$$\boxed{\Phi_{\alpha(2s)} \sim \frac{M}{r^{s+1}} \left(u^+ u^-\right)^s_{\alpha(2s)}} \qquad \textit{(Didenko, Vasiliev)}$$

- The spin-2 element coincides with the full AdS-Schwarzschild Weyl tensor.

This follows from the Kerr-Schild property of bhs in gravity: they solve both the linearized and the nonlinear eqs. In gravity, the above are local hallmark of bhs.

Massless particle modes

- Massless particle modes build up unitary $\mathfrak{so}(3,2)$ LW modules.

Unfolded Weyl 0-form equations, i.e., reformulation of the Bargmann-Wigner eqs. via a covariant constancy condition on the twisted adjoint module,

$$\Phi(x|Y) \;=\; L^{-1}(x) \star \Phi' \star \pi(L)(x)$$

show that particle modes can be encoded into specific algebraic elements: operators on singleton Fock space, non-polynomial functions of Y with definite eigenvalues under the Cartan subalgebra (*E*,*J*) of $\mathfrak{so}(3,2)$,

$$\Phi'(Y) \;\in\; \mathcal{M} \;=\; \bigoplus_{\mathbf{n},\mathbf{m}} \mathbf{C} \otimes P_{\mathbf{n}|\mathbf{m}}$$

$$P_{\mathbf{n}|\mathbf{n}'} \star P_{\mathbf{m}|\mathbf{m}'} \;=\; \delta_{\mathbf{n}',\mathbf{m}} P_{\mathbf{n}|\mathbf{m}'}\;, \qquad P_{\mathbf{n}|\mathbf{m}'} \sim |\mathbf{n}><\mathbf{m}|\;, \qquad \mathbf{n},\,\mathbf{m} \equiv (n_1,n_2),(m_1,m_2)$$

$$E \star P_{\mathbf{n}|\mathbf{m}} \;=\; \frac{n_1+n_2}{2} P_{\mathbf{n}|\mathbf{m}}\;, \qquad J \star P_{\mathbf{n}|\mathbf{m}} \;=\; \frac{n_2-n_1}{2} P_{\mathbf{n}|\mathbf{m}}$$

- Modules built by solving LW conditions $[L^{-}_{r}, P_{n|m}]_\pi = 0$ and then acting with L^{+}_{r}.

- This offers a simple way of solving for all the AdS-massless particle modes.

Massless scalar particle modes

- For example, the rotationally-invariant scalar field modes are encoded by projectors |n> <n|

$$\mathcal{P}_n(E) \;=\; 4(-)^{n-\frac{1+\varepsilon}{2}}\, e^{-4E} L^{(1)}_{n-1}(8E) \;=\; 2(-)^{n-\frac{1+\varepsilon}{2}} \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-4\eta E}.$$

- Indeed, using the simple AdS gauge function *L* (*L-gauge*) we reconstruct exactly the Breitenlohner-Freedman scalar modes,

$$\Phi'(Y) \;=\; \Phi'_{s=0}(Y) \;=\; \sum_n \widetilde{\nu}_n\, \mathcal{P}_n(E)\ , \qquad (\widetilde{\nu}_n)^* \;=\; \widetilde{\nu}_{-n}$$

$$\Phi_{s=0}(x|Y) \;=\; L^{-1}(x)\star\Phi'_{s=0}\star\pi(L)(x) \;=\; (1-x^2)\sum_n \mathcal{N}_n\widetilde{\nu}_n \oint_{C(\epsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \frac{e^{iy^\alpha M_\alpha{}^{\dot\beta}(x,\eta)\bar y_{\dot\beta}}}{1-2i\eta x_0+\eta^2 x^2}$$

- For instance, the LW element n=1 ($\Phi' = 4e^{-4E}$) gives rise to the ground state of the D(1,0) scalar, as expected:

$$4\widetilde{\nu}_1\,\frac{1-x^2}{1-2ix_0+x^2} \;\sim\; \widetilde{\nu}_1\,\frac{e^{-it}}{(1+r^2)^{1/2}}$$

(C.I., P. Sundell)

Twisted projectors

- By examining the nonlinear corrections we will find out that there exists an enveloping-algebra realizations of bh/type-D modes, too.

- The Z-space connection (and the gauge fields) receive non-linear corrections of all orders. In the U=0 gauge, they appear as powers of $\Psi = \Phi' \star \kappa_y$.
  → Injecting massless particles into Φ' results in the appearance of the *twisted projectors* $\mathcal{P}_n \star \kappa_y$ in Ψ,

$$\widetilde{\mathcal{P}}_n \;:=\; \mathcal{P}_n \star \kappa_y \;=\; 4\pi(-)^{n-\frac{1+\varepsilon}{2}} \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \delta^2(y - i\eta\sigma_0\bar{y})$$

  and <u>all non-linear corrections can be expanded over this basis</u>, due to the generalized projector algebra

$$\boxed{\begin{array}{ll} \mathcal{P}_n \star \mathcal{P}_m = \delta_{nm}\mathcal{P}_n \,, & \widetilde{\mathcal{P}}_n \star \widetilde{\mathcal{P}}_m = \delta_{n,-m}\mathcal{P}_n \\ \mathcal{P}_n \star \widetilde{\mathcal{P}}_m = \delta_{nm}\widetilde{\mathcal{P}}_n \,, & \widetilde{\mathcal{P}}_n \star \mathcal{P}_m = \delta_{n,-m}\widetilde{\mathcal{P}}_n \end{array}} \quad \Longrightarrow$$

$$\widetilde{\mathcal{P}}^{\star n} = \begin{cases} \mathcal{P} \,, & n = 2k \,, \\ \widetilde{\mathcal{P}} \,, & n = 2k+1 \end{cases} \quad \rightarrow \quad F'^{\star k} = \left(\sum_n \widetilde{\nu}_n \widetilde{\mathcal{P}}_n\right)^{\star k} = \sum_n \left(\nu_n^{(k)} \mathcal{P}_n + \widetilde{\nu}_n^{(k)} \widetilde{\mathcal{P}}_n\right)$$

Twisted projectors and HSBH

- Projectors and twisted projectors form a subalgebra of the star-product algebra. The star-multiplication with κ_y induces a change of sign of the E-eigenvalue, so the twisted projectors →

$$\widetilde{\mathcal{P}}_n \simeq |n/2;0\rangle\langle -n/2;0| \in \mathcal{D}_0 \otimes \widetilde{\mathcal{D}}_0^*$$

and correspond, via twisted-adjoint action, to states with zero energy, *static* → *soliton*-like solutions.

(C.I., P. Sundell)

- Indeed, dressing with the gauge function the spacetime behaviour of individual fields shows that they are spherically-symmetric HS black holes! If $\Phi'(Y)$ is expanded in twisted projectors,

$$\Phi'(Y) = \Phi'_{\rm bh}(Y) = \sum_n \nu_n \mathcal{P}_n(E) \star \kappa_y = \sum_n \nu_n \widetilde{\mathcal{P}}_n(Y) \, , \qquad \nu_n = i^n \mu_n$$

$$\Phi_{\rm bh}(x|Y) = \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \underbrace{L^{-1}(x) \star e^{-4\eta E} \star L(x) \star \kappa_y}$$

Coeffs. of the Y-expansion → a tower of type-D Weyl tensors of all spins (+ derivatives):

$$\Phi^{(n)}_{\alpha(2s)} \sim \frac{i^{n-1}\mu_n}{r^{s+1}} (u^+u^-)^s_{\alpha(2s)}$$

Twisted projectors and HSBH

- Each individual Weyl tensor has a curvature singularity in r=0. At the master-field level, this converts into the statement that in the r → 0 limit the Weyl zero-form becomes a delta function in Y,

$$\boxed{\Phi_{\rm bh}(x|Y) \ \sim \ \mathcal{O}_n \, \frac{1}{\eta r} \, \exp i \frac{\widetilde{y}^+\widetilde{y}^-}{r} \qquad \xrightarrow[r\to 0]{} \qquad \mathcal{O}_n 2\pi \delta^2(\widetilde{y})}$$

$$\mathcal{O}_n \ := \ \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n \ , \qquad \widetilde{y}_\alpha \ := \ y_\alpha - i\eta(\sigma_0 \bar{y})_\alpha$$

- HS symmetry forces all such static solutions to appear together in a infinite-dim. multiplet, packed into the Y expansion of the Weyl zero-form. At this level the spacetime singularities have a more readable meaning: *r* appears as the parameter of a delta sequence in Y, so effectively unfolding trades the spacetime singularities for a distributional behaviour in Y.

- This is a more tractable problem: a delta function of non-commutative variables can be considered smooth, in the sense that it is well-behaved under star product (and is in fact part of the associative algebra that governs such solutions).

Solutions with 6 killing vectors

- The factorized Ansatz enables one to impose symmetries involving translations directly at full level.
  This is a consequence of the (star-)factorization (first-order exact Weyl zero-form and Z-dependece solved universally): one can impose symmetries on the full Φ' via *undeformed* generators, and the *same* generators will correspond to symmetries of V_α!

$$\delta_{\rm gl}\Phi'(Y) \;=\; [\epsilon^{(0)}_{\rm gl}(Y),\Phi'(Y)]_\pi \;=\; 0 \quad\Rightarrow\quad [\epsilon^{(0)}_{\rm gl}(Y),\Psi(Y)]=0 \quad\Rightarrow\quad \delta_{\rm gl}V'_\alpha(Y) \;=\; [\epsilon^{(0)}_{\rm gl}(Y),V'_\alpha(Y)]_\star \;=0$$

  → a chance to look for domain-wall-like and FRW-like solutions within the factorized Ansatz, and possibly to study fluctuations over them.

- Possible to embed $\mathfrak{g}_6$ symmetries inside $\mathfrak{so}(3,2)$ or $\mathfrak{so}(4,1)$ in different ways:

$$\boxed{M_{rs}=L_r{}^aL_s{}^bM_{ab}\;,\qquad T_r=L_r{}^a\left(\alpha M_{ab}L^b+\beta P_a\right)}$$

$$L_r{}^aL_s{}^b\eta_{ab}=\eta_{rs}=(++,-\epsilon)\;;\quad L^aL_a=\epsilon\;,\quad L_r{}^aL_a=0\;,\qquad \epsilon=\pm1$$

$$[M_{rs},M_{pq}]=i\eta_{sp}M_{rq}+3\text{ more}\;,\qquad [M_{rs},T_p]=2i\eta_{p[s}T_{r]}\;,$$

$$[T_r,T_s]=-i(\epsilon\alpha^2-\lambda^2\beta^2)M_{rs}$$

Solutions with 6 killing vectors

- Candidate domain-wall and FRW solutions follow from imposing

$$[M_{rs}, \Phi']_\pi = 0 \ , \qquad [T_r, \Phi']_\pi = 0$$

$$\Phi' = \Phi'(P) \ , \qquad P := L^a P_a \qquad \qquad \left(-\frac{\epsilon\beta\lambda^2}{8}\frac{d^2}{dP^2} + i\epsilon\alpha\frac{d}{dP} + 2\beta\right)\Phi'(P) = 0$$

- Can solve by "Laplace"-transforming:

$$\boxed{\Phi' = \oint_C \frac{d\eta}{2\pi i}\widetilde{\Phi}'(\eta)\exp(-4\eta\lambda^{-1}P) \equiv \mathcal{O}\exp(-4\eta\lambda^{-1}P)}$$

→ The characteristic eq. has roots $\eta_\pm = -\gamma \pm \sqrt{\epsilon + \gamma^2} \ , \qquad \gamma := \frac{i\alpha}{\lambda\beta} \ , \qquad \eta_+\eta_- = -\epsilon$

→ Laplace transform:

$$\mathfrak{o}(1,3) \ : \ \widetilde{\Phi}' = \frac{\nu_+}{\eta - \eta_+} + \frac{\nu_-}{\eta - \eta_-} \ ,$$

$$\mathfrak{iso}(1,2) \ , \ \mathfrak{iso}(3) \ : \ \widetilde{\Phi}' = \frac{\nu}{\eta + \sqrt{-\epsilon}} + \frac{\sqrt{-\epsilon}\widetilde{\nu}}{(\eta + \sqrt{-\epsilon})^2} \ ,$$

$$\mathfrak{o}(4) \ , \ \mathfrak{o}(2,2) \ : \ \widetilde{\Phi}' = \frac{\mu}{\eta - \eta_+} + \frac{\bar{\mu}}{\eta - \eta_-} \ .$$

Solutions with 6 killing vectors

- Candidate domain-wall and FRW solutions:

Type	M_3	$\mathfrak{g}_6$	ϵ	λ^2	Condition on (α,β) for $\mathfrak{g}_6$ closure	$\gamma := \frac{i\alpha}{\lambda\beta}$ (modulo G_{10})	(η_+,η_-) $\eta_\pm \equiv -\gamma \pm \sqrt{\epsilon+\gamma^2}$	Φ' , $\mu \in \mathbb{C}$, $\nu,\tilde{\nu},\nu_\pm \in \mathbb{R}$ $P = L^a P_a$, $(\lambda^{-1}P)^\dagger = \lambda^{-1}P$
$\mathrm{DW}_+^{(\mathrm{dS})}$	dS_3	$\mathfrak{o}(1,3)$	$+1$	<0	$\alpha^2-\lambda^2\beta^2>0\,,\ \beta\neq 0$	$\gamma=0$	$(1,-1)$	$\nu_+e^{-4\lambda^{-1}P}+\nu_-e^{4\lambda^{-1}P}$
FLRW_+	S^3	$\mathfrak{o}(4)$	-1	<0	$-\lambda^2\beta^2>\alpha^2$	$\gamma=0$	$(i,-i)$	$\mu e^{-4i\lambda^{-1}P}+\bar{\mu}e^{4i\lambda^{-1}P}$
FLRW_0	$Eucl_3$	$\mathfrak{iso}(3)$	-1	<0	$-\lambda^2\beta^2=\alpha^2>0$	$\gamma=1$	$(-1,-1)$	$(\nu-4\tilde{\nu}\lambda^{-1}P)e^{4\lambda^{-1}P}$
$\mathrm{FLRW}_-^{(\mathrm{dS})}$	H_3	$\mathfrak{o}(1,3)$	-1	<0	$\alpha^2>-\lambda^2\beta^2$	$\gamma>1$	$\eta_-<-1<\eta_+<0$	$\nu_+e^{-4\eta_+\lambda^{-1}P}+\nu_-e^{-4\eta_-\lambda^{-1}P}$
I	dS_3, H_3	$\mathfrak{o}(1,3)$	±1	$\neq 0$	$\alpha^2>0$, $\beta=0$	$\gamma=\infty$	$(0,\infty)$	ν
$\mathrm{DW}_+^{(\mathrm{AdS})}$	dS_3	$\mathfrak{o}(1,3)$	$+1$	>0	$\alpha^2>\lambda^2\beta^2$	$-i\gamma>1$	$0<i\eta_-<1<i\eta_+$	$\nu_+e^{-4\eta_+\lambda^{-1}P}+\nu_-e^{-4\eta_-\lambda^{-1}P}$
DW_0	$Mink_3$	$\mathfrak{iso}(1,2)$	$+1$	>0	$\lambda^2\beta^2=\alpha^2>0$	$-i\gamma=1$	$(-i,-i)$	$(\nu-4i\tilde{\nu}\lambda^{-1}P)e^{4i\lambda^{-1}P}$
DW_-	AdS_3	$\mathfrak{o}(2,2)$	$+1$	>0	$\lambda^2\beta^2>\alpha^2$	$\gamma=0$	$(1,-1)$	$\mu e^{-4\lambda^{-1}P}+\bar{\mu}e^{4\lambda^{-1}P}$
$\mathrm{FLRW}_-^{(\mathrm{AdS})}$	H_3	$\mathfrak{o}(1,3)$	-1	>0	$\alpha^2+\lambda^2\beta^2>0,\ \beta\neq 0$	$\gamma=0$	$(i,-i)$	$\nu_+e^{-4i\lambda^{-1}P}+\nu_-e^{4i\lambda^{-1}P}$

Table 1: $\mathfrak{g}_6$-invariant M_3-foliations arising in the minimal bosonic models, with I standing for instantons, and FLRW_k and DW_k, respectively, standing for FLRW-like solutions ($\epsilon=-1$) and domainwalls ($\epsilon=+1$) with foliates with curvatures of sign $k=\mathrm{sign}(\epsilon\alpha^2-\lambda^2\beta^2)$. The embeddings of $\mathfrak{g}_6$ into the isometry algebra of the $(A)dS_4$ vacua are governed by a vector L^a with $L^2=\epsilon$ and two real parameters $\alpha,\beta>0$. The last column contains the corresponding $\mathfrak{g}_6$-invariant initial data for the Weyl zero-form. Two families of foliations with $k=-1$ interpolate between the cases with $k=0$ and the instantons.

Solutions with 6 killing vectors

- The contour integral realization is instrumental in having finite $(F')^{*k}$:

$$F' \star F' \; = \; \Phi' \star \pi(\Phi') \; = \left\{ \begin{array}{cc} \mathfrak{o}(4) \;, \mathfrak{o}(2,2) : & \frac{\mu^2+\bar\mu^2}{4} \;, \\ \mathfrak{iso}(3) \;, \mathfrak{iso}(1,2) : & 0 \;, \\ \mathfrak{o}(1,3) : & \frac{(\nu_+)^2}{(1+\epsilon(\eta_+)^2)^2} + \frac{(\nu_-)^2}{(1+\epsilon(\eta_-)^2)^2} \;, \end{array} \right.$$

following from the star-product rule

$$\boxed{e^{-4\eta\lambda^{-1}P} \star e^{-4\eta'\lambda^{-1}P} \; = \; \frac{1}{(1-\epsilon\eta\eta')^2} \exp\left(-4\frac{\eta+\eta'}{1-\epsilon\eta\eta'}\lambda^{-1}P\right)}$$

after which one takes the contour integration.

- From this product rule it follows that $\mathfrak{g}_6$-states enlarge the particle+bh algebra with the "fusion rules"

$$\begin{aligned} \mathfrak{g}_6 \star \mathfrak{g}_6 \; &= \; \mathfrak{g}_6 \\ \mathfrak{g}_6 \star \text{particle} \; &= \; \text{bh} \\ \mathfrak{g}_6 \star \text{bh} \; &= \; \text{particle} \end{aligned}$$

→ the coupling of $\mathfrak{g}_6$-modes to particles and bh modes generates pt and bh modes, which then generate an ideal, i.e., cannot generate new $\mathfrak{g}_6$.

Iso(3)-invariant solution

- The Weyl zero-form in L-gauge contains only a scalar field. In planar coordinates,

$$ds^2 = -dt^2 + e^{2t} \sum_i (dy^i)^2$$

its profile is

$$\boxed{\phi(x) = (\nu + \widetilde{\nu})\, e^{-t} - \widetilde{\nu} e^{-2t}}$$

- At first order the metric remains dS_4, but deviations due to the scalar are expected at second order in the deformation parameters.

- The internal connection is

$$\begin{aligned}
\widehat{V}^{(L)}_{\alpha} \;&=\; -\frac{ib\,\mathcal{C}}{\pi} z_\alpha \int_{-1}^{+1} \frac{d\tau}{(\tau+1)^2} \int_0^1 ds \sqrt{\frac{1-s}{s}}\, e^{-\omega\xi}\ \sinh\left(\frac{b\mathcal{C}s}{2}\log\tau^2\right) \\
&+\; \mathcal{O}\frac{ib}{\pi} u_\alpha{}^\beta \tilde{y}_\beta \frac{e^{i\tilde{y}^\alpha z_\alpha}}{\det A} \int_{-1}^{+1} \frac{d\tau}{(\tau-1)^2} \int_0^1 ds \sqrt{\frac{1-s}{s}} \exp\left(\frac{i}{\xi}\tilde{y}^+\tilde{y}^-\right) \cosh\left(\frac{b\mathcal{C}s}{2}\log\tau^2\right)
\end{aligned}$$

Conclusions and outlook

- The construction of exact solutions to the Vasiliev equations offers many insights into several open questions and challenges related to HS gravity.

- Many peculiar features of HS gravity are at work in this study: natural theoretical lab to test some of our expectations, and to get inspiration for new ideas (e.g., insisting on encoding spacetime physics in the fibre → enlarging the class of functions of oscillators, looking for more efficient ways of solving the eqs. → studying effects of changing ordering prescriptions, …)

- Many interesting open questions to investigate:

  - HS bhs or bh microstates?
  - proper formulation of boundary value problem?
  - multi-soliton solutions?
  - HS geometry
  - …