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Rectangular W-algebras, extended higher spin gravity and dual coset CFTs

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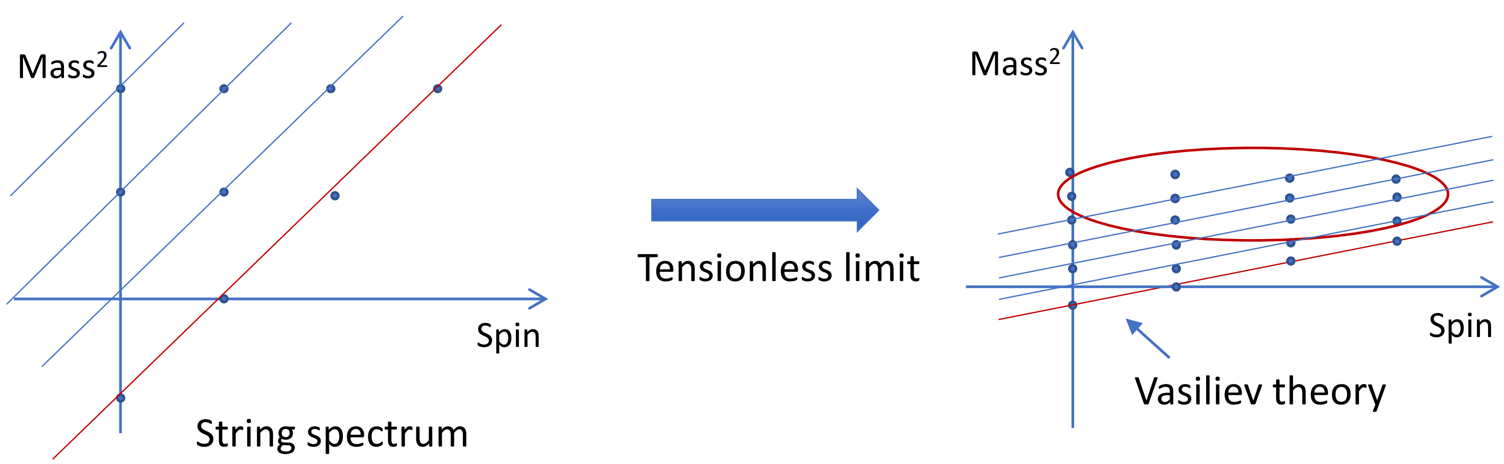
Strings from higher spins

- Leading Regge trajectory of strings ⇔ Vasiliev theory
- How about higher Regge trajectories?
  - A characteristic of string theory
  - $M \times M$ matrix extension of Vasiliev theory is supposed to be useful (cf. Tensor extensions [Vasiliev'18])



Higher spins from dual CFTs

- ABJ triality supports the claim [Chang-Minwalla-Sharma-Yin'12]
  - HS side: 4d Vasiliev theory with $M \times M$ fields
  - CFT side: 3d ABJ(M) theory ($U(N) \times U(M)$ CSM theory)
  - String side: Superstrings on AdS4 x CP3
- 3d HS theory can be analyzed with the infinite dim. symmetry of 2d CFT [Creutzig-YH-Rønne'13]
  - HS side: 3d Prokushkin-Vasiliev theory with $M \times M$ fields
  - CFT side: 2d Grassmannian-like coset

$$\frac{su(N+M)_k}{su(N)_k \oplus u(1)_{kNM(N+M)}}$$

  - String side: Superstrings on AdS3 x M7 [Creutzig-YH-Rønne'14]

Full quantum corrections in 3d extended HS theory

- Higher spin holography [Creutzig-YH-Rønne'13]
  - HS side: 3d Prokushkin-Vasiliev theory with $M \times M$ fields
  - CFT side: 2d Grassmannian-like coset

$$\frac{su(N+M)_k}{su(N)_k \oplus u(1)_{kNM(N+M)}}$$

- Evidence:
  - The holography reduces to known one for $M = 1$ [Gaberdiel-Gopakumar'10; Creutzig-YH-Rønne'11]
  - The matches of spectrum and low spin symmetry for large N
- The aim of this talk
  - Examine the asymptotic symmetry of the 3d extended HS theory with full quantum corrections

Plan of this talk

- Introduction
- Classical asymptotic symmetry
- Quantum asymptotic symmetry
- Relation to dual CFT symmetry
- Conclusion

Chern-Simons description

- 3d pure AdS gravity can be described by $sl(2)$ Chern-Simons (CS) gauge theory
  [Achucarro-Townsend'86; Witten'88]

$$S = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}]$$

$$S_{\rm CS}[A] = \frac{k_{\rm CS}}{4\pi} \int \mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) , \; k_{\rm CS} = \frac{\ell}{4G}$$

- A higher spin gravity can be constructed from g Chern-Simons theory with a gravitational $sl(2)$

Ex: $g = sl(n)$ with the principal embedding of $sl(2)$

$$sl(n) = sl(2) \oplus \left(\oplus_{s=3}^{n} g^{(s)} \right)$$

← Spin of gauge fields

Extended higher spin algebra

- The gauge algebra of 3d Prokushkin-Vasiliev theory

$$hs[\lambda] = B[\lambda] \ominus \mathbb{1}, \;\; B[\lambda] = \frac{U(sl(2))}{\langle C_2 - \frac{1}{4}(\lambda^2 - 1)\mathbb{1}\rangle}$$

  - It can be truncated to $sl(n)$ at $\lambda = n$ $(n = 2,3, \dots)$
- Multiplying $M \times M$ matrix algebra

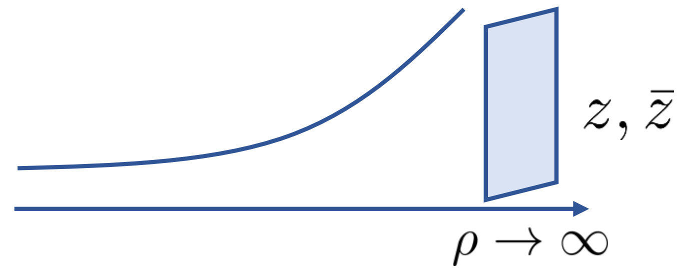
(cf. [Gaberdiel-Gopakumar'13; Creutzig-YH-Rønne'13; Joung-Kim[2]-Rey'17])

$$\begin{aligned} hs_M[\lambda] &= gl(M) \otimes B[\lambda] \ominus \mathbb{1}_M \otimes \mathbb{1} \\ &= sl(M) \otimes \mathbb{1} \oplus \mathbb{1}_M \otimes hs[\lambda] \oplus sl(M) \otimes hs[\lambda] \end{aligned}$$

  - It can be truncated at $\lambda = n$

$$sl(Mn) = sl(M) \otimes \mathbb{1}_n \oplus \mathbb{1}_M \otimes \underset{\substack{\Cap \\ \text{Gravitational } sl(2)}}{sl(n)} \oplus sl(M) \otimes sl(n)$$

Asymptotic AdS condition



- Solutions to the equations of motion for the CS theory are

$$A = e^{-\rho(\mathbb{1}_M \otimes V_0^2)} a(z) e^{\rho(\mathbb{1}_M \otimes V_0^2)} dz + (\mathbb{1}_M \otimes V_0^2) d\rho$$

- Asymptotic AdS condition is assigned
  [Henneaux-Rey'10; Campoleoni-Fredenhagen-Pfenninger-Theisen'10]

$$(A - A_{\text{AdS}})|_{\rho\to\infty} = \mathcal{O}((e^\rho)^0) \qquad (a_{\text{AdS}} = \mathbb{1}_M \otimes V_1^2)$$

  - The condition restricts the form of gauge field with

$$sl(Mn) = sl(M) \otimes \mathbb{1}_n \oplus \mathbb{1}_M \otimes sl(n) \oplus sl(M) \otimes sl(n)$$

$$a(z) = \mathcal{J}_a(z)(t^a \otimes \mathbb{1}_n) + a_{\text{AdS}} + \sum_{s=2}^{n} \mathcal{W}^{(s)}(z)(\mathbb{1}_M \otimes V^s_{-s+1})$$
$$+ \sum_{s=2}^{n} \mathcal{Q}_a^{(s)}(z)(t^a \otimes V^s_{-s+1})$$

Classical asymptotic symmetry

- Asymptotic symmetry is obtained as a Hamiltonian reduction of $sl(Mn)$ with the $sl(2)$ embedding
  [Henneaux-Rey'10; Campoleoni-Fredenhagen-Pfenninger-Theisen'10]
- The embedding can be classified by a partition

$Mn = n + n + \cdots + n \Leftrightarrow$ Rectangular Young diagram

- Generators of rectangular W-algebra come from gauge field

$$a(z) = \mathcal{J}_a(z)(t^a \otimes 1\!\!1_n) + a_{\text{AdS}} + \sum_{s=2}^{n} \mathcal{W}^{(s)}(z)(1\!\!1_M \otimes V^s_{-s+1}) + \sum_{s=2}^{n} \mathcal{Q}_a^{(s)}(z)(t^a \otimes V^s_{-s+1})$$

Generators of $sl(M)$

Higher spin generators $(T \equiv \mathcal{W}^{(2)})$

Charged higher spin generators

Quantum asymptotic symmetry

- The constraint is realized as BRST cohomology at quantum level
- The central charge c and the level k of $sl(M)$ can be computed from $sl(Mn)$ with level t
(e.g. [Kac-Wakimoto'03])

$$\begin{cases} c = \dfrac{t(n^2M^2-1)}{t+Mn} - Mtn(n^2-1) + M^2(n-1)(1-(n-1)^2(n+1)) \\ k = tn + Mn(n-1) \end{cases}$$

Shifts from BRST ghosts

$$\Rightarrow \quad c = -\frac{(k^2-1)n^2M}{k+nM} + kM - 1$$

- OPEs among generators are uniquely obtained for $n = 2$ by requiring their associativity
(cf. classical analysis [Joung-Kim²-Rey '17])

OPEs among generators

- Generators of the algebra
  - J^a: $su(M)$ currents with level k
  - T: Energy-momentum tensor with central charge c
  - Q^a: Spin 2 currents in the adjoint rep of $su(M)$
- OPEs among Q^a
  - Operator products generate composites of J^a, Q^a

$$Q^a \times Q^b = c_1 \delta^{ab}[\mathbb{1}] + c_2 f^{ab}{}_c[J^c] + C^{ab}_{3,cd}[J^{(c}J^{d)}] + C^{ab}_{4,cd}[J^{[c}\partial J^{d]}]$$
$$+ C^{ab}_{5,cde}[J^{(c}J^d J^{e)}] + c_6 d^{ab}{}_c[Q^a] + C^{ab}_{7,cd}[J^c Q^d]$$

  - There is only one parameter, say, k
    ➔ The central charge c is related to k as

$$c = -\frac{4(k^2-1)M}{k+2M} + kM - 1$$

Symmetry of dual coset

- The extended higher spin theory was proposed to be dual to the coset [Creutzig-YH-Rønne'13]

$$\frac{su(N+M)_k}{su(N)_k \oplus u(1)_{kNM(N+M)}}$$

- Holography suggest that the symmetry of the coset should be identified as the rectangular W-algebra
- The symmetry of the coset algebra has
  - $su(M)$ sub-algebra with level k
  - Central charge c

$$c_{\text{coset}}(k,N,M) = \frac{((N+M)^2-1)k}{k+N+M} - \frac{(N^2-1)k}{k+N} - 1$$

Coset algebra ⇔ W-algebra

- For the identification, we need the match of the central charge c & the level k of $su(M)$

$$c_{\text{coset}}(k,N,M) = c_{\text{W}}(t,n,M), \;\; k = tn + Mn(n-1)$$

$$\Rightarrow \quad \lambda \equiv \frac{k}{k+N} = n \quad \text{or} \quad \lambda' \equiv -\frac{k}{k+N+M} = n$$

Duality of coset (inherited from $hs_M[\lambda] = hs_M[-\lambda]$)

- OPEs among generators can be reproduced from the coset for $n = 2$
  - J^a come from $su(N+M)$
  - T is obtained by the standard coset construction
  - Q^a are constructed by requiring the OPEs with J^a, T

Duality of the coset CFT

- The coset symmetry algebra can be decomposed as

$$\frac{\mathrm{su}(N+M)_k}{\mathrm{su}(N)_k \oplus \mathrm{u}(1)} \supset \frac{\mathrm{su}(N+M)_k}{\mathrm{su}(N+M-1)_k \oplus \mathrm{u}(1)} \oplus \cdots \oplus \frac{\mathrm{su}(N+1)_k}{\mathrm{su}(N)_k \oplus \mathrm{u}(1)} \oplus (M-1)\mathrm{u}(1)$$

- A component coset is related to $W_\infty^c[\lambda]$

$$\frac{su(K+1)_L}{su(K)_L \oplus u(1)} \Leftrightarrow \frac{su(L)_K \oplus su(L)_1}{su(L)_{K+1}} \Leftrightarrow W_\infty^c[\lambda]$$

  - $W_\infty^c[\lambda]$ can be truncated to W_N at $\lambda = N$
  - Triality relation [Gaberdiel-Gopakumar'12]

$$\lambda = \frac{L}{L+K} \text{ or } \lambda = -\frac{L}{L+K+1} \text{ or } \lambda = L$$

- Component cosets are connected such as to have extra symmetry currents

$$\mathbb{Z}_3 \times \cdots \times \mathbb{Z}_3 \Rightarrow \mathbb{Z}_2$$: Duality of coset

(cf. Yangian description [Procházka-Rapčák'18; Gaberdiel,Li,Peng,Zhang'17;'18])

Summary
and further developments

- The asymptotic symmetry of extended 3d HS theory is identified as the rectangular W-algebra
- Its full quantum corrections were examined
  - Full expressions of c,k with s=1,2,…,n
  - Commutation relations among generators for n=2
  - Identified with the symmetry of dual CFT
- More general rectangular W-algebras are constructed from $so(Mn)$ or $sp(2mn)$ algebra and with $N = 1{,}2$ SUSY
  [in progress w/ Creutzig & Uetoko; see also Eberhardt-Gaberdiel-Rienacker'18]
- Degenerate representations of the W-algebra are studied in several ways and check consistency among them
  [in progress w/ Creutzig]