Université de Mons

complexys

Observables and boundary conditions in Vasiliev's higher-spin theory

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R. Bonezzi, N. Boulanger, D.D.F. and P. Sundell : arXiv :1705.03928
D.D.F., C. Iazeolla, P. Sundell : to appear

Centre
Algèbre,
Géométrie et
Interactions
Fondamentales

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Outline

Vasiliev's equations

- Non linear equations in extended $(x + Z)$-space
- Solve Z-dependent equations and gauge conditions
- Get interacting equations in x-space

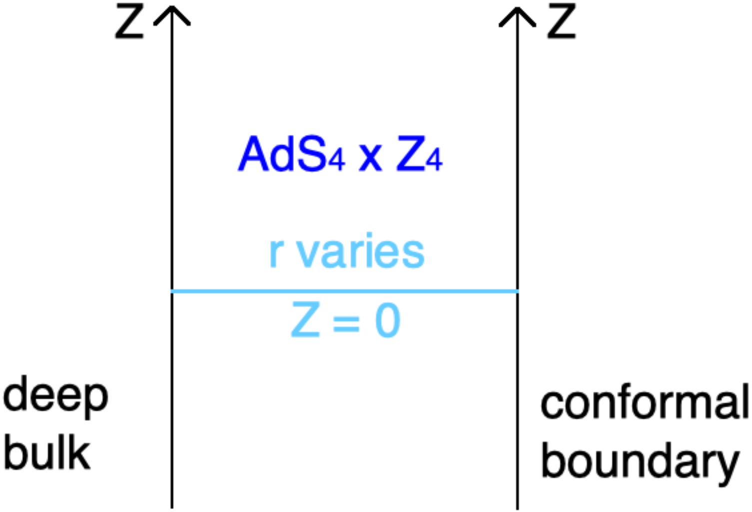
Using Z-space to extract observables
[*Bonezzi, Boulanger, D.D.F. & Sundell, 2017*]

- Wilson Lines closed in x-space and open in Z-space
- All correlators of free $U(N)$ and $O(N)$ vector models
- Matching at the level of cyclic structures

Resolving Z-space to impose boundary conditions
[*D.D.F., Iazeolla & Sundell, to appear*]

- Start from exact solutions
- Regular prescription compatible with Fronsdal fields
- Proposal for *ALAdS* boundary conditions

Resolving Z-space



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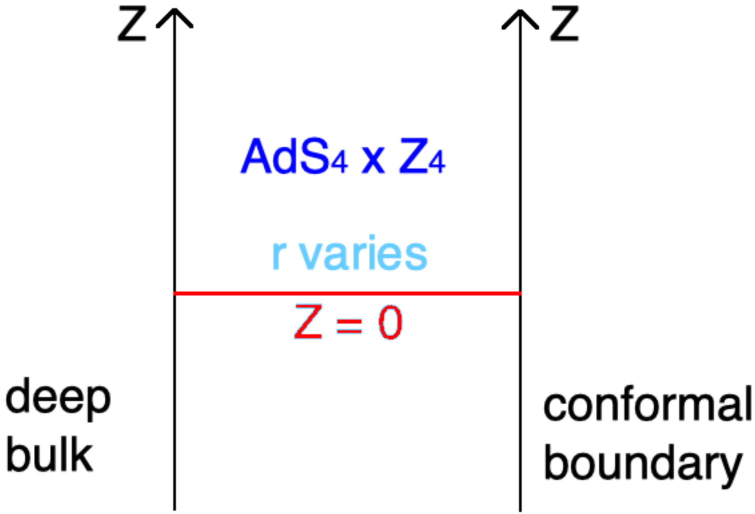

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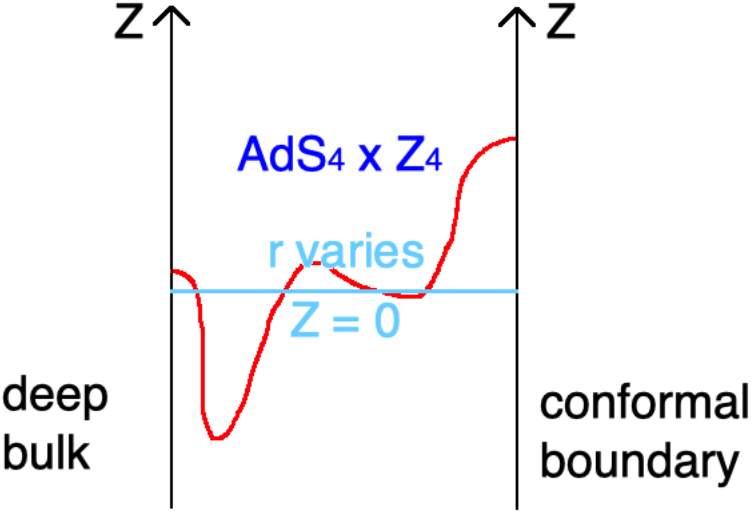

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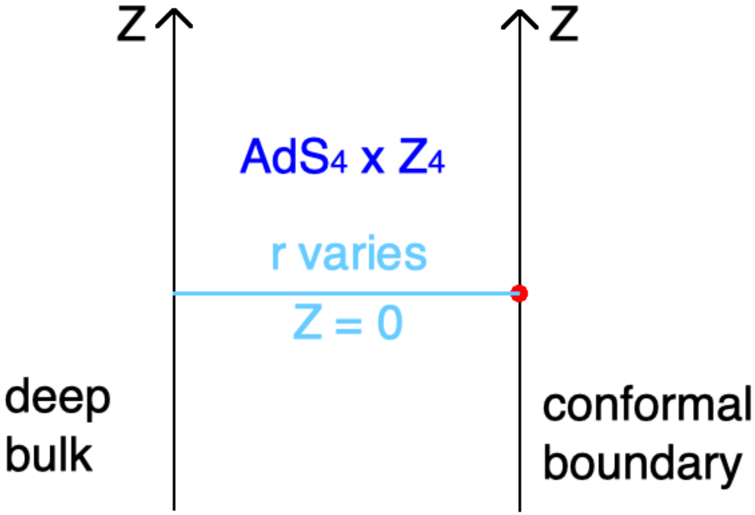

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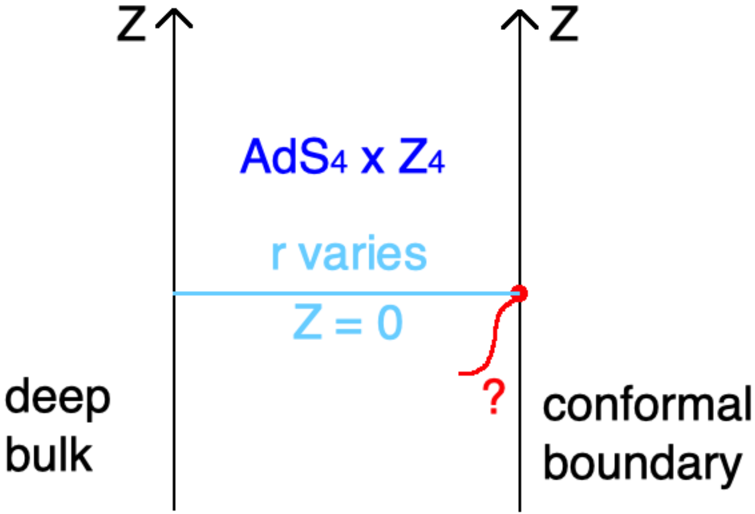

1. Vasiliev's equations
2. Observables à la Non-Commutative Yang-Mills
3. Holographic interpretation of observables
4. Solving around AdS_4
5. Asymptotic unfolded Fronsdal fields from exact solutions

Oscillator realization of the HS algebra

Higher-spin gauge fields on AdS_4

- $W(x;M,P) := \sum_{s,t} W^{a(s-1),b(t)}(x)(M_{ab})^t(P_a)^{s-t-1}$
- $[M_{ab},M_{cd}] = i(\eta_{ad}M_{bc}+\eta_{bc}M_{ad}-\eta_{ac}M_{bd}-\eta_{bd}M_{ac})$
  $[M_{ab},P_c] = i\left(\eta_{bc}P_a-\eta_{ca}P_b\right)$
  $[P_a,P_b] = i\lambda^2 M_{ab}$

Weyl algebra $(Y_{\underline{\alpha}}\,,\star)$

- $Y_{\underline{\alpha}} = (y_\alpha,\bar{y}_{\dot{\alpha}})$
- $[y_\alpha,y_\beta]_\star = 2i\epsilon_{\alpha\beta} \quad [y_\alpha,\bar{y}_{\dot{\beta}}]_\star = 0 \quad [\bar{y}_{\dot{\alpha}},\bar{y}_{\dot{\beta}}]_\star = 2i\epsilon_{\dot{\alpha}\dot{\beta}}$
- $M_{ab} = -\frac{1}{16}\left(\left(\sigma_{ab}\right)^{\alpha\beta}\{y_\alpha,y_\beta\}_\star + \left(\bar{\sigma}_{ab}\right)^{\dot{\alpha}\dot{\beta}}\{\bar{y}_{\dot{\alpha}},\bar{y}_{\dot{\beta}}\}_\star\right)$
  $P_a = -\frac{\lambda}{8}\left(\sigma_a\right)^{\alpha\dot{\alpha}}\{y_\alpha,\bar{y}_{\dot{\alpha}}\}_\star$

Higher-spin gauge fields

- $W(x;Y) := \sum_{m,\bar{m}} W_{\alpha(m)\dot{\alpha}(\bar{m})}(x)\left(y^\alpha\right)^{\star m}\star\left(\bar{y}^{\dot{\alpha}}\right)^{\star\bar{m}}$
- Bosonic projection $W(x;Y) = W(x;-Y)$

Master fields

Master Fields on $\mathcal{X}_4 \times \mathcal{Z}_4 \times \mathcal{Y}_4$

- Connection $A(x, Z; Y; \mathrm{d}x, \mathrm{d}Z) = \mathrm{d}x^\mu U_\mu + \mathrm{d}Z^{\underline{\alpha}} V_{\underline{\alpha}}$
- Zero-form $\Phi(x, Z; Y)$

Physical fields on $\mathcal{X}_4 \times \mathcal{Y}_4$

- $Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot{\alpha}})$ has to be eliminated
- Higher spin connection $W(x, Y) \sim A|_{Z=\mathrm{d}Z=0}$
- Weyl zero-form $C(x, Y) \sim \Phi|_{Z=0}$

Commutations relations

- $[Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha\beta}}$
- $[Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha\beta}}$
- $[Y_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = 0$

Weyl order among Y and among Z

- $Y_{\underline{\alpha}} \star Y_{\underline{\beta}} = Y_{\underline{\alpha}} Y_{\underline{\beta}} + iC_{\underline{\alpha\beta}}$
- $Z_{\underline{\alpha}} \star Z_{\underline{\beta}} = Z_{\underline{\alpha}} Z_{\underline{\beta}} - iC_{\underline{\alpha\beta}}$

Normal-order between $Y - Z$ and $Y + Z$

- $Y_{\underline{\alpha}} \star Z_{\underline{\beta}} = Y_{\underline{\alpha}} Z_{\underline{\beta}} - iC_{\underline{\alpha\beta}}$
- $Z_{\underline{\beta}} \star Y_{\underline{\alpha}} = Z_{\underline{\beta}} Y_{\underline{\alpha}} - iC_{\underline{\alpha\beta}}$

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Physical fields on $\mathcal{X}_4 \times \mathcal{Y}_4$

- $Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot{\alpha}})$ has to be eliminated
- Higher spin connection $W(x, Y) = \mathcal{P}_{H(\mathrm{d}_z)} A$
- Weyl zero-form $C(x, Y) = \mathcal{P}_{H(\mathrm{d}_z)} \Phi$

Commutations relations

- $[Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha}\underline{\beta}}$
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(Anti)automorphisms

Kinematical contraints

- Reality condition $A^\dagger = -A$ $\Phi^\dagger = \pi(\Phi)$
- Bosonic projection $\pi\bar{\pi}(A) = A$ $\pi\bar{\pi}(\Phi) = \Phi$
- Minimal bosonic projection $\tau(A) = -A$ $\tau(\Phi) = \pi(\Phi)$

Hermitian conjugation

- $(f(x, z, \bar{z}; y, \bar{y}; \mathrm{d}x, \mathrm{d}z, \mathrm{d}\bar{z}))^\dagger = f^*(x, \bar{z}, z; \bar{y}, y; \mathrm{d}x, \mathrm{d}\bar{z}, \mathrm{d}z)$
- Star product is however defined in terms of real oscillators Y and Z

π**-map**

- $\pi(f(x, z, \bar{z}; y, \bar{y}; \mathrm{d}x, \mathrm{d}z, \mathrm{d}\bar{z})) = f(x; -z, \bar{z}; -y, \bar{y}; \mathrm{d}x; -\mathrm{d}z, \mathrm{d}\bar{z})$
- $\bar{\pi}(f(x; z, \bar{z}; y, \bar{y}; \mathrm{d}x; \mathrm{d}z, \mathrm{d}\bar{z})) = f(x; z, -\bar{z}; y, -\bar{y}; \mathrm{d}x; \mathrm{d}z, -\mathrm{d}\bar{z})$

τ**-map**

- $\tau(f(x; z, \bar{z}; y, \bar{y}; \mathrm{d}x; \mathrm{d}z, \mathrm{d}\bar{z})) = f(x; -iz, -i\bar{z}; iy, i\bar{y}; \mathrm{d}x; -i\mathrm{d}z, -i\mathrm{d}\bar{z})$

Gauge transformations

Kinematical contraints on the master fields

- Reality condition $A^\dagger = -A$ $\Phi^\dagger = \pi(\Phi)$
- Bosonic projection $\pi\bar{\pi}(A) = A$ $\pi\bar{\pi}(\Phi) = \Phi$
- Minimal bosonic projection $\tau(A) = -A$ $\tau(\Phi) = \pi(\Phi)$

Gauge symmetry

- $M(x, Z; Y)$
- $A \to M^{\star -1} \star \mathrm{d}M + M^{\star -1} \star A \star M$
- $\Phi \to M^{\star -1} \star \Phi \star \pi(M)$

Kinematical contraints on the gauge parameters

- Reality condition $M^\dagger = M^{\star -1}$
- Bosonic projection $\pi\bar{\pi}(M) = M$
- Minimal bosonic projection $\tau(M) = M^{\star -1}$

Vasiliev's Equations

[*Vasiliev, 1989*]

Field equations

- $\mathrm{d}A + A \star A = -\Phi \star (J + \bar{J})$
- $\mathrm{d}\Phi + A \star \Phi - \Phi \star \pi(A) = 0$

Inner kleinians and source

- $\kappa := e^{iyz}$
- $J = -\frac{ib}{4}\kappa\, \mathrm{d}^2 z$
- $f \star J = J \star \pi(f)$
- $\bar{\kappa} := e^{-i\bar{y}\bar{z}}$
- $\bar{J} = -\frac{i\bar{b}}{4}\bar{\kappa}\, \mathrm{d}^2 \bar{z}$
- $f \star \bar{J} = \bar{J} \star \bar{\pi}(f)$

Gauge symmetry

- $A \to M^{\star -1} \star \mathrm{d}M + M^{\star -1} \star A \star M$
- $\Phi \to M^{\star -1} \star \Phi \star \pi(M)$
- Captures all x-dependence of master fields

AdS_4 **solution**

- $\Phi^{(0)} = V^{(0)} = 0$
- $U^{(0)} = \Omega(x, Y) = e^a_{AdS} P_a + \frac{1}{2}\omega^{ab}_{AdS} M_{ab}$

Non-commutative gauge theories

In Non-Commutative Yang Mills theory
[*Ambjørn, Makeenko, Nishimura & Szabo, 2000*]
[*Gross, Hashimoto & Ithzaki, 2000*]

In Vasiliev's theory

Non-commutative gauge theories

In Non-Commutative Yang Mills theory
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- $[x^\mu, x^\nu]_\star = i\theta^{\mu\nu}$

In Vasiliev's theory

- $[x^\mu, x^\nu]_\star = 0 \qquad [x^\mu, Z_{\underline{\alpha}}]_\star = 0 \qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha}\underline{\beta}}$

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- $[x^\mu, x^\nu]_\star = i\theta^{\mu\nu}$
- Associative algebra $U(N)$ with Tr

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- Associative algebra $\mathcal{F}(\mathcal{Y}_4)$ with $\mathrm{Tr}_Y := \int \mathrm{d}^4 Y$

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- $[x^\mu, x^\nu]_\star = i\theta^{\mu\nu}$
- Associative algebra $U(N)$ with Tr
- *NCYM* action

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- Associative algebra $\mathcal{F}(\mathcal{Y}_4)$ with $\mathrm{Tr}_Y := \int \mathrm{d}^4 Y$
- Vasiliev's equations

Non-commutative gauge theories

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[*Gross, Hashimoto & Ithzaki, 2000*]

- $[x^\mu, x^\nu]_\star = i\theta^{\mu\nu}$
- Associative algebra $U(N)$ with Tr
- *NCYM* action
- $A \longrightarrow g \star \mathrm{d}g^{-1} + g \star A \star g^{-1}$

In Vasiliev's theory

- $[x^\mu, x^\nu]_\star = 0 \qquad [x^\mu, Z_{\underline{\alpha}}]_\star = 0 \qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha}\underline{\beta}}$
- Associative algebra $\mathcal{F}(\mathcal{Y}_4)$ with $\mathrm{Tr}_Y := \int \mathrm{d}^4 Y$
- Vasiliev's equations
- $A \to M^{\star-1} \star \mathrm{d}M + M^{\star-1} \star A \star M$
- $\Phi \to M^{\star-1} \star \Phi \star \pi(M)$

Wilson Lines

Curve

- $\mathcal{C} : [0, 1] \to \mathcal{X}_4 : \sigma \to \xi^\mu(\sigma)$

Wilson Line

- $W_{\mathcal{C}}(x; Y) := P \exp_\star \int_0^1 \mathrm{d}\sigma \dot{\xi}^\mu(\sigma) U_\mu(\sigma)$
  $A(\sigma) := A(x + \xi(\sigma); Y)$
- $W_{\mathcal{C}}(x; Y) \longrightarrow M(x; Y) \star W_{\mathcal{C}}(x; Y) \star M(x + \xi(1); Y)^{\star -1}$

Closed Wilson Lines

$$\widetilde{I}_{\mathcal{C}}(x) := \mathrm{Tr}_Y \left[W_{\mathcal{C}}(x; Y) \right]$$

Curve

- $\mathcal{C} : [0,1] \to \mathcal{X}_4 : \sigma \to \xi^\mu(\sigma)$
- $\xi^\mu(0) = \xi^\mu(1) = 0$

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Closed Wilson Lines

$$\widetilde{O}_{\mathcal{C}}(x) := \mathrm{Tr}_Y \left[O(x;Y) \star W_{\mathcal{C}}(x;Y) \right]$$

Curve

- $\mathcal{C} : [0,1] \to \mathcal{X}_4 : \sigma \to \xi^\mu(\sigma)$
- $\xi^\mu(0) = \xi^\mu(1) = 0$

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- $W_{\mathcal{C}}(x;Y) := P \exp_\star \int_0^1 \mathrm{d}\sigma \dot{\xi}^\mu(\sigma) U_\mu(\sigma)$
  $A(\sigma) := A(x + \xi(\sigma); Y)$
- $W_{\mathcal{C}}(x;Y) \longrightarrow M(x;Y) \star W_{\mathcal{C}}(x;Y) \star M(x+\xi(1);Y)^{\star -1}$

Adjoint operator

- $O(x;Y)$
- $O \longrightarrow M \star O \star M^{\star -1}$

Closed Wilson Lines

$$\widetilde{O}_{\mathcal{C}}(x) := \int d^4 Z \ \mathrm{Tr}_Y \left[O(x, Z; Y) \star W_{\mathcal{C}}(x, Z; Y) \right]$$

Curve

- $\mathcal{C} : [0,1] \to \mathcal{X}_4 \times \mathcal{Z}_4 : \sigma \to (\xi^{\mu}(\sigma), \xi^{\underline{\alpha}}(\sigma))$
- $\xi^{\mu}(0) = \xi^{\mu}(1) = 0 \quad \xi^{\underline{\alpha}}(0) = 0 \quad \xi^{\underline{\alpha}}(1) = 0$

Wilson Line

- $W_{\mathcal{C}}(x, Z; Y) := P \exp_{\star} \int_0^1 d\sigma \left(\dot{\xi}^{\mu}(\sigma) U_{\mu}(\sigma) + \dot{\xi}^{\underline{\alpha}}(\sigma) V_{\underline{\alpha}}(\sigma) \right)$
  $A(\sigma) := A(x + \xi(\sigma), Z + \xi(\sigma); Y)$
- $W_{\mathcal{C}}(x, Z; Y) \longrightarrow$
  $M(x, Z; Y) \star W_{\mathcal{C}}(x, Z; Y) \star M(x + \xi(1), Z + \xi(1); Y)^{\star -1}$

Adjoint operator

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Open Wilson Lines

$$\widetilde{O}_{\mathcal{C}}(x) := \int d^4Z \ \mathrm{Tr}_Y \left[O(x,Z;Y) \star W_{\mathcal{C}}(x,Z;Y) \star e^{iMZ} \right]$$

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Wilson Line

- $W_{\mathcal{C}}(x,Z;Y) := P \exp_\star \int_0^1 d\sigma \left(\dot{\xi}^\mu(\sigma) U_\mu(\sigma) + \dot{\xi}^{\underline{\alpha}}(\sigma) V_{\underline{\alpha}}(\sigma) \right)$
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Adjoint operator

- $O(x,Z;Y)$
- $O \longrightarrow M \star O \star M^{\star -1}$

Translation

- $f(x, Z + 2M; Y) \star e^{iMZ} = e^{iMZ} \star f(x,Z;Y)$

Open Wilson Lines

$$\widetilde{O}_{\mathcal{C}}(x) := \int d^4 Z\, d^4 Y\, \left[O(x, Z; Y) \star W_{\mathcal{C}}(x, Z; Y) \star e^{iMZ} \right]$$

Curve

- $\mathcal{C} : [0,1] \to \mathcal{X}_4 \times \mathcal{Z}_4 : \sigma \to (\xi^{\mu}(\sigma), \xi^{\underline{\alpha}}(\sigma))$
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- $f(x, Z + 2M; Y) \star e^{iMZ} = e^{iMZ} \star f(x, Z; Y)$

Straight Wilson Lines

$$\widetilde{O}_{\mathcal{C}}(x) := \int d^4Z\, d^4Y \left[O(x, Z; Y) \star W_{\mathcal{C}}(x, Z; Y) \star e^{iMZ} \right]$$

Straight Wilson Lines

$$\widetilde{O}_{\mathcal{C}}(x) := \int \mathrm{d}^4 Z\ \mathrm{d}^4 Y\ \left[O(x, Z; Y) \star W_{\mathcal{C}}(x, Z; Y) \star e^{iMZ} \right]$$

Straight Line

- Information about the shape of $\mathcal{C}$ can be absorbed in O
- $L_{2M} : [0,1] \to \mathcal{X}_4 \times \mathcal{Z}_4 : \sigma \to (0, 2\sigma M^{\underline{\alpha}})$
- $W_{L_{2M}}(x, Z; Y) \star \exp(iMZ) = \exp\left(iM\left(Z - 2iA\right)\right)$

Adjoint operators on-shell

- $(\Phi \star \kappa)^{\star n_0}$
- U_μ is pure gauge
- $S_{\underline{\alpha}} := Z_{\underline{\alpha}} - 2iV_{\underline{\alpha}}$
  - $(S_{\underline{\alpha}})^{\star K} \sim (-i\partial^M_{\underline{\alpha}})^{\star K}$
  - $[S_{\underline{\alpha}}, S_{\underline{\beta}}]_\star = -2i\epsilon_{\alpha\beta}(1 - b\widehat{\Phi} \star \kappa)$
- $(\kappa\bar{\kappa})^{\star t} \qquad t = 0, 1$

Zero-Form Charges

$$\mathcal{I}_{n_0,t}(M) := \int d^4Y\, d^4Z\ \left[(\Phi \star \kappa)^{\star n_0} \star (\kappa\bar{\kappa})^{\star t} \star e^{iMS}\right]$$

On shell action from zero form charges
[*Colombo & Sundell, 2012*]

- $\mathcal{I}_{n_0,t}(0)$ was considered a priori
- e^{iMS} was introduced artificially as a regulator
- Building blocks for on-shell action

Perturbation around *AdS*

- Background $\Phi^{(0)} = 0 \qquad S^{(0)}_{\underline{\alpha}} = Z_{\underline{\alpha}}$
- First order $\Phi^{(1)} = C(x;Y)$

$$\mathcal{I}^{(n_0)}_{n_0,t}(M) = \int d^4Z\, d^4Y\ \left[\left(\Phi^{(1)} \star \kappa\right)^{\star n_0} \star (\kappa\bar{\kappa})^{\star t} \star e^{iMZ}\right]$$

Higher spin geometry

Gauge symmetry

- $A \to M^{\star -1} \star \mathrm{d}M + M^{\star -1} \star A \star M$
- $\Phi \to M^{\star -1} \star \Phi \star \pi(M)$

Structure group G **with** $\mathrm{Lie}(G) \subset \mathcal{A}(\mathcal{Y}_4, \mathcal{Z}_4)$
[*Sezgin & Sundell, 2012*]

- Splitting of connection $A = \Gamma + E$
  - Γ is a connection
  - E is a section
- Splitting of gauge parameters M
  - M_Γ are Cartan gauge parameters
  - « small » M_E are gauge parameters
  - « large » M_E are dressing fields

More observables

- Left invariant by M_Γ and small M_E, affected by large M_E
- Might have « space-time dependence »
- Example : $\int \mathrm{d}^4x \;\; \mathrm{Tr}\left(E^{\star 4}\right)$

Bulk to boundary propagator

Poincaré coordinates

- $ds^2 = \frac{1}{r^2}\eta_{\mu\nu}dx^\mu dx^\nu$
- Conformal boundary $r = 0$
- Reference bulk point (r_0, x_0)
- Boundary points $(0, x_i)$

Propagator of the master field Φ
[*Giombi & Yin, 2009*]

$$\mathcal{K}_i(x_0, x_i, \chi_i|Y) := \frac{r_0}{(x_{0,i})^2} e^{iy\Sigma_i\bar{y}} \sum_{\sigma_i=\pm1} \left(b e^{i\sigma_i \bar{\nu}_i \bar{\Sigma}_i y} + \bar{b} e^{i\sigma_i \nu_i \Sigma_i \bar{y}} \right)$$

- Polarization spinor χ_i and $(\chi_i)^\dagger = \bar{\chi}_i = \bar{\sigma}^r \chi_i$
- $\Sigma_i := \sigma^r - \frac{2r_0}{(x_{0,i})^2} x_{0,i}$
- $\nu_i := \frac{\sqrt{2r_0}}{(x_{0,i})^2} \Sigma_i \, x_{0,i} \, \chi_i$ and $(\nu_i)^\dagger = \bar{\nu}_i = -\bar{\Sigma}_i \nu_i$

Pre-amplitudes

$$\mathcal{I}^{(n_0)}_{n_0,t}(M) = \int \mathrm{d}^4 Z\, \mathrm{d}^4 Y \left[\left(\Phi^{(1)} \star \kappa \right)^{\star n_0} \star (\kappa\bar{\kappa})^{\star t} \star e^{iMZ} \right]$$

Towards amplitudes

- We have propagators $\mathcal{K}_i$
- $\mathcal{I}^{(n_0)}_{n_0,t}(M)$ needs legs to plug $\mathcal{K}_i$

Quasi-amplitudes

[*Colombo & Sundell, 2012*]

- $\mathcal{Q}^{(n)}_{n_0,t}(\mathcal{K}_i|M)\Big|_{\mathcal{K}_1=\ldots=\mathcal{K}_n=\Phi} = \mathcal{I}^{(n)}_{n_0,t}(M)$
- Invariant under permutations of $\mathcal{K}_i$

Pre-amplitudes

- $\mathcal{A}_{n_0,t}(\mathcal{K}_i|M) := \int \mathrm{d}^4 Z\, \mathrm{d}^4 Y \;\bigstar_{i=1}^{n_0} (\mathcal{K}_i \star \kappa) \star (\kappa\bar{\kappa})^{\star t} \star e^{iMZ}$
- $\mathcal{Q}_{n_0,t}(\mathcal{K}_i|M) = \sum_{\mathrm{perm.}\mathcal{K}_j} \mathcal{A}_{n_0,t}(\mathcal{K}_i|M)$
- Invariant under cyclic permutations of $\mathcal{K}_i$

Pre-amplitudes

$$\mathcal{I}^{(n_0)}_{n_0,t} = \int d^4M \int d^4Z\, d^4Y \left[\left(\Phi^{(1)} \star \kappa\right)^{\star n_0} \star (\kappa\bar{\kappa})^{\star t} \star e^{iMZ}\right]$$

Towards amplitudes

- We have propagators $\mathcal{K}_i$
- $\mathcal{I}^{(n_0)}_{n_0,t}(M)$ needs legs to plug $\mathcal{K}_i$

Quasi-amplitudes

[*Colombo & Sundell, 2012*]

- $\mathcal{Q}^{(n)}_{n_0,t}(\mathcal{K}_i|M)\Big|_{\mathcal{K}_1=\ldots=\mathcal{K}_n=\Phi} = \mathcal{I}^{(n)}_{n_0,t}(M)$
- Invariant under permutations of $\mathcal{K}_i$

Pre-amplitudes

- $\mathcal{A}_{n_0,t}(\mathcal{K}_i|M) := \int d^4Z\, d^4Y\ \bigstar_{i=1}^{n_0} (\mathcal{K}_i \star \kappa) \star (\kappa\bar{\kappa})^{\star t} \star e^{iMZ}$
- $\mathcal{Q}_{n_0,t}(\mathcal{K}_i|M) = \sum_{\text{perm.}\mathcal{K}_j} \mathcal{A}_{n_0,t}(\mathcal{K}_i|M)$
- Invariant under cyclic permutations of $\mathcal{K}_i$

Correlators from Open Wilson Line

$$\mathcal{A}_{n_0,t}\left(\mathcal{K}_i\right) = \beta_{n_0,t}\ \exp\left(-\frac{i}{4}\sum_{i=1}^{n_0} Q_i\right)\left(\prod_{i=1}^{n_0}\frac{1}{|x_{i,i+1}|}\right)$$
$$\times\left(\left(\frac{b+\bar{b}}{2}\right)^{n_0}\prod_{i=1}^{n_0}\cos\left(\frac{1}{2}P_{i,i+1}\right)-(-1)^t\left(\frac{b-\bar{b}}{2i}\right)^{n_0}\prod_{i=1}^{n_0}\sin\left(\frac{1}{2}P_{i,i+1}\right)\right)$$

Parity-invariant models

- $b = 1$: Type A
- $b = i$: Type B

Our method

- Reproduces [*Didenko & Skvortsov, 2012*]
- Is amenable to Z-space-induced interactions
- Includes the $(\kappa\bar{\kappa})$ insertion in $\mathcal{I}^{(n_0)}_{n_0,t}$

Conformal structures

- $P_{i,i+1} := (x_{i,i+1})^{-2}\chi_i\,\sigma^r\,\bar{x}_{i,i+1}\,\chi_{i+1}$
- $Q_i := \chi_i\,\sigma^r\,\left((x_{i,i+1})^{-2}\bar{x}_{i,i+1}-(x_{i,i-1})^{-2}\bar{x}_{i,i-1}\right)\,\chi_i$

Free U(N) vector model

Free U(N) vector model

Propagators

- $\langle\phi^i(x)\phi^j(y)\rangle = 0 = \langle\phi^*_i(x)\phi^*_j(y)\rangle \quad \langle\phi^i(x)\phi^*_j(y)\rangle = \delta^i{}_j\,|x-y|^{\frac{1}{d-2}}$

Conserved currents

[*Craigie, Dobrev & Todorov 1985*] [*Giombi & Yin 2009*]

- $\sum_{s=0}^{\infty} a_s J_{\mu(s)}(x)\,(\epsilon^\mu)^s = J(x,\epsilon) = \phi^*_i(x) f\left(\epsilon, \overleftarrow{\partial}, \overrightarrow{\partial}\right)\phi^i(x)$
- $f(\epsilon,u,v) = \sum_{k,\ell,m,p,q} f_{k,\ell,m,p,q}(\epsilon\cdot u)^k(\epsilon\cdot v)^\ell((u\cdot v)\epsilon^2)^m(u^2\epsilon^2)^p(v^2\epsilon^2)^q$
- Traceless : $\partial_\epsilon^2 f(\epsilon,u,v) = 0$
- Conserved modulo $\Box\phi$: $\partial_\epsilon\cdot(\partial_u+\partial_v)f(\epsilon,u,v)|_{u^2=v^2=0} = 0$

Ambiguities

- $(\epsilon^\mu)^s \to (\epsilon^\mu)^s + (\eta^{\mu(2)})^\ell(\epsilon^2)^\ell(\epsilon^\mu)^{s-2\ell}$
- Normalisation of $J_{\mu(s)}(x)\,(\epsilon^\mu)^s$

$$f(u,v,\epsilon) = \sum_{s,k}\binom{s}{k}\frac{\Gamma(\frac{d-2}{2})}{s!\,\Gamma(k+\frac{d-2}{2})\Gamma(s-k+\frac{d-2}{2})}\left(\frac{i}{4}\right)^s(-\epsilon\cdot u)^k(\epsilon\cdot v)^{s-k}$$

Correlator of the free U(N) vector model

Connected n-point function

- $\langle J_1, \ldots, J_{n_0}\rangle_{\text{conn.}} = \sum_{\text{perm.}J_i} \langle J_1, \ldots, J_{n_0}\rangle_{\text{cyclic}}$
- $\langle J_1, \ldots, J_{n_0}\rangle_{\text{cyclic}} :=$
  $\frac{1}{N} \prod_{i=1}^{n_0} f\left(\partial_{x_i'}, \partial_{x_i}, \epsilon_i\right) \prod_{j=1}^{n_0} \langle \phi^{i_j}(x_j)\phi^*_{i_{j+1}}(x'_{j+1})\rangle \Big|_{x_k'=x_k \forall k}$
- $\langle J_1, \ldots, J_{n_0}\rangle_{\text{cyclic}} = \prod_{i=1}^{n_0} \exp\left(-\frac{i}{4} Q_i\right) \sum_{c_i} \frac{\left(\frac{i}{4} P_{i,i+1}\right)^{2c_i}}{c_i! \, \Gamma(c_i + \frac{d-2}{2})} |x_{i,i+1}|^{2-d}$

Conformal structures

- $Q_i = 2\epsilon_i \cdot ((x_{i-1,i})^{-2} x_{i-1,i} + (x_{i,i+1})^{-2} x_{i,i+1})$
- $P^2_{i,i+1} = 4(x_{i,i+1})^{-4} \left((\epsilon_i \cdot x_{i,i+1})(\epsilon_{i+1} \cdot x_{i,i+1}) - \frac{1}{2}((\epsilon_i \cdot \epsilon_{i+1}))x^2_{i,i+1}\right)$

Correlator of the free U(N) vector model

Connected n-point function

- $\langle J_1, \ldots, J_{n_0} \rangle_{\text{conn.}} = \sum_{\text{perm.} J_i} \langle J_1, \ldots, J_{n_0} \rangle_{\text{cyclic}}$
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  $\frac{1}{N} \prod_{i=1}^{n_0} f\left(\partial_{x_i'}, \partial_{x_i}, \epsilon_i\right) \prod_{j=1}^{n_0} \langle \phi^{i_j}(x_j) \phi^*_{i_{j+1}}(x'_{j+1}) \rangle \Big|_{x'_k = x_k \forall k}$
- $\langle J_1, \ldots, J_{n_0} \rangle_{\text{cyclic}} = \prod_{i=1}^{n_0} \exp\left(-\frac{i}{4} Q_i\right) \sum_{c_i} \frac{\left(\frac{i}{4} P_{i,i+1}\right)^{2c_i}}{c_i!\, \Gamma(c_i + \frac{d-2}{2})} \left|x_{i,i+1}\right|^{2-d}$

Conformal structures in 3 dimensions

- $Q_i = \chi_i\, \sigma^r \left((x_{i,i+1})^{-2} \bar{x}_{i,i+1} - (x_{i,i-1})^{-2} \bar{x}_{i,i-1} \right) \chi_i$
- $P^2_{i,i+1} = (x_{i,i+1})^{-4} \left(\chi_i\, \sigma^r\, \bar{x}_{i,i+1}\, \chi_{i+1}\right)^2$

Spinorial language in 3 dimensions

- Polarisation vector : $(\chi_i)_\alpha\, (\bar{\chi}_i)_{\dot\alpha} = \epsilon_i^\mu\, (\sigma_\mu)_{\alpha\dot\alpha}$

Correlator of the free U(N) vector model

In d dimensions

- $\langle J_1, \dots, J_{n_0} \rangle_{\text{cyclic}} = \prod_{i=1}^{n_0} \exp\left(-\frac{i}{4} Q_i\right) \sum_{c_i} \frac{\left(\frac{i}{4} P_{i,i+1}\right)^{2c_i}}{c_i!\,\Gamma(c_i + \frac{d-2}{2})} \left|x_{i,i+1}\right|^{2-d}$

In the literature

- [*Giombi & Yin, 2009*] : 3-point in 3 dimensions
- [*Sleight & Taronna, 2016*] : 3-point in any dimension
- [*Gelfond & Vasiliev, 2016*] : n-point in terms of spinors

In 3 dimensions

- $\langle J_1, \dots, J_{n_0} \rangle_{\text{cyclic}} = \prod_{i=1}^{n_0} \frac{1}{\sqrt{\pi}} \exp\left(-\frac{i}{4} Q_i\right) \cos\left(\frac{1}{2} P_{i,i+1}\right) \frac{1}{\left|x_{i,i+1}\right|}$

Holographic correspondence

- At linear order in perturbation
- In type A model
- Cyclic : supports Topological Open String interpretation [*Engquist & Sundell, 2006*]

Free $O(N)$ model and minimal bosonic projection

In the free U(N) vector model

- $\langle \phi^i(y)\phi^*_j(y)\rangle = \delta^{ij}\,|x-y|^{\frac{1}{d-2}}$
- $J(x,\epsilon) = \phi^*_i(x) f\left(\epsilon, \overleftarrow{\partial}, \overrightarrow{\partial}\right)\phi^i(x)$

Free $O(N)$ model and minimal bosonic projection

In the free U(N) vector model

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- $J(x,\epsilon) = \phi_i(x) f\left(\epsilon, \overleftarrow{\partial}, \overrightarrow{\partial}\right)\phi^i(x)$
  This symmetrizes $\overleftarrow{\partial}$ and $\overrightarrow{\partial}$
- Previously : $f(\epsilon, v, u) = f(-\epsilon, u, v)$
  One has to project the correlator on the part which is even in all ϵ_i

Free $O(N)$ model and minimal bosonic projection

In the free O(N) vector model

- $\langle \phi^i(y)\phi^j(x)\rangle = \delta^{ij}\,|x-y|^{\frac{1}{d-2}}$
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- Previously : $f(\epsilon, v, u) = f(-\epsilon, u, v)$
  One has to project the correlator on the part which is even in all ϵ_i

In Vasiliev's theory

- $\Phi = \pi\tau\Phi \qquad \tau(A) = -A \qquad \tau(M) = M^{\star -1}$
- $\tau(f(x;z,\bar{z};y,\bar{y};\mathrm{d}x;\mathrm{d}z,\mathrm{d}\bar{z})) = f(x;-iz,-i\bar{z};iy,i\bar{y};\mathrm{d}x;-i\mathrm{d}z,-i\mathrm{d}\bar{z})$
- $\pi\tau\mathcal{K}_i(x_0,x_i,\chi_i|Y) = \mathcal{K}_i(x_0,x_i,i\chi_i|Y)$
- Polarisation vector : $(\chi_i)_\alpha\,(\bar{\chi}_i)_{\dot{\alpha}} = \epsilon_i^\mu\,(\sigma_\mu)_{\alpha\dot{\alpha}}$
- One has to project the preamplitude on the part which is even in all ϵ_i

Vasiliev's equations

Field equations on $\mathcal{X}_4 \times \mathcal{Z}_4$

- $\mathrm{d}A + A \star A = -\Phi \star (J + \bar{J})$
- $\mathrm{d}\Phi + A \star \Phi - \Phi \star \pi(A) = 0$

Splitting geometric objects

- Differential $\mathrm{d} = \mathrm{d}_x + \mathrm{d}_z$
- Connection $A(x, Z; Y; \mathrm{d}x, \mathrm{d}Z) = \mathrm{d}x^\mu U_\mu + \mathrm{d}Z^{\underline{\alpha}} V_{\underline{\alpha}}$

Field equations on $\mathcal{X}_4$ and on $\mathcal{Z}_4$

- $\mathrm{d}_z \Phi + V \star \Phi - \Phi \star \pi(V) = 0$
- $\mathrm{d}_x \Phi + V \star \Phi - \Phi \star \pi(V) = 0$
- $\mathrm{d}_z V + V \star V = -\Phi \star (J + \bar{J})$
- $\mathrm{d}_z U + \mathrm{d}_x V + U \star V + V \star U = 0$
- $\mathrm{d}_x U + U \star U = 0$

Linearized equations

AdS_4 background

- $\Phi^{(0)} = V^{(0)} = 0$
- $U^{(0)} = \Omega(x, Y) = \frac{1}{4i}\left(\omega^{(0)\alpha\beta} y_\alpha y_\beta + \bar{\omega}^{(0)\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}} + 2e^{(0)\alpha\dot{\beta}} y_\alpha \bar{y}_{\dot{\beta}}\right)$

Covariant derivatives

- $\mathcal{D}_{\text{ad}} := \text{d}_x + \Omega \star \bullet - \bullet \star \Omega$
- $\mathcal{D}_{\text{tw}} := \text{d}_x + \Omega \star \bullet - \bullet \star \pi(\Omega)$

Expanding around AdS_4

- $\text{d}_z \, \Phi^{(1)} = 0$
- $\mathcal{D}_{\text{tw}} \Phi^{(1)} = 0$
- $\text{d}_z \, V^{(1)} = -\Phi^{(1)} \star (J + \bar{J})$
- $\text{d}_z \, U^{(1)} = -\mathcal{D}_{\text{ad}} V^{(1)}$
- $\mathcal{D}_{\text{ad}} U^{(1)} = 0$

Linearized equations

AdS_4 background

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- $U^{(0)} = \Omega(x, Y) = \frac{1}{4i}\left(\omega^{(0)\alpha\beta} y_\alpha y_\beta + \bar{\omega}^{(0)\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}} + 2e^{(0)\alpha\dot{\beta}} y_\alpha \bar{y}_{\dot{\beta}}\right)$

Covariant derivatives

- $\mathcal{D}_{\mathrm{ad}} = \mathrm{d}_x + \Omega^{\underline{\alpha\beta}} Y_{\underline{\alpha}} \partial^Y_{\underline{\beta}} - i\Omega^{\underline{\alpha\beta}} \partial^Y_{\underline{\alpha}} \partial^Z_{\underline{\beta}}$
- $\mathcal{D}_{\mathrm{tw}} = \mathrm{d}_x + \omega(Y, \partial^Y) - i\omega(\partial^Y, \partial^Z) - \frac{i}{2} e(Y, Y) - e(Y, \partial^Z) + \frac{i}{2} e(\partial^Y, \partial^Y) + \frac{i}{2} e(\partial^Z, \partial^Z)$

Expanding around AdS_4

- $\mathrm{d}_z \Phi^{(1)} = 0$
- $\mathcal{D}_{\mathrm{tw}} \Phi^{(1)} = 0$
- $\mathrm{d}_z V^{(1)} = -\Phi^{(1)} \star (J + \bar{J})$
- $\mathrm{d}_z U^{(1)} = -\mathcal{D}_{\mathrm{ad}} V^{(1)}$
- $\mathcal{D}_{\mathrm{ad}} U^{(1)} = 0$

Resolution operators

Equation

- $\mathrm{d}_z f = g$

General solution

- $g = q^{*(A)} f + \mathrm{d}_z \, \epsilon^{(A)} + C^{(A)}$
  - Particular solution $q^{*(A)} f$
  - Gauge parameter $\epsilon^{(A)}$
  - Cohomological element $C^{(A)}$

Cohomology of d_z

[*Didenko, Misuna & Vasiliev, 2015*]

- Consist of Z-space 0-form in (unextended) Vasiliev's system
- Can be reached via $\mathcal{P}^{(A)} := 1 - \mathrm{d}_z \, q^{*(A)} - q^{*(A)} \mathrm{d}_z$

Solution in (A)-gauge

- $g^{(A)} = q^{*(A)} f + C^{(A)}$

Solving free equations

Zero-form $\Phi^{(1)}$

- $\mathrm{d}_z\, \Phi^{(1)} = 0 \Rightarrow \Phi^{(1)} = C(x,Y)$
- $\mathcal{D}_{\mathrm{tw}}\, C = 0$ encodes the Bargmann-Wigner equations
- $C(x,Y) = \phi + F_{\underline{\alpha\alpha}}(Y^{\underline{\alpha}})^{\star 2} + C_{\underline{\alpha\alpha\alpha\alpha}}(Y^{\underline{\alpha}})^{\star 4} + \ldots$

Internal connection : $\deg_Z V = 1$

- $\mathrm{d}_z\, V^{(1)} = -\Phi^{(1)} \star (J + \bar{J})$
- $V^{(1,A)} = -q^{*(A)}\left(\Phi^{(1)} \star (J + \bar{J})\right)$
- Can be (partially) specified by a gauge condition $\mathcal{O}_{(A)} V^{(1,A)} = 0$

Spacetime connection : $\deg_Z U = 0$

- $\mathrm{d}_z\, U^{(1)} = -\mathcal{D}_{\mathrm{ad}}\, V^{(1)}$
- $U^{(1,A)} = q^{*(B)}\mathcal{D}_{\mathrm{ad}}\, q^{*(A)}\left(\Phi^{(1)} \star (J + \bar{J})\right) + W^{(1,A,B)}$
- $\mathcal{D}_{\mathrm{ad}}\, U^{(1)} = 0$
- $\mathcal{D}_{\mathrm{ad}}\, W^{(1,A,B)} = -\mathcal{D}_{\mathrm{ad}}\, q^{*(B)}\mathcal{D}_{\mathrm{ad}}\, q^{*(A)}\left(\Phi^{(1)} \star (J + \bar{J})\right)$

Homotopy contractions

Poincaré lemma

- No nontrivial d_z cohomology in (unextended) Vasiliev's model
- $q^{*(0)}\left[g(Z;\mathrm{d}Z)\right] := Z^{\underline{\alpha}}\partial^{\mathrm{d}Z}_{\underline{\alpha}}\int \frac{dt}{t} g(tZ;t\mathrm{d}Z)$

Central On Mass Shell Theorem (COMST) from homotopy

[*Vasiliev, 1989*]

- $q^{*(A)} = q^{*(B)} = q^{*(0)}$
- $\mathcal{D}_{\rm ad} W^{(1,0,0)} = -\frac{ib}{4} e^{\alpha\dot\alpha} e_{\alpha}{}^{\dot\alpha} \partial^{\bar y}_{\dot\alpha}\partial^{\bar y}_{\dot\alpha} C|_{y=0} - \frac{i\bar b}{4} e^{\alpha\dot\alpha} e^{\alpha}{}_{\dot\alpha} \partial^{y}_{\alpha}\partial^{y}_{\alpha} C|_{\bar y=0}$
- Describes unfolded Fronsdal fields around AdS_4

Interactions from homotopy

[*Boulanger, Kessel, Skvortsov & Taronna, 2015*]

- Locality issues if $q^{*(0)}$ is used beyond free theory

Shifted homotopies

[*Didenko, Gelfond, Korybut, Misuna & Vasiliev, since 2016*]

- $q^{*(V)}\left[g(Z;\mathrm{d}Z)\right] := (Z^{\underline{\alpha}} + V^{\underline{\alpha}})\partial^{\mathrm{d}Z}_{\underline{\alpha}}\int \frac{dt}{t} g(t(Z+V)-V;t\mathrm{d}Z)$
- Minimal non-locality proposal

Other resolutions

AdS_4 gauge function L

- Encodes AdS_4 space as $\Omega = L^{\star-1} \star \mathrm{d}_x L$
- $q^{*(L)} f := L^{\star-1} \star q^{*(0)} \left(L \star f \star L^{\star-1}\right) \star L = q^{*(iM(x)\partial^Y)}$
- With specific initial datum $C = \nu L \star \pi(L)$, gives all orders solution
- This is the solution of [*Sezgin & Sundell, 2005*]

Ordering prescriptions

- Reordered field $[f]_{\mathrm{ord.}} = \hat{\tau}_{\mathrm{ord.}} f$ for some ordering ord.
- Vasiliev's equations are written in terms of $\star$
- Reordered homotopy $q^{*(\mathrm{ord.})} := \hat{\tau}_{\mathrm{ord.}}^{-1} q^{*(0)} \hat{\tau}_{\mathrm{ord.}}$

ϵ-ordering

- $[f]_\epsilon (Y, Z) := \int \frac{\mathrm{d}^4 Y' \ \mathrm{d}^4 Z'}{(2\pi(\epsilon-1))^4} \exp\left(\frac{i}{\epsilon-1}(Y - Y')(Z - Z')\right) f(Y', Z')$
- $Y_{\underline{\alpha}} \star_\epsilon Z_{\underline{\beta}} = Y_{\underline{\alpha}} \star Z_{\underline{\beta}} - i\epsilon\, \epsilon_{\underline{\alpha\beta}}$
- $q^{*(\epsilon-\mathrm{ordering})} := \hat{\tau}_\epsilon^{-1} q^{*(0)} \hat{\tau}_\epsilon = q^{*(i(1-\epsilon)\partial^Y)}$

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$\epsilon = 1$: normal ordering

- $f(Y,Z) \equiv \int \lim_{\xi\to 0} \frac{\mathrm{d}^4 Y' \mathrm{d}^4 Z'}{(2\pi\xi)^4} \exp\left(\frac{i}{\xi}(Y - Y')(Z - Z') \right) f(Y', Z')$
- $Y_{\underline{\alpha}} \star Z_{\underline{\beta}} = Y_{\underline{\alpha}} \star Z_{\underline{\beta}} - i\epsilon_{\underline{\alpha\beta}}$
- $q^{*(N)} \equiv q^{*(0)}$

Other resolutions

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- $Y_{\underline{\alpha}} \star_\epsilon Z_{\underline{\beta}} = Y_{\underline{\alpha}} \star Z_{\underline{\beta}} - i\epsilon\, \epsilon_{\underline{\alpha}\underline{\beta}}$
- $q^{*(\epsilon-\mathrm{ordering})} := \hat{\tau}_\epsilon^{-1} q^{*(0)} \hat{\tau}_\epsilon = q^{*(i(1-\epsilon)\partial^Y)}$

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- Encodes AdS_4 space as $\Omega = L^{\star -1} \star \mathrm{d}_x L$
- $q^{*(L)} f := L^{\star -1} \star q^{*(0)} \left(L \star f \star L^{\star -1} \right) \star L = q^{*(iM(x)\partial^Y)}$
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- Reordered field $[f]_{\mathrm{ord.}} = \hat{\tau}_{\mathrm{ord.}} f$ for some ordering ord.
- Vasiliev's equations are written in terms of $\star$
- Reordered homotopy $q^{*(\mathrm{ord.})} := \hat{\tau}^{-1}_{\mathrm{ord.}} q^{*(0)} \hat{\tau}_{\mathrm{ord.}}$

$\epsilon = 0$: **Weyl ordering**

- $[f]_W (Y, Z) := \int \frac{\mathrm{d}^4 Y'\mathrm{d}^4 Z'}{(2\pi)^4} \exp\left(-i(Y - Y')(Z - Z')\right) f(Y', Z')$
- $Y_{\underline{\alpha}} \star_W Z_{\underline{\beta}} = Y_{\underline{\alpha}} \star Z_{\underline{\beta}}$
- $q^{*(W)} := \hat{\tau}_0^{-1} q^{*(0)} \hat{\tau}_0 = q^{*(i\partial^Y)}$

Factorisation of linearised equations

Weyl order and homotopy

- $[f(Y) \star g(Z)]_W (Y, Z) = f(Y) \star_W g(Z) = f(Y)g(Z)$
- $q^{*(W)} (f(Y) \star g(Z)) = f(Y) \star \left(q^{*(0)} g(Z)\right)$

Factorisation of Klein operator

[*Didenko & Vasiliev, 2009*]

- $\kappa = \kappa_y \star \kappa_z$
- $\Phi =: \Psi \star \kappa_y$
- $\bar\kappa = \bar\kappa_y \star \bar\kappa_z$
- $\Phi =: \bar\Psi \star \bar\kappa_y$

Linearized analysis

- $\mathrm{d}_z \Phi = 0$
- $\mathcal{D}_{\mathrm{tw}} \Phi = 0$
- $\mathrm{d}_z V^{(1)}_{\mathrm{hol}} = -\Psi \star j_z$
- $\mathrm{d}_z U^{(1,W)}_{\mathrm{hol}} = -\mathcal{D}_{\mathrm{ad}} V^{(1,W)}_{\mathrm{hol}} = 0$
- $\mathcal{D}_{\mathrm{ad}} U^{(1,W)}_{\mathrm{hol}} = 0$
- $\Phi^{(1)} \star J = \Psi(Y) \star j_z$
- $\mathcal{D}_{\mathrm{ad}} \Psi = 0$
- $V^{(1,W)}_{\mathrm{hol}} = -\Psi \star q^{*(0)} j_z$
- $U^{(1,W)}_{\mathrm{hol}} = W^{(1,1)}_{\mathrm{hol}}$

Factorisation of linearised equations

Weyl order and homotopy

- $[f(Y) \star g(Z)]_W(Y,Z) = f(Y) \star_W g(Z) = f(Y)g(Z)$
- $q^{*(W)}(f(Y) \star g(Z)) = f(Y) \star \left(q^{*(0)} g(Z)\right)$

Factorisation of Klein operator

[*Didenko & Vasiliev, 2009*]

- $\kappa = \kappa_y \star \kappa_z$
- $\Phi =: \Psi \star \kappa_y$
- $\bar{\kappa} = \bar{\kappa}_y \star \bar{\kappa}_z$
- $\Phi =: \bar{\Psi} \star \bar{\kappa}_y$

Linearized analysis

- $\mathrm{d}_z \Phi = 0$
- $\mathcal{D}_{\mathrm{tw}} \Phi = 0$
- $\mathrm{d}_z V^{(1)}_{\mathrm{hol}} = -\Psi \star j_z$
- $\mathrm{d}_z U^{(1,W)}_{\mathrm{hol}} = -\mathcal{D}_{\mathrm{ad}} V^{(1,W)}_{\mathrm{hol}} = 0$
- $\mathcal{D}_{\mathrm{ad}} U^{(1,W)}_{\mathrm{hol}} = 0$
- $\Phi^{(1)} \star J = \Psi(Y) \star j_z$
- $\mathcal{D}_{\mathrm{ad}} \Psi = 0$
- $V^{(1,W)}_{\mathrm{hol}} = -\Psi \star q^{*(0)} j_z$
- $U^{(1,W)}_{\mathrm{hol}} = W^{(1,1)}_{\mathrm{hol}}$
- $U^{(1,W)}_{\mathrm{hol}} = 0$

Factorised solution

Second order analysis

- $\mathrm{d}_z\,\Phi^{(2,W)} = -[V^{(1,W)}_{\mathrm{hol}}, \Psi]_\star \star \kappa_y + \mathrm{h.c.} = 0$
- $\mathcal{D}_{\mathrm{tw}}\Phi^{(2,W)} = 0$
- $[V^{(2,W)}_{\mathrm{hol}}, V^{(2,W)}_{\mathrm{a-hol}}]_\star = 0$
- $\mathrm{d}_z\,V^{(2,W)}_{\mathrm{hol}} = -V^{(1,W)}_{\mathrm{hol}} \star V^{(1,W)}_{\mathrm{hol}}$
- $\mathrm{d}_z\,U^{(2,W)} = \mathcal{D}_{\mathrm{ad}}U^{(2,W)} = 0$

- $\Phi^{(2,W)} = 0$
- $V^{(2,W)}_{\mathrm{hol}} = \Psi^{\star 2} \star V_2(z)$
- $U^{(2,W)}$

All order solution

- $\Phi = \Psi \star \kappa_y = \bar{\Psi} \star \bar{\kappa}_y$
- $V = \sum_{n=1}^{\infty} \Psi^{\star n} \star V_n(z) + \sum_{n=1}^{\infty} \bar{\Psi}^{\star n} \star \bar{V}_n(z)$
- $U = 0$

Zero form charges

- $$\mathcal{I}_{n_0,t}(M) = \int \mathrm{d}^4Z\,\mathrm{d}^4Y\,\left[\left(\Phi^{(1)} \star \kappa\right)^{\star n_0} \star (\kappa\bar{\kappa})^{\star t} \star e^{iMS}\right]$$
  $$= \sum_{n=0}^{\infty} \rho_{n_0,n,t}(M) \int \mathrm{d}^4Y\,\Psi^{\star n_0+n} \star (\kappa_y\bar{\kappa}_y)^{\star t}$$

Factorised solution

Recursive solution

- $V_1(z) = -q^{*(0)} j_z$
- $V_{n+1} = -q^{*(0)} \sum_{m=1}^{n} V_m \star V_n$
- $q^{*(0)} [g(Z; \mathrm{d}Z)] := Z^{\underline{\alpha}} \partial^{\mathrm{d}Z}_{\underline{\alpha}} \int \frac{dt}{t} g(tZ; t\mathrm{d}Z)$

Factorised solution

Recursive solution

- $V_1(z) = -q^{*(0)} j_z$
- $V_{n+1} = -q^{*(0)} \sum_{m=1}^{n} V_m \star V_n$
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Factorisation of Klein operator

- $\kappa = \kappa_y \star \kappa_z = 2\pi\delta^2(y) \star \delta^2(z)$
- $j_z = -\frac{ib}{4} \kappa_z dz^\alpha \mathrm{d}z_\alpha$
- $q^{*(0)} j_z = \frac{ib}{2} \int_0^1 \frac{dt}{t} \mathrm{d}z^\alpha z_\alpha \delta^2(z) = +\infty.0$

Regularised factorised solution

Recursive solution

- $V_1(z) = -q^{*(0)} j_z$
- $V_{n+1} = -q^{*(0)} \sum_{m=1}^{n} V_m \star V_n$
- $q^{*(0)}\left[g(Z; \mathrm{d}Z)\right] := Z^{\underline{\alpha}} \partial_{\underline{\alpha}}^{\mathrm{d}Z} \int \frac{dt}{t} g(tZ; t\mathrm{d}Z)$

Factorisation of Klein operator

- $\kappa = \kappa_y \star \kappa_z = 2\pi\delta^2(y) \star \delta^2(z)$
- $j_z = -\frac{ib}{4}\kappa_z dz^\alpha \mathrm{d}z_\alpha$
- $q^{*(0)} j_z = \frac{ib}{2}\int_0^1 \frac{dt}{t} \mathrm{d}z^\alpha z_\alpha \delta^2(z) = +\infty.0$

Integral δ-sequence

- Symmetric matrix $D_{\alpha\beta}$, $D_\alpha{}^\beta D_\beta{}^\gamma = \delta_\alpha{}^\gamma$
- $\kappa_z = 2\int \frac{ds}{1+s}\delta(1+s) \exp\left(\frac{i}{2}\frac{1-s}{1+s} zDz\right)$

Factorised solution

- $V_n(z) = -\frac{ib}{2}\frac{(2(n-1))!}{n!(n-1)!^2}\mathrm{d}z^\alpha z_\alpha \int \frac{ds}{1+s}\left(\log\left(\frac{1}{s^2}\right)\right)^{n-1} \exp\left(\frac{i}{2}\frac{1-s}{1+s} zDz\right)$
- Solution of [*Iazeolla & Sundell, 2011*]

Changing frame in linearised solution

Solution in some (A)-gauge

- Zero form $\Phi^{(1)}(x, Y)$ satisfies $\mathcal{D}_{\text{tw}}\Phi = 0$
- $V^{(1,A)} = -q^{*(A)}\left(\Phi^{(1)} \star (J + \bar{J})\right)$
- $U^{(1,A)} = q^{*(B)}\mathcal{D}_{\text{ad}}q^{*(A)}\left(\Phi^{(1)} \star (J + \bar{J})\right) + W^{(1,A,B)}$

Gauge transforming the internal connection $V^{(1,A)}$ to $V^{(1,A')}$

- Gauge transformation $V^{(1,A')} = V^{(1,A)} + \mathrm{d}_z H^{(1,A\to A')}$
- Gauge condition $\mathcal{O}_{A'}V^{(1,A')} = 0$
- $H^{(1,A\to A')} = h^{(1,A\to A')}(x; Y) - \frac{1}{\mathcal{O}_{A'}\mathrm{d}_z}\mathcal{O}_{A'}V^{(1,A)}$
- From integrability : $U^{(1,A')} = U^{(1,A)} + \mathcal{D}_{\text{ad}}H^{(1,A\to A')}$

Field redefining the spacetime connection

- Projection $W^{(1,A',B')} := \mathcal{P}^{(B')}U^{(1,A')}$
- $\mathcal{D}_{\text{ad}}W^{(1,A',B')} = \mathcal{D}_{\text{ad}}\mathcal{P}^{(B')}U^{(1,A)} - \mathcal{D}_{\text{ad}}\mathcal{P}^{(B')}\mathcal{D}_{\text{ad}}\frac{1}{\mathcal{O}_{A'}\mathrm{d}_z}\mathcal{O}_{A'}V^{(1,A)}$

COMST from changing frame in linearised solution

Solution in some (A)-gauge

- Zero form $\Phi^{(1)}(x,Y)$ satisfies $\mathcal{D}_{\mathrm{tw}}\Phi = 0$
- $V^{(1,A)} = -q^{*(A)}\left(\Phi^{(1)} \star (J+\bar{J})\right)$
- $U^{(1,A)} = q^{*(B)}\mathcal{D}_{\mathrm{ad}}q^{*(A)}\left(\Phi^{(1)} \star (J+\bar{J})\right) + W^{(1,A,B)}$
- $U^{(1,A)}$ and $V^{(1,A)}$ are analytic in Z

Gauge transforming the internal connection $V^{(1,A)}$ to $V^{(1,G)}$

- Gauge transformation $V^{(1,G)} = V^{(1,A)} + \mathrm{d}_z H^{(1,A\to G)}$
- Strong Vasiliev gauge condition $Z^{\underline{\alpha}}V^{(1,G)}_{\underline{\alpha}} = 0$
- $H^{(1,A\to G)} = h^{(1,A\to G)} - \int_0^1 dt\, Z^{\underline{\alpha}}V_{\underline{\alpha}}(tZ)$
- From integrability : $U^{(1,G)} = U^{(1,A)} + \mathcal{D}_{\mathrm{ad}}H^{(1,A\to G)}$

Field redefining the spacetime connection

- Projection $W^{(1)} := U^{(1,G)}|_{Z=0}$
- $\mathcal{D}_{\mathrm{ad}}W^{(1)} = -\frac{ib}{4}e^{\alpha\dot{\alpha}}e_{\alpha}{}^{\dot{\alpha}}\partial^{\bar{y}}_{\dot{\alpha}}\partial^{\bar{y}}_{\dot{\alpha}}C|_{y=0} - \frac{i\bar{b}}{4}e^{\alpha\dot{\alpha}}e^{\alpha}{}_{\dot{\alpha}}\partial^{y}_{\alpha}\partial^{y}_{\alpha}C|_{\bar{y}=0}$

COMST from changing frame in linearised solution

Solution in some (A)-gauge

- Zero form $\Phi^{(1)}(x,Y)$ satisfies $\mathcal{D}_{\rm tw}\Phi = 0$
- $V^{(1,A)} = -q^{*(A)}\left(\Phi^{(1)} \star (J+\bar{J})\right)$
- $U^{(1,A)} = q^{*(B)}\mathcal{D}_{\rm ad}q^{*(A)}\left(\Phi^{(1)} \star (J+\bar{J})\right) + W^{(1,A,B)}$
- $U^{(1,A)}$ and $V^{(1,A)}$ are analytic in Z

Gauge transforming the internal connection $V^{(1,A)}$ to $V^{(1,G)}$

- Gauge transformation $V^{(1,G)} = V^{(1,A)} + \mathrm{d}_z H^{(1,A\to G)}$
- Relaxed Vasiliev gauge condition $Z^{\underline{\alpha}}V^{(1,G)}_{\underline{\alpha}} = O(Z^2)$
- $H^{(1,A\to G)} = h^{(1,A\to G)} - \int_0^1 dt\, Z^{\underline{\alpha}}V_{\underline{\alpha}}(tZ) + O(Z^2)$
- From integrability : $U^{(1,G)} = U^{(1,A)} + \mathcal{D}_{\rm ad}H^{(1,A\to G)}$

Field redefining the spacetime connection

- Projection $W^{(1)} := U^{(1,G)}|_{Z=0}$
- $\mathcal{D}_{\rm ad}W^{(1)} = -\frac{ib}{4}e^{\alpha\dot{\alpha}}e_{\alpha}{}^{\dot{\alpha}}\partial^{\bar{y}}_{\dot{\alpha}}\partial^{\bar{y}}_{\dot{\alpha}}C|_{y=0} - \frac{i\bar{b}}{4}e^{\alpha\dot{\alpha}}e^{\alpha}{}_{\dot{\alpha}}\partial^{y}_{\alpha}\partial^{y}_{\alpha}C|_{\bar{y}=0}$

Exact solutions to 4D higher spin equations

Solutions based on specific initial data
[*Iazeolla, Sezgin & Sundell, 2005-2008*]
Black Hole-like initial data
[*Didenko & Vasiliev, 2009*]

- Used Killing symmetries and factorisation of Klein operator
- $W^{(1)}$ generically non-trivial
- C is first order exact

[*Bourdier & Drukker, 2014*]

- Particular gauge where DV-solution has $W^{(1)} = 0$

[*Iazeolla, Sundell & Yin, since 2011*]

- Factorised solution
- Algebraically meaningful input in spacetime independent gauge

Factorisation with various initial data
[*Aros, Iazeolla, Noreña, Sezgin, Sundell & Yin, since 2017*]

Regular prescription

Initial data in spacetime independent gauge
[*Iazeolla & Sundell, 2017*]

- $C = \sum_n L^{\star -1} \star \left(\nu_n \tilde{\mathcal{P}}_n + \tilde{\nu}_n \mathcal{P}_n\right) \star \pi(L)$
- Particle mode : $\mathcal{P}_n = \alpha_n \oint_{\mathrm{sign}(n)} \frac{d\eta}{2i\pi} \left(\frac{\eta+1}{\eta-1}\right)^n \exp\left(\eta y \sigma_0 \bar{y}\right)$
- BH mode : $\tilde{\mathcal{P}}_n = 2\pi\alpha_n \oint_{\mathrm{sign}(n)} \frac{d\eta}{2i\pi} \left(\frac{\eta+1}{\eta-1}\right)^n \delta^2(y - i\eta\sigma_0\bar{y})$
- $\tilde{\mathcal{P}}_n = \mathcal{P}_n \star \kappa_y$

Regular prescription

Initial data in spacetime independent gauge
[*Iazeolla & Sundell, 2017*]

- $C^X = \sum_n L^{\star -1} \star \left(\nu_n(\partial_X)\tilde{\mathcal{P}}_n{}^X + \tilde{\nu}_n(\partial_X)\mathcal{P}_n^X\right) \star \pi(L)$
- Particle mode : $\mathcal{P}_n^X = \alpha_n \oint_{\mathrm{sign}(n)} \frac{d\eta}{2i\pi} \left(\frac{\eta+1}{\eta-1}\right)^n \exp\left(\eta y \sigma_0 \bar{y} + \chi y + \bar{\chi}\bar{y}\right)$
- BH mode : $\tilde{\mathcal{P}}_n^X = 2\pi\alpha_n \oint_{\mathrm{sign}(n)} \frac{d\eta}{2i\pi} \left(\frac{\eta+1}{\eta-1}\right)^n \delta^2(y - i\eta\sigma_0\bar{y} + i\chi) \exp(\bar{\chi}\bar{y})$
- $\tilde{\mathcal{P}}_n^X = \mathcal{P}_n^X \star \kappa_y$

Regular prescription

1. Take star-products
2. Take traces
3. Do parametric integrals (η, s, t)

Regular prescription

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[*Iazeolla & Sundell, 2017*]

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- $\tilde{\mathcal{P}}_n^X = \mathcal{P}_n^X \star \kappa_y$

Regular prescription

1. Take star-products
2. Take traces
3. Do parametric integrals (η,s,t)

Compatible with COMST

- From factorised gauge
- From Didenko-Vasiliev gauge (for BH)

Regular prescription

Initial data in spacetime independent gauge
[*Iazeolla & Sundell, 2017*]

- $C^X = \sum_n L^{\star -1} \star \left(\nu_n(\partial_X)\tilde{\mathcal{P}}_n{}^X + \tilde{\nu}_n(\partial_X)\mathcal{P}_n^X\right) \star \pi(L)$
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- BH mode : $\tilde{\mathcal{P}}_n^X = 2\pi\alpha_n \oint_{\mathrm{sign}(n)} \frac{d\eta}{2i\pi} \left(\frac{\eta+1}{\eta-1}\right)^n \delta^2(y - i\eta\sigma_0\bar{y} + i\chi) \exp(\bar{\chi}\bar{y})$
- $\tilde{\mathcal{P}}_n^X = \mathcal{P}_n^X \star \kappa_y$

Regular prescription

1. Take star-products
2. Take traces : gauge functions do not break the cyclicity of the trace
3. Do parametric integrals (η, s, t)

Compatible with COMST

- From factorised gauge
- From Didenko-Vasiliev gauge (for BH)

Beyond linearisation

Quadratic solution

- $\Phi^{(2,G)} = \Phi^{(2,A)} - [H^{(1,A_1\to G)}, \Phi^{(1)}]_\pi + C^{(2)}(x,Y)$
- $V^{(2,G)} = V^{(2,A)} - [H^{(1,A\to G)}, V^{(1,A)}]_\star - H^{(1,A\to G)} \star \mathrm{d}_z H^{(1,A\to G)} + \mathrm{d}_z H^{(2,A\to G)}$
- $U^{(2,G)} = U^{(2,A)} - [H^{(1,A\to G)}, U^{(1,A)}]_\star - H^{(1,A\to G)} \star \mathcal{D}_{\mathrm{ad}} H^{(1,A\to G)} + \mathcal{D}_{\mathrm{ad}} H^{(2,A\to G)}$

Proposed perturbative scheme

- $\tilde{\Phi}, \tilde{V}, \tilde{U}$ solve the linear equation
- Glued at the boundary
  - $\Phi = \tilde{\Phi} + O\left(\frac{1}{r}\right)$
  - $V = \tilde{V} + O\left(\frac{1}{r}\right)$
  - $U = \tilde{U} + O\left(\frac{1}{r}\right)$

Fixes at most

- Integration constants $C^{(n)}$
- Boundary gauge function

Beyond linearisation

Quadratic solution

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- $U^{(2,G)} = U^{(2,A)} - [H^{(1,A\to G)}, U^{(1,A)}]_\star - H^{(1,A\to G)} \star \mathcal{D}_{\mathrm{ad}} H^{(1,A\to G)} + \mathcal{D}_{\mathrm{ad}} H^{(2,A\to G)}$

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  - $V = \tilde{V} + O\left(\frac{1}{r}\right)$
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- $U^{(2,G)} = U^{(2,A)} - [H^{(1,A \to G)}, U^{(1,A)}]_\star - H^{(1,A \to G)} \star \mathcal{D}_{\mathrm{ad}} H^{(1,A \to G)} + \mathcal{D}_{\mathrm{ad}} H^{(2,A \to G)}$

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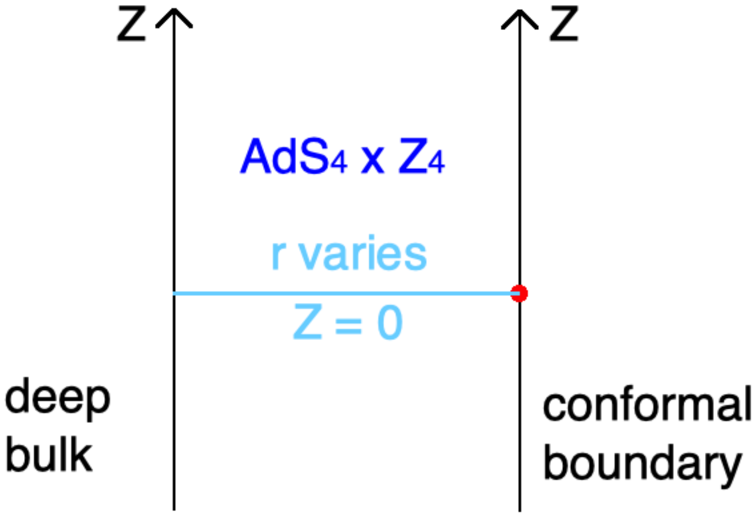
Fixes at most

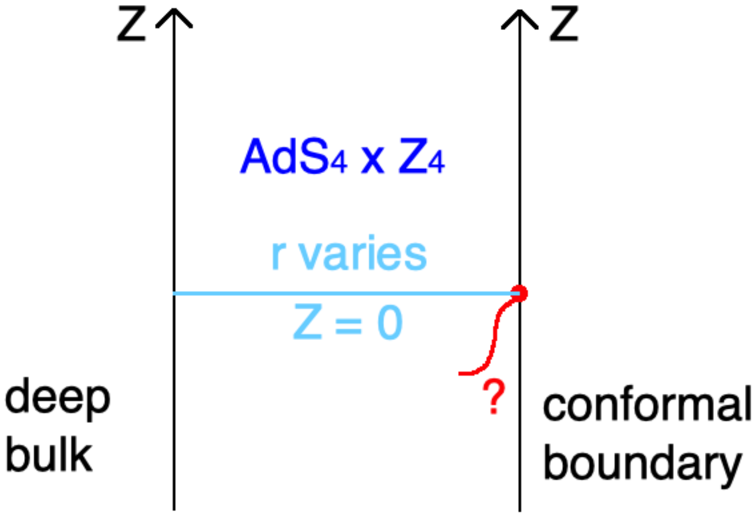
- Integration constants $C^{(n)}$
- Boundary gauge function

Hence all observables

Resolving Z-space



Resolving Z-space



Outline

Vasiliev's equations

- Non linear equations in extended $(x+Z)$-space
- Solve Z-dependent equations and gauge conditions
- Get interacting equations in x-space

Using Z-space to extract observables
[*Bonezzi, Boulanger, D.D.F. & Sundell, 2017*]

- Wilson Lines closed in x-space and open in Z-space
- All correlators of free $U(N)$ and $O(N)$ vector models
- Matching at the level of cyclic structures

Resolving Z-space to impose boundary conditions
[*D.D.F., Iazeolla & Sundell, to appear*]

- Start from exact solutions
- Regular prescription compatible with Fronsdal fields
- Proposal for *ALAdS* boundary conditions

Outlook

Wilson Lines

- Different boundary conditions [*Didenko, Mei & Skvortsov, 2013*]
- Parity breaking term
- $(d+1)$ dimensions
- Extensions
  - Supersymmetry
  - Frobenius-Chern-Simons
  - Multiparticles
- Corrections from interactions(in different schemes ?)

Asymptotic boundary conditions

- Implement the scheme
- Better understand the class of functions
- Expand around non-trivial backgrounds

Outlook

Thank you for your attention !

Wilson Lines

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