EPR=ER in LLM

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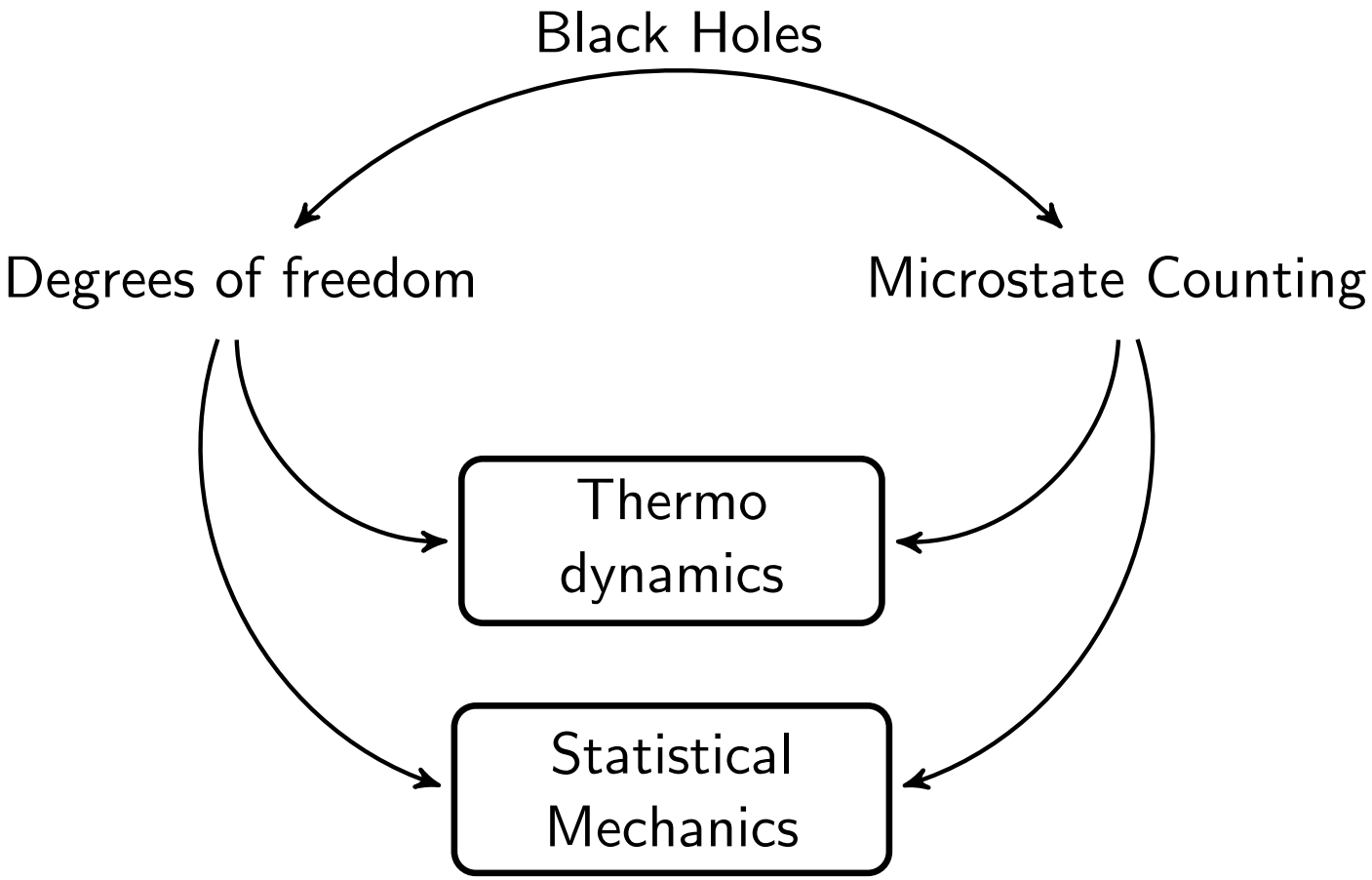
Outline

- Short (& limited) review
  - to provide some perspective to non-experts
- Revisit LLM
  - EPR=ER in two boundaries
  - Entanglement in R-charge space
  - Comments on entangled "black holes"

Black holes & entropy



Wins & losses

- in susy scenarios, dof (D-branes) are identified and counted, but not in the regime of parameters where BHs exist
- extremality, in many situations, leads to some CFT where Cardy's formula reproduces Hawking-Bekenstein

Despite huge success,

- emergence of locality
- information paradox and consistency with quantum mechanics

remain unanswered $\implies$ study of string theory dynamics led to a new framework

A new framework : AdS/CFT

Given same kinematic symmetries, assume quantum gravity = QFT

$\exists$ weakly coupled semiclassical gravity

$$I_{\text{gravity}} \sim \frac{1}{G_N} \int (R - 2\Lambda) \,, \quad G_N \sim \ell_p^{d-2} \,, \quad g_{\text{eff}}^2 \sim \left(\frac{\ell_p}{L}\right)^{d-2} \ll 1$$

- Any CFT has a stress tensor with 2-pt function

$$\langle T_{\mu\nu}(x)\, T_{\alpha\beta}(y) \rangle \sim \mathcal{N}\, g(x,y)$$

the natural semi-classical gravity calculation involves

$$\frac{\delta}{\delta g_{\mu\nu}} \frac{\delta}{\delta g_{\alpha\beta}} e^{-I_{\text{gravity}}} \sim \frac{R_{\text{AdS}}^{d-2}}{G_N} \equiv \frac{1}{g_{\text{eff}}^2} \gg 1$$

Hence, $\mathcal{N} \sim \frac{1}{g_{\text{eff}}^2} \gg 1$

A new framework : AdS/CFT

Reproducing gravity spectrum

- In perturbative Einstein-like gravity, the only particles running in loops are gravitons. To ensure the same property in CFT, we require

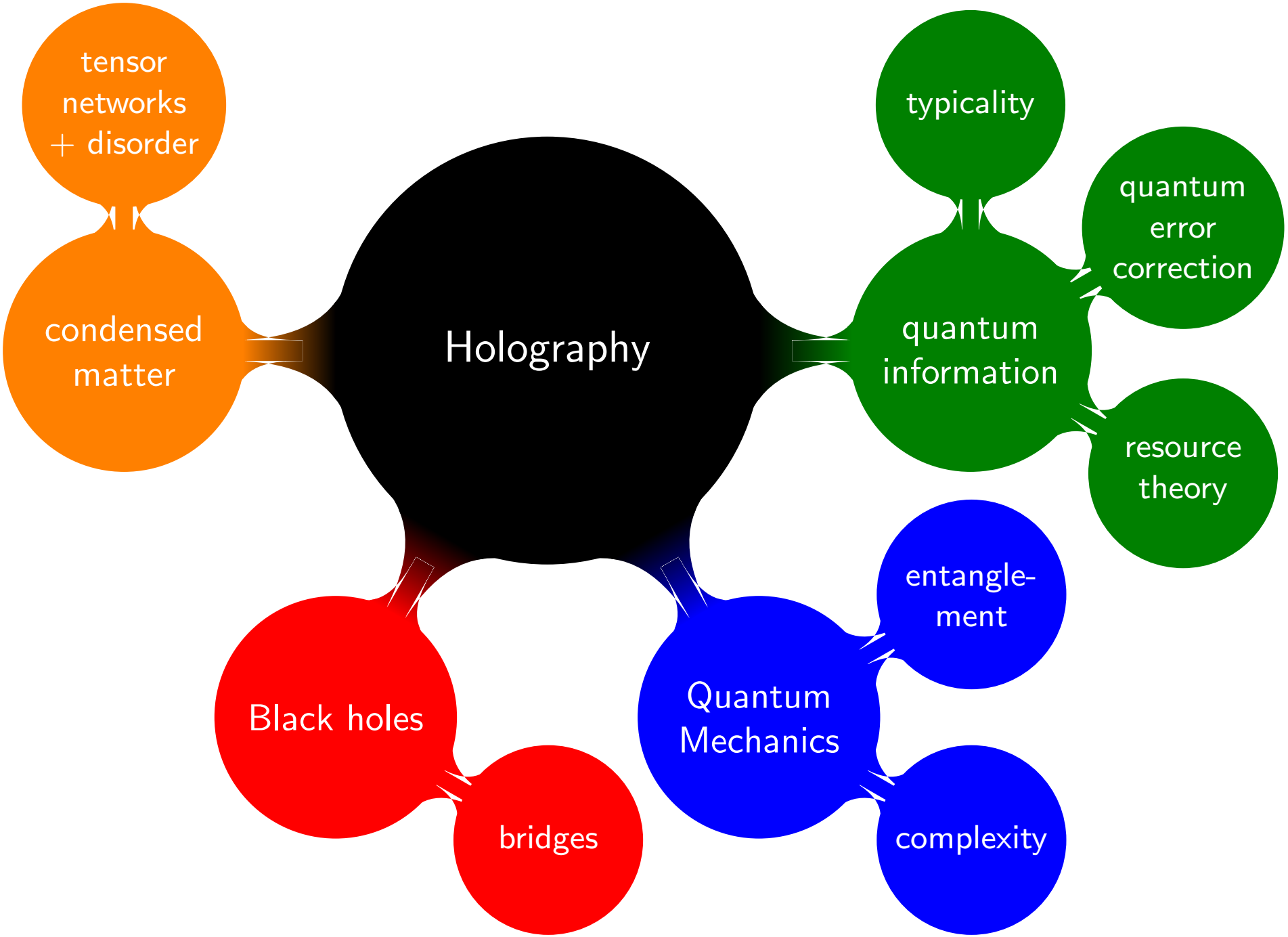
$$\Delta_{s>2} \gg 1$$

i.e., large anomalous dimensions due to strong interactions

$\exists$ some evidence : CFTs satisfying

$$\boxed{\mathcal{N} \gg 1 \ \& \quad \text{very strongly coupled}}$$

have gravity duals


tensor networks + disorder
condensed matter
Holography
typicality
quantum error correction
quantum information
resource theory
entangle-ment
Quantum Mechanics
complexity
Black holes
bridges

Holographic lessons

1. Connected 2-pt correlation function of a heavy operator

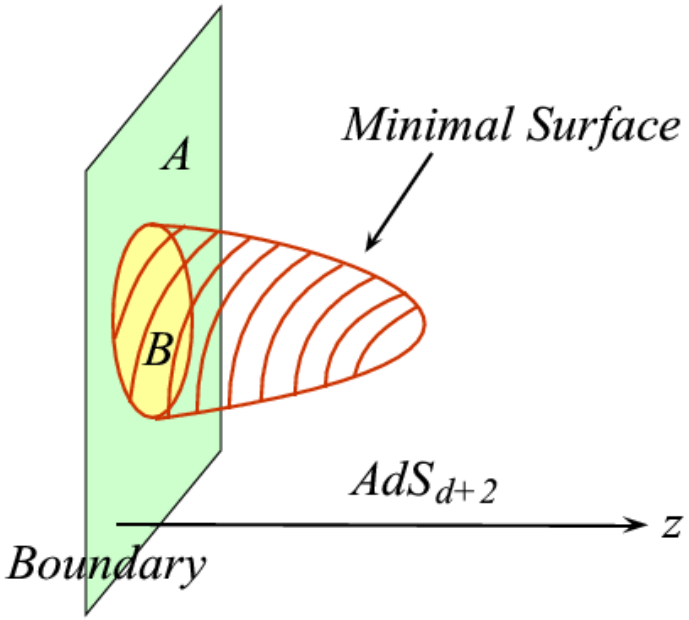
$$\langle \mathcal{O}_A(x_a)\mathcal{O}_B(x_b)\rangle \sim e^{-mL_{\text{bulk}}(x_a,x_b)}$$

$L_{\text{bulk}}(x_a, x_b)$ bulk geodesic distance between boundary x_a and x_b.

2. Entanglement entropy (RT)

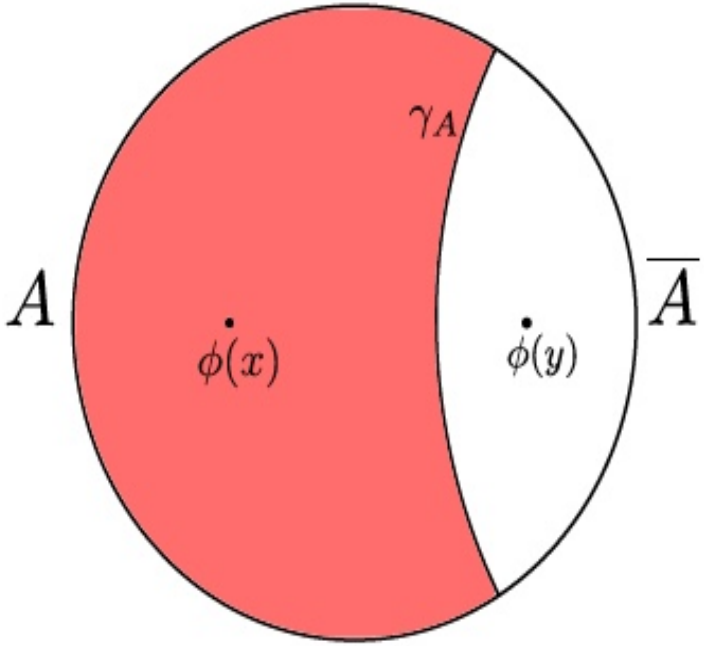
$$S(\rho_B) = \frac{\text{Area}(\Sigma_{\text{bulk}})}{4G_N}$$

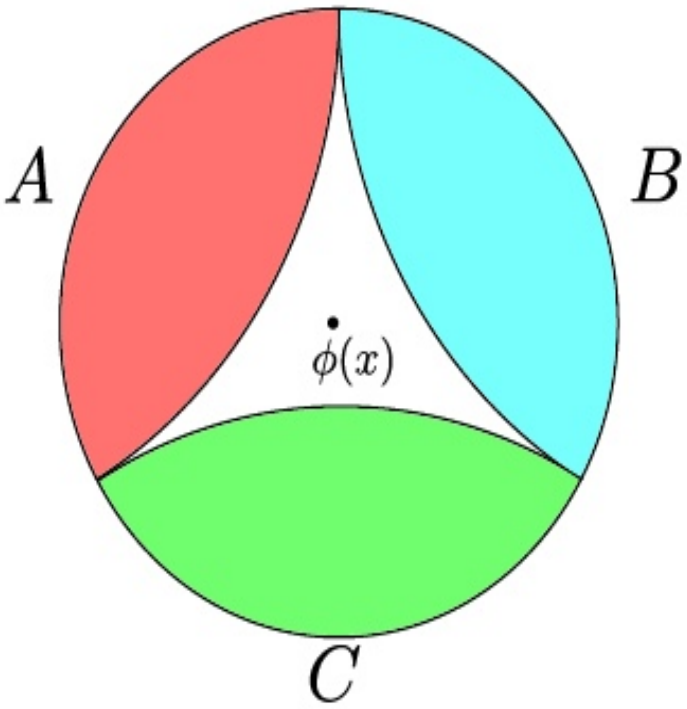
Σ_{bulk} is a bulk minimal surface anchored to ∂B



Holographic lessons

3. Subregion duality : any bulk operator $\phi(x)$ in the entanglement wedge of A can be reconstructed from its boundary data





Notice $\phi(x)$ can not be reconstructed from A, B or C alone, but it does have multiple representations in AB, AC and BC

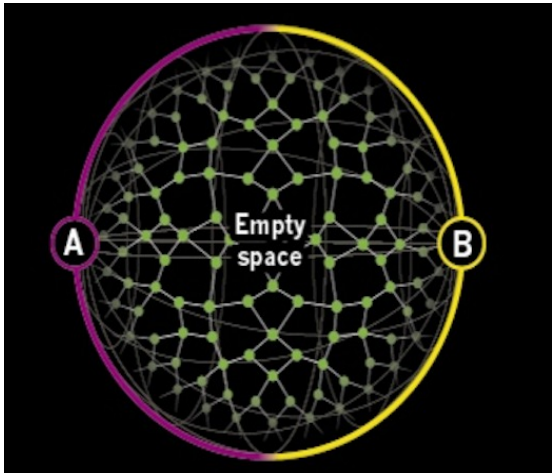
Entanglement vs Spacetime Connectivity

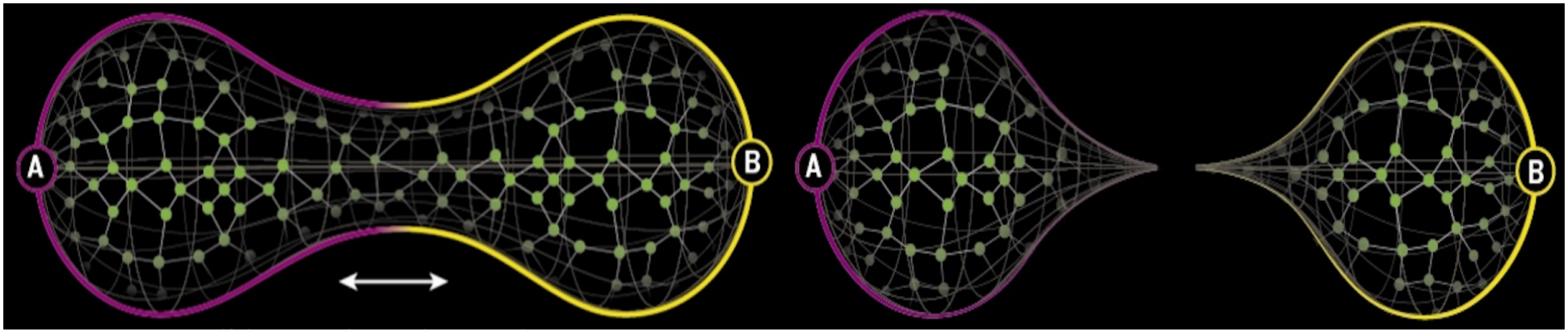
Mutual information bounds the amount of correlation

$$I(A:B) \geq \frac{(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle)^2}{2\|\mathcal{O}_A\|^2 \|\mathcal{O}_B\|^2}$$

Sending entanglement to zero, requires :

1. Proper bulk distance to infinity
2. Area of the common boundary to zero $\Rightarrow$ pinching





EPR-ER

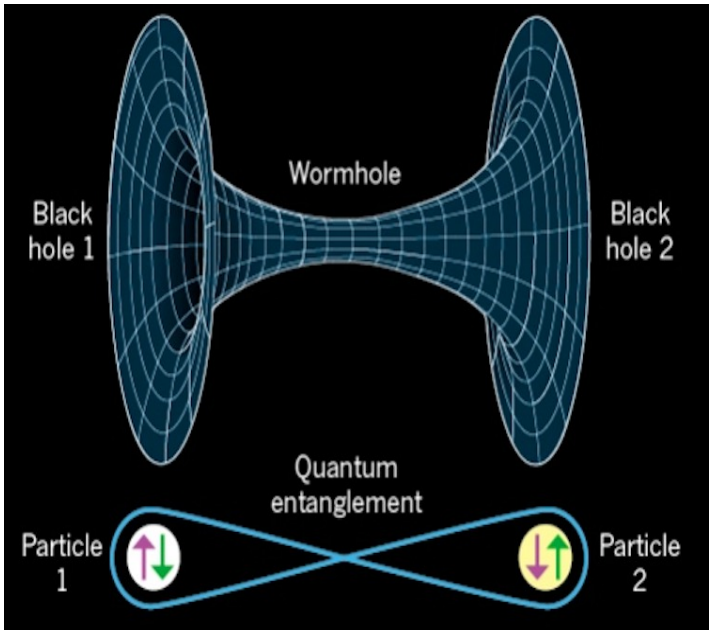
Avoiding its connection to the firewall discussion

There is no fundamental difference in the quantum state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle + |-\rangle|+\rangle\right) ,$$

$$|\Psi'\rangle = \frac{1}{\sqrt{Z}}\sum_n e^{-\beta E_n/2}|n\rangle|n\rangle$$

except



- Hilbert space : dimensionality, spectrum & dynamics (holographic)
- Entropy of the state

The wormhole interpretation

Consider the 4d Schwarzschild black hole metric

$$ds^2 = -e^{2\Phi}dt^2 + \frac{dr^2}{1 - B/r} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Study a fixed t slice at $\theta = \pi/2$:

$$ds^2\big|_\Sigma = \frac{dr^2}{1 - B/r} + r^2 d\phi^2$$

View this section as a surface $z(r)$ in one higher euclidean dimension

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2 = \left(1 + (z')^2\right) dr^2 + r^2 d\phi^2$$

$$z(r) = \pm 2B\left(r/B - 1\right)^{1/2}$$

This is a non-traversable wormhole, but it illustrates that black holes can be reinterpreted in terms of Einstein-Rosen (ER) bridges (wormholes)

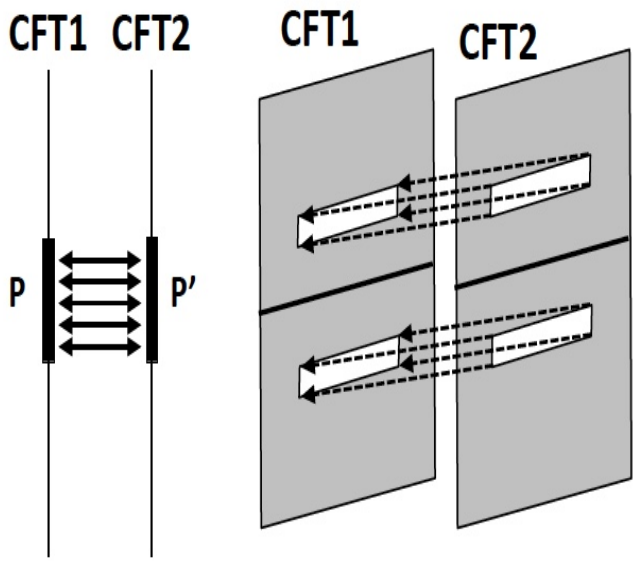
Path integral perspective

Consider some partial entangling between two CFTs through the projection

$$\mathcal{P} = \prod_{x\in P}\left(\sum_{n_x}|n_x\rangle_1|n_x\rangle_2\right)\left(\sum_{m_x}|m_x\rangle_1|m_x\rangle_2\right)\otimes\prod_{x\in P^c}\left(\mathbb{I}_x^2\otimes\mathbb{I}_x^2\right)$$

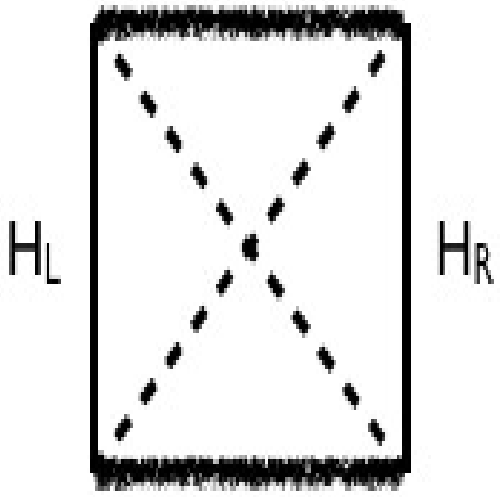
Path integral requires

- slit along interval P in each CFT
- gluing of path integrals across P
- regularisation



Eternal AdS BH revisited

1. Classical maximal extension of the eternal AdS BH
2. Connectedness through BH event horizon



For certain observables and low energies, an observer in $\mathcal{H}_R$ measures a thermal state :

$$\rho_{\text{BH}} = \frac{1}{Z(\beta)} \sum_i e^{-\beta E_i} \left| E_i \right\rangle \left\langle E_i \right| , \qquad \left| E_i \right\rangle \in \mathcal{H}_R$$

Can we interpret ρ_{BH} as a reduced density matrix ? (Maldacena)

$$\rho_{\text{BH}} = \text{tr}_{\mathcal{H}_L} |\Psi\rangle\langle\Psi| \ \text{ with } \ |\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_i e^{-\beta E_i/2} |E_i\rangle \otimes |E_i\rangle \in \mathcal{H}_L \otimes \mathcal{H}_R$$

Quantum entanglement is responsible for the existence of correlations

EPR = ER (Maldacena & Susskind)

Eternal black hole re-interpreted

1. Non-vanishing correlators between $\mathcal{H}_L$ and $\mathcal{H}_R$ are due to quantum entanglement (EPR)
2. These correlations are holographically captured by the bulk geodesic distance between opposite boundaries $\Rightarrow$ length of the ER bridge
3. Entanglement entropy = black hole entropy $\Rightarrow$ maximal cross-section of the ER bridge

EPR=ER conjecture

In short, it takes the above picture and states it is always correct

One problem : there is no quantum analogue of what an ER bridge is

One question : can we check this proposal in the semi-classical regime ?

Adding perturbations

This EPR-ER picture holds

- when perturbing the eternal black hole/thermofield double
  - bulk : shock wave
  - boundary : insertion of local operator

  This set-up
  - made more precise the notion of scrambling

$$\tau_\star \sim \beta \log S$$

  - gave rise to out-ot-time-order correlators to put bounds on quantum chaos
- It is compatible with arguments to avoid quantum cloning in the presence of a horizon

Wormhole traversability

In GR, traversability requires to violate null energy condition

Gao, Jafferis & Wall turned on some interaction between $\mathcal{H}_L$ and $\mathcal{H}_R$

- under some conditions, the 1-loop stress tensor has negative average null energy condition
- Null geodesics can connect both boundaries

Given some interaction $e^{ig\mathcal{O}_L\,\mathcal{O}_R}$, if we turn on a perturbation $e^{i\varepsilon_R\phi_R}$ in $\mathcal{H}_R$ a probe of traversability is :

$$\langle e^{-i\varepsilon_R\phi_R(-t)}\,\hat{\phi}_L(t)\,e^{i\varepsilon_R\phi_R(-t)}\rangle = \langle\hat{\phi}_L(t)\rangle - g\,\varepsilon_R\,[\phi_R(-t)\,,\mathcal{O}_R]\,[\mathcal{O}_L\,,\phi_L(t)] + \dots$$

N=4 SYM : half-BPS sector

$\Delta = J$ states $\Leftrightarrow \prod_i (\mathrm{tr} Z^{m_i})^{n_i}$, $\quad \sum_i m_i n_i = \Delta = J$

Fermion description

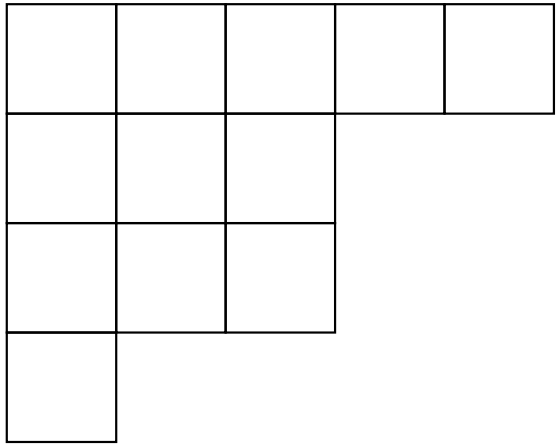
SYM dimensionally reduced on 3-sphere + complex adjoint matrix Z
Diagonalisation : $Z \to \mathrm{diag}\,(\lambda_1, \dots \lambda_N)$
Change in measure : Van der Monde determinant $\mu = \prod_{i \neq j} (\lambda_i - \lambda_j)$
N free fermions in a 1-d harmonic oscillator

$$\psi(n_1) \sim H_{n_1}(\lambda_1)\, e^{-\lambda_i^2/2}$$

$$\psi(n_i, \dots) \sim \mathrm{Slater}[H_{n_i}, \dots]\, e^{-\sum_i \lambda_i^2/2}$$

$$r_i = \frac{1}{\hbar}(E_i - E_i^{\mathrm{fs}}) = n_i - i + 1$$

$\{5,3,3,1\}$



Interpretation of states

- $\Delta \sim \mathcal{O}(1)$ pointlike gravitons spinning on the 5-sphere : multitrace
- $\Delta \sim \mathcal{O}(N)$ giant gravitons
  - Myers' effect : gravitons expand into spinning D3-branes with size

$$\sin^2\theta = \frac{J}{N}$$

  - subdeterminants, as operators

$$\det_k Z = \frac{1}{k!}\varepsilon_{i_1\ldots i_k a_1\ldots a_{N-k}}\varepsilon^{j_1\ldots j_k a_1\ldots a_{N-k}} Z^{i_1}_{j_1}\ldots Z^{i_k}_{j_k}$$

- $\Delta \sim \mathcal{O}(N^2)$ bound states of giant gravitons (superstars), with distribution

$$\frac{dn}{d\theta} = N_c \sin 2\theta\,, \quad N_c \sim N$$

N_c is the number of excited columns in the Young tableau

Gravity : classical moduli space

$\mathbb{R} \times SO(4) \times SO(4)$ + susy type IIB metric & RR 5-form

$$ds^2 = -\frac{y}{\sqrt{\frac{1}{4} - z^2}}(dt + V_i dx^i)^2 + \frac{\sqrt{\frac{1}{4} - z^2}}{y}(dy^2 + dx^i dx^i)$$

$$+ y\sqrt{\frac{\frac{1}{2} - z}{\frac{1}{2} + z}} d\Omega_3^2 + y\sqrt{\frac{\frac{1}{2} + z}{\frac{1}{2} - z}} d\tilde{\Omega}_3^2$$

characterised by a single scalar function droplet data

$$z(y;\, x_1,\, x_2) = \frac{y^2}{\pi} \int dx_1'\, dx_2'\, \frac{z(0;\, x_1',\, x_2')}{[(x - x')^2 + y^2]^2} \ .$$

uniquely determined by droplet data on the y=0 plane

Smooth configurations

$$z(0; x_1, x_2) = \pm\frac{1}{2}$$

Natural to introduce $u(0;\, x_1,\, x_2) = \frac{1}{2} - z(0;\, x_1,\, x_2)$.

LLM : dictionary

Energy

$$\Delta = J = \int_{\mathbb{R}^2} \frac{d^2x}{2\pi\hbar}\, \frac{1}{2}\, \frac{x_1^2 + x_2^2}{\hbar}\, u(0;\, x_1,\, x_2) - \frac{1}{2} \left(\int_{\mathbb{R}^2} \frac{d^2x}{2\pi\hbar}\, u(0;\, x_1,\, x_2) \right)^2$$

Flux

$$N = \int_{\mathbb{R}^2} \frac{d^2x}{2\pi\hbar}\, u(0;\, x_1,\, x_2)\,, \qquad \hbar = 2\pi\ell_p^4$$

Holographic dictionary

- $y = 0$ LLM plane as phase space of single fermion
- $u(x_1, x_2; 0)$ phase space density

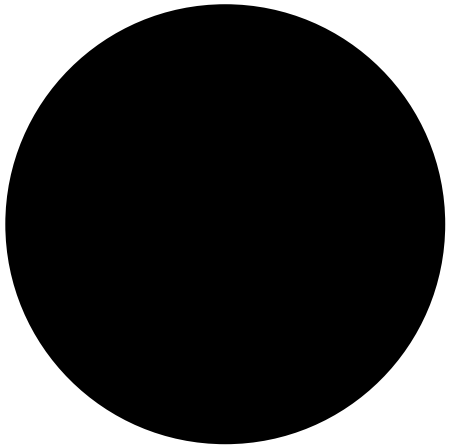
Hamiltonian eigenstates

Global AdS : black droplet of radius $r_{\rm fs} = R^2_{\rm AdS}$
Working in polar coordinates : $(x_1,\, x_2) \to (r, \phi)$, map to global AdS

$$y = R^2_{\rm AdS}\, \sinh\rho \sin\theta\,, \quad r = R^2_{\rm AdS}\, \cosh\rho \cos\theta\,,$$
$$\tilde{\phi} = \phi + t$$

Wigner semi-circle distribution

$$\rho(x) = 2\int_0^{\sqrt{R^4_{\rm AdS} - x^2}} \frac{dx_2\, dx_1}{2\pi\hbar}$$
$$= \frac{1}{\pi\hbar}\sqrt{R^4_{\rm AdS} - x^2}$$



Hamiltonian eigenstates

Excited states : rings
large number ($\sim N$) of fermions with same excitation

Excitations set scales of annulus sizes

Superstar ensemble

Set of BPS states $\Delta = J$ describing, at most, N_c giant gravitons

Superstar ensemble : infinite effective temperature $\Longrightarrow$ all configurations are equiprobable

$$\rho_{\text{superstar}} = \frac{1}{Z} \sum_A |\Psi_A\rangle\langle\Psi_A| \,, \qquad Z = \binom{N+N_c}{N}$$

where A labels N fermion states whose Young tableau have $\leq N_c$ columns.

Limit curve

Averaged number of columns of length j, $\langle c_j \rangle \to \frac{N_c}{N}$ as $N \to \infty$

Approximate the Young tableau by a continuous curve :

$$r_i \to y(x) = \int_{N-x}^{N} di \, \langle c_j \rangle = \frac{N_c}{N} x$$

where x labels the fermion and $y(x)$ its excitations, i.e. the limit curve $y(x)$ describes the shape of the

Superstar : matching gravity

Using the phase space interpretation :

$$\frac{u(0;r^2)}{2\hbar}\,dr^2 = dx\,, \qquad \frac{r^2\,u(0;r^2)}{4\hbar^2}\,dr^2 = (y(x)+x)\,dx$$

$$\Longrightarrow \quad u(0;r^2) \equiv u_{\text{superstar}} = \frac{1}{1+y'} = \frac{1}{1+N_c/N}\,,$$

matching the singular boundary condition defining the superstart configuration (naked singularity)

Derivation of the giant graviton distribution

Phase space density of giants $u_{\text{giants}} = N_c/N$
Giants are located at $\rho = 0$, hence using the LLM map

$$r|_{\rho=0} = R^2_{\text{AdS}} \sin\theta\,,$$

inserting in the phase space measure and integrating over the angular variable

$$dn = \frac{rdr}{\hbar}\,u_{\text{giants}} = N_c\,\sin 2\theta\,d\theta$$

LLM vs EPR=ER

Consider half-BPS states (potentially entangled) in two non-interacting N=4 SYM

Preliminary expectations for the superstar thermofield double

- $\exists$ naked singularity $\Longrightarrow$ absence of an infinite throat in this extremal situation
- entanglement entropy on one copy equals thermodynamic entropy

$$S_L \ll N^2 \quad \Longrightarrow \quad \text{quantum bridges}$$

Proposal : to glue two LLM geometries through droplet regions where correlations exist

- NO new classical geometries
- different quantum states will not be distinguished by a single observer

Product states

Two global AdS

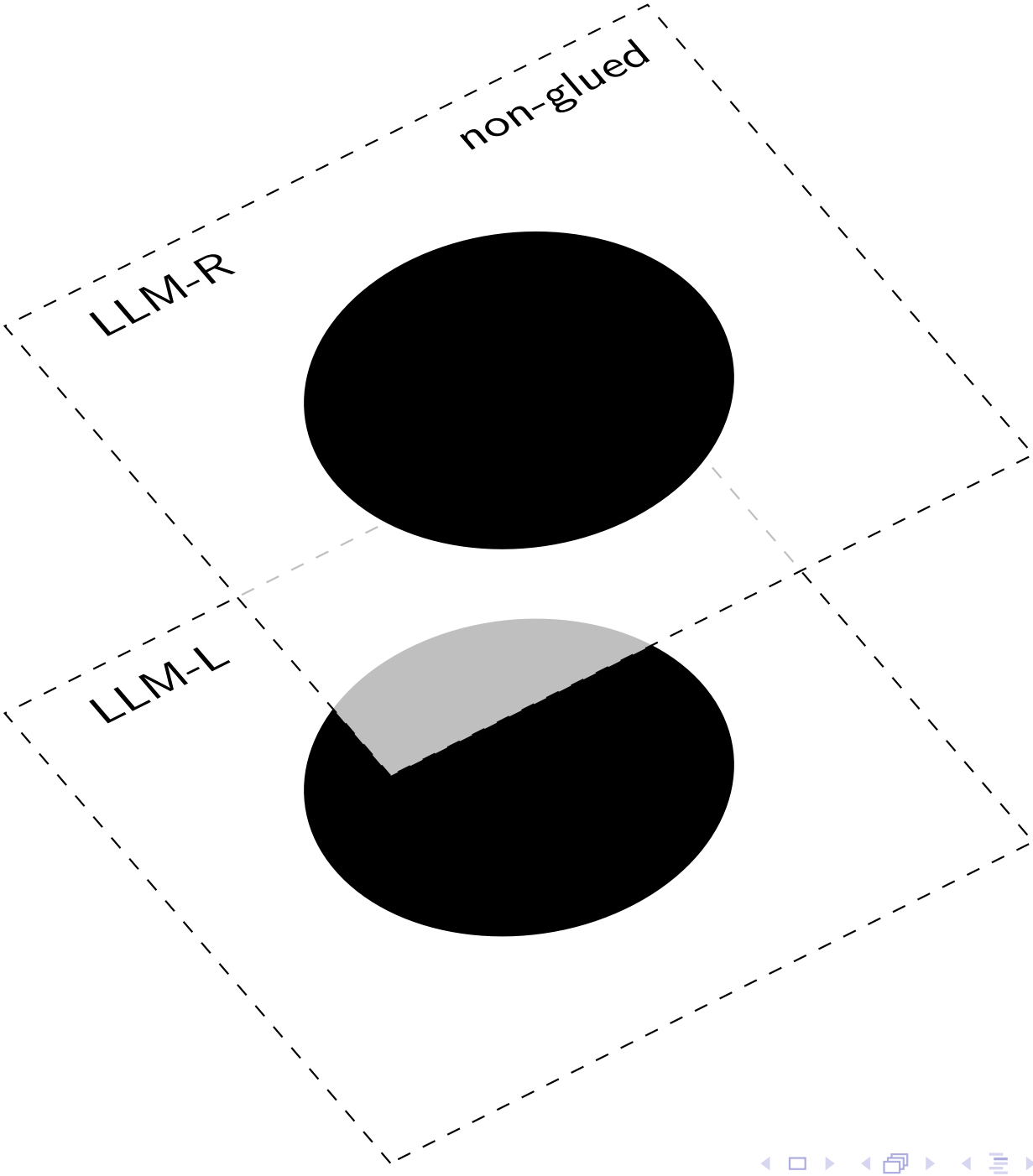
$|0\rangle_L \otimes |0\rangle_R$: product of 2 global AdS spaces

- all 2-sided correlators will factorise $\Longrightarrow$ absence of correlations
- gravity dual : two independent LLM geometries

Two non-entangled states

$|n\rangle_L \otimes |n\rangle_R$: product of 2 global AdS spaces

- all 2-sided correlators will factorise $\Longrightarrow$ absence of correlations
- if $|n\rangle$ has a gravity dual : two independent LLM geometries with relevant droplets


non-glued
LLM-R
LLM-L

Superstar

Consider a maximally correlated state

$$|\Psi\rangle = \sum_A \frac{1}{\sqrt{Z}} |\Psi_A\rangle \otimes |\Psi_A\rangle \,, \quad Z = \binom{N+N_c}{N}$$

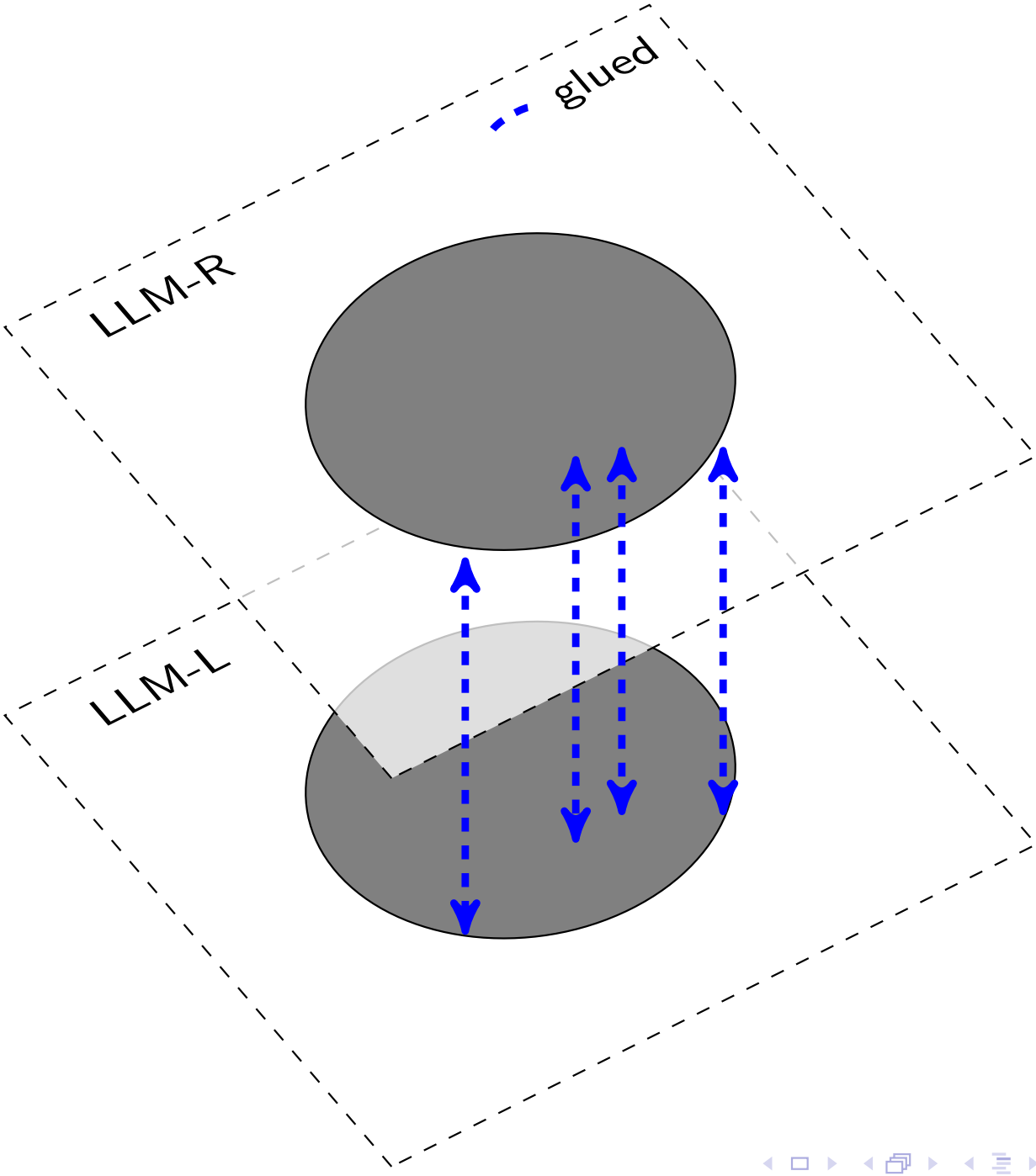
Notice $\rho_L = \rho_R$ equal the superstar density matrix. Furthermore,

$$\rho_L^{(1)} = \frac{1}{1+y'} \sum_{i=1}^{N+N_c} |i\rangle\langle i| = u_{\text{superstar}} \sum_{i=1}^{N+N_c} |i\rangle\langle i| \,,$$

2-sided correlators for one-particle operators

$$\langle\Psi|\mathcal{O}_L^{(1)}\,\mathcal{O}_R^{(1)}|\Psi\rangle = \frac{N\,N_c}{(N+N_c)(N+N_c-1)} \sum_{k\neq j} \langle j|\mathcal{O}_L^{(1)}|k\rangle\langle j|\mathcal{O}_R^{(1)}|k\rangle$$

$$+ \frac{N_c}{N}\,\frac{1}{(N+N_c)^2} \left\{ \sum_i^{N+N_c} \langle i|\mathcal{O}_L^{(1)}|i\rangle\,\langle i|\mathcal{O}_R^{(1)}|i\rangle - \frac{1}{N+N_c-1} \sum_{i\neq j} \langle i|\mathcal{O}_L^{(1)}|i\rangle\,\langle j|\mathcal{O}_R^{(1)}|j\rangle \right\}$$


glued
LLM-R
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Modified superstar

To stress where the gluing occurs, consider

- N_1 fermions in the Fermi sea
- remaining $N_2 = N - N_1$ in all possible excited states with equal weight
- N_c giant gravitons

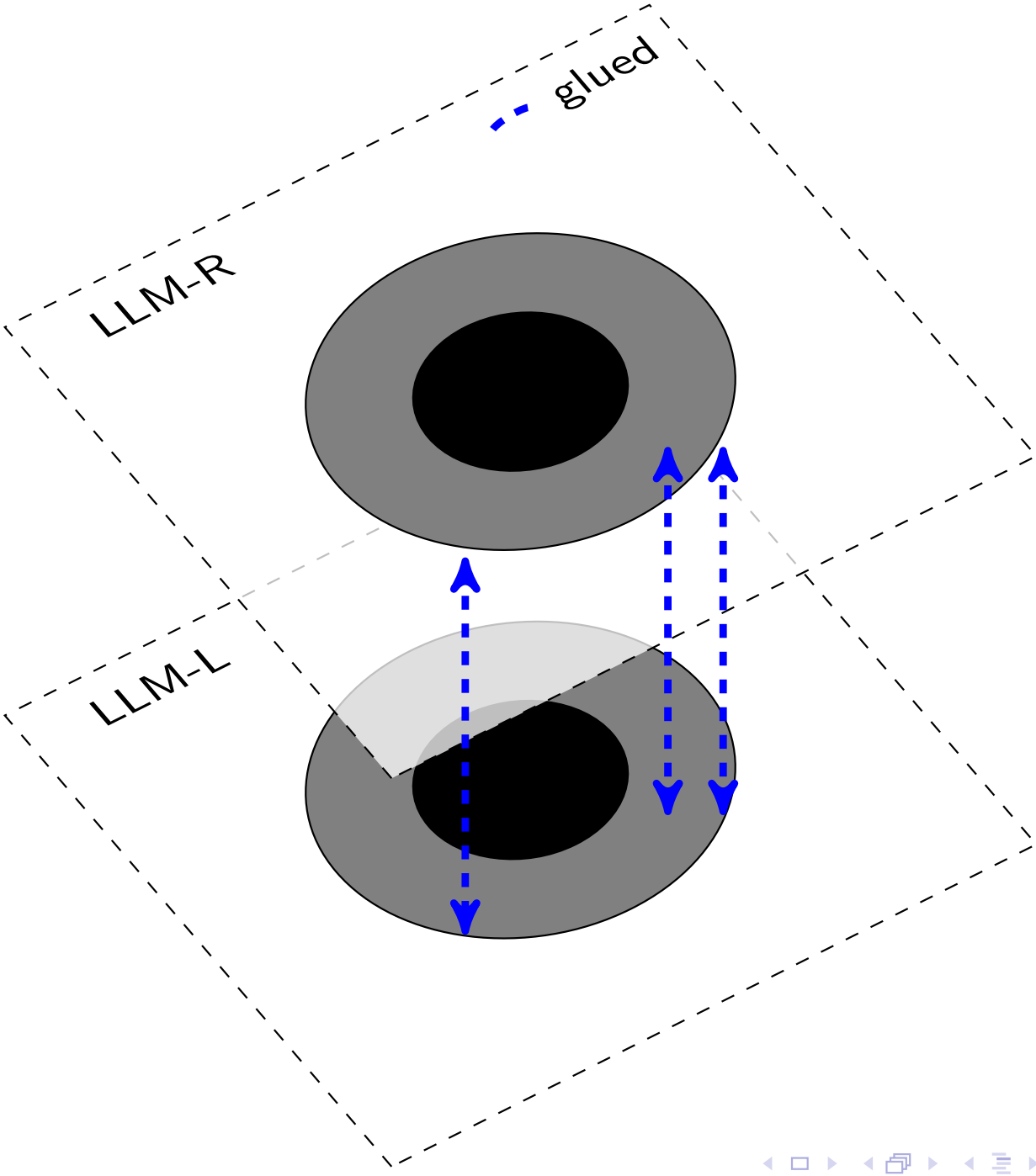
Maximally correlated state :

$$|\Psi\rangle = \frac{1}{\sqrt{Z_a}} \sum_{A \in \mathcal{H}} |\Psi_A\rangle |\Psi_A\rangle \,, \quad \text{with} \quad Z_a = \binom{N_2 + N_c}{N_2} \,.$$

By construction, 2-sided correlator is as before, up to

- $N \to N_2$
- sum over states $\sum_{A \in \mathcal{H}}$

Hence, correlations between L-R are only supported in the region of phase space describing the $N_2 - N_1$ excited fermions. Semiclassically, this corresponds to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LLM-R
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Correlation design

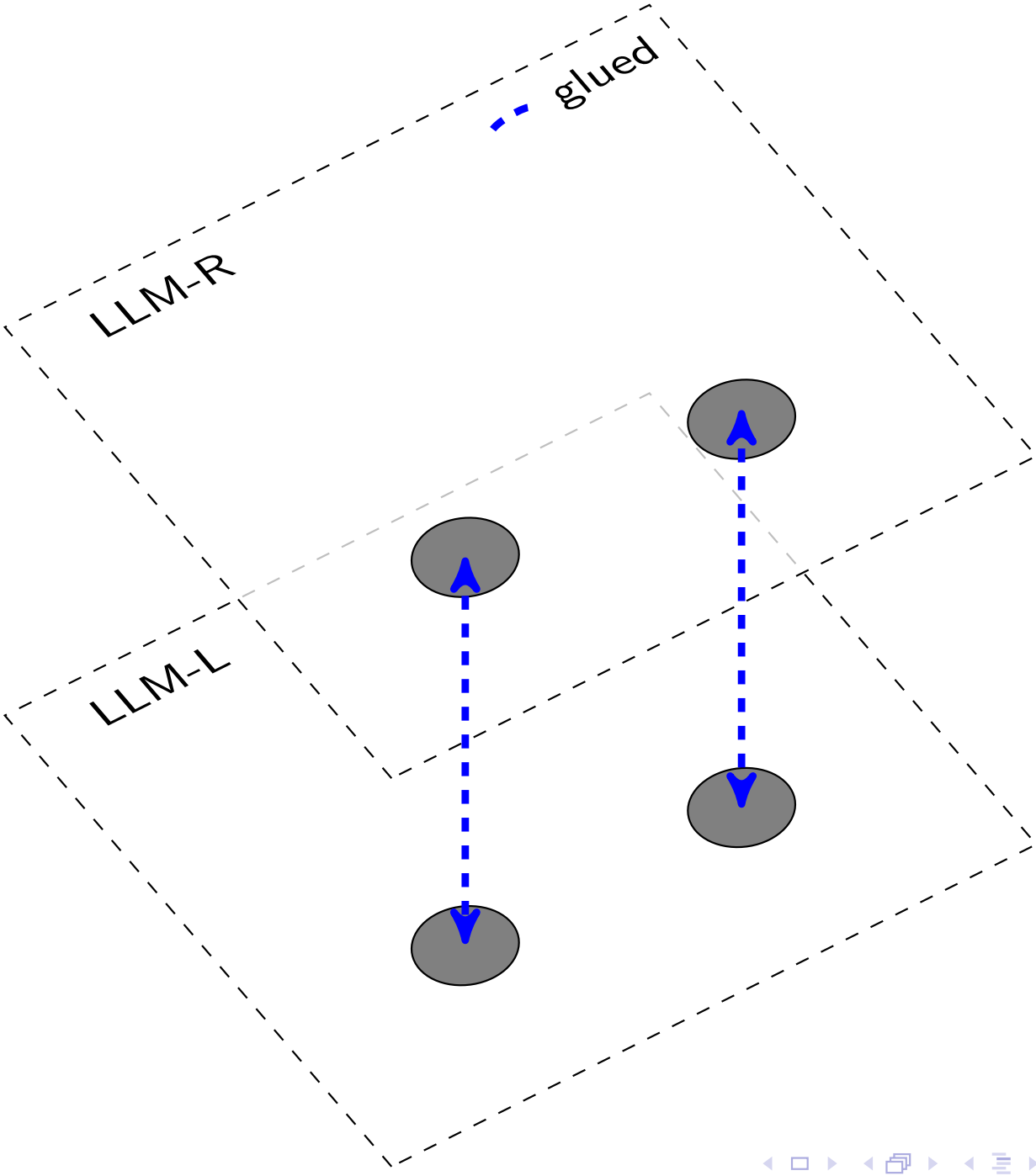
Consider maximally correlated coherent states of fermions, i.e.

$$
\begin{aligned}
|\Psi\rangle &= p\,|\Psi_1,\Psi_1\rangle + \sqrt{1-p^2}\,|\Psi_2,\Psi_2\rangle\,,\\
|\Psi_1\rangle &= A\,\left(|\alpha_1,\alpha_2\rangle - |\alpha_2,\alpha_1\rangle\right)\,,\\
|\Psi_2\rangle &= B\,\left(|\alpha_1,\alpha_3\rangle - |\alpha_3,\alpha_1\rangle\right)\,.
\end{aligned}
$$

with $\alpha_n \equiv \frac{x_n+iy_n}{\sqrt{2\hbar}}$. Coherent states are not orthogonal

$$
\begin{aligned}
\langle\alpha_1|\alpha_2\rangle &= e^{-(x_1-x_2)^2/(4\hbar)}\,e^{-(y_1-y_2)^2/(4\hbar)}\,e^{i(x_1y_2-y_1x_2)/(2\hbar)}\\
&\to 0 \quad \text{when} \quad x_1-x_2,\,y_1-y_2 \sim N\sqrt{\hbar}\,, \quad \hbar\to 0
\end{aligned}
$$

Hence, these tails are subleading for large semiclassical distances and 2-sided correlators still controlled by the same terms as before


glued
LLM-R
LLM-L

LLM & entanglement entropy in R-charge space

Half-BPS operators are delocalised in the N=4 SYM 3-sphere, but they have non-trivial correlations

- Reproduced by LLM phase space calculations & AdS/CFT methods

$\Longrightarrow$ $\exists$ entanglement entropy in R-charge space

Proposal : this is the entanglement among the fermions, i.e. we can ask for the entanglement in some region A of the real line where the harmonic potential acts.

Condensed matter aside

This is relevant in the field of optically trapped ultra-cold atomic gases given the experimental possibility to measure this entanglement

Computing fermion entanglement entropy

Gaussian systems + Wick theorem $\implies \rho_A \propto e^{-\sum_{i,j} H_{ij} c_i^\dagger c_j}$

Overlap formalism

Renyi entropies are given by

$$S_q = \sum_{i=1}^{N} e_q(a_i)\,, \qquad e_q(x) \equiv \frac{1}{1-q}\log[x^q + (1-x)^q]\,,\ \ q > 1$$

$$S_1 = \sum_{i=1}^{N} H(a_i)\,, \qquad H(x) = -x\log x - (1-x)\log(1-x)\,.$$

where a_i are the eigenvalues of the matrices

$$A_{nm} = \int_A dz\, \phi_n^\star(z)\, \phi_m(z) \qquad n, m = 1, \ldots, N$$

$$\mathcal{C}_A(x,y) = I_A(x)\langle c^\dagger(x)\, c(y)\rangle I_A(y)$$

CFT effective approximation

$\exists$ intermediate scale ℓ where inhomogeneous systems allow a continuous description $\rho^{-1} \ll \ell \ll \rho|\partial_x \rho|^{-1}$

- ρ^{-1} controls the microscopic scale
- $\rho|\partial_x \rho|^{-1}$ scale on which physical quantities vary macroscopically

Connection to CFT : consider the ground state propagator at large distances, i.e. linearise the dispersion relation around Fermi points

$$\langle c^\dagger(x,y)c(0,0)\rangle = \int_{-k_F}^{k_F} \frac{dk}{2\pi}\, e^{-i[kx+i\varepsilon(k)\frac{\tau}{\hbar}]} \simeq \int_{-\infty}^{k_F} \frac{dk}{2\pi}\, e^{-i[kx+i(k-k_F)v_F\,\tau]}$$
$$+ \int_{-k_F}^{\infty} \frac{dk}{2\pi}\, e^{-i[kx-i(k+k_F)v_F\,\tau]} = \frac{i}{2\pi}\left[\frac{e^{-ik_F x}}{x+iv_F\tau} - \frac{e^{ik_F x}}{x-iv_F\tau}\right]$$

where $v_F = \left.\frac{d\varepsilon(k)}{\hbar dk}\right|_{k_F}$

This looks like the propagator of a massless Dirac fermion in a non-trivial metric !! (Calabrese et al)

CFT framework

For some potential $V(x)$, this non-trivial background is

$$ds^2 = e^{2\sigma} dz d\bar{z}\,,$$
$$z(x,y) = \int^x \frac{1}{v_F(x')} dx' + i\tau\,, \quad e^{\sigma} = v_F(x)\,.$$

Using $\hbar = 1$ for a trapped potential, the semicircle Wigner distribution is non-zero for $x \in [-L, L]$ where $L = \sqrt{2N}$. Away from the edges, we can use the approach above giving rise to

$$z(x,y) = \arcsin \frac{x}{L} + i\tau\,, \qquad e^{\sigma} = v_F = \sqrt{L^2 - x^2}\,.$$

The coordinate $z \in [-\pi/2, \pi/2] \times \mathbb{R}$ lives on an infinite strip.

Application

Idea :

- Use Weyl rescaling (to deal with $e^{2\sigma}$)
- Map the strip to the upper half plane (to use correlation functions of twist operators)
- Identify the UV cut-off with $k_F(x)$ (to capture the validity of the effective CFT approach)

Renyi entropies :

$$S_n = \frac{n+1}{12n} \log \left[k_F(x)\, e^{\sigma} \left| \frac{dg}{dz} \right|^{-1} \mathrm{Im}(g(z)) \right] = \frac{n+1}{12n} \log[N(1-x^2/L^2)^{3/2}] \,,$$

It reproduces the behaviour of the exact diagonalisation in the overlap formalism.

Further comments

Renyi entropy for the Fermi sea is captured by the variance in the number of particles

$$S_q \approx \frac{\pi^2}{6}\left(1+\frac{1}{q}\right) V_{N_A}\,, \quad V_{N_A} = \langle N_A^2\rangle - \langle N_A\rangle^2$$

Holographic comments

- $V_{N_A} \equiv$ variance in LLM charge in phase space region anchored by A
  - relevance of the RR 5-form flux
- Classical gravity dual depends on the phase space density; covariant quantisation reproduces fermionic picture (Maoz-Rychkov)
  - suggests some quantum effect (matching subleading behaviour in G_N)
- $c_{\text{eff}} = 1 \implies$ highly curved geometry (this may be tangential)

Prospects : small excitations (CFT) + giant gravitons entanglement, ...