

Spectral Flow and Flat Space Holography in 3D

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Black Holes, Quantum Information and the Firewall Paradox
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Based on [R. Basu, S. Detournay, M.R; 170x.xxxxx]



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Introduction

The Quest for Quantum Gravity

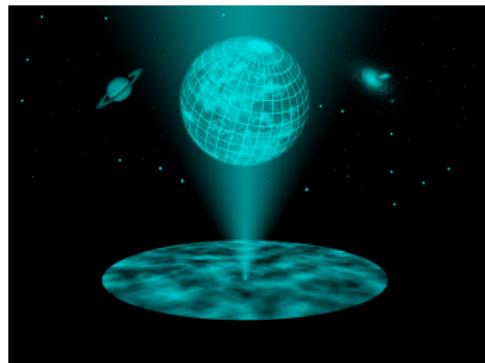
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Introduction

The Holographic Principle

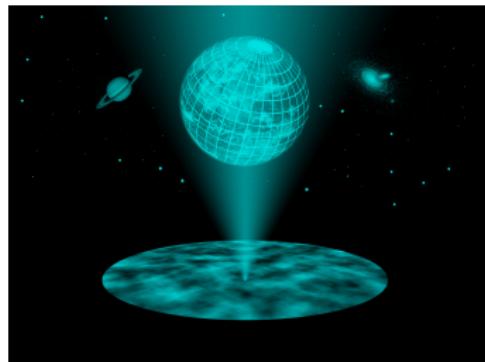
- ▶ Gravity $(d+1) \Leftrightarrow$ QFT (d)



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The Holographic Principle

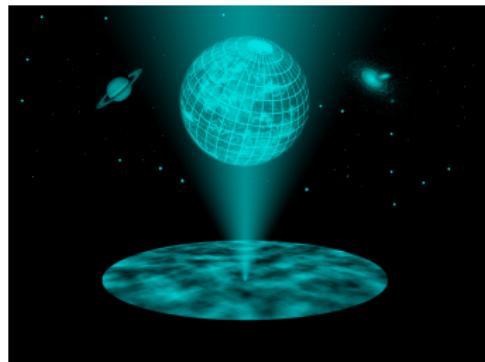
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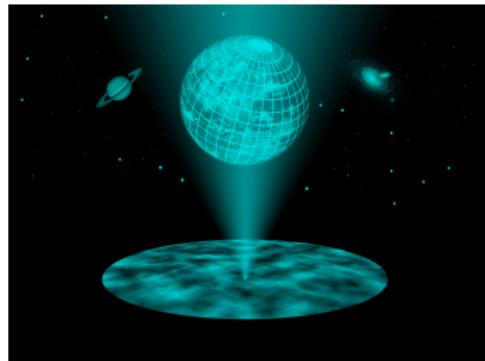
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- ▶ AdS/CFT [J. Maldacena; 9711200]
- ▶ Strong \Leftrightarrow Weak



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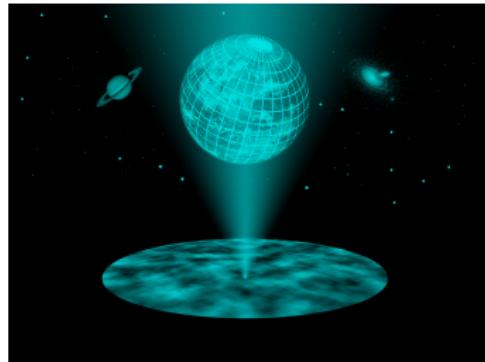


How general is Holography?

Introduction

The Holographic Principle

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- ▶ AdS/CFT [J. Maldacena; 9711200]
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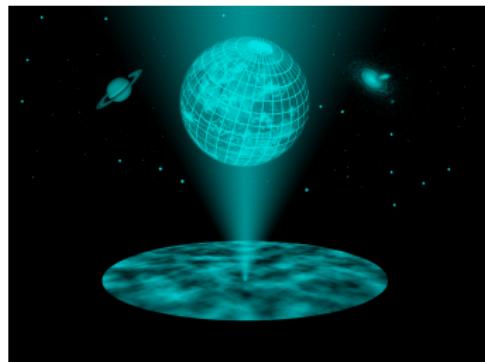
How general is Holography?

- ▶ String theory?

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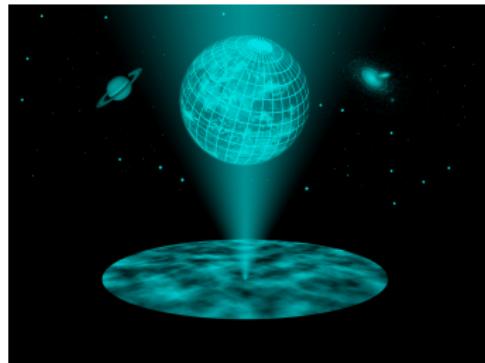
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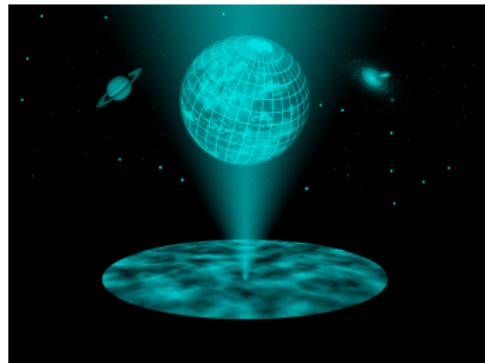
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- ▶ AdS?
- ▶ Higher-spin symmetries?

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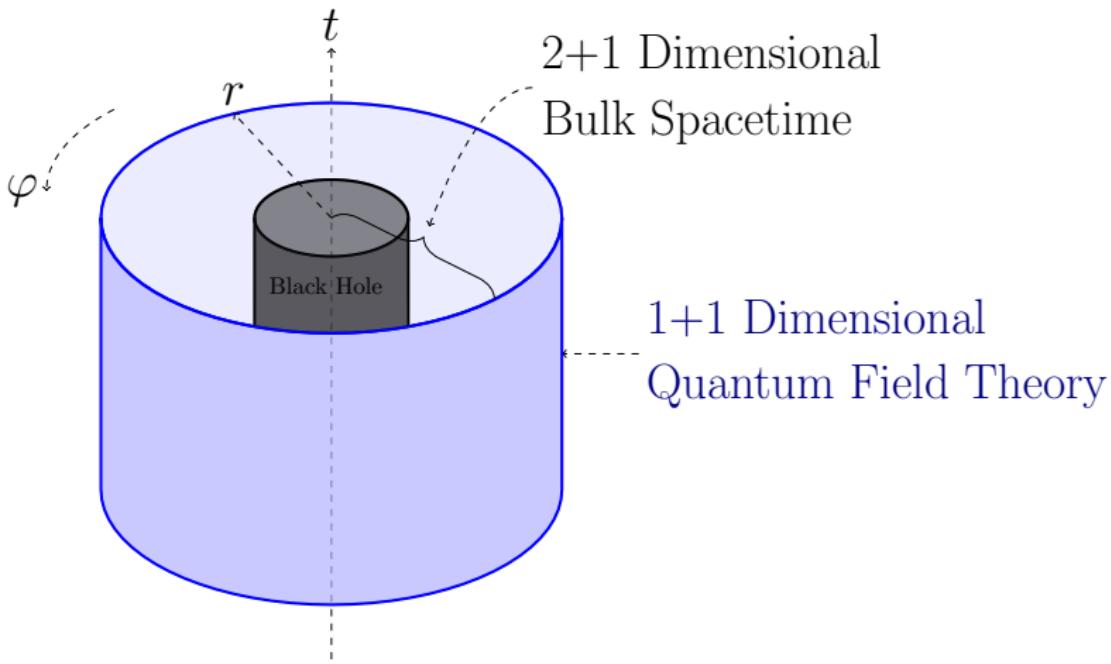


How general is Holography?

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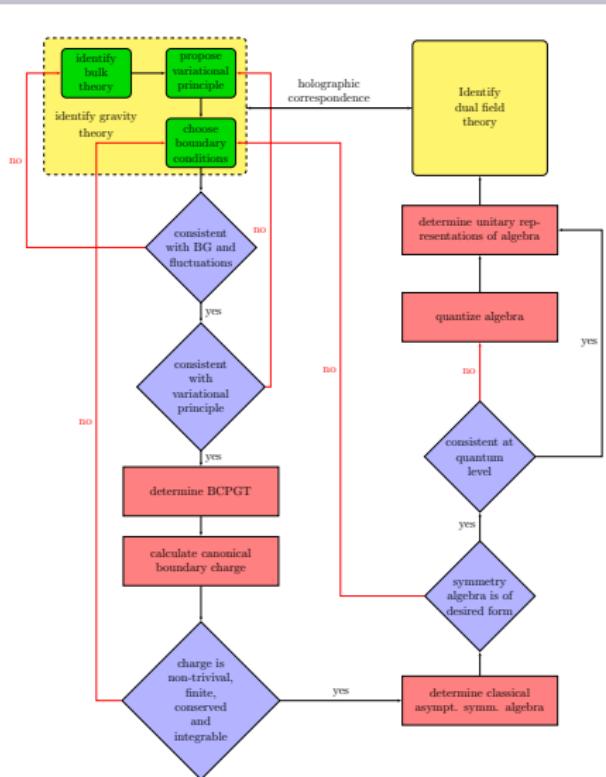
Introduction

Holography in 2+1 Spacetime Dimensions



Introduction

Canonical Analysis and Flat Space Holography



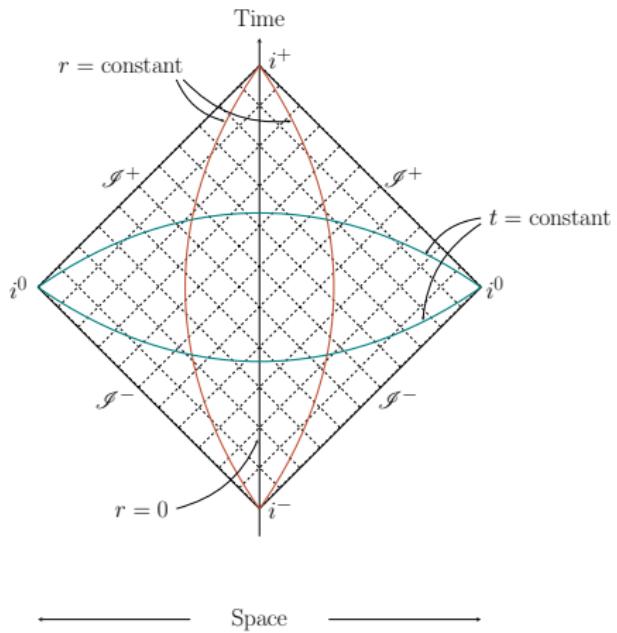
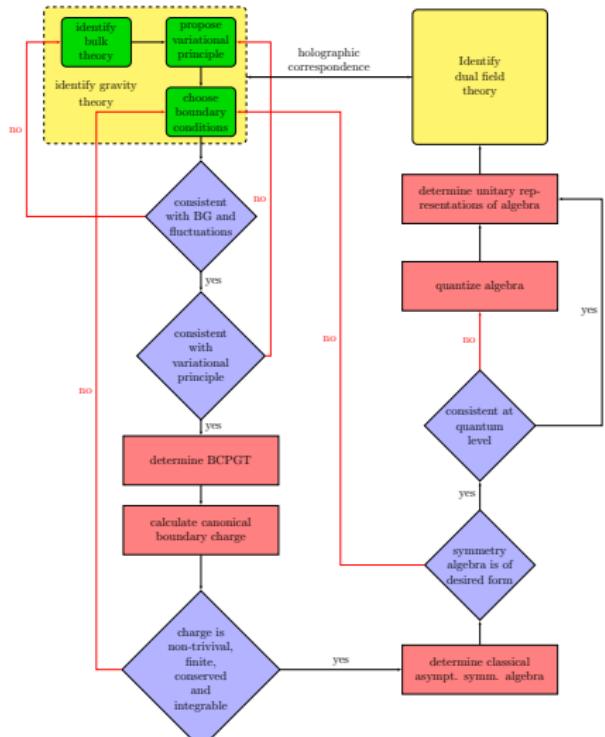
Non-AdS Higher-Spin Holography

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Flat Space Holography

Introduction

Canonical Analysis and Flat Space Holography



Non-AdS Higher-Spin Holography

Flat Space Holography

Motivation

Barnich-Compére Boundary Conditions

$$ds^2 = \mathcal{M} du^2 + 2\mathcal{N} du d\varphi - 2 dr du + r^2 d\varphi^2$$

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- ▶ $\mathcal{M} > 0, \mathcal{N} \neq 0$: Flat Space Cosmologies

Motivation

Barnich-Compére Boundary Conditions

$$ds^2 = \mathcal{M} du^2 + 2\mathcal{N} du d\varphi - 2 dr du + r^2 d\varphi^2$$

- ▶ $\mathcal{M} > 0, \mathcal{N} \neq 0$: Flat Space Cosmologies
- ▶ $\mathcal{M} = -1, \mathcal{N} = 0$: Global Flat Space

$$ds^2 = \mathcal{M} du^2 + 2\mathcal{N} du d\varphi - 2drdu + r^2 d\varphi^2$$

- ▶ $\mathcal{M} > 0, \mathcal{N} \neq 0$: Flat Space Cosmologies
- ▶ $\mathcal{M} = -1, \mathcal{N} = 0$: Global Flat Space
- ▶ $\mathcal{M} = \mathcal{N} = 0$: Null-Orbifold

$$ds^2 = \mathcal{M} du^2 + 2\mathcal{N} du d\varphi - 2drdu + r^2 d\varphi^2$$

[G. Barnich, G. Compére; 0610130]

Asymptotic Symmetries

$$[L_n, L_m] = (n - m)L_{n+m}$$

$$[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12}n(n^2 - 1)\delta_{n+m,0}$$

with $c_M = \frac{3}{G_N}$

Motivation

Barnich-Compére Boundary Conditions

$$ds^2 = \mathcal{M} du^2 + 2\mathcal{N} du d\varphi - 2drdu + r^2 d\varphi^2$$

[G. Barnich, G. Compére; 0610130]

Thermal Entropy

$$S_{\text{Th}} = \frac{A_H^0}{4G_N} = \frac{\pi}{2G_N} \frac{L_0}{\sqrt{M_0}},$$

Motivation

New Flat Space Boundary Conditions

$$ds^2 = \mathcal{M} du^2 + 2\mathcal{N} du d\varphi - 2e^{\alpha} (dr + \beta d\varphi) du + \left[e^{2\alpha} r^2 + \beta (2\mathcal{N} - \mathcal{M}\beta) \right] d\varphi^2$$

[S. Detournay, M. R; 1612.00278]

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Asymptotic Symmetries

$$[\mathbf{L}_n, \mathbf{L}_m] = (n - m)\mathbf{L}_{n+m}$$

$$[\mathbf{L}_n, \mathbf{M}_m] = (n - m)\mathbf{M}_{n+m} + \frac{c_M}{12} n^3 \delta_{n+m,0}$$

$$[\mathbf{L}_n, \mathbf{J}_m] = -m\mathbf{J}_{n+m}$$

$$[\mathbf{L}_n, \mathbf{P}_m] = -m\mathbf{P}_{n+m}$$

$$[\mathbf{M}_n, \mathbf{J}_m] = -m\mathbf{P}_{n+m}$$

$$[\mathbf{J}_n, \mathbf{P}_m] = \frac{\kappa_P}{2} n \delta_{n+m,0}$$

with $c_M = \frac{3}{G_N}$ and $\kappa_P = -\frac{1}{2G_N}$

$$ds^2 = \mathcal{M} du^2 + 2\mathcal{N} du d\varphi - 2e^{\alpha} (dr + \beta d\varphi) du + \left[e^{2\alpha} r^2 + \beta (2\mathcal{N} - \mathcal{M}\beta) \right] d\varphi^2$$

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Thermal Entropy

$$S_{\text{Th}} = \frac{A_H^0}{4G_N} = \frac{\pi}{2G_N} \frac{L_0 + f(J, P, \kappa_P)}{\sqrt{M_0 + g(J, P, \kappa_P)}},$$

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Thermal Entropy

$$S_{\text{Th}} = \frac{A_H^0}{4G_N} = \frac{\pi}{2G_N} \frac{L_0}{\sqrt{M_0}},$$

General \mathfrak{bms}_3 and two $\hat{\mathfrak{u}}(1)$ Algebras

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_L}{12}n^3\delta_{n+m,0}$$

$$[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12}n^3\delta_{n+m,0}$$

$$[L_n, J_m] = -mJ_{n+m}$$

$$[L_n, P_m] = -mP_{n+m}$$

$$[M_n, J_m] = -mP_{n+m}$$

$$[J_n, J_m] = \frac{\kappa_J}{2} n\delta_{n+m,0}$$

$$[J_n, P_m] = \frac{\kappa_P}{2} n\delta_{n+m,0}$$

Spectral Flow

$$\tilde{L}_n := L_n + \textcolor{red}{u} J_n + \textcolor{blue}{v} P_n + \frac{2\textcolor{red}{u}\textcolor{blue}{v}\kappa_P + \textcolor{red}{u}^2\kappa_J}{4} \delta_{n,0}$$

$$\tilde{M}_n := M_n + \textcolor{red}{u} P_n + \frac{\textcolor{red}{u}^2}{4} \kappa_P \delta_{n,0}$$

$$\tilde{J}_n := J_n + \frac{\textcolor{red}{u}\kappa_J + \textcolor{blue}{v}\kappa_P}{2} \delta_{n,0}$$

$$\tilde{P}_n := P_n + \frac{\textcolor{red}{u}}{2} \kappa_P \delta_{n,0}$$

Motivation

Follow Up Questions



- ▶ Models that exhibit spectral flow?

Motivation

Follow Up Questions



- ▶ Models that exhibit spectral flow?
- ▶ Derive thermal entropy via a partition function?

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- ▶ Models that exhibit spectral flow? ✓
- ▶ Derive thermal entropy via a partition function? ✓
- ▶ Supersymmetry? (✓)

$\mathcal{N} = 4$ Extended Super- \mathfrak{bms}_3

⋮

$$[L_n, G_r^\pm] = (\frac{n}{2} - r) G_{n+r}^\pm$$

$$[L_n, R_r^\pm] = (\frac{n}{2} - r) R_{n+r}^\pm$$

$$[M_n, G_r^\pm] = (\frac{n}{2} - r) R_{n+r}^\pm$$

$$[J_n, G_r^\pm] = \pm G_{n+r}^\pm$$

$$[J_n, R_r^\pm] = \pm R_{n+r}^\pm$$

$$[P_n, G_r^\pm] = \pm R_{n+r}^\pm$$

$$\{G_r^\pm, G_s^\mp\} = 2L_{r+s} \pm (r-s)J_{r+s} + \frac{c_L}{3} \left(r^2 - \frac{1}{4}\right) \delta_{r+s,0}$$

$$\{G_r^\pm, R_s^\mp\} = 2M_{r+s} \pm (r-s)P_{r+s} + \frac{c_M}{3} \left(r^2 - \frac{1}{4}\right) \delta_{r+s,0}$$

Spectral Flow

$$\tilde{\mathbf{L}}_n := \mathbf{L}_n + \textcolor{red}{u}\mathbf{J}_n + \textcolor{blue}{v}\mathbf{P}_n + \frac{2\textcolor{red}{u}\textcolor{blue}{v}\kappa_P + \textcolor{red}{u}^2\kappa_J}{4}\delta_{n+m,0}$$

$$\tilde{\mathbf{M}}_n := \mathbf{M}_n + \textcolor{red}{u}\mathbf{P}_n + \frac{\textcolor{red}{u}^2}{4}\kappa_P\delta_{n,0}$$

$$\tilde{\mathbf{J}}_n := \mathbf{J}_n + \frac{\textcolor{red}{u}\kappa_J + \textcolor{blue}{v}\kappa_P}{2}\delta_{n,0}$$

$$\tilde{\mathbf{P}}_n := \mathbf{P}_n + \frac{\textcolor{red}{u}}{2}\kappa_P\delta_{n,0}$$

$$\tilde{\mathbf{G}}_r^\pm = \mathbf{G}_{r\pm\textcolor{red}{u}}^\pm + ?(\textcolor{blue}{v})$$

$$\tilde{\mathbf{R}}_r^\pm = \mathbf{R}_{r\pm\textcolor{red}{u}}^\pm + ?(\textcolor{blue}{v})$$

Spectral Flow

$$\tilde{\mathbf{L}}_n := \mathbf{L}_n + \textcolor{red}{u} \mathbf{J}_n + \frac{\textcolor{red}{u}^2 \kappa_J}{4} \delta_{n,0}$$

$$\tilde{\mathbf{M}}_n := \mathbf{M}_n + \textcolor{red}{u} \mathbf{P}_n + \frac{\textcolor{red}{u}^2}{4} \kappa_P \delta_{n,0}$$

$$\tilde{\mathbf{J}}_n := \mathbf{J}_n + \frac{\textcolor{red}{u} \kappa_J}{2} \delta_{n,0}$$

$$\tilde{\mathbf{P}}_n := \mathbf{P}_n + \frac{\textcolor{red}{u}}{2} \kappa_P \delta_{n,0}$$

$$\tilde{\mathbf{G}}_r^\pm = \mathbf{G}_{r\pm}^\pm \textcolor{red}{u}$$

$$\tilde{\mathbf{R}}_r^\pm = \mathbf{R}_{r\pm}^\pm \textcolor{red}{u}$$

Simple Models

Einstein Gravity Coupled to Two $u(1)$ Gauge Fields

$$I_{EH+CS} = \frac{k}{4\pi} \int R + \frac{\kappa_P}{16\pi} \int \langle \mathcal{C} \wedge d\mathcal{C} \rangle$$

Simple Models

Einstein Gravity Coupled to Two $\mathfrak{u}(1)$ Gauge Fields

$$I_{\text{EH+CS}} = \frac{k}{4\pi} \int R + \frac{\kappa_P}{16\pi} \int \langle \mathcal{C} \wedge d\mathcal{C} \rangle$$

$$ds^2 = \left(\mathcal{M} - \frac{2\pi}{\kappa_P} \mathcal{P}^2 \right) du^2 + \left(\mathcal{N} - \frac{4\pi}{\kappa_P} \mathcal{J}\mathcal{P} \right) du d\varphi - 2 dr du + r^2 d\varphi^2$$

$$ds^2 = \left(\mathcal{M} - \frac{2\pi}{\kappa_P} \mathcal{P}^2\right) du^2 + \left(\mathcal{N} - \frac{4\pi}{\kappa_P} \mathcal{J}\mathcal{P}\right) du d\varphi - 2 dr du + r^2 d\varphi^2$$

Asymptotic Symmetries

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Einstein Gravity Coupled to Two $u(1)$ Gauge Fields

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Thermal Entropy

$$S_{\text{Th}} = \frac{A_H^0}{4G_N} = \frac{\pi}{6} \frac{c_M \left(L_0 - 2 \frac{J_0 P_0}{\kappa_P}\right)}{\sqrt{\frac{c_M}{6} \left(M_0 - \frac{P_0^2}{\kappa_P}\right)}}$$

Thermal Entropy

$$S_{\text{Th}} = \frac{A_H^0}{4G_N} = \frac{\pi}{6} \frac{c_M \left(L_0 - 2 \frac{J_0 P_0}{\kappa_P}\right)}{\sqrt{\frac{c_M}{6} \left(M_0 - \frac{P_0^2}{\kappa_P}\right)}} = \frac{\pi}{6} \frac{c_M \left(\tilde{L}_0 - 2 \frac{\tilde{J}_0 \tilde{P}_0}{\kappa_P}\right)}{\sqrt{\frac{c_M}{6} \left(\tilde{M}_0 - \frac{\tilde{P}_0^2}{\kappa_P}\right)}}$$

Spectral Flow

$$\tilde{L}_n := L_n + \textcolor{red}{u} J_n + \textcolor{blue}{v} P_n + \frac{\textcolor{red}{u} \textcolor{blue}{v}}{2} \kappa_P \delta_{n,0}$$

$$\tilde{M}_n := M_n + \textcolor{red}{u} P_n + \frac{\textcolor{red}{u}^2}{4} \kappa_P \delta_{n,0}$$

$$\tilde{J}_n := J_n + \frac{\textcolor{blue}{v} \kappa_P}{2} \delta_{n,0}$$

$$\tilde{P}_n := P_n + \frac{\textcolor{red}{u}}{2} \kappa_P \delta_{n,0}$$

Thermal Entropy and Partition Functions

Spectral Flow and \mathfrak{bms}_3 Modular Transformations



$$Z(\rho, \eta, \mu, \nu) = \text{Tr } e^{2\pi i (\rho M_0 + \eta L_0 + \mu P_0 + \nu J_0)}$$

Thermal Entropy and Partition Functions

Spectral Flow and \mathfrak{bms}_3 Modular Transformations



$$Z(\rho, \eta, \mu, \nu) = \text{Tr } e^{2\pi i(\rho M_0 + \eta L_0 + \mu P_0 + \nu J_0)}$$

Spectral Flow

$$\textcolor{red}{u} = \frac{\nu}{\eta}, \quad \textcolor{blue}{v} = \frac{\eta\mu - \nu\rho}{\nu^2}$$

Thermal Entropy and Partition Functions

Spectral Flow and \mathfrak{bms}_3 Modular Transformations



$$Z(\rho, \eta, \mu, \nu) = e^{(\dots)} \text{Tr } e^{2\pi i (\rho \tilde{M}_0 + \eta \tilde{L}_0)}$$

Spectral Flow

$$\textcolor{red}{u} = \frac{\nu}{\eta}, \quad \textcolor{blue}{v} = \frac{\eta\mu - \nu\rho}{\nu^2}$$

Thermal Entropy and Partition Functions

Spectral Flow and \mathfrak{bms}_3 Modular Transformations



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\mathfrak{bms}_3 Modular Transformations

$$\rho \rightarrow \frac{\rho}{\eta^2}, \quad \eta \rightarrow -\frac{1}{\eta}$$

Thermal Entropy and Partition Functions

Spectral Flow and \mathfrak{bms}_3 Modular Transformations

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$$Z(\rho, \eta, \mu, \nu) = e^{(\dots)} \text{Tr} e^{-\frac{2\pi i}{\eta} (\frac{\rho}{\eta} \tilde{M}_0 - \tilde{L}_0)}$$

[A. Bagchi, S. Detournay, R. Fareghbal, J. Simon; 1208.4372]

\mathfrak{bms}_3 Modular Transformations

$$\rho \rightarrow \frac{\rho}{\eta^2}, \quad \eta \rightarrow -\frac{1}{\eta}$$

Thermal Entropy and Partition Functions

Spectral Flow and \mathfrak{bms}_3 Modular Transformations

ULB

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$$Z(\rho, \eta, \mu, \nu) = e^{(\dots)} \text{Tr } e^{-4\pi^2 \left(\frac{\beta_I}{\Omega^2} \tilde{M}_0 + \frac{i}{\Omega} \tilde{L}_0 \right)}$$

[A. Bagchi, S. Detournay, R. Fareghbal, J. Simon; 1208.4372]

\mathfrak{bms}_3 Modular Transformations

$$\rho \rightarrow \frac{\rho}{\eta^2}, \quad \eta \rightarrow -\frac{1}{\eta}$$

Thermal Entropy and Partition Functions

Spectral Flow and \mathfrak{bms}_3 Modular Transformations



$$Z(\rho, \eta, \mu, \nu) = e^{(\dots)} \text{Tr } e^{-4\pi^2 \left(\frac{\beta_I}{\Omega^2} \tilde{M}_0 + \frac{i}{\Omega} \tilde{L}_0 \right)}$$

[A. Bagchi, S. Detournay, R. Fareghbal, J. Simon; 1208.4372]

High-Temperature Limit

$$\tilde{M}_0^{\min} = h_M^\nu \quad \tilde{L}_0^{\min} = h_L^\nu$$

Thermal Entropy and Partition Functions

Spectral Flow and \mathfrak{bms}_3 Modular Transformations

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$$\log Z(\rho, \eta, \mu, \nu) \approx \frac{2\pi i}{\eta} \left(\frac{\rho}{\eta} h_M^\nu - h_L^\nu - \frac{\nu^2}{4} \kappa_J - \frac{2\eta\mu\nu - \nu^2\rho}{4\eta} \kappa_P \right)$$

High-Temperature Limit

$$\tilde{M}_0^{\min} = h_M^\nu \quad \tilde{L}_0^{\min} = h_L^\nu$$

Thermal Entropy and Partition Functions

Thermal Entropy



$$h_L = \frac{1}{2\pi i} \partial_\eta \log Z$$

$$h_M = \frac{1}{2\pi i} \partial_\rho \log Z$$

$$j = \frac{1}{2\pi i} \partial_\nu \log Z$$

$$p = \frac{1}{2\pi i} \partial_\mu \log Z$$

Thermal Entropy and Partition Functions

Thermal Entropy

$$h_L = \frac{1}{2\pi i} \partial_\eta \log Z$$

$$j = \frac{1}{2\pi i} \partial_\nu \log Z$$

$$h_M = \frac{1}{2\pi i} \partial_\rho \log Z$$

$$p = \frac{1}{2\pi i} \partial_\mu \log Z$$

Thermal Entropy

$$S = (-1 + \eta \partial_\eta + \rho \partial_\rho + \nu \partial_\nu + \mu \partial_\mu) \log Z = 4\pi i \frac{\eta h_L^\nu - \rho h_M^\nu}{\eta^2}$$

Thermal Entropy and Partition Functions

Thermal Entropy

$$h_L = \frac{1}{2\pi i} \partial_\eta \log Z$$

$$j = \frac{1}{2\pi i} \partial_\nu \log Z$$

$$h_M = \frac{1}{2\pi i} \partial_\rho \log Z$$

$$p = \frac{1}{2\pi i} \partial_\mu \log Z$$

$$h_L^\vee = -\frac{c_L}{24}$$

$$h_M^\vee = -\frac{c_M}{24}$$

Thermal Entropy

$$S = \frac{\pi}{6} \frac{c_M \left(h_L - 2 \frac{jp}{\kappa_P} + \frac{\kappa_J}{\kappa_P^2} p^2 \right) + c_L \left(h_M - \frac{p^2}{\kappa_P} \right)}{\sqrt{\frac{c_M}{6} \left(h_M - \frac{p^2}{\kappa_P} \right)}}$$

Thermal Entropy and Partition Functions

Logarithmic Corrections



$$\mathcal{Z}_{ab} = \frac{1}{2\pi i} \partial_a \partial_b \log Z$$

Thermal Entropy and Partition Functions

Logarithmic Corrections



$$\mathcal{Z}_{ab} = \frac{1}{2\pi i} \partial_a \partial_b \log Z$$

Logarithmic Corrections

$$\Delta S = -\frac{1}{2} \log \det \mathcal{Z}$$

Thermal Entropy and Partition Functions

Logarithmic Corrections

$$\mathcal{Z}_{ab} = \frac{1}{2\pi i} \partial_a \partial_b \log Z$$

Logarithmic Corrections

$$\Delta S = -\frac{1}{2} \log \left[\frac{(h_M^v)^2 \kappa_P^2}{\eta^8} \right] = \log \left[\frac{c_M}{24 \kappa_P \left(M_0 - \frac{P_0^2}{\kappa_P} \right)^2} \right]$$

Simple Models

$\mathcal{N} = 4$ Supergravity



$$I_{\text{CS}}[\mathcal{A}] = \frac{k}{4\pi} \int_{\mathcal{M}} \langle \mathcal{A} \wedge d\mathcal{A} + \frac{2}{3}\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \rangle$$

$$I_{\text{CS}}[\mathcal{A}] = \frac{k}{4\pi} \int_{\mathcal{M}} \langle \mathcal{A} \wedge d\mathcal{A} + \frac{2}{3}\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \rangle$$

$$ds^2 = \left(\mathcal{M} + \frac{\pi}{4k} \mathcal{P}^2 \right) du^2 + \left(\mathcal{N} + \frac{\pi}{2k} \mathcal{J}\mathcal{P} \right) du d\varphi - 2 dr du + r^2 d\varphi^2$$



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Asymptotic Symmetries

$$[L_n, L_m] = (n - m)L_{n+m}$$

$$[L_n, M_m] = (n - m)M_{n+m} + \frac{c_M}{12} n(n^2 - 1)\delta_{n+m,0}$$

⋮

$$\{G_r^\pm, G_s^\mp\} = 2L_{r+s} \pm (r - s)J_{r+s},$$

$$\{G_r^\pm, R_s^\mp\} = 2M_{r+s} \pm (r - s)P_{r+s} + \frac{c_M}{3} \left(r^2 - \frac{1}{4}\right) \delta_{r+s,0},$$

$$[J_n, P_m] = \frac{c_M}{3} n \delta_{n+m,0}$$

Thermal Entropy

First Law of Flat Space Cosmologies

$$\delta S_{\text{Th}} = \beta_T (-\delta M + \Omega \delta J - \Phi_{\mathcal{J}} \delta Q_{\mathcal{J}} - \Phi_{\mathcal{P}} \delta Q_{\mathcal{P}})$$

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Charges

$$\delta M := \delta Q[\partial_u] = \frac{k}{2\pi} \int d\varphi \langle \mathcal{A}_u \delta \mathcal{A}_\varphi \rangle = \delta M_0$$

$$\delta J := \delta Q[\partial_\varphi] := \frac{k}{2\pi} \int d\varphi \langle \mathcal{A}_\varphi \delta \mathcal{A}_\varphi \rangle = \delta L_0$$

$$\delta Q_{\mathcal{J}} := \frac{k}{2\pi} \int d\varphi \langle J \delta \mathcal{A}_\varphi \rangle = \delta J_0$$

$$\delta Q_{\mathcal{P}} = \frac{k}{2\pi} \int d\varphi \langle P \delta \mathcal{A}_\varphi \rangle = \delta P_0$$



$$\delta S_{\text{Th}} = \left\langle -\frac{\beta_T}{2\pi} \left(\int d\varphi a_u - \Omega \int d\varphi a_\varphi + \Phi_{\mathcal{J}} \int d\varphi J + \Phi_{\mathcal{P}} \int d\varphi P \right) \delta \mathcal{A}_\varphi \right\rangle$$

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Thermal Entropy

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Holonomy

$$h = -\frac{\beta_T}{2\pi} \left(\int d\varphi a_u - \Omega \int d\varphi a_\varphi + \Phi_{\mathcal{J}} \int d\varphi J + \Phi_{\mathcal{P}} \int d\varphi P \right)$$

Thermal Entropy

First Law of Flat Space Cosmologies



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Thermal Entropy

$$S_{\text{Th}} = \frac{\pi}{6} \frac{c_M \left(L_0 - 3 \frac{J_0 P_0}{c_M} \right)}{\sqrt{\frac{c_M}{6} \left(M_0 - \frac{3P_0^2}{2c_M} \right)}}$$

Thermal Entropy and Partition Functions

bms₃ Modular Transformations

ULB

15

$$Z(\rho, \eta, \mu, \nu) = \text{Tr}_{RR}(-1)^F e^{2\pi i (\rho M_0 + \eta L_0 + \mu P_0 + \nu J_0)}$$

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$$\rho \rightarrow \frac{\rho}{\eta^2}, \quad \eta \rightarrow -\frac{1}{\eta}, \quad \mu \rightarrow \frac{\eta\mu - \nu\rho}{\eta^2}, \quad \nu \rightarrow \frac{\nu}{\eta}$$

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$$Z(\rho, \eta, \mu, \nu) = e^{-i\pi \left(\frac{\nu^2}{2\eta} \kappa_J + \frac{2\eta\mu\nu - \nu^2\rho}{2\eta^2} \kappa_P \right)} Z \left(\frac{\rho}{\eta^2}, -\frac{1}{\eta}, \frac{\eta\mu - \nu\rho}{\eta^2}, \frac{\nu}{\eta} \right)$$

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High-Temperature Limit

$$\tilde{M}_0^{\min} = h_M^\nu \quad \tilde{L}_0^{\min} = h_L^\nu, \quad j^\nu = p^\nu = 0$$

Thermal Entropy and Partition Functions

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$$Z(\rho, \eta, \mu, \nu) \approx e^{-i\pi \left(\frac{\nu^2}{2\eta} \kappa_J + \frac{2\eta\mu\nu - \nu^2\rho}{2\eta^2} \nu \kappa_P \right)} e^{2\pi i \left(\frac{\rho}{\eta^2} h_M^\nu - \frac{1}{\eta} h_L^\nu \right)}$$

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Density of States

$$Z(\rho, \eta, \mu, \nu) = \int d(h_L, h_M, j, p) e^{2\pi i (\rho h_M + \eta h_L + \mu p + \nu j)}$$

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Density of States

$$d(\dots) = \int e^{-i\pi \left(\frac{\nu^2}{2\eta} \kappa_J + \frac{2\eta\mu\nu - \nu^2\rho}{2\eta^2} \kappa_P \right)} e^{2\pi i \left(\frac{\rho}{\eta^2} h_M^\nu - \frac{1}{\eta} h_L^\nu - \rho h_M - \eta h_L - \mu p - \nu j \right)}$$

Thermal Entropy and Partition Functions

bms3 Modular Transformations

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Thermal Entropy

$$S = \log d_{(0)} = -\pi \frac{4h_M^v \left(h_L - 2\frac{jp}{\kappa_P} + \frac{\kappa_J p^2}{\kappa_P^2} \right) + 4h_L^v \left(h_M - \frac{p^2}{\kappa_P} \right)}{\sqrt{-4h_M^v \left(h_M - \frac{p^2}{\kappa_P} \right)}}$$

Thermal Entropy and Partition Functions

bms3 Modular Transformations

ULB

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Vacuum Weights

$$h_L^V = -\frac{c_L}{24}, \quad h_M^V = -\frac{c_M}{24}$$

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Thermal Entropy and Partition Functions

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$$h_L^V = -\frac{c_L}{24}, \quad h_M^V = -\frac{c_M}{24}$$

Thermal Entropy

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- ▶ How does the spectral flow influence the spectrum?
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Thank you for your attention!

