Symmetry protected entanglement between gravity and matter

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Introduction – QM & QG

Problems common to both QM & QG:

- *Nonlocality* – entanglement-based in QM vs. dynamical in QG (superpositions of different spacetimes and their respective causal orders)

- *Quantum-to-classical transition* and the related *measurement problem*

Introduction – QM & QG

The measurement problem – gravitationally-induced *objective collapse*:

- Fluctuating gravity field due to position uncertainty of massive objects (Karolyhazy, Diósi, etc.)
- Impossibility to (exactly) identify the points from different macroscopically superposed spacetimes (Penrose)

Introduction – QM & QG

Emergence of classicality – *decoherence* of matter by gravity:

- Gravitational time dilation by purely classical gravity (Brukner et al, etc.)
- Perturbative quantum gravity (Kay, Anastopoulos & Hu, Oniga & Wang, etc.)
- Holographic entanglement between the degrees of freedom (*both* gravitational and matter) on the two sides of the black-hole horizon (Hawking, Susskind, Maldacena, etc.)

Introduction – QM & QG

Previous approaches:

- The "for all practical purposes" inevitable and fast interaction-induced decoherence from initially product to final entangled states between gravity and matter (*dynamical* decoherence)

Our approach:

- The symmetry requirements allow for only the entangled states between gravity and matter as physical ones (*symmetry-protected* decoherence)

General Relativity and Local Symmetry

The principle of **General Relativity (GR)**:

All reference frames (i.e., choices of coordinates on $\mathcal{M}_4$) are equivalent.

Implementation of **GR**:

The theory is invariant with respect to $Diff(\mathbb{R}^4)$ Lie group, by localising the translational sub-group $\mathbb{R}^4$ of the Poincaré group $P(4) = \mathbb{R}^4 \ltimes SO(3,1)$.

Group $Diff(\mathbb{R}^4)$ has four generators at each point x of the spacetime:

- The *scalar constraint* $C(x)$ (time translation generator)
- The *3-diffeomorphism* constraints $C_i(x)$, with $i = 1, 2, 3$ (three space translation generators)

Additional local *Lorentz (Gauss) constraints* $C_{ab}(x)$ come from the localisation of the $SO(3,1)$ group, implementing the $c = 1$ postulate (six local Lorentz generators, three rotations and three boosts).

Quantising Gravity

Kinematical Hilbert space: $\mathcal{H}_{\text{kin}} = \mathcal{H}_G \otimes \mathcal{H}_M$

Postulating the principle of General Relativity at the quantum level, we pass from $\mathcal{H}_{\text{kin}}$ to its gauge invariant subspace $\mathcal{H}_{\text{phys}}$ of diffeomorphism invariant states.

In particular, the *scalar constraint* is given as (the minimal coupling with matter, C^M, is prescribed by the **Strong Equivalence Principle**):

$$\hat{\mathcal{C}}|\Psi\rangle \equiv \left[\mathcal{C}^G(\hat{g},\hat{\pi}_g) + \mathcal{C}^M(\hat{g},\hat{\pi}_g,\hat{\phi},\hat{\pi}_\phi)\right]|\Psi\rangle = 0\,.$$

The result: the physical Hilbert space contains *no separable states*, with respect to the gravity-matter factorisation.

Covariant formalism – Hartle-Hawking state

General state from the kinematical Hilbert space $\mathcal{H}_{\text{kin}} = \mathcal{H}_G \otimes \mathcal{H}_M$:

$$|\Psi\rangle = \int \mathcal{D}g \int \mathcal{D}\phi \, \Psi[g,\phi] \, |g\rangle \otimes |\phi\rangle$$

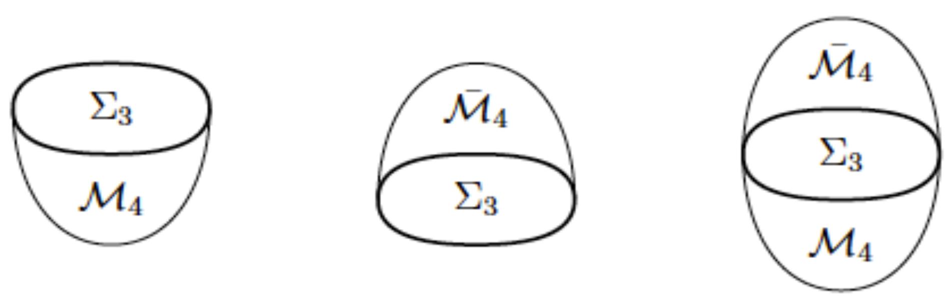
Hartle-Hawking state from $\mathcal{H}_{\text{phys}}$ (satisfies the scalar constraint):

$$\Psi_{\text{HH}}[g,\phi] = \mathcal{N} \int \mathcal{D}G \int \mathcal{D}\Phi \, e^{iS_{\text{tot}}[g,\phi,G,\Phi]}$$

The action: $S_{\text{tot}}[g,\phi,G,\Phi] = S_G[g,G] + S_M[g,\phi,G,\Phi]$

Partial state of matter: $\hat{\rho}_M = \int \mathcal{D}\phi \int \mathcal{D}\phi' \, Z[\phi,\phi'] \, |\phi\rangle \otimes \langle\phi'|$

$$Z[\phi,\phi'] \equiv \int \mathcal{D}g \, \Psi_{\text{HH}}[g,\phi] \Psi^*_{\text{HH}}[g,\phi']$$



Covariant formalism – entanglement criterion

Standard QM – Schmidt bi-orthogonal form: $|\Psi\rangle_{12} = \sum_i \sqrt{r_i} |\alpha_i\rangle_1 \otimes |\beta_i\rangle_2$

Partial state: $\hat{\rho}_1 = \sum_i r_i |\alpha_i\rangle_1 \otimes \langle\alpha_i|_1$

Squaring: $\hat{\rho}_1^2 = \sum_i r_i^2 |\alpha_i\rangle_1 \otimes \langle\alpha_i|_1$

Separable state has only one term with $r_1 = 1$: $\mathrm{Tr}\,\hat{\rho}_1^2 = \mathrm{Tr}\,\hat{\rho}_1 = 1$

Entangled state has more than one term: $\mathrm{Tr}\,\hat{\rho}_1^2 = \sum_i r_i^2 < \sum_i r_i = \mathrm{Tr}\,\hat{\rho}_1 = 1$

Field theory: $\mathrm{Tr}_M\,\hat{\rho}_M^2 = \int \mathcal{D}\phi \int \mathcal{D}\phi' \,\left|Z[\phi,\phi']\right|^2$

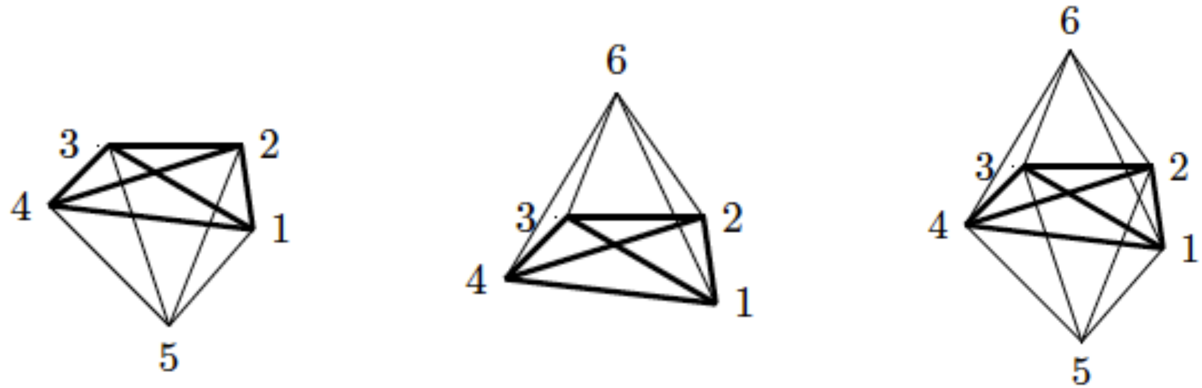
Covariant formalism – Regge model

State sum: $$\int \mathcal{D}\phi \, Z[\phi,\phi] = Z \equiv \int \mathcal{D}G \int \mathcal{D}\Phi \, e^{iS[G,\Phi]}$$

Regge state sum on triangulation $T(\mathcal{M}_4)$: $$Z_T = \int \mathcal{D}L \int \mathcal{D}\Phi \, e^{i(S_R[L]+S_M[L,\Phi])}$$

$$\mathrm{Tr}_M \, \hat{\rho}_M^2 = \frac{\int \prod_{v\in\partial T} \mathrm{d}\varphi_v \int \prod_{v\in\partial T} \mathrm{d}\varphi'_v \left| \int \prod_{\epsilon\in T\cup\bar{T}\cup\partial T} \mathrm{d}L_\epsilon \, \mu(L) \int \prod_{v\in T\cup\bar{T}} \mathrm{d}\Phi_v \, e^{iS_{\mathrm{tot}}[\varphi,\varphi',L,\Phi]} \right|^2}{\left(\int \prod_{\epsilon\in T\cup\bar{T}\cup\partial T} \mathrm{d}L_\epsilon \, \mu(L) \int \prod_{v\in T\cup\bar{T}\cup\partial T} \mathrm{d}\Phi_v \, e^{iS_{\mathrm{tot}}[L,\Phi]} \right)^2}$$

Regge model – a toy example



$$\mathrm{Tr}_M\,\hat{\rho}_M^2 = \frac{\int \mathrm{d}\varphi_1 \mathrm{d}\varphi_3 \mathrm{d}\varphi_1' \mathrm{d}\varphi_3' \left| \int \mathrm{d}^7 L\, \mu(L) \int \mathrm{d}\Phi_5 \mathrm{d}\Phi_6\, e^{iS_{\mathrm{tot}}[\varphi,\varphi',L,\Phi]} \right|^2}{\left(\int \mathrm{d}^7 L\, \mu(L) \int \mathrm{d}^4\Phi\, e^{iS_{\mathrm{tot}}[L,\Phi]} \right)^2},$$

$$\mathrm{d}^7 L \equiv \mathrm{d}l_{12}\mathrm{d}l_{13}\mathrm{d}l_{34}\mathrm{d}L_{15}\mathrm{d}L_{16}\mathrm{d}L_{35}\mathrm{d}L_{36}\,,$$

$$\mathrm{d}^4\Phi \equiv \mathrm{d}\varphi_1\mathrm{d}\varphi_3\mathrm{d}\Phi_5\mathrm{d}\Phi_6\,.$$

$$\mathrm{Tr}_M\,\hat{\rho}_M^2 = 0.979 \pm 0.005$$

Canonical formalism

Canonical quantisation of GR (LQG, WdW, etc.):

- Poisson brackets promoted into commutators
- Classical fields g, ϕ become operator fields $\hat{g}, \hat{\phi}$

Scalar constraint: $\hat{\mathcal{C}}|\Psi\rangle \equiv \left[\mathcal{C}^G(\hat{g}, \hat{\pi}_g) + \mathcal{C}^M(\hat{g}, \hat{\pi}_g, \hat{\phi}, \hat{\pi}_\phi)\right]|\Psi\rangle = 0$

Matter part: $\mathcal{C}^M(\hat{g}, \hat{\pi}_g, \hat{\phi}, \hat{\pi}_\phi) = \hat{\pi}_{\phi A}\hat{\nabla}_\perp{}^A{}_B\hat{\phi}^B - \frac{1}{N}\mathcal{L}_M(\hat{g}, \hat{\pi}_g, \hat{\phi}, \hat{\pi}_\phi)$

Canonical formalism – matter Lagrangians

Scalar field: $\mathcal{L}_M(\hat{g}, \hat{\varphi}, \partial\hat{\varphi}) = \frac{1}{2}\hat{e}\left[\hat{g}^{\mu\nu}(\partial_\mu\hat{\varphi})(\partial_\nu\hat{\varphi}) - m^2\hat{\varphi}^2 + U(\hat{\varphi})\right]$

Dirac (spinor) field: $\mathcal{L}_M(\hat{e}, \hat{\omega}, \hat{\psi}, \hat{\bar{\psi}}) = \hat{e}\left(\frac{i}{2}\hat{\bar{\psi}}\gamma^a\hat{e}^\mu{}_a\hat{\overleftrightarrow{\nabla}}_\mu\hat{\psi} - m\hat{\bar{\psi}}\hat{\psi}\right)$

EM (gauge) field: $\mathcal{L}_M(\hat{g}, \hat{A}, \partial\hat{A}) = -\frac{1}{4}\hat{e}\,\hat{g}^{\mu\rho}\hat{g}^{\nu\sigma}\hat{F}_{\mu\nu}\hat{F}_{\rho\sigma}$

None of the above is separable!

Conclusions

- **The matter *does not* decohere, it is by default *decohered*** – a possibly deeper fundamental explanation of the quantum-to-classical transition.
- The impact to the decoherence programme: gravity (environment $\mathcal{E}$) is generically entangled with the *whole* matter (both the system $\mathcal{S}$ and the apparatus $\mathcal{A}$), that way allowing for non-trivial tripartite system-apparatus-environment *effective* interaction of the form $\mathcal{H}_{\mathcal{SAE}}$, explicitly excluded in Zurek 1981: *(the assumption of pairwise interactions) "is customary and clear, even though it may prevent one from even an approximate treatment of the gravitational interaction beyond its Newtonian pairwise form."* **Symmetry-protected gravity-matter entanglement violates the the stability criterion of a faithful measurement.**
- Symmetry-protected gravity-matter entanglement as an **"effective interaction"** (such as exchange interactions due to particle statistics, etc.) – possible **corrections to Einstein's weak equivalence principle.**
- The effective interaction between gravity and matter forbids the existence of a single background spacetime: one cannot talk of "matter in a point of space", confirming the conjecture that **spacetime is an "emergent phenomenon"**. A possible criterion for a plausible candidate theory of quantum gravity (in tune with relational approach to physics).

The Outlook

- Detailed numerical study of Tr $(\rho_{\mathrm{M}})^2$ and the entropy of entanglement $S(\rho_{\mathrm{M}})$.
- The study of tripartite gravity-matter-EM fields: possible qualitatively new results.
- Checking how other QG candidates incorporate the general gravity constraints regarding the entanglement with matter, in particular the string theory (formulated by manifestly breaking the gauge symmetry – a consequence of perturbative expansion of the gravitational field).
- The analysis of the entanglement between different spacetime regions induced by the symmetry-protected gravity-matter entanglement: comparison with the AdS/CFT correspondence and the holographic principle.
- Experimental detection