Gravitational Anomalies (in) matter

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arXiv:1610.04413 **[hep-th]**
arXiv:1703.10682 **[cond-mat.mtrl-sci]**
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Outline

- Anomalies
- Quantum Currents
- Applications: NMR and NTMR in WSMs
- Conclusions

Anomalies

Early 20th century: 2 key concepts of physics

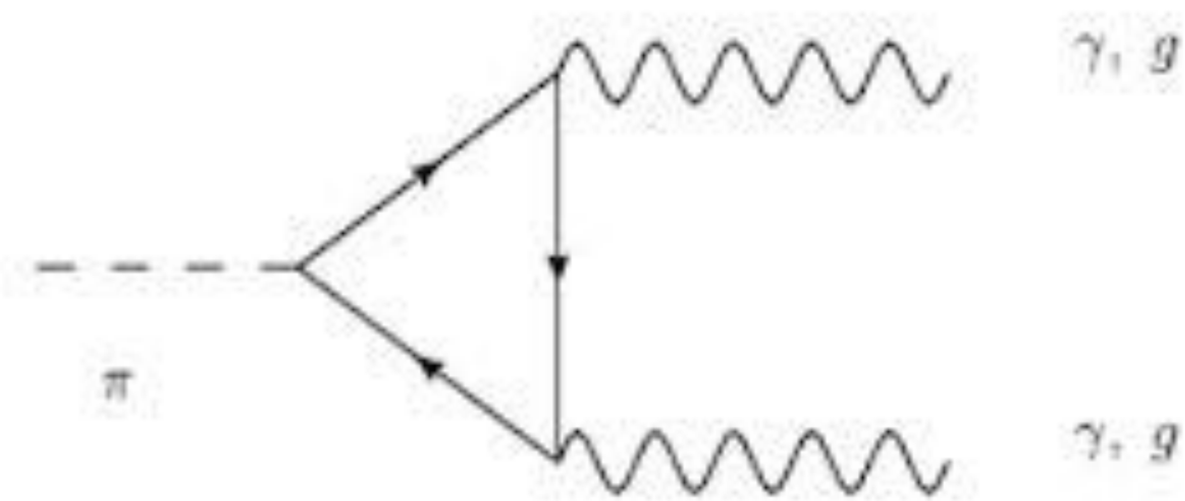
- Symmetry (Noether)
- Quantum Mechanics

Second half of 20th century: not always compatible

- Chiral Anomalies (Adler-Bell-Jackiw)

Anomalies

High energy physics: decay of neutral pion



Symmetries suggest process is strongly suppressed

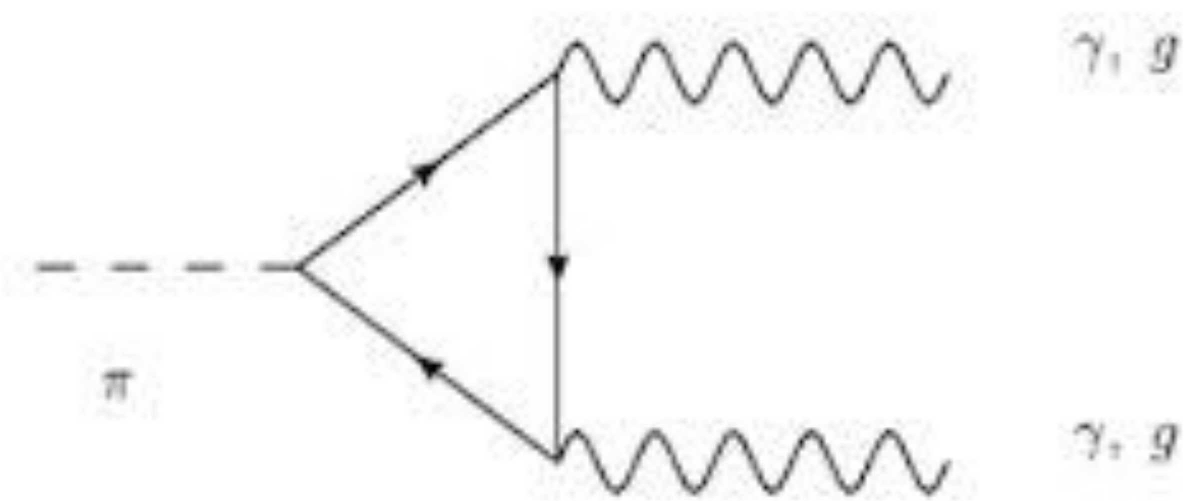
$$\partial_\mu J_5^\mu = f_\pi m_\pi^2 \pi^0 - \frac{\alpha}{8\pi} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$$

Quantum corrections destroy symmetry

[Adler], [Bell,Jackiw] 1969

Anomalies

Decay of neutral pion into gravitons



$$\partial_\mu J_5^\mu = \frac{1}{384\pi^2}\epsilon^{\mu\nu\rho\lambda} R^a{}_{b\mu\nu} R^b{}_{a\rho\lambda}$$

Suppressed by Planck scale, impossible to detect at LHC + successors

[Kimura] 1969, [Delbourgo, Salam] 1972, [Eguchi,Freund] 1976

Anomalies

$$D_\mu J^\mu_a = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{abc}}{96\pi^2}F^b_{\mu\nu}F^c_{\rho\lambda} + \frac{b_a}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right)$$

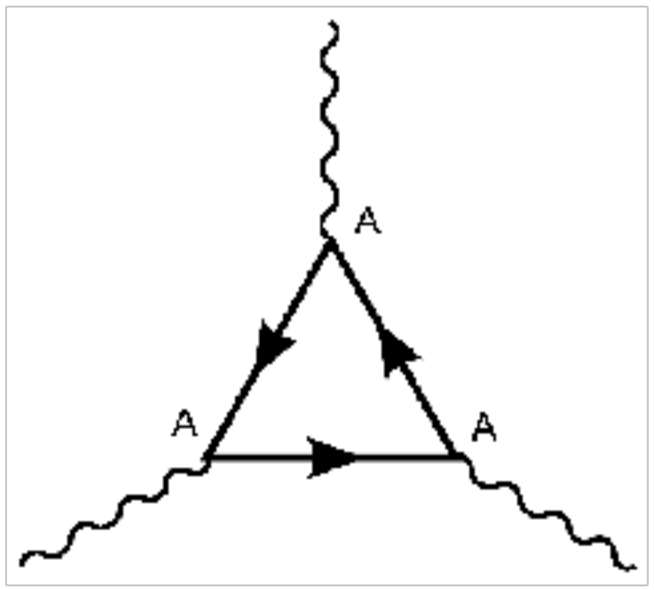
$$d_{abc} = \sum_r q^r_a q^r_b q^r_c - \sum_l q^l_a q^l_b q^l_c$$

$$b_a = \sum_r q^r_a - \sum_l q^l_a$$

- Anomaly coefficients detect presence of anomalies
- In 3+1 no pure gravitational anomaly
- Anomalies can always be shifted into currents

Anomalies

Lead to important consistency conditions



$$\int A_\mu J^\mu = \int \partial_\mu \theta J^\mu = -\int \theta \partial_\mu J^\mu$$

- Pure gauge field configuration does not decouple from the physical states!
- Unphysical states (longitudinal) can scatter into physical sector (transverse)
- HEP: Most basic principle of model building
- COND-MAT: Classification of possible edge theories of top. Insulators

New: collective dynamics induced by anomaly (CME, CVE)

Anomalies

The basic Rule

- Anomaly in gauge current = BAD!
- Anomaly in global current = GOOD!

Anomalies

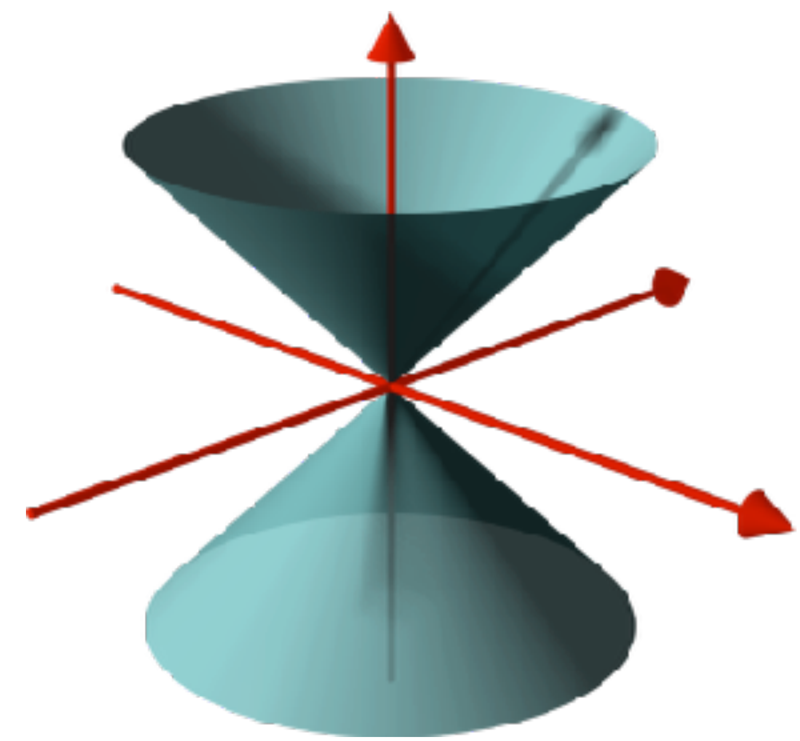
-- Do you ever run across a fellow that even you can't understand?"

"Yes," says he.

"This well make a great reading for the boys down at the office," says I. "Do you mind releasing to me who he is?"

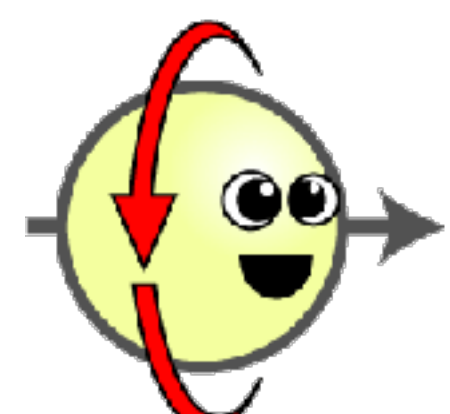
"Weyl," says he. (ROUNDY INTERVIEWS PROFESSOR DIRAC)

Weyl fermions (massless Dirac):



$$\boxed{H = \pm\vec{\sigma}\vec{p}}$$

Spin is either aligned or anti-aligned with momentum

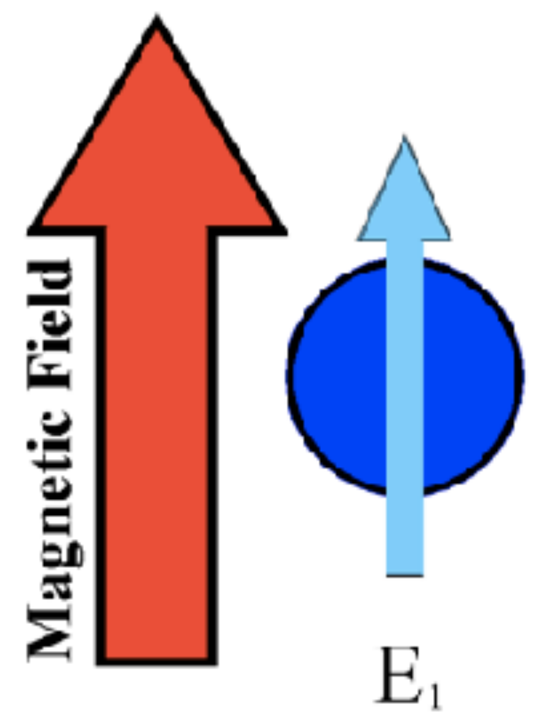


Anomalies and Landau levels

Chiral fermions in magnetic field:

Landau Levels: n>0

$$E_n = \sqrt{p_z^2 + 2nB}$$



Lowest Landau Level : n=0

$$E_0 = \pm p_z$$

Sign is determined by chirality

Degeneracy of LL: $\dfrac{eB}{2\pi}$

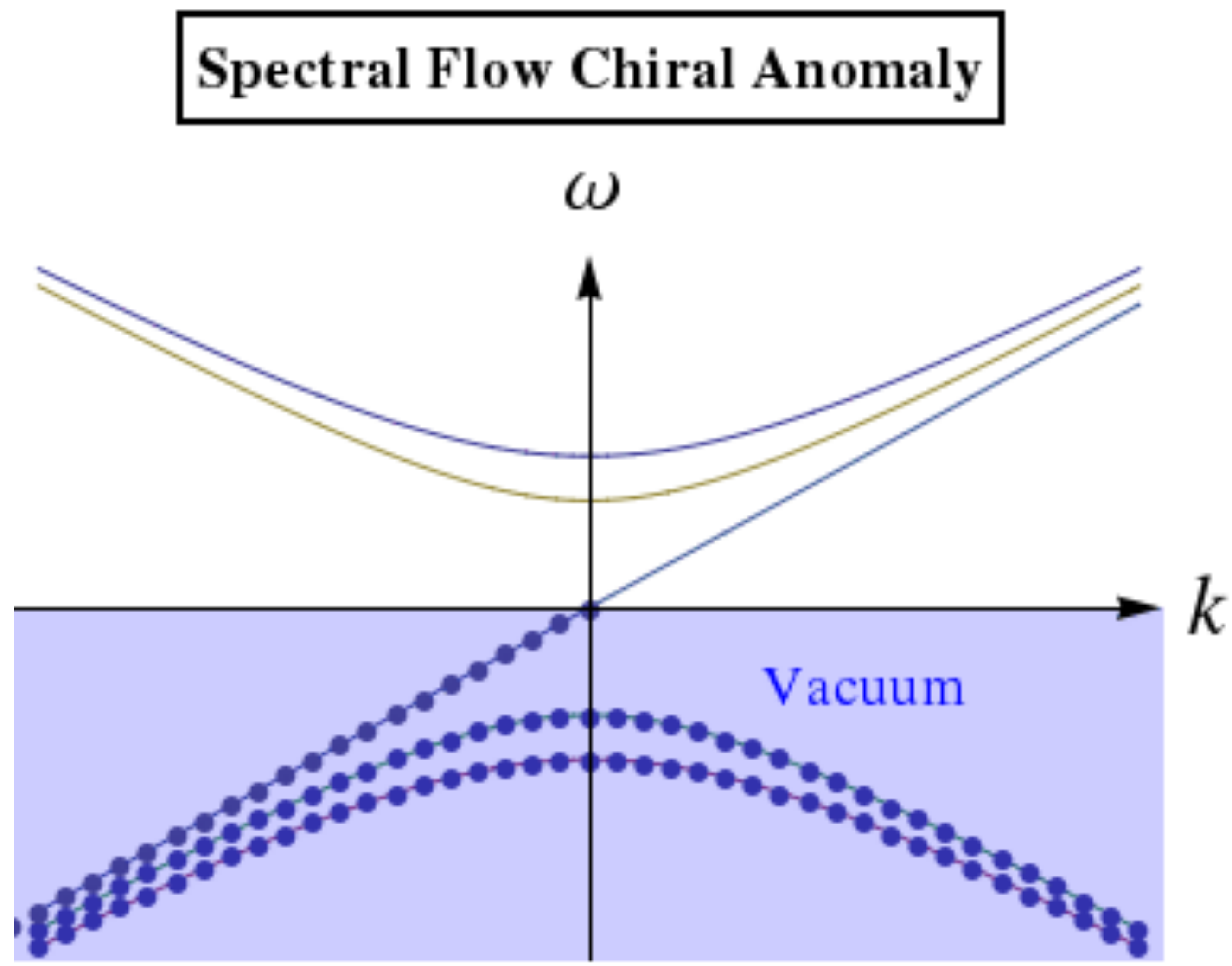
Anomalies

Chiral Fermion in electric and magnetic fields: Landau levels

$$\dot{p} = E$$

$$2\pi d\rho = dp$$

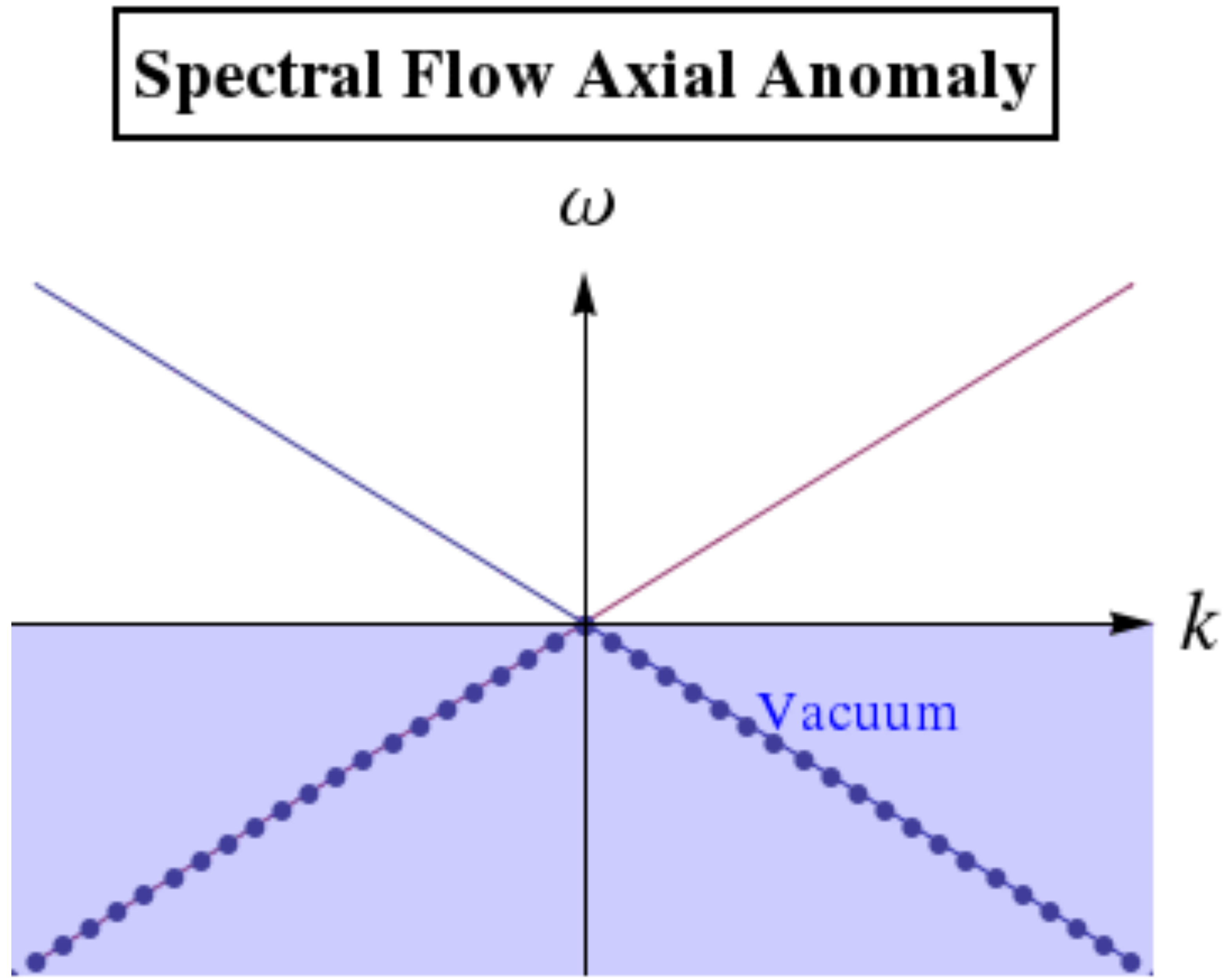
$$\boxed{\dot{\rho}_{\mathrm{R,L}} = \pm\frac{1}{4\pi^2}\vec{E}.\vec{B}}$$



Anomalies

Axial Anomaly: $J_5^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi = J_R^\mu - J_L^\mu$

$$\dot{\rho}_5 = \frac{1}{2\pi^2}\vec{E}\vec{B}$$



Anomalies

Dirac Fermion in electric and magnetic fields

$$\boxed{\dot{\rho}_5 = \pm\frac{1}{2\pi^2}(\vec{E}.\vec{B} + \vec{E}_5.\vec{B}_5)}$$

Axial Anomaly

But also violation of basic rule:

$$\boxed{\dot{\rho} = \pm\frac{1}{2\pi^2}(\vec{E}.\vec{B}_5 + \vec{E}_5.\vec{B})}$$

Not really an Anomaly

To say nature knows no axial field is not good enough.

WSM: B_5 = Strain

[Cortijo, Ferreiros, K.L, Vozmediano], [Cortijo, Kharzeev, K.L., Vozmediano]
[Pikulin, Chen, Franz] [Grushin, Venderbos, Vishwanath, Ilan]

Quantum Current

Dirac Fermion in electric and magnetic fields

$$\partial_\mu J^\mu = \frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\lambda}F^5_{\mu\nu}F_{\rho\lambda}$$

Anomaly is total derivative: ***consistent conserved*** current

$$\partial_\mu \left(J^\mu - \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}A^5_\nu F_{\rho\lambda}\right) = 0$$

Physical field !

No anomaly but **quantum current**

Mathematics of Anomalies: Consistency conditions, covariant anomalies, descent equation, cohomology [Wess, Zumino], [Bardeen], [Bardeen,Zumino] [Stora]

Quantum Current

$$\boxed{J^\mu = \frac{1}{4\pi^2}\epsilon^{\mu\nu\rho\lambda}A^5_\nu F_{\mu\nu}}$$

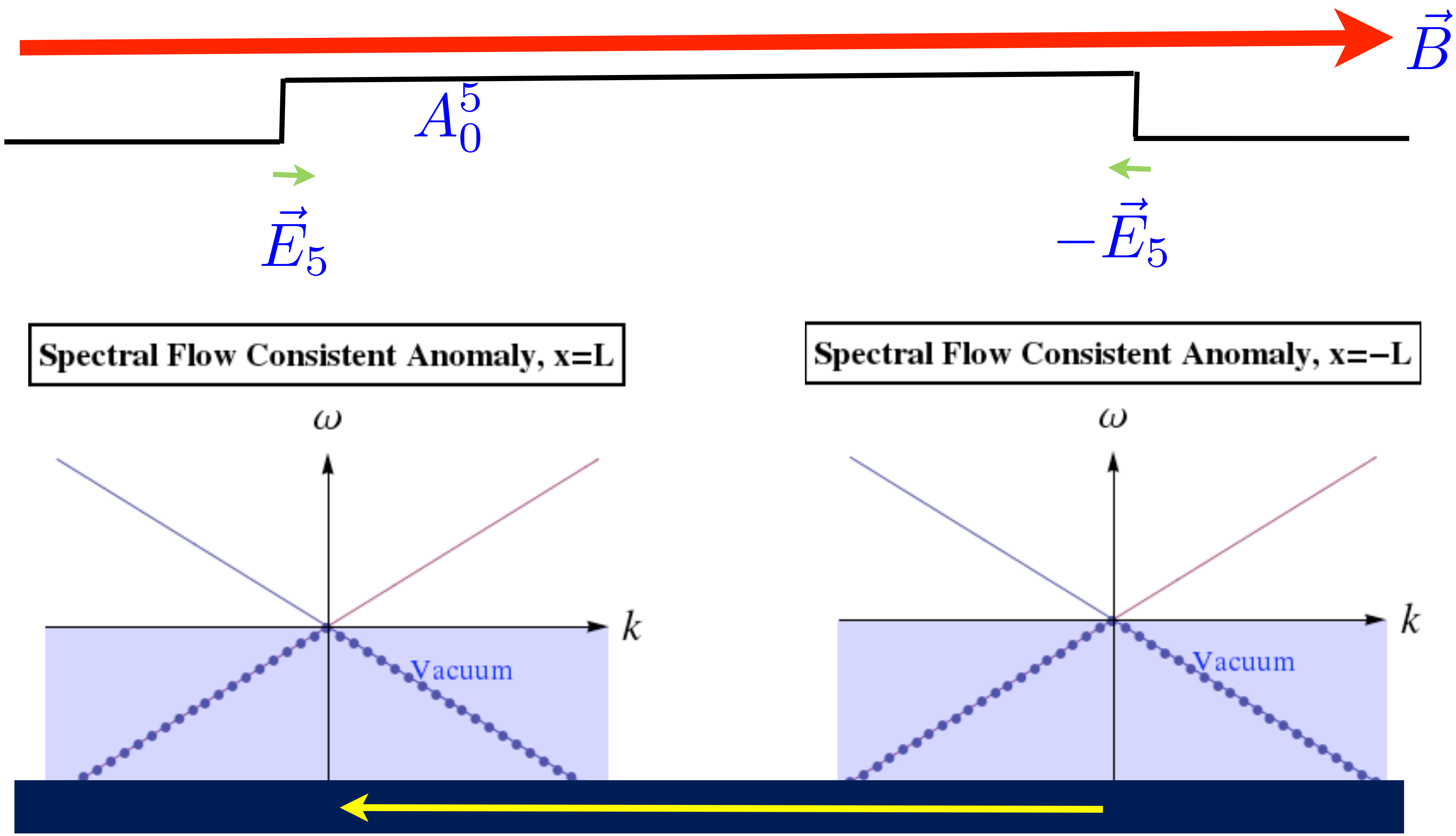
Anomalous Hall Current: $\vec{J} = \frac{1}{2\pi^2}\vec{A}_5 \times \vec{E}$

Anomalous Hall Charge: $J^0 = \frac{1}{2\pi^2}\vec{A}_5 . \vec{B}$

Chiral Magnetic Current: $\vec{J} = -\frac{1}{2\pi^2}A^0_5 . \vec{B}$

?

Anomalies: Covariant vs. Consistent:



$$\boxed{\vec{J}_{C.S} = -\frac{1}{2\pi^2} A_0^5 \vec{B}}$$

Chern-Simons current via boundary conditions at cutoff

Landau Levels and Transport

Landau levels of chiral fermion in magnetic field

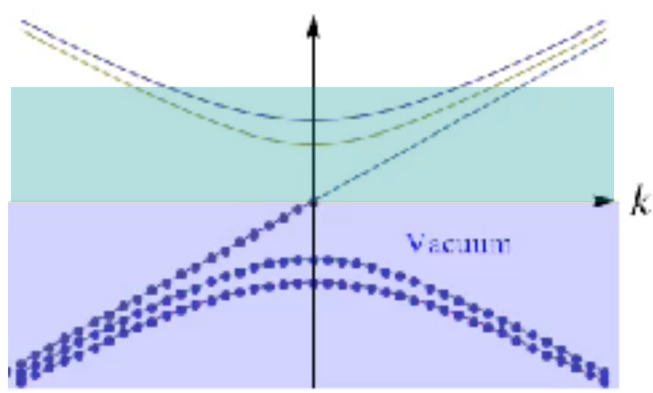
HLLs: $$E_n = \sqrt{p_z^2 + 2nB}$$

LLL: $$E_0 = \pm p_z$$

Current = charge x velocity

HLLs: $$\int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \frac{\partial E_n}{\partial p_z} \left(\frac{1}{e^{\frac{E_n - \mu}{T}} + 1} - \frac{1}{e^{\frac{E_n + \mu}{T}} + 1} \right) = 0$$

LLL: $$\int_{0}^{\infty} \frac{dp_z}{2\pi} (\pm 1) \left(\frac{1}{e^{\frac{E_0 - \mu}{T}} + 1} - \frac{1}{e^{\frac{E_0 + \mu}{T}} + 1} \right) = \frac{\mu}{2\pi}$$



Taking degeneracy of LLL into account :
Chiral Magnetic Effect (CME)

$$\boxed{\vec{J}_{L,R} = \pm \frac{\mu}{4\pi^2} \vec{B}}$$

[Vilenkin], [Alekseev, Chainanov, Frohlich]
[Giovannini, Shaposhnikov]
[Kharzeev,Fukushima,Warringa],…

Landau Levels and Transport

Energy transport = E . v = p = Momentum density

$$\vec{P}_{L,R} = \vec{J}_{\epsilon,L,R} = \pm \int_0^\infty \frac{dp_z}{2\pi} \left(\frac{p}{e^{\frac{p-\mu}{T}}+1} + \frac{p}{e^{\frac{p+\mu}{T}}+1} \right)$$

$$= \pm \frac{T^2}{2\pi} \left[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T}) \right]$$

Jonquiere inversion relations: Bernoulli polynomials

Chiral Magnetic Effect (CME) in energy current

$$\boxed{\vec{J}_{\epsilon,L,R} = \pm \left(\frac{\mu_{L,R}^2}{8\pi^2} + \frac{T^2}{24} \right) \vec{B}}$$

Rotation

Heuristic relation magnetic field and rotation

$$\vec{F} = \vec{v} \times q\vec{B}$$

Lorentz force

$$\vec{F} = \vec{v} \times 2m\vec{\Omega}$$

Coriolis force

$$q\vec{B} \to 2E\vec{\Omega}$$

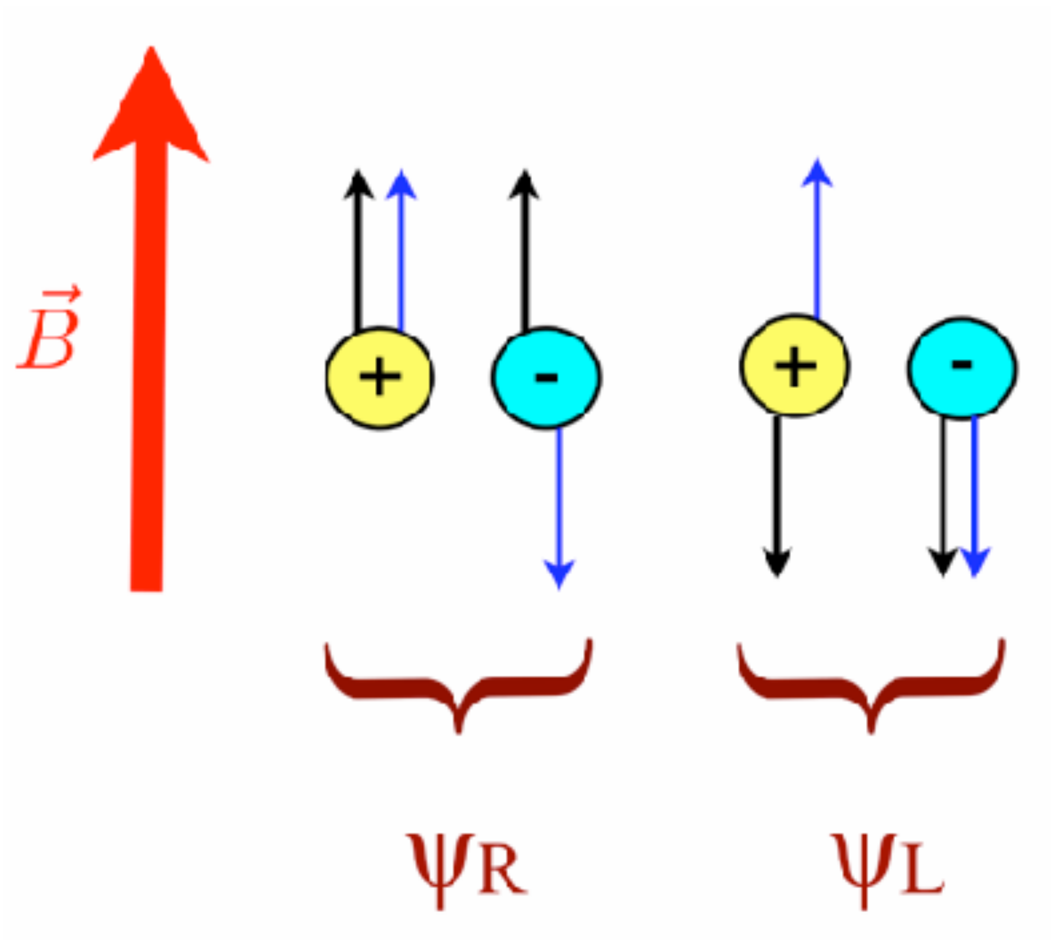
Quantum currents induced by rotation:

Chiral Vortical Effect

$$\begin{aligned} \vec{J} &= \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)\vec{\Omega} \\ \vec{J}_\epsilon &= \left(\frac{\mu^3}{6\pi^2} + \frac{\mu T^2}{6}\right)\vec{\Omega} \end{aligned}$$

[Vilenkin],[Son, Zhitnitsky]
[Kharzeev, Zhitnitsky]
[Erdmenger, Haack, Kaminski, Yarom],
[Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganayagam, Surowka],

Subtlety about CME



$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$

$$\mu_5 = \frac{1}{2}(\mu_R - \mu_L)$$

But there is also the "topological" contribution due to charge conservation:

$$\boxed{\vec{J} = \frac{\mu_5 - A_0^5}{2\pi^2}\vec{B}}$$

[A. Gynther, K.L., F. Pena-Benitez, A. Rebhan]

Temperature dependence

$$\vec{J} = \frac{T^2}{12}\vec{\Omega}$$

$$\vec{J}_\epsilon = \frac{T^2}{24}\vec{B} + \frac{\mu T^2}{12}\vec{\Omega}$$

For a more general collection of chiral fermions with different charges

$$\vec{J}_\epsilon = \frac{T^2}{24}\left(\sum_R q_R - \sum_L q_L\right)\vec{B}$$

Coefficient of gravitational anomaly!

[K.L., E. Megias, F. Pena-Benitez]

Gravitational Anomaly

- Mixed between chiral and general covariance
- General covariance can always be saved (cfg. electricity)

$$\boxed{D_\mu J^\mu = \frac{b}{768\pi^2}\epsilon^{\mu\nu\rho\lambda} R^\alpha{}_{\beta\mu\nu} R^\beta{}_{\alpha\rho\lambda}}$$

- Very similar to usual anomaly

$$F \sim dA + A \wedge A \quad , \quad R \sim \partial\Gamma + \Gamma \wedge \Gamma$$

- BUT also very different $\Gamma \sim g^{-1}\partial g$

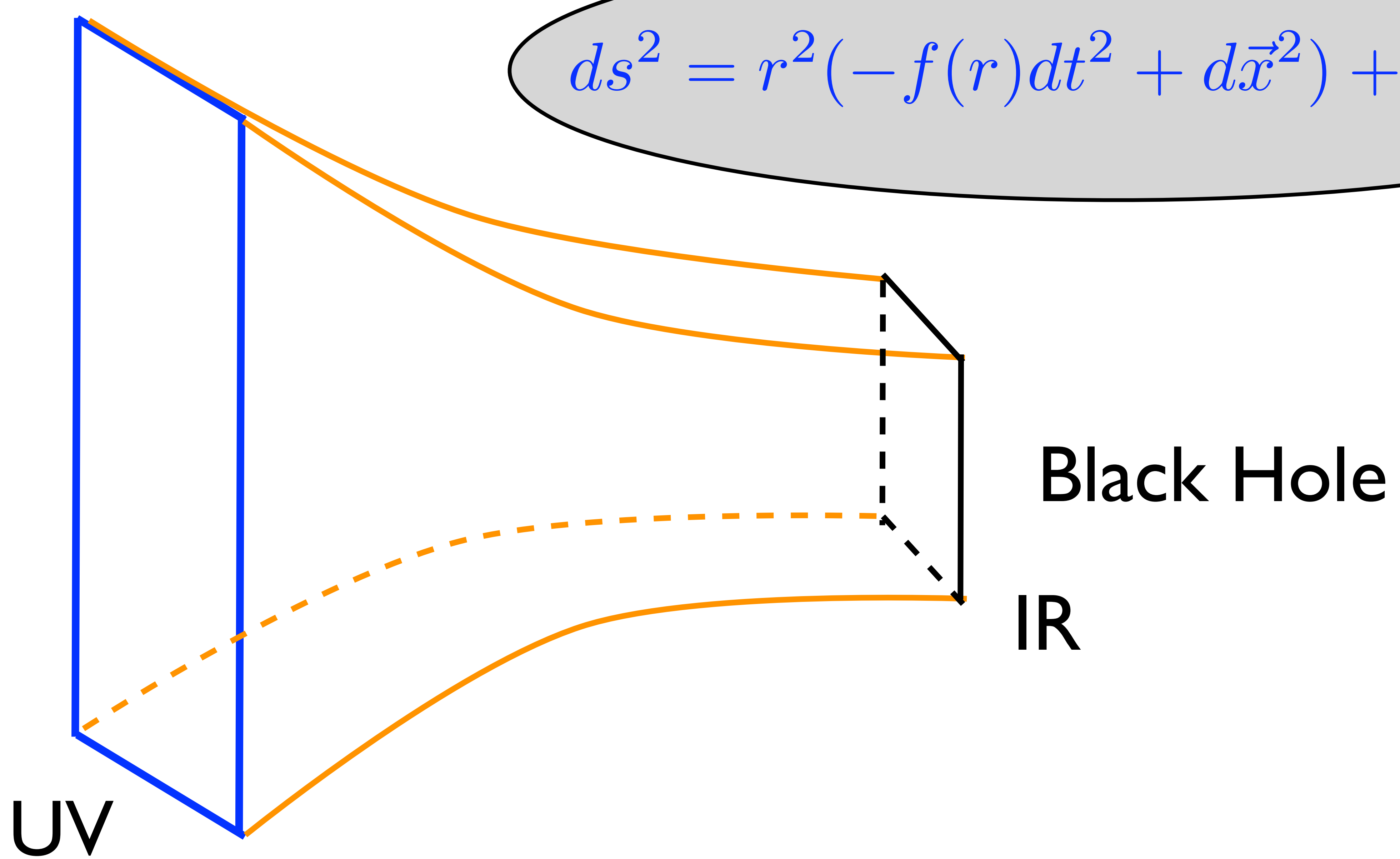
- HOW is this possible: $\vec{J} = \frac{b}{12} T^2 \vec{\omega}$ **!!????!!**

[K.L., E. Megias, L. Melgar, F. Pena-Benitez]

Quantum Currents from 5D

Strongly coupled QFT = gravity in 5D

$$ds^2 = r^2(-f(r)dt^2 + d\vec{x}^2) + \frac{dr^2}{f(r)r^2}$$



Quantum Currents from 5D

- Anomaly = dynamics of Chern-Simons terms

$$\int_{\mathrm{AdS}} d^5x A \wedge F \wedge F \to \int_{\partial(\mathrm{AdS})} d^4x \theta(F \wedge F)$$

- Grav. Anomaly = dynamics of grav. Chern-Simons terms

$$\int_{\mathrm{AdS}} d^5x A \wedge R^{(5)} \wedge R^{(5)} \to \int_{\partial(\mathrm{AdS})} d^4x\, \theta(\hat{R}^{(4)} \wedge \hat{R}^{(4)} + \nabla K \wedge \nabla K)$$

- "Extrinsic curvature" K appears
- QFT anomaly knows only 4D curvature
- K-terms does not contribute at r=∞ (UV)

Quantum Currents from 5D

- BUT: at the black hole horizon

$$ds^2 = r^2 f(r)(-dt^2 + 2\vec{A}d\vec{x}dt) + r^2 d\vec{x}^2 + \frac{dr^2}{r^2 f(r)}$$

$$\nabla K \wedge \nabla K \propto r_H^4 f'(r_H)^2 \frac{\partial \vec{A}}{\partial t}(\nabla \times \vec{A})$$

- Following Hawking

$$\nabla K \wedge \nabla K = 64\pi^2 T^2 \vec{E}_g . \vec{B}_g$$

- Local phenomenon in space time!
- No obvious restriction to (near) equilibrium!

Quantum Currents from 5D

- Holography suggests in deep IR $\partial_\mu J^\mu = \frac{T^2}{12}\vec{E}_g.\vec{B}_g$
- A new anomaly ??? $\partial_\mu J^\mu = \frac{e^2}{4\pi^2}\vec{E}.\vec{B}$
- NO! not anomaly but quantum current (Luttinger)

$$\frac{T^2}{12}\underbrace{\vec{E}_g}_{-\frac{\vec{\nabla}T}{T}}.\underbrace{\vec{B}_g}_{\vec{\nabla}\times\vec{v}} = \vec{\nabla}\vec{J}_q$$

$$\boxed{\vec{J}_q = \frac{b}{12}T^2\vec{\omega}}$$

- The Chiral Vortical Effect (CVE) !!

[Azayanagi, Loganayagam, Ng, Rodriguez],[Chapman, Neiman, Oz],[Jensen,Loganayagam,Yarom], [Grozdanov, Poovuttikul] global anomaly: [Gorkar, Sethi]

Quantum Currents

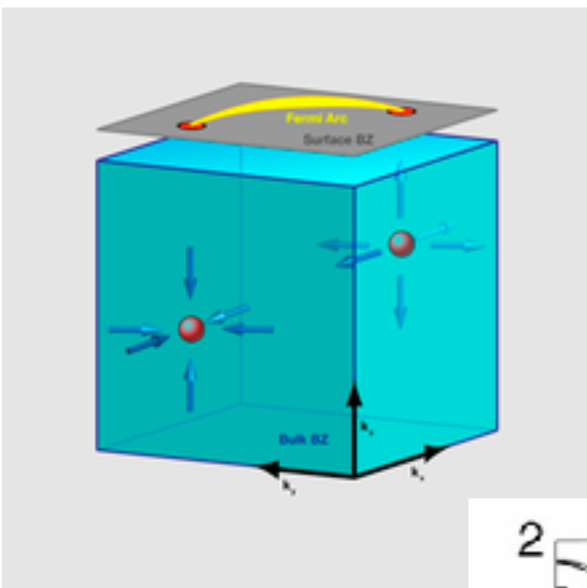
$$\vec{J} = \frac{\mu}{2\pi^2}\vec{B} \qquad \vec{J} = (\frac{\mu^2}{2\pi^2} + \frac{T^2}{12})\vec{\Omega}$$

$$\vec{J}_\epsilon = (\frac{\mu^2}{4\pi^2} + \frac{T^2}{24})\vec{B} \qquad \vec{J}_\epsilon = (\frac{\mu^3}{6\pi^2} + \frac{\mu T^2}{6})\vec{\Omega}$$

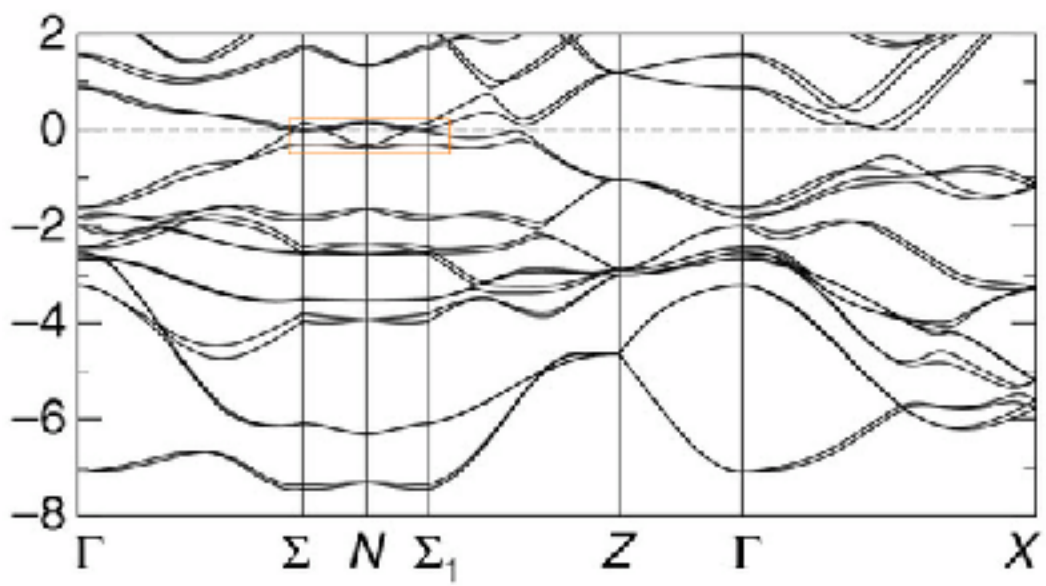
No entropy is produced: dissipationless

Applications: WeylSemiMetal

physicsworld
TOP 10
BREAKTHROUGH
2015

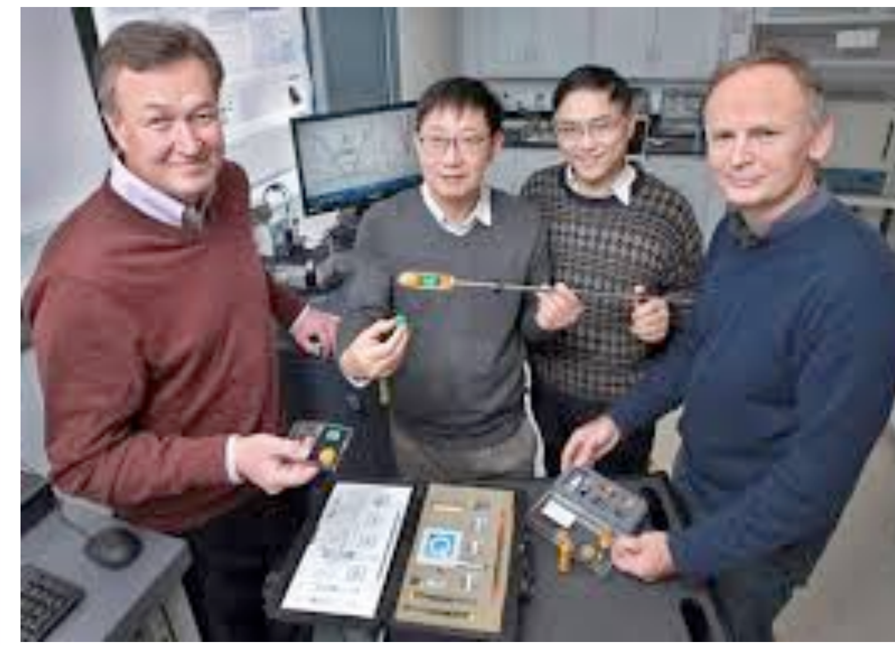


Wikipedia

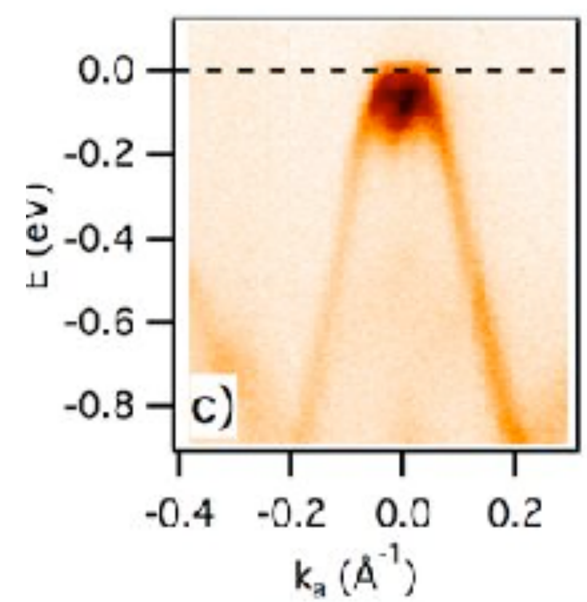


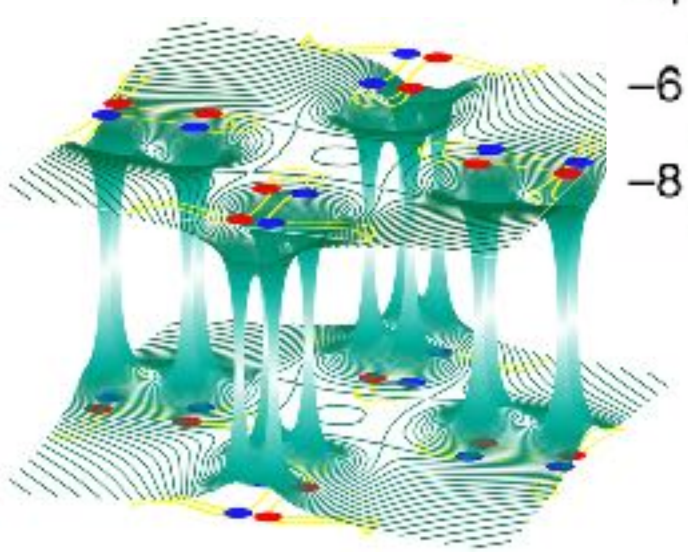
TaAs

[Huang, Xu, Belopolski,Hasan] Nature Comm.



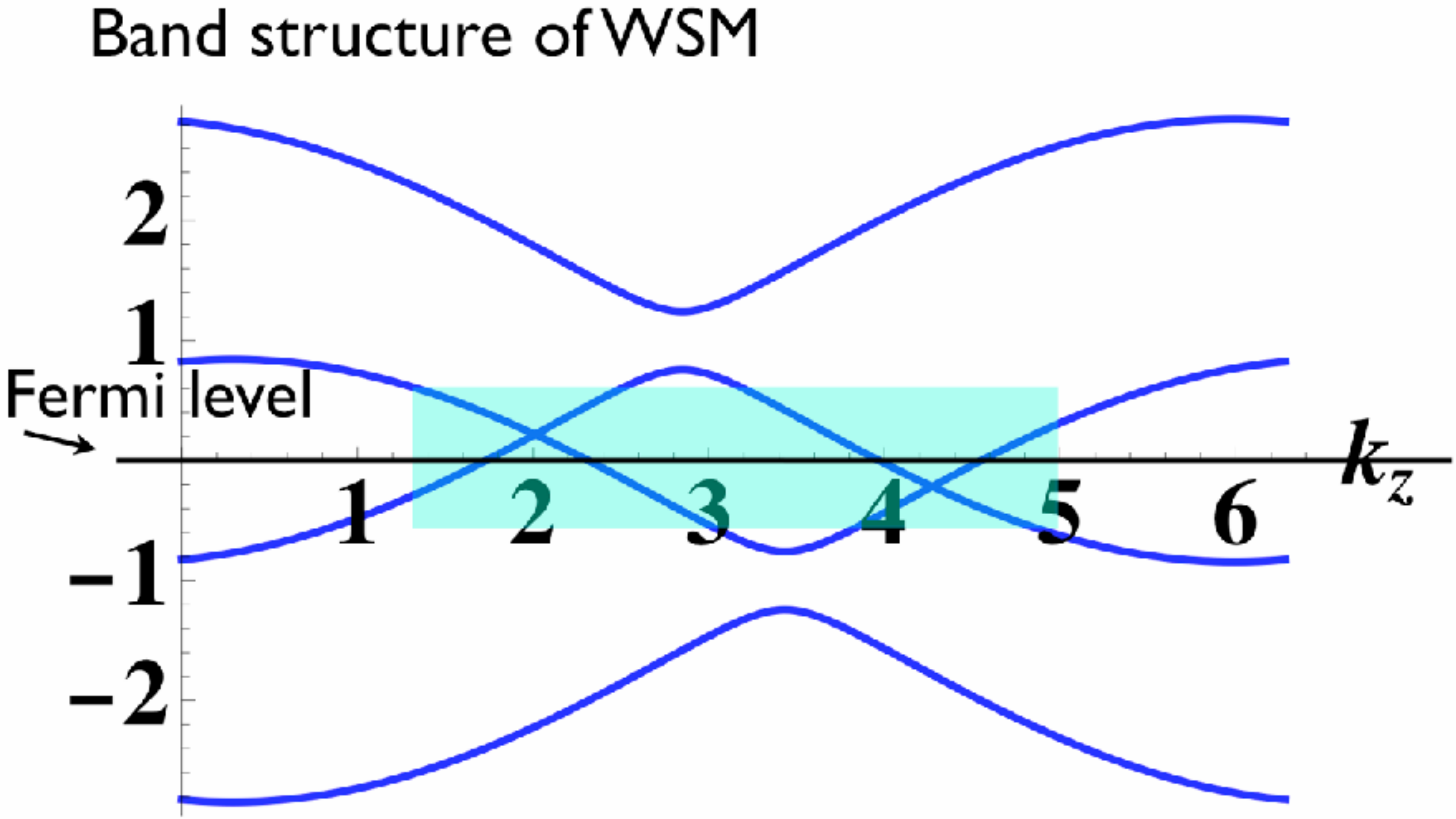
Qiang Li,Dmitri E. Kharzeev,Cheng Zhang,Yuan Huang,I. Pletikosić,A. V. Fedorov,R. D. Zhong,J. A. Schneeloch,G. D. Gu& T. Valla
Nature Physics 12, 550–554 (2016)





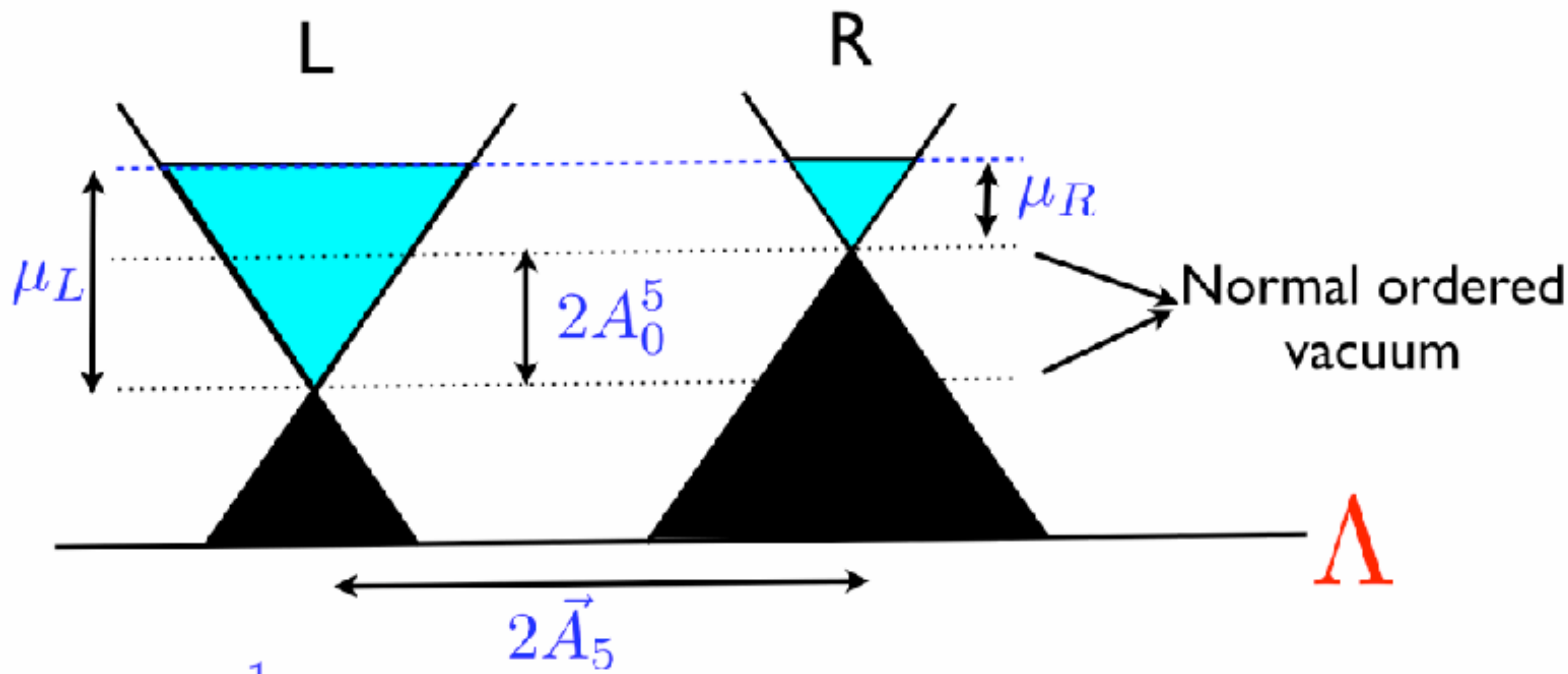
Hiroyuki Inoue, András Gyenis, Zhijun Wang, Jian Li, Seong Woo Oh, Shan Jiang,
Ni Ni, B. Andrei Bernevig,and Ali Yazdani, was published in the March 11, 2016 issue of the journal *Science*

Applications: WSM



$$\gamma^\mu \left(iD_\mu + \gamma_5 A_\mu^5 \right) \Psi = 0$$

CME in WSM



$$\mu_5 = \frac{1}{2}(\mu_L - \mu_R) \qquad \mu = \frac{1}{2}(\mu_R + \mu_L)$$

CME: $$\boxed{\vec{J} = \frac{1}{2\pi^2}\left(\mu_5 - A_0^5\right)\vec{B} = 0}$$

CME vanishes in equilibrium in WSM "Bloch theorem"

NMR and NTMR in WSM

NMR = Negative Magnetoresistivity

In equilibrium CME vanishes,
Induce non-equilibrium steady state

$$\dot{\rho}_5 = \frac{1}{2\pi^2}\vec{E}.\vec{B} - \frac{1}{\tau_5}\rho_5$$

$$\rho_5 = \chi_5 \mu_5 \qquad \vec{J} = \sigma \vec{E} + \frac{\mu_5}{2\pi^2}\vec{B}$$

$$\boxed{\vec{J} = \left(\sigma + \frac{\tau_5 B^2}{(2\pi^2)\chi_5}\right)\vec{E}}$$

Test grav. Anomaly ?

$$\vec{J}_\epsilon = \frac{b_5 T^2}{24} \vec{B}_5$$

[K.L., Yan Liu]

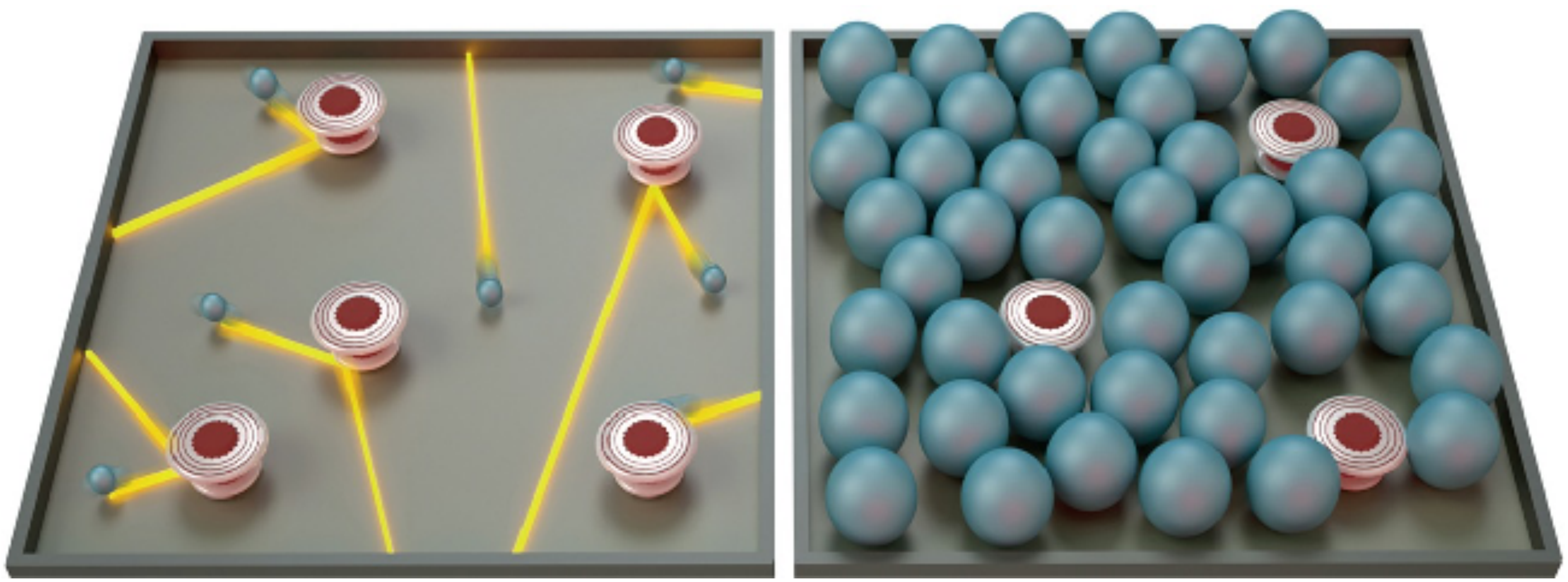
- No effect expected due to usual magnetic field

ρ_5 Not conserved because of anomaly

ϵ_5 Not conserved at all

BUT :

NMR and NTMR in WSM

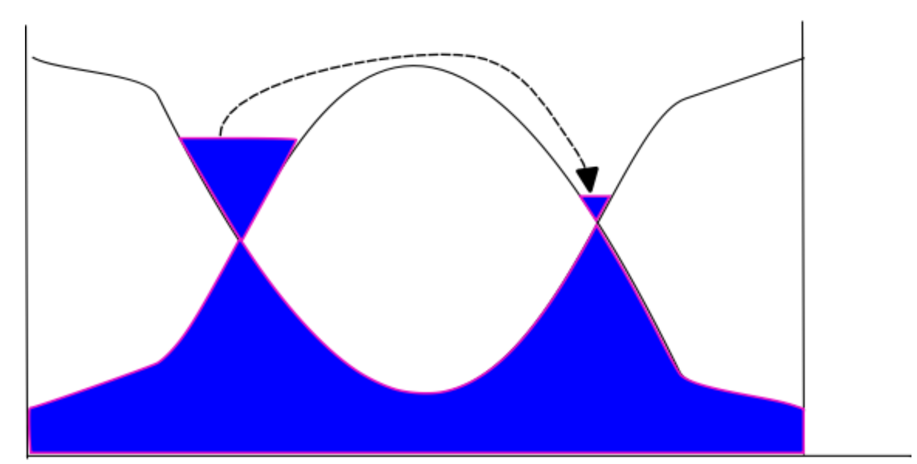


[J. Zaanen, "Electrons go with the flow in exotic materials", *Science* Vol. 351, 6277]

If WSM is not strongly coupled,
hierarchy of scattering times

$$\tau_{\text{inner}} < \tau_{\text{inter}} < \tau_{ee}$$

Kills $\vec{P}$ Kills ρ_5, ϵ_5 Is irrelevant



NTMR via CME

$$\dot{\epsilon} + \vec{\nabla}\vec{J}_\epsilon = \vec{E}.\vec{J}$$

$$\dot{\rho} + \vec{\nabla}\vec{J} = a_\chi \vec{E}.\vec{B}$$

$$\vec{J}_\epsilon = \left(\frac{a_\chi}{2}\mu^2 + a_g T^2\right)\vec{B}$$

$$\vec{J} = a_\chi \mu \vec{B}$$

Grad T pumps chiral energy into the system

$$\dot{\epsilon} = -\vec{\nabla}\vec{J}_\epsilon = -2a_g T\vec{\nabla}T.\vec{B}$$

Electric field (-grad μ) pumps chiral charge and energy into the system

$$\dot{\rho} = a_\chi(\vec{E} - \vec{\nabla}\mu).\vec{B}$$

$$\dot{\epsilon} = a_\chi \mu \left(\vec{E} - \vec{\nabla}\mu\right).\vec{B}$$

NTMR via CME

$$\begin{pmatrix} \dot{\epsilon} \\ \dot{\rho} \end{pmatrix} = -i\omega \frac{\partial(\epsilon, \rho)}{\partial(T, \mu)} \begin{pmatrix} \delta T \\ \delta\mu \end{pmatrix}$$

$$\begin{pmatrix} \dot{\epsilon} \\ \dot{\rho} \end{pmatrix} = \begin{pmatrix} -2a_g T & a_\chi \mu \\ 0 & a_\chi \end{pmatrix} \begin{pmatrix} \vec{\nabla} T \\ \vec{E} \end{pmatrix}$$

The chiral pumping is stopped due to internally scattering

$$\omega \to \frac{i}{\tau_5}$$

NTMR via CME

Calculate increase in T,μ and re-insert into chiral currents

$$\vec{J} = G_E \vec{E} + G_T \vec{\nabla} T$$

$$G_E = \tau_5 \frac{a_\chi^2}{\det(\Xi)} \left(\frac{\partial \epsilon}{\partial T} - \mu \frac{\partial \rho}{\partial T} \right) B^2$$

$$G_T = \tau_5 \frac{2 a_g a_\chi}{\det(\Xi)} \frac{\partial \rho}{\partial T} B^2$$

Supplementary material to arXiv:1703.10682

Large B (ultraquantum limit):

- G_E linear in B
- G_T vanishes

$$\rho = \frac{|B|}{4\pi^2} \mu$$

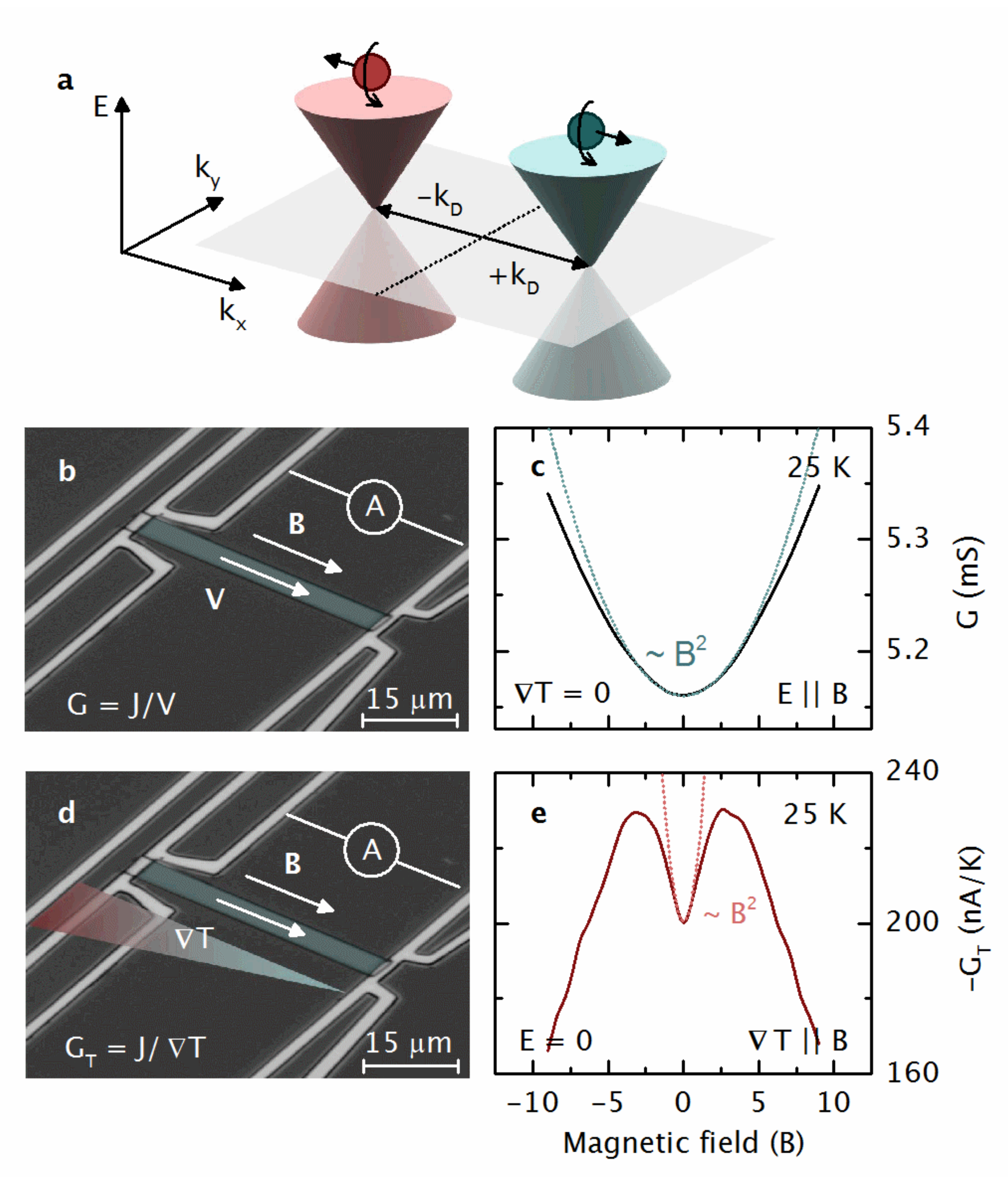
[Spivak, Andreev], [Lundgren, Laurell, Fiete] *kinetic theory*
[Lucas, Davison, Sachdev] *chiral fluids*

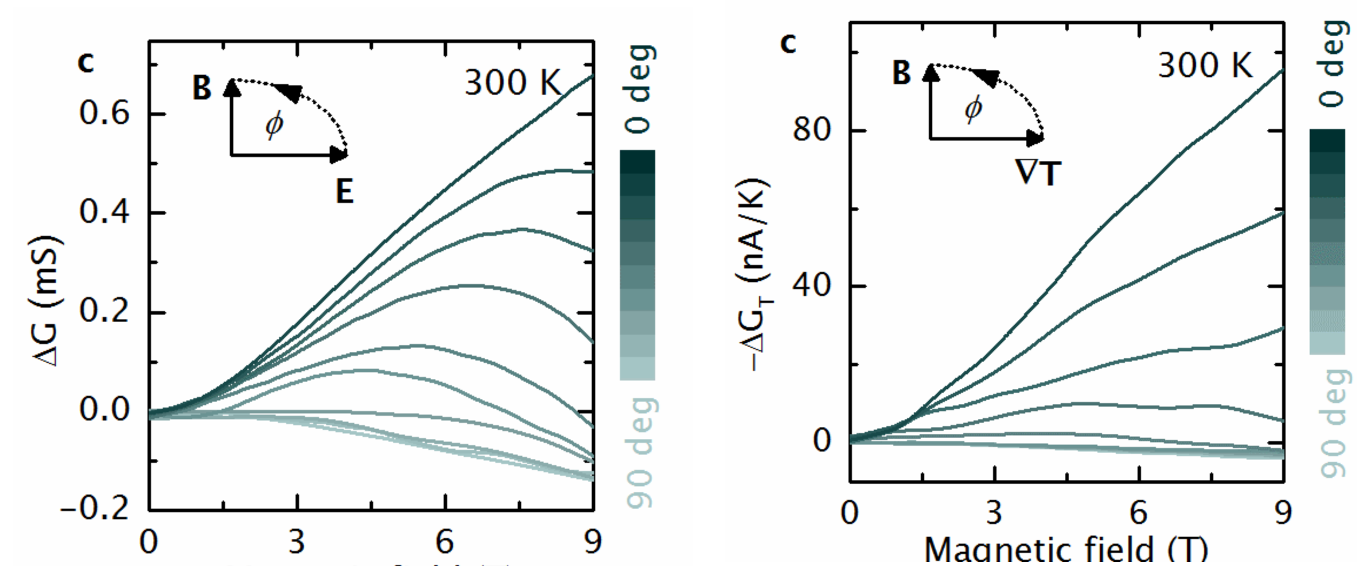
NMR and NTMR in NbP

Experimental signatures of the mixed axial-gravitational anomaly in the Weyl semimetal NbP

Johannes Gooth, Anna Corinna Niemann, Tobias Meng, Adolfo G. Grushin, Karl Landsteiner, Bernd Gotsmann, Fabian Menges, Marcus Schmidt, Chandra Shekhar, Vicky Süß, Ruben Huehne, Bernd Rellinghaus, Claudia Felser, Binghai Yan, Kornelius Nielsch

arXiv:1703.10682 [cond-mat.mtrl-sci]





- ☑ Angle dependence
- ☑ NMR and NTMR show B^2 at small B
- ☑ NMR ~ linear for large B field
- ☑ NTMR vanishes for large B field

Oida !!!!!

Summary

- Anomalies have moved from HEP-TH to Cond-Mat
- Rich anomaly induced transport phenomenology (CME, CVE, CSE, NMR, AHE, and now NTMR)
- Universal: QGP, WSM but also Cosmo (primordial magnetic fields), Astro (Neutron star kicks, Supernova explosions, ...)
- WSMs promise experimental observation of effect of gravitational anomaly

Thank You!