BRST-BV treatment of Vasiliev's four-dimensional higher-spin gravity

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Outline

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- Poisson sigma models
- Their BRST quantization
- Adaptation to Vasiliev's HSGR
- Comparison with dual CFT
- Conclusions

Abstract

We provide Vasiliev's 4D HSGR with a classical Batalin – Vilkovisky (BV) master action using an adaptation of the Alexandrov – Kontsevich – Schwarz – Zaboronsky (AKSZ) implementation of the (BV) field-anti-field formalism to the case of differential algebras on non-commutative manifolds.

- Vasiliev's equations follow via the variational principle from a Poisson sigma model (PSM) on a non-commutative manifold (see talk by Nicolas Boulanger which we shall also review below)
- AKSZ procedure: classical PSM on commutative manifold is turned into BV action for "minimal" set of fields and anti-fields by substituting each classical differential form by a "vectorial superfield" of fixed total degree given by form degree plus ghost number
- Apply to Vasiliev's HSGR by adapting the AKSZ procedure to PSMs on non-commutative manifolds part of a more general story!

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Many motivations for HSGR

Existence of gauge theories are non-trivial facts, mathematically as well as physically once the dynamics is interpreted properly.

In the case of HSGR, a benchmark is set by Vasiliev's equations:

- AdS/CFT correspondence:
 - weak/weak-coupling approaches
 - physically realistic AdS/CMT dualities
 - anti-holographic duals of as. free QFTs
- formal developments of QFT:
 - unfolding
 - generally covariant quantization
 - twistorization
- co-existence of HSGR and string theory:
 - interesting for stringy de Sitter physics and cosmology
 - new phenomenologically viable windows to quantum gravity
 - new perspectives on the cosmological constant problem and long-distance gravity (*e.g.* dark matter vs scalar hairs)

Why the PSM off shell formulation

- Thinking of Vasiliev's 4D equations as "toy" for quantum gravity:
 - Many symmetries may suppress UV divergencies …
 - ... but also introduce higher time derivatives already at the classical level
 - Find suitable generalization of canonical quantization?
- Perturbative expansion around AdS4:
 - Linearized spectrum \sim square of free conformal scalar/fermion
 - \blacktriangleright ... suggests dual CFT3 \sim large-N free field theory
 - No loop corrections at all! Tree-level exact or perfect cancellations?

The situation in 4D is cleaner than in 3D where the PSM formulation essentially amounts to BF-models in the case of HSCS theory and BFCG-models in the case of Prokushkin – Vasiliev HSGR — we shall not discuss the latter models any further here but they are for sure very interesting and actually in a certain sense more complicated than those arising for 4D HSGR.

Fradkin – Vasiliev cubic action

- Intrinsically 4D action \sim free Fronsdal plus interactions:
 - \blacktriangleright Fradkin Vasiliev cubic action \sim free first-order action for Fronsdal fields plus cubic interactions
 - ... but it requires extra auxiliary fields to be eliminated via subsidiary constraints on extra linearized higher spin curvatures
 - Non-abelian higher spin connection and curvature

$$A\equiv A_{\rm dyn}+A_{\rm Extra}$$

$$\begin{split} F &\equiv F_{\rm Fronsdal\,EoM} + F_{\rm Extra} + F_{\rm Weyl} \\ \delta S_{\rm FV}^{(2)} \propto F_{\rm Fronsdal\,EoM}^{(1)} , \qquad F_{\rm Extra}^{(1)} \propto A_{\rm Extra} + \nabla^{(0)} \cdots \nabla^{(0)} A_{\rm dyn} \\ F_{\rm Weyl} \propto J^{(1)}(e,e;\Phi) \end{split}$$

- Beyond cubic order, non-abelian corrections mix equations of motion with subsidiary constraints
- Completion of Fradkin Vasiliev action on shell as generating functional for tree diagrams?

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Lagrange multipliers

• Introduce Weyl zero-form Φ and Lagrange multipliers V and U

$$S_{ ext{tot}} = \int \mathcal{V}_{ ext{FV}}[A, \Phi] + \int \operatorname{Tr}\left[V \star (F + J(A, A; \Phi)) + U \star (D\Phi + P(A; \Phi))\right]$$

assuming that (note direction of $\Rightarrow)$

$$\delta \int \mathcal{V}_{\rm FV} \approx 0 \quad \Leftarrow \quad F + J \approx 0 \ , \quad D\Phi + P \approx 0$$

- Shortcomings:
 - Apparent "conflict of interest" between kinetic terms in $\mathcal{V}_{\rm FV}$ and in $\int \text{Tr} \left[V \star dA + U \star d\Phi \right]$ resolved in negative fashion as FV term can redefined away modulo total derivative!
 - The generation of quantum corrections to A and Φ becomes problematic

Higher dimensional "BF-like" models (first run)

Embed 4D spacetime into the boundary of higher dimensional "bulk" manifold $\mathcal{M}:$

- "duality extend" (A, B; U, V) into all form degrees mod 2 and add bulk Hamiltonian $\mathcal{H}(B; U, V)$
- \bullet natural boundary conditions: $\mathit{U}|_{\partial \mathit{M}} \approx 0$ and $\mathit{V}|_{\partial \mathcal{M}} \approx 0$
- original equations of motion are recuperated on ∂M without need to fix any gauges (*N.B.* interpolations between inequivalent 4D configurations on different boundaries *c.f.* Hawking's no-boundary proposal and transitions between complete 4D histories)
- perturb around free bulk theory \rightsquigarrow correlators on $\partial \mathcal{M}$ restricted by conservation of form degree
- add "topological vertex operators" $\int_{\partial \mathcal{M}} \mathcal{V}[A, B; dA, dB]$ such that $\delta \int \mathcal{V}$ vanishes on the bulk shell \rightsquigarrow more vertices \rightsquigarrow additional loop corrections on $\partial \mathcal{M}$
- $\Rightarrow \int \mathcal{V}_{FV}$ remains tree-level exact deformation

Poisson sigma models on commutative manifolds

- Topological models on manifolds \mathcal{M} , say of dimension $\hat{p} + 1$.
- The fundamental fields are locally defined differential forms X^{α} (α label internal indices) and their canonical momenta P_{α} (non-linear Lagrange multipliers) obeying

$$\deg(X^{\alpha}) + \deg(P_{\alpha}) = \hat{p}$$

- Physical degrees of freedom are captured by topological vertex operators ∮_{Mi} V[X, dX]
- In particular, local degrees of freedom enter via X^{α} of degree zero and topologically broken gauge functions of degree zero, captured by
 - zero-form invariants which are topological vertex operators that can be inserted at points
 - topological vertex operators depending on the topologically broken gauge fields (generalized vielbeins)

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Target space and generalized Hamiltonian

- Target space: phase space $T[\hat{p}]\mathcal{N}$ of graded Poisson manifold \mathcal{N} equipped with:
 - ▶ a nilpotent vector field $Q \equiv \Pi_{(1)} = Q^{lpha}(X)\partial_{lpha}$ of degree 1
 - compatible multi-vector fields $\Pi_{(r)}$ of ranks r and degrees $1 + (1 r)\hat{p}$

$$\{\Pi_{(r)}, \Pi_{(r')}\}_{\text{Schouten}} \equiv 0$$
.

• In canonical coordinates, the classical bulk Lagrangian is of the generalized Hamiltonian form

$$\mathcal{L}^{\mathrm{cl}}_{\mathrm{bulk}} = P_{lpha} \wedge dX^{lpha} - \mathcal{H}(P, X)$$

where $\mathcal{H} = \sum_{r} P_{\alpha_1} \cdots P_{\alpha_r} \Pi^{\alpha_1 \cdots \alpha_r}(X)$ obeys the structure equation

$$0 \ \equiv \ \{ {\cal H}, {\cal H} \}_{{\rm P.B.}} \ \sim \ \partial_\alpha {\cal H} \wedge \partial^\alpha {\cal H} \ .$$

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Gauge invariance, structure group and topological symmetry breaking

• Structure equation \Rightarrow under the gauge transformations

$$\delta_{\epsilon,\eta}(X^{\alpha}, \mathcal{P}_{\alpha}) := d(\epsilon^{\alpha}, \eta_{\alpha}) + (\epsilon^{\alpha}\partial_{\alpha} + \eta_{\alpha}\partial^{\alpha})(\partial^{\alpha}, \partial_{\alpha})\mathcal{H} ,$$

the classical Lagrangian transforms into a total derivative, viz.

$$\delta_{\epsilon,\eta} \mathcal{L}^{\mathrm{cl}}_{\mathrm{bulk}} \; = \; \textit{d}(\epsilon^{\alpha} \partial_{\alpha} (1 - \textit{P}_{\beta} \partial^{\beta}) \mathcal{H} + \eta_{\alpha} (\textit{d} \textit{X}^{\alpha} - \partial^{\alpha} \mathcal{H}))$$

 Globally defined classical topological field theory with graded structure group generated by gauge parameters (t^α, 0) obeying

$$t^{lpha}\partial_{lpha}(1-P_{eta}\partial^{eta})\mathcal{H} = 0$$

 Remaining gauge parameters and corresponding fields are glued together across chart boundaries by means of transitions from the structure group (*c.f.* separate treatment of local translations and Lorentz rotations in Einstein – Cartan gravity)

Boundary data and Cartan integrability

The degrees of freedom reside in the boundary data:

- if $\mathcal{H}|_{P=0} = 0$ the variational principle holds with $P_{\alpha}|_{\partial \mathcal{M}} = 0$ so P_{α} can be taken to vanish on-shell in the case of a single boundary component
- integration constants C^{α} for the X^{α} of degree zero
- values of topologically broken gauge functions, λ^{α} say, on boundaries of the boundary $\partial \mathcal{M}$ (noncompact)
- windings in transitions, monodromies etc

N.B. Given C^{α} and λ^{α} , the local field configuration on $\partial \mathcal{M}$ given on shell by Cartan's integration formula:

$$X^{\alpha}_{\mathcal{C},\lambda} \approx \left[\exp((d\lambda^{\beta} + \lambda^{\gamma}\partial_{\gamma}Q^{\alpha})\partial_{\beta})X^{\alpha} \right] \Big|_{X=\mathcal{C}}$$

where we recall that $\mathcal{H} = P_{\alpha}Q^{\alpha}(X) + O(P^2)$

Intrinsically defined observables on shell versus topological vertex operators off shell

 Classically, the physical information in initial/boundary data is captured by various classes of intrinsically defined observables O[X] obeying

$$\delta_{t'} \mathcal{O} \equiv 0, \qquad \delta_{\epsilon} \mathcal{O} \approx 0$$

- ► t' parameters of chosen structure group (need not be same as t)
- structure group not unique by any general means
- $\blacktriangleright \ larger/smaller \ structure \ group \leftrightarrow fewer/more \ observables$
- notions of unbroken phase and various broken phases
- ▶ topologically broken symmetries (incl. diffeos) resurface on shell
- Certain *O* ≈ ∮ *V* with *V*[*X*, *dX*] being topological vertex operator defined off shell such that

$$\delta_{t'} \mathcal{V} \equiv 0, \qquad \delta \mathcal{V} \approx 0$$

N.B. On shell $\oint \mathcal{V}$ only depends on the homotopy class of its cycle.

Total action

• PSM characterized by the total action

$$S^{\mathrm{cl}}_{\mathrm{tot}} = \int_{\mathcal{M}} (P_{lpha} dX^{lpha} - \mathcal{H}(X, P)) + \sum_{i} g_{i} \oint_{\mathcal{M}_{i}} \mathcal{V}^{i}(X, dX)$$

- \blacktriangleright ${\cal H}$ breaks Cartan gauge algebra down to maximal structure group
- ▶ topological vertex operators \mathcal{V}^i (may) cause further breaking
- all Cartan gauge symmetries restored on shell
- \blacktriangleright the on shell values \mathcal{V}^i identified as semi-classical generating functions for amplitudes
- for a field theory with an "ordinary" action principle S_D in D dimensions, one may choose $\mathcal{V} = S_D$ in general, the does *not* reproduce the standard S-matrix/holographic correlation functions
- in the case of HSGR, we propose to take ${\cal V}$ to be the yet-to-be found completion of $S_{\rm FV}$

Gauge fixing and Batalin–Vilkovisky field-anti-field formalism

The first step of the gauge fixing procedure is to exhibit all gauge-for-gauge symmetries:

extend the classical fields $(X^{\alpha}, P_{\alpha}) \equiv (X_{[p_{\alpha}]}^{\alpha,\langle 0 \rangle}, P_{\alpha,[\hat{p}-p_{\alpha}]}^{\langle 0 \rangle}) \longrightarrow$ finite towers of ghosts $(X_{[p_{\alpha}-q]}^{\alpha,\langle q \rangle}, P_{\alpha,[\hat{p}-p_{\alpha}-q']}^{\langle q' \rangle})$ with ghost numbers $q = 1, \ldots, p_{\alpha}$ and $q' = 1, \ldots, \hat{p} - p_{\alpha}$

As the Cartan gauge symmetries do not in general close off-shell, the BRST operator needs to be constructed via a "minimal" master action

 $S^{\rm BV}[\phi^i,\phi^*_i]$

depending on

- "fields" ϕ^i comprising the classical fields and the ghost towers
- "anti-fields" ϕ_i^* obeying

$$\mathrm{gh}\phi^i + \mathrm{gh}\phi^*_i = -1 , \qquad \mathrm{deg}\phi^i + \mathrm{deg}\phi^*_i = \hat{p} + 1$$

The space of fields and anti-fields is equipped by a natural symplectic structure corresponding to the BV bracket

$$(A,B) := \int_{\rho \in \mathcal{M}} \delta_i(\rho) A \, \delta^i_*(\rho) B , \qquad \mathrm{gh}(\cdot, \cdot) = 1 ,$$

where $\delta_i(p)$ denotes the functional derivative with respect to ϕ^i at the point p idem $\delta^i_*(p)$.

Classical and quantum master equation

• If the anti-fields are eliminated by means of a canonical transformation then the path-integral over the remaining Lagrangian submanifold is does not depend on the c.t. provided that

$$(S,S)+rac{i}{2}\hbar\Delta S \equiv 0$$

where the BV Laplacian Δ is the slightly singular operator defined by

$$\Delta \ := \ \int_{oldsymbol{p}\in\mathcal{M}} \delta_i(oldsymbol{p}) \delta^i_\star(oldsymbol{p}) \ , \qquad \mathrm{gh}\Delta \ = \ 1 \ .$$

• Δ is formally nilpotent but does not act as a differential; rather

$$\Delta(AB) - \Delta(A)B - A\Delta(B) \equiv (A,B) .$$

• The nilpotent BRST differential s is given by

$$sA := (S, A)$$
.

• s is generated by a BRST current only if $\Delta S = 0$.

AKSZ vectorial superfields

AKSZ: Minimal quantum master action of the PSM obtained by extending the classical differential forms into unconstrained vectorial superfields of fixed total degree p given by the sum of form degrees and ghost numbers as follows:



AKSZ quantum master action

Ultra-local functionals $F = F(\mathbf{X}, \mathbf{P})$ and F' obey

$$(\int_{\mathcal{M}} \mathbf{P}_{\alpha} d\mathbf{X}^{\alpha}, F) = dF , \qquad (\int_{\mathcal{M}} F, F') = \{F, F'\}_{\text{P.B.}} ,$$

$$\Delta \int_{\mathcal{M}} F \equiv 0 .$$

Thus, since $\{\mathcal{H},\mathcal{H}\}_{|\textit{rmP}.B.}\equiv 0,$ it follows that

$$\mathcal{S}_{ ext{bulk}}^{ ext{AKSZ}} \; := \; \int_{\mathcal{M}} \left[\mathbf{P}_{lpha} d \mathbf{X}^{lpha} - \mathcal{H}(\mathbf{X}, \mathbf{P})
ight)
ight]$$

obeys

$$(S_{\mathrm{bulk}}^{\mathrm{AKSZ}}, S_{\mathrm{bulk}}^{\mathrm{AKSZ}}) = 0 = \Delta S_{\mathrm{bulk}}^{\mathrm{AKSZ}}$$

modulo boundary terms that vanish for suitable assignments of cross-chart transformation properties and boundary conditions (here the general theory involves sophisticated generalizations of bundle theory).

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HSGR: non-commutative base and target spaces

- $\mathcal{M} \supset \partial \mathcal{M}$ (X and Z coordinates) and $\mathcal{T}^*[\hat{p}]\mathcal{N}$ (spaces of functions of Y) non-commutative; denote the combined associative product by \star .
- The generalized Hamiltonian bulk action $(Z^i = (A, B, U, V))$

$$S^{\mathrm{cl}}_{\mathrm{bulk}}[Z^{i};J] = \int_{\mathcal{M}} \left[U \star DB + V \star \left(F + \mathcal{F}(B;J) + \widetilde{\mathcal{F}}(U;J)\right) \right] ,$$

where $DB := dB + [A, B]_{\star}$, $F := dA + A \star A$ and J are d-closed central elements (of even form degrees).

- If $\hat{p} = 2n$, the fields decompose under form degree as follows:
 - $A = A_{[1]} + A_{[3]} + \dots + A_{[2n-1]} , \qquad B = B_{[0]} + B_{[2]} + \dots + B_{[2n-2]} ,$ $U = U_{[2]} + U_{[4]} + \dots + U_{[2n]} , \qquad V = V_{[1]} + V_{[3]} + \dots + V_{[2n-1]} .$
- Variational principle $\Rightarrow R^i := dZ^i + Q^i(Z^j) \approx 0$ and homogeneous boundary conditions on (U, V) such that on ∂M

$$F + \mathcal{F} \approx 0$$
, $DB \approx 0$.

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Adaptation of BV to non-commutative setting

• *-functional differentiation defined naturally via

$$\delta F[X,P] \equiv F[X + \delta X, P + \delta P] =: \int_{\mathcal{M}} (\delta X^{lpha} \star \delta_{lpha} F + \delta P_{lpha} \star \delta^{lpha} F) .$$

N.B. The \star -functional derivatives act as differentials on ultra-local functionals and cyclic derivatives on local functionals. The former action induces a suitable non-commutative generalization of the Dirac delta function (which in a certain sense is less singular than the commuting ditto).

 Introduce fields and master fields; the gauge fixing procedure (independence of choice of Lagrangian submanifold) → BV Laplacian given by double *-functional derivatives of the above types.

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Adaptation of AKSZ to non-commutative setting

- Extend $Z^i \rightarrow \mathbf{Z}^i = (\mathbf{A}, \mathbf{B}, \mathbf{U}, \mathbf{V})$
- The AKSZ action

$$S_{\text{bulk}}^{\text{AKSZ}}[\mathbf{Z}^{i}; J] = \sum_{\xi} \int_{\mathbf{M}_{\xi}} \text{Tr} \left[\mathbf{U} \star D\mathbf{B} + \mathbf{V} \star \left(\mathbf{F} + \mathcal{F}(\mathbf{B}; J) + \widetilde{\mathcal{F}}(\mathbf{U}; J) \right) \right]$$

generates BRST transformations

$$(S_{\text{bulk}}^{\text{AKSZ}}, \mathbf{Z}^i) = \mathbf{R}^i := d\mathbf{Z}^i + Q^i(\mathbf{Z}^j; J)$$

It follows that the AKSZ action obeys the classical master equation

$$(S_{\text{bulk}}^{\text{AKSZ}}, S_{\text{bulk}}^{\text{AKSZ}}) = 0$$

modulo boundary terms that vanish if $(\mathbf{U}, \mathbf{V}) = 0$ on $\partial \mathcal{M}$ and (\mathbf{U}, \mathbf{V}) form sections with bold-faced transition functions from the unbroken gauge algebra associated to (\mathbf{A}, \mathbf{B}) .

Quantum master action and deformations

- Whether the bulk action also obeys the quantum master equation is under investigation.
- As is well-known from ordinary topological field theories, gauge fixing actually requires "non-minimal" extensions of the AKSZ action → cancellation of all vacuum bubbles (modulo topological terms) → bulk partition function as sum over topologies.
- There exists various topological impurities $\mathcal{V}[\mathbf{X}, d\mathbf{X}; J]$ obeying $(S, \mathcal{V}) = 0 = (\mathcal{V}, \mathcal{V})$ and $\Delta \mathcal{V} = 0$.
- In particular, in certain broken phase there exist a topological two-form (surface operator) and four-form; the relation between the latter and the on shell FV action is under investigation.

Correspondence with free O(N) vector model

- Integrate out Vasiliev's Z-variables →→ classical formulation on commutative *M* times internal *Y*-space with perturbatively defined *Q*-structure
- Duality extend $B = B_{[0]} + B_{[2]}$; $A = A_{[1]} + A_{[3]}$; $U = U_{[2]} + U_{[4]}$ and $V = V_{[1]} + V_{[3]}$
- for any H(U, V; B) the boundary correlators
 ⟨B_[0](p₁) ··· B_[p_n]A_[1](p_{n+1}) ··· A_[1](p_{n+m})⟩|_{p_i∈∂M} are given by their
 semi-classical limits (assume vacuum bubbles cancel)
- assume the existence of a perturbative completion $\int_{\partial \mathcal{M}} \mathcal{V}_{\rm FV}(B_{[0]}, dB_{[0]}; A_{[1]}, dA_{[1]}) \text{ of Fradkin - Vasiliev action; add it as}$ deformation to be treated as vertices (including kinetic terms!)
- $\rightsquigarrow \langle \exp(g \int_{\partial \mathcal{M}} \mathcal{V}_{\mathrm{FV}}) \rangle$ tree-level exact which matches the ultra-violet fixed point of the O(N)-vector model
- Still miss correspondence at the level of the vacuum value of the on shell action (free energy)

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1/N corrections from $\Delta = 2$ boundary conditions?

- Klebanov Polyakov: infra-red fixed point of double-trace deformation \leftrightarrow bulk theory with $\Delta = 2$ boundary conditions
- add Gibbons-Hawking term $\int_{\partial^2 M} \phi \partial_n \phi$ and treat as extra vertex:
 - Giombi Yin: Modified scalar two-point functions

$$G_{\Delta=1}(p;r,r') + |p|K_{\Delta=1}(p;r)K_{\Delta=1}(p;r') \equiv G_{\Delta=2}(p;r,r')$$

- \blacktriangleright Sew together pairs of external scalar legs of $\Delta=1$ tree diagrams \rightsquigarrow scalar loops touching the boundary
- ► HS completion of the GH term ~→ HS fields starts running in loops touching the boundary?

Broken higher spin gauge symmetries and flat limit?

- Girardello Porrati Zaffaroni: spin-(s-1) Goldstone modes for s = 4, 6, ... identified as scalar-spin-(s-2) composite
- multi-particle states start behaving as "fundamentals" → stringy spectrum with many trajectories but problem with large-N normalization of higher-point functions (impose connectedness?)
- flat double-scaling limit: $\Lambda \to 0$ and N small but not really a limit as N quantized

Conclusions

- Quantize 4D HS fields via topological Poisson sigma models in higher dimensions
- 4D spacetime meant to arise as submanifold where on shell action a lá Fradkin Vasiliev can be added as a deformation.
- Assumptions of the structure of the FV action imply that holographic correlators with $\Delta=1$ BC agree with free O(N) vector model but free energy problematic still
- Assumptions on the Gibbons Hawking terms imply qualitative fitting to the interacting O(N) vector model

Thanks for your attention!

As far as Schrödinger's cat is concerned, I was thinking of something funny to say but then I thought that I better leave it to the audience :)