On String Theory and Higher Spins

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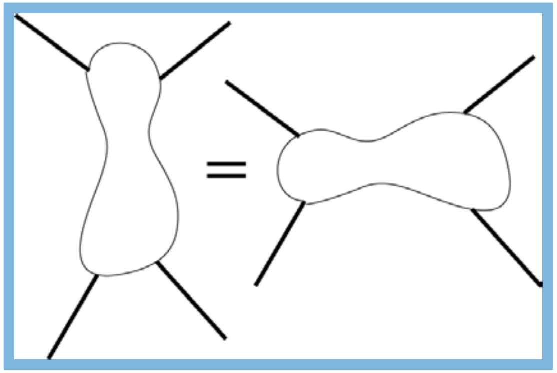
String Theory & HS

- **Originally:** an S-matrix with (planar) duality

$$\sum_n \frac{R_n(t)}{\alpha(s)-n} = \sum_n \frac{R_n(s)}{\alpha(t)-n}$$



- **Rests crucially** on the presence of ∞ **massive modes**
- **Massive modes:** mostly Higher Spins (HS)

(Dirac, Fierz, Pauli, 1936 - 39)

4D: $A_\mu \longrightarrow \phi_{\mu_1 \ldots \mu_s}$

[Symmetric]

Dirac-Fierz-Pauli (DFP) conditions:

$$\begin{aligned}(\Box - M^2)\varphi_{\mu_1\ldots\mu_s} &= 0\\ \partial^{\mu_1}\varphi_{\mu_1\ldots\mu_s} &= 0\\ \varphi^{\mu_1}{}_{\mu_1\ldots\mu_s} &= 0\end{aligned}$$

[D>4: $A_\mu \longrightarrow \phi_{\mu_1^{(1)}\ldots\mu_{s_1}^{(1)};\ldots;\mu_1^{(N)}\ldots\mu_{s_N}^{(N)}}$]

[Mixed (multi-symmetric)]

String Theory & HS

- [Vacuum stability: OK with SUSY]
- Key addition: low-energy effective SUGRA
  (2D data translated via RG into space-time notions)

$$S_2 = \int \sqrt{\gamma}\gamma^{ab}\partial_a x^\mu \partial_b X^\nu G_{\mu\nu}(X) + \int \epsilon^{ab}\partial_a x^\mu \partial_b X^\nu B_{\mu\nu}(X) + \int \alpha' \sqrt{\gamma} R^{(2)} \Phi(X) + ..$$

$$S_D = \frac{1}{2k_D^2} \int d^D X \sqrt{-G} e^{-\Phi} \left(R - \frac{1}{12} H^2 + 4(\partial\Phi)^2 \right) + ..$$

- Deep conceptual problems are inherited from (SU)GRA.
- String Field Theory: field theory combinatorics for amplitudes.
  Massive modes included. Background (in)dependence?

Are strings really at the heart of String Theory?

Lessons from (and for) (massive) HS?

Plan

- **Key properties of HS fields:**
  - Symmetric HS fields, triplets and HS geometry;
- **HS interactions:**
  - External currents and the vDVZ discontinuity;
  - Limiting string 3-pt functions and gauge symmetry;
  - Conserved HS currents & exchanges;
  - 4-point functions and beyond.

Free Symmetric HS

Fronsdal (1978): natural extension of s=1,2 cases **(BUT with CONSTRAINTS)**

$$\mathcal{F}_\mu \equiv \Box A_\mu - \partial_\mu \partial \cdot A = 0$$
$$\mathcal{F}_{\mu\nu} \equiv \Box h_{\mu\nu} - (\partial_\mu \partial \cdot h_\nu + (\mu \leftrightarrow \nu)) + \partial_\mu \partial_\nu h' = 0$$
$$\ldots$$
$$\mathcal{F}_{\mu_1 \ldots \mu_s} \equiv \Box \varphi_{\mu_1 \ldots \mu_s} - (\partial_{\mu_1} \partial \cdot \varphi_{\mu_2 \ldots \mu_s} + \ldots) + (\partial_{\mu_1} \partial_{\mu_2} \varphi'_{\mu_3 \ldots \mu_s} + \ldots) = 0$$

$$\delta \mathcal{F}_\mu = 0 \ , \quad \delta \mathcal{F}_{\mu\nu} = 0$$
but
$$\delta \mathcal{F}_{\mu_1 \ldots \mu_s} = 3 \partial_{(\mu_1} \partial_{\mu_2} \partial_{\mu_3} \Lambda'_{\mu_4 \ldots \mu_s)}$$

("primes" = traces)

Can simplify notation (and algebra) **hiding** space-time indices:

$$\varphi_{\mu_1 \ldots \mu_s} \rightarrow \varphi \ , \quad \varphi'_{\mu_3 \ldots \mu_s} \rightarrow \varphi'$$
$$\partial_{\mu_1} \varphi_{\mu_2 \ldots \mu_{s+1}} + \ldots \rightarrow \partial \varphi$$
$$\mathcal{F}_{\mu_1 \ldots \mu_s} \rightarrow \mathcal{F} \equiv \Box \varphi - \partial \partial \cdot \varphi + \partial^2 \varphi'$$
$$\delta \mathcal{F} = 3 \partial^3 \Lambda'$$

⇒ $\Lambda' = 0$

1st Fronsdal constraint

Free Symmetric HS

Bianchi identity :

$$\partial \cdot \mathcal{F} - \frac{1}{2}\partial \mathcal{F}' = -\frac{3}{2}\partial^3 \varphi''$$

$$\mathcal{L} = \varphi \left(\mathcal{F} - \frac{1}{2}\eta \mathcal{F}' \right)$$

$$\delta \mathcal{L} = -s\Lambda \left(\partial \cdot \mathcal{F} - \frac{1}{2}\partial \mathcal{F}' \right)$$

$$\varphi'' = 0$$

2nd Fronsdal constraint

Natural to try and forego these "trace" constraints:

- BRST (non minimal) *(Buchbinder, Burdik, Pashnev, Tsulaia, , 1998-)*
- Minimal compensator form *(Francia, AS, Mourad, 2002 -)*

$$\mathcal{F} = 0 \rightarrow \mathcal{A} \equiv \mathcal{F} - 3\partial^3 \alpha = 0$$

$$\partial \cdot \mathcal{A} - \frac{1}{2}\partial \mathcal{A}' = -\frac{3}{2}\partial^3 \mathcal{C}$$

$$\delta \alpha = \Lambda'$$

$$\mathcal{C} = \varphi'' - 4\partial \cdot \alpha + \partial \alpha'$$

s=3 (Schwinger)

$$\mathcal{L} = \varphi \left(\mathcal{A} - \frac{1}{2}\eta \mathcal{A}' \right) - \frac{3}{4}\binom{s}{3} \alpha \partial \cdot \mathcal{A}' + 3\binom{s}{4} \beta \mathcal{C}$$

Unconstrained Lagrangian

[2-derivative : (Buchbinder et al, 2007; Francia, 2007)]

Free HS Geometry

What are we gaining ?

$$\mathcal{F}^{(n+1)} = \mathcal{F}^{(n)} + \frac{1}{(n+1)(2n+1)} \frac{\partial^2}{\Box} \mathcal{F}'^{(n)} - \frac{1}{n+1} \frac{\partial}{\Box} \partial \cdot \mathcal{F}^{(n)}$$

$$\mathcal{F}^{(k)} = (2k+1) \frac{\partial^{2k+1}}{\Box^{k-1}} \alpha^{[k-1]}$$

After some iterations: NON-LOCAL gauge invariant equation for φ ONLY

s = 2: $$\delta h_{\mu\nu} = \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu \rightarrow \delta\Gamma^{\mu}_{\nu\rho} = \partial_\nu \partial_\rho \Lambda^\mu \rightarrow \delta R^{\alpha}_{\mu\nu\rho} = 0$$

s > 2: Hierarchy of connections and curvatures (de Wit and Freedman, 1980)

$$\Gamma_{\mu;\nu_1\ldots\nu_s}, \ldots, \Gamma_{\mu_1\ldots\mu_{s-1};\nu_1\ldots\nu_s} ; \mathcal{R}_{\mu_1\ldots\mu_s;\nu_1\ldots\nu_s}$$

NON LOCAL geometric equations :

$$s = 2n+1 : \qquad \frac{1}{\Box^n} \partial_\mu \mathcal{R}^{\mu[n];\nu_1\ldots\nu_s} = 0$$

$$s = 2n : \qquad \frac{1}{\Box^{n-1}} \mathcal{R}^{[n];\nu_1\ldots\nu_s} = 0$$

(Francia and AS, 2002)

String Theory & Free HS

$$\mathcal{Q}\,|\Psi\rangle = 0$$
$$\delta\,|\Psi\rangle = \mathcal{Q}\,|\Lambda\rangle$$

(Kato and Ogawa, 1982; Witten; Neveu, West et al, 1985,,...)

BRST equations for "contracted" Virasoro:

$$L_k = \frac{1}{2}\sum_{l=-\infty}^{+\infty} \alpha^{\mu}_{k-l}\,\alpha_{\mu l}$$
$$[L_k, L_l] = (k-l)\,L_{k+l} + \frac{\mathcal{D}}{12}\,m\,(m^2-1)\,\delta_{k+l,0}$$

$\alpha' \rightarrow \infty$

$$\ell_0 = p^2$$
$$\ell_k = p\cdot\alpha_k$$
$$[\ell_m, \ell_n] = m\,\ell_0\,\delta_{m+n,0}$$

First open bosonic Regge trajectory → TRIPLETS

Propagate: s,s-2,s-4, ...

(A. Bengtsson, 1986; Henneaux, Teitelboim, 1987)
(Pashnev, Tsulaia, 1998; Francia, AS, 2002; Bonelli, 2003; AS, Tsulaia, 2003)

$$\Box\,\varphi = \partial\,C\,,$$
$$\partial\cdot\varphi - \partial\,D = C$$
$$\Box\,D = \partial\cdot C$$

On-shell truncation :

(Francia, AS, 2002)

$$\varphi' - 2\,D = \partial\,\alpha$$

$$\mathcal{F} = 3\,\partial^{\,3}\,\alpha$$
$$\varphi'' = 4\partial\cdot\alpha + \partial\alpha'$$
$$\delta\varphi = \partial\Lambda\,, \qquad \delta\alpha = \Lambda'$$

Off-shell truncation : (Buchbinder, Krykhtin, Reshetnyak 2007)

Geometric form:

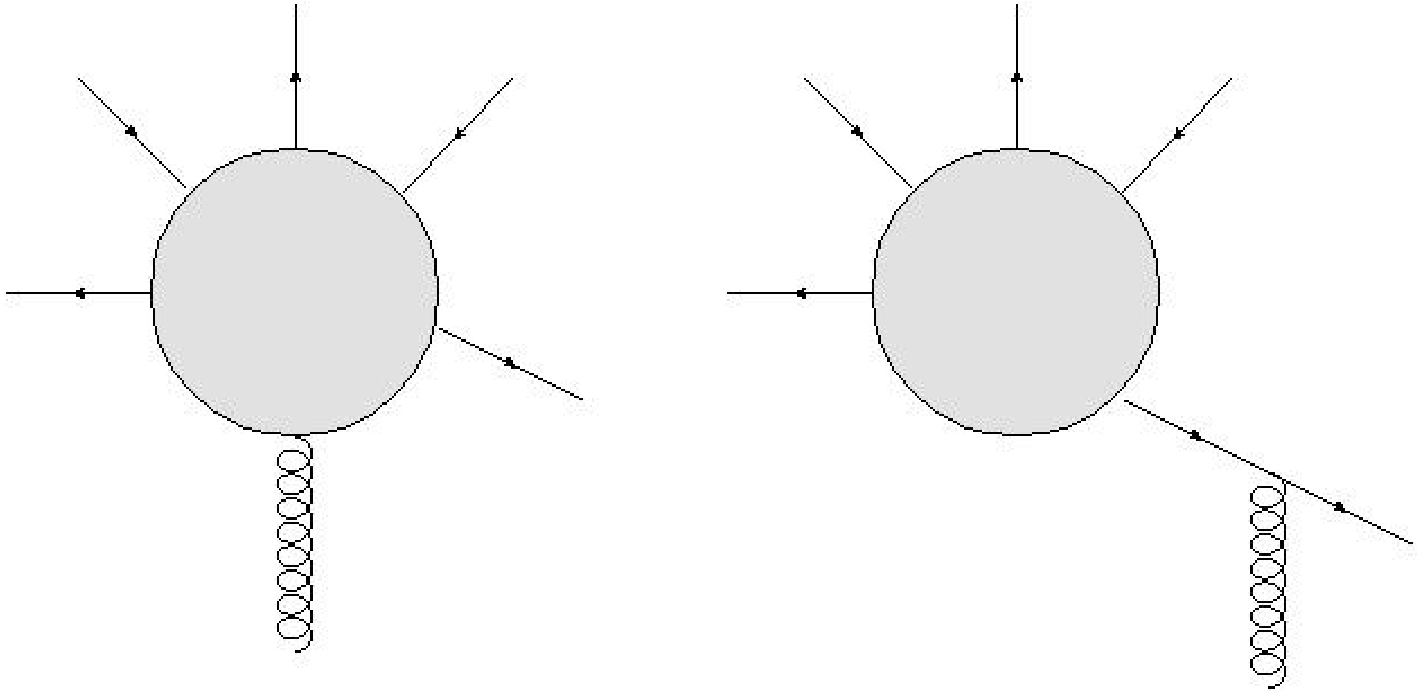
(Francia, 2010)

$$\mathcal{L} \sim \mathcal{R}^{[s]\;\mu_1\ldots\mu_s}\,\frac{1}{\Box^{s-1}}\,\mathcal{R}^{[s]}{}_{\mu_1\ldots\mu_s}$$

HS Interactions

Problems :

- Aragone – Deser problem (with "minimal" gravity coupling)
- Weinberg – Witten
- Coleman - Mandula
- Velo - Zwanziger problem
- Weinberg's 1964 S-matrix argument
- ....



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- Weinberg's 1964 S-matrix argument
- ……………………………

But:

- (Light-cone or covariant) 3-vertices

  [higher derivatives]

Berends, Burgers, van Dam, 1982)
(Bengtsson^2, Brink, 1983)
(Boulanger et al, 2001 -)
(Metsaev, 2005,2007)
(Buchbinder,Fotopoulos, Irges, Petkou, Tsulaia, 2006)
(Boulanger, Leclerc, Sundell, 2008)
(Zinoviev, 2008)

- (Scalar) scattering via current exchanges

(Manvelyan, Mkrtchyan, Ruhl, 2009)
(Bekaert, Mourad, Joung, 2009)

- String contact terms resolve Velo-Zwanziger

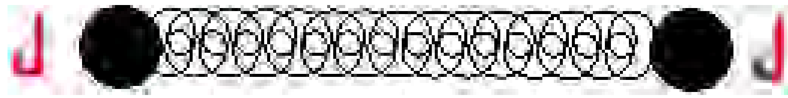
(Argyres, Nappi, 1989)
(Porrati, Rahman, 2009)
(Porrati, Rahman, AS, 2010)

Vasiliev eqs:

- Deformed low-derivative with $\Lambda \neq 0$
- Infinitely many fields

(Vasiliev, 1990, 2003)
(Sezgin, Sundell, 2001)

External Currents



(Fronsdal, 1978)
(Francia, Mourad, AS, 2007, 2008)

NOTICE:
- Static sources: Coulomb-like
- Residues: degrees of freedom

- e.g. s=1 :

$$p^2 A_\mu - p_\mu \, p \cdot A = J_\mu$$

$$p^2 J^\mu A_\mu = J^\mu J_\mu$$

$$J_i \, J_i$$

- All s :

$$\mathcal{A} - \frac{1}{2} \eta \, \mathcal{A}' + \eta^2 \mathcal{B} = J$$

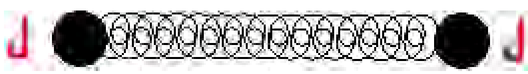
$$\partial \cdot \mathcal{A}' - (2 \partial + \eta \, \partial \cdot) \mathcal{B} = 0$$

$$\varphi'' - 4 \partial \cdot \alpha - \partial \alpha' = 0$$

… unique NON-LOCAL Lagrangian
(with proper current exchange)
E.g. for s=3:

$$\mathcal{A} = \frac{1}{\Box} \partial \cdot \mathcal{R}' + \frac{1}{2} \frac{\partial^2}{\Box^2} \partial \cdot \mathcal{R}''$$

VD-V-Z (Dis)continuity for HS



$$m = 0 \; : \quad T_{\mu\nu}T^{\mu\nu} \; - \; \frac{1}{2}(T')^2$$

(van Dam, Veltman; Zakharov, 1970)

$$m \neq 0 \; : \quad T_{\mu\nu}T^{\mu\nu} \; - \; \frac{1}{3}(T')^2$$

(conserved $T_{\mu\nu}$)

∀s and D, m=0 :

$$\sum_{n=0}^{N} \rho_n(D-2,s)\, \frac{s!}{n!\,(s-2n)!\,2^n}\, J^{[n]} \cdot J^{[n]}$$

$$\rho_{n+1}(D,s) \; = \; - \, \frac{\rho_n(D,s)}{D+2(s-n-2)}$$

$$s = 2 \; : \quad T_{\mu\nu}T^{\mu\nu} \; - \; \frac{1}{D-2}(T')^2$$

- **VDVZ discontinuity:** comparing **D** and **(D+1)**-dim exchanges
- **∀s:** can describe massive fields a' la Scherk-Schwarz from (D+1) dimensions :
  [e.g. for s=2 : h_{AB} → (h_{ab} cos(my) , A_a sin(my), φ cos(my))]

$$\sum_{n=0}^{N} \rho_n(D-2,s)\, \frac{s!}{n!\,(s-2n)!\,2^n}\, J^{[n]} \cdot J^{[n]} \quad \Longleftrightarrow \quad \sum_{n=0}^{N} \rho_n(D-1,s)\, \frac{s!}{n!\,(s-2n)!\,2^n}\, J^{[n]} \cdot J^{[n]}$$

(A)dS Current Exchanges

Two types of deformations:

- a finite dS radius L
- a mass M

s=2 : $$(J_{\mu\nu})^2 \;-\; \frac{1}{D-1}\,\frac{(ML)^2-(D-1)}{(ML)^2-(D-2)}\,(J\,')^2$$

(Higuchi, 1987,2002)
(Porrati, 2001)
(Kogan, Mouslopoulos, Papazoglou, 2001)

Rational function of $(ML)^2$:
- massless result as $(ML)\to 0$
- massive result as $(ML)\to\infty$

Questions: How to extend to **all s** ? Origin of the **poles** ?

E.g.: massive s=2 in dS → in general **NO** gauge symmetry, **BUT** if $(ML)^2 = D-2$

"Partial" gauge symmetry : $$\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \zeta + \frac{M^2}{D-2}\, g_{\mu\nu}\,\zeta$$

(Deser, Nepolmechie, 1984)
(Deser, Waldron, 2001)

NOTE: the coupling $J_{\mu\nu}h^{\mu\nu}$ is **NOT** "partially gauge invariant"
unless $J_{\mu\nu}$ is **CONSERVED AND TRACELESS**

(A)dS Current Exchanges

$$
\begin{aligned}
\mathcal{K}(x,u^a) \;&=\; J_s + \frac{u^2}{4(\frac{5}{2}-\zeta)}\,\frac{(ML)^2+2(\frac{5}{2}-\zeta)}{(ML)^2-2(\zeta-3)}\,{J_s}'\\
&+\; \frac{(u^2)^2}{32}\,\frac{(ML)^4+8(ML)^2(\frac{7}{2}-\zeta)+12\left(\frac{5}{2}-\zeta\right)_2}{\left(\frac{5}{2}-\zeta\right)_2[(ML)^2-2(\zeta-3)][(ML)^2-6(\zeta-4)]}\,{J_s}^{[2]}\\
&+\; \frac{(u^2)^3}{384}\,\frac{(ML)^6-(ML)^4(18\zeta-77)+92(ML)^2\left(\frac{7}{2}-\zeta\right)_2+120\left(\frac{5}{2}-\zeta\right)_3}{\left(\frac{5}{2}-\zeta\right)_3[(ML)^2-2(\zeta-3)][(ML)^2-6(\zeta-4)][(ML)^2-10(\zeta-5)]}\,{J_s}^{[3]}\\
&+\; \ldots + (u^2)^n\,\frac{\mathcal{N}_n}{\mathcal{D}_n}\,J_s^{[n]} + \ldots \qquad \left[\zeta = \frac{D}{2}+s\right]
\end{aligned}
$$

$$
\mathcal{D}_n = 2^{3n} n! \left(\frac{5}{2}-\zeta\right)_n \frac{1}{2^{2n}} \prod_{j=0}^{n-1}\left[(ML)^2+2(2j+1)(j+3-\zeta)\right]
$$

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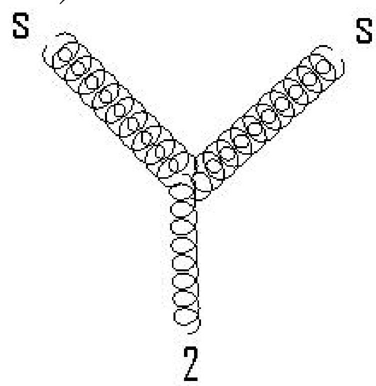
HS Cubic Couplings

Closer look at old difficulties (for definiteness s-s-2 case)



- **Aragone-Deser:** NO "standard" gravity coupling for **massless** HS around **flat** space;

- **Fradkin-Vasiliev** : OK with also higher-derivative terms around
- **Metsaev** (2007) light-cone
- **Boulanger, Leclercq, Sundell** (2008): non-Abelian **CUBIC "SEEDS"**

HS around flat space; → MORE DERIVATIVES

Naively: **2 derivatives,**

BUT ACTUALLY: **4 derivatives!**

$$\begin{aligned}\mathcal{L}_3 \quad \sim \quad & \tilde{w}_{\alpha\beta\gamma\delta}\Big[2\,\widetilde{\phi}'_{\mu}\partial^{\beta}\partial^{\delta}\widetilde{\phi}^{\alpha\gamma\mu} + \widetilde{\phi}^{\alpha\gamma}{}_{\mu}\partial^{\delta}\partial^{\mu}\widetilde{\phi}'^{\beta} - 3\,\widetilde{\phi}'^{\alpha}\partial^{\delta}\partial^{\mu}\widetilde{\phi}^{\beta\gamma}{}_{\mu} \\ & +2\,\widetilde{\phi}^{\alpha}{}_{\mu\nu}\partial^{(\delta}\partial^{\nu)}\widetilde{\phi}^{\beta\gamma\mu} + \partial_{\mu}\widetilde{\phi}^{\alpha\gamma\mu}\partial_{\nu}\widetilde{\phi}^{\beta\delta\nu} - \widetilde{\phi}^{\alpha\gamma\mu}\partial_{\mu}\partial_{\nu}\widetilde{\phi}^{\beta\delta\nu} \\ & -2\,\partial^{(\mu}\widetilde{\phi}^{\nu)\alpha\gamma}\partial_{\mu}\widetilde{\phi}^{\beta\delta}{}_{\nu} - 2\,\widetilde{\phi}^{\alpha\gamma}{}_{\mu}\partial^{\delta}\partial^{\nu}\widetilde{\phi}^{\beta\mu}{}_{\nu} + \widetilde{\phi}'^{\alpha}\partial^{\beta}\partial^{\delta}\widetilde{\phi}'^{\gamma} - \widetilde{\phi}^{\alpha}{}_{\mu\nu}\partial^{\beta}\partial^{\delta}\widetilde{\phi}^{\gamma\mu\nu}\Big]\end{aligned}$$

DEFORMING the highest vertex to (A)dS one can recover consistent "minimal" couplings **WITH higher-derivative tails**, with a singular limit as $\Lambda \to 0$

String Amplitudes & HS

(AS, Taronna, 2010)

Chan-Paton factors

Gauge fixed Polyakov path integral → Koba-Nielsen amplitudes

$$S^{open}_{j_1\cdots j_n} = \int_{\mathbb{R}^{n-3}} dy_4 \cdots dy_n \, |y_{12}y_{13}y_{23}| \times \langle \mathcal{V}_{j_1}(\hat{y}_1)\mathcal{V}_{j_2}(\hat{y}_2)\mathcal{V}_{j_3}(\hat{y}_3)\cdots\mathcal{V}_{j_n}(y_n)\rangle Tr(\Lambda^{a_1}\cdots\Lambda^{a_n})$$

$y_{ij} = y_i - y_j$

Vertex operators ←→ asymptotic states

Virasoro → Fierz-Pauli

$$\begin{aligned}(L_0-1)\,|\Psi\rangle &= 0\\ L_1\,|\Psi\rangle &= 0\\ L_2\,|\Psi\rangle &= 0\end{aligned} \quad\Longrightarrow\quad \begin{aligned}(\Box - M^2)\varphi_{\mu_1\ldots\mu_s} &= 0\\ \partial^{\mu_1}\varphi_{\mu_1\ldots\mu_s} &= 0\\ \varphi^{\mu_1}{}_{\mu_1\ldots\mu_s} &= 0\end{aligned}$$

HERE massive HS, but …

Generating Functions

$$Z[J] = i(2\pi)^d \delta^{(d)}(J_0)\mathcal{C}\, \exp\left(- \frac{1}{2}\int d^2\sigma d^2\sigma'\, J(\sigma)\cdot J(\sigma')\, G(\sigma,\sigma')\right)$$

"symbols"

$$Z(\xi_i^{(n)}) \sim exp\Big(\sum \xi_i^{(n)} A_{ij}^{nm}(y_l)\, \xi_j^{(m)} + \xi_i^{(n)} \cdot B_i^n(y_l;\, p_l) + \alpha'\, p_i \cdot p_j \ln |y_{ij}|\Big)$$

For symmetric open-string states

(1st Regge trajectory)

$$\phi_i(p_i, \xi_i) = \frac{1}{n!}\, \phi_{i\,\mu_1 \cdots \mu_n}\, \xi_i^{\mu_1} \ldots \xi_i^{\mu_n}$$

$$Z \sim \exp\left[-\frac{1}{2}\sum_{i\neq j}^{n} \alpha'\, p_i \cdot p_j\, \ln |y_{ij}| - \sqrt{2a'}\, \frac{\xi_i \cdot p_j}{y_{ij}} + \frac{1}{2}\frac{\xi_i\cdot\xi_j}{y_{ij}^2}\right]$$

3-point Amplitudes

(AS, Taronna, 2010)

❖ **Virasoro constraints** directly in generating function

$$-p_1^2=\frac{s_1-1}{\alpha'}\qquad -p_2^2=\frac{s_2-1}{\alpha'}\qquad -p_3^2=\frac{s_3-1}{\alpha'}$$

- **L_0 constraint:** mass
- **L_1 constraint:** transversality
- **L_2 constraint:** vanishing trace

→ **DFP conditions**

❖ **KEY SIMPLIFICATION OF 3-POINT FUNCTIONS:**

$$\mathbf{Z}_{phys}\sim\exp\left\{\sqrt{\frac{\alpha'}{2}}\left(\xi_1\cdot p_{23}\left\langle\frac{y_{23}}{y_{12}y_{13}}\right\rangle+\xi_2\cdot p_{31}\left\langle\frac{y_{13}}{y_{12}y_{23}}\right\rangle+\xi_3\cdot p_{12}\left\langle\frac{y_{12}}{y_{13}y_{23}}\right\rangle\right)+(\xi_1\cdot\xi_2+\xi_1\cdot\xi_3+\xi_2\cdot\xi_3)\right\}$$

→ **Signs (twist)**

"On-shell" couplings → star- product with symbols of fields

$$\mathcal{A}_\pm=\phi_1\left(p_1,\,\frac{\partial}{\partial\xi}\pm\sqrt{\frac{\alpha'}{2}}\,p_{31}\right)\phi_2\left(p_2,\,\xi+\frac{\partial}{\partial\xi}\pm\sqrt{\frac{\alpha'}{2}}\,p_{23}\right)\phi_3\left(p_3,\,\xi\pm\sqrt{\frac{\alpha'}{2}}\,p_{12}\right)\Bigg|_{\xi=0}$$

Some Examples

- **0-0-s:**

Conserved (massless ϕ)

(Berends, Burger, van Dam, 1986)

$$\mathcal{A}^{\pm}_{0-0-s} = \left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s} \phi_1\,\phi_2\,\phi_3 \cdot p_{12}^{\,s}$$

$$J^{\pm}(x,\xi) = \Phi\left(x \pm i\sqrt{\frac{\alpha'}{2}}\,\xi\right)\,\Phi\left(x \mp i\sqrt{\frac{\alpha'}{2}}\,\xi\right)$$

Wigner Function

- **1-1-(s≥2):**

$$\mathcal{A}^{\pm}_{1-1-s} = \left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s-2} s(s-1)\,A_{1\,\mu}\,A_{2\,\nu}\,\phi^{\mu\nu\cdots}\,p_{12}^{\,s-2}$$
$$+\left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s}\Big[A_1\cdot A_2\;\phi\cdot p_{12}^{\,s} + s\,A_1\cdot p_{23}\,A_{2\,\nu}\,\phi^{\nu\cdots}\,p_{12}^{\,s-1}$$
$$+\,s\,A_2\cdot p_{31}\,A_{1\,\nu}\,\phi^{\nu\cdots}\,p_{12}^{\,s-1}\Big]$$
$$+\left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s+2} A_1\cdot p_{23}\,A_2\cdot p_{31}\,\phi\cdot p_{12}^{\,s}\;,$$

String HS Couplings

(AS, Taronna, 2010)

- **OLD IDEA:** String Theory broken phase of "something"

- **AMPLITUDES:** can spot extra "debris" that drops out in the "massless" limit, where one ought to recover couplings based on conserved currents.

❖ A gauge invariant pattern shows up!

$$\mathcal{A}_{\pm} = \exp\left\{\sqrt{\frac{\alpha'}{2}}\Big[(\partial_{\xi_1}\cdot\partial_{\xi_2})(\partial_{\xi_3}\cdot p_{12}) + (\partial_{\xi_2}\cdot\partial_{\xi_3})(\partial_{\xi_1}\cdot p_{23}) + (\partial_{\xi_3}\cdot\partial_{\xi_1})(\partial_{\xi_2}\cdot p_{31})\Big]\right\}$$
$$\times\ \phi_1\left(p_1;\xi_1+\sqrt{\frac{\alpha'}{2}}\,p_{23}\right)\phi_2\left(p_2;\xi_2+\sqrt{\frac{\alpha'}{2}}\,p_{31}\right)\phi_3\left(p_3;\xi_3+\sqrt{\frac{\alpha'}{2}}\,p_{12}\right)\Bigg|_{\xi_i=0}$$

G operator: builds a CASCADE of lower-derivative terms

HS Conserved Currents

(AS, Taronna, 2010)

The **limiting couplings** are induced by **conserved currents**: $\mathcal{J} \cdot \phi$

$$\mathcal{J}^{\pm}(x\,;\xi) = \exp\left(\mp i\sqrt{\frac{\alpha'}{2}}\,\xi_\alpha\left[\partial_{\zeta_1}\cdot\partial_{\zeta_2}\,\partial_{12}^{\alpha} - 2\,\partial_{\zeta_1}^{\alpha}\,\partial_{\zeta_2}\cdot\partial_1 + 2\,\partial_{\zeta_2}^{\alpha}\,\partial_{\zeta_1}\cdot\partial_2\right]\right)$$
$$\times\,\phi_1\left(x \mp i\sqrt{\frac{\alpha'}{2}}\,\xi\,,\,\zeta_1 \mp i\sqrt{2\alpha'}\,\partial_2\right)\,\phi_2\left(x \pm i\sqrt{\frac{\alpha'}{2}}\,\xi\,,\,\zeta_2 \pm i\sqrt{2\alpha'}\,\partial_1\right)\Bigg|_{\zeta_i=0}$$

- **GENERALIZED WIGNER FUNCTIONS, conserved** up to massless Klein-Gordon, divergences and traces. **Unique extension to both Fronsdal and compensator cases (with divergences and traces), conserved up to complete eqs.**
- **CORRESPONDING GAUGE INVARIANT 3-POINT FUNCTIONS:**

$$\mathcal{A}_{\pm} = \exp\left\{\sqrt{\frac{\alpha'}{2}}\left[(\partial_{\xi_1}\cdot\partial_{\xi_2}+1)(\partial_{\xi_3}\cdot p_{12}) + (\partial_{\xi_2}\cdot\partial_{\xi_3}+1)(\partial_{\xi_1}\cdot p_{23}) + (\partial_{\xi_3}\cdot\partial_{\xi_1}+1)(\partial_{\xi_2}\cdot p_{31})\right]\right\}$$
$$\times\,\phi_1(p_1;\xi_1)\,\phi_2(p_2;\xi_2)\,\phi_3(p_3;\xi_3)\Big|_{\xi_i=0}$$

Related work (even s): Manvelyan, Mkrtchyan, Ruhl, 2010

HS Conserved Currents

(AS, Taronna, 2010)

A natural guess for **gauge invariant FFB couplings in (type-0) superstrings:**

$$\mathcal{A}_F^{[0]\,\pm} = \exp(\pm\mathcal{G})\,\bar{\psi}_1\left(p_1\,,\,\xi_1 \pm \sqrt{\frac{\alpha'}{2}}\,p_{23}\right)[1+\partial\!\!\!/_{\xi_3}]\,\psi_2\left(p_2\,,\,\xi_2 \pm \sqrt{\frac{\alpha'}{2}}\,p_{31}\right) \times\ \phi_3\left(p_3\,,\,\xi_3 \pm \sqrt{\frac{\alpha'}{2}}\,p_{12}\right)\Big|_{\xi_i=0}\,,$$

Determines corresponding **(Bose and Fermi) conserved HS currents:**

$$J_{FF}^{[0]\,\pm}(x\,;\,\xi) = \exp\left(\mp i\,\sqrt{\frac{\alpha'}{2}}\,\xi_\alpha\left[\partial_{\zeta_1}\cdot\partial_{\zeta_2}\,\partial_{12}^\alpha - 2\,\partial_{\zeta_1}^\alpha\,\partial_{\zeta_2}\cdot\partial_1 + 2\,\partial_{\zeta_2}^\alpha\,\partial_{\zeta_1}\cdot\partial_2\right]\right) \times\bar{\Psi}_1\left(x \mp i\,\sqrt{\frac{\alpha'}{2}}\,\xi\,,\,\zeta_1 \mp i\sqrt{2\,\alpha'}\,\partial_2\right)\left[1+\xi\!\!\!/\right]\Psi_2\left(x \pm i\,\sqrt{\frac{\alpha'}{2}}\,\xi\,,\,\zeta_2 \pm i\sqrt{2\,\alpha'}\,\partial_1\right)\Big|_{\zeta_i=0}\,.$$

$$J_{BF}^{[0]\,\pm}(x\,;\,\xi) = \exp\left(\mp i\,\sqrt{\frac{\alpha'}{2}}\,\xi_\alpha\left[\partial_{\zeta_1}\cdot\partial_{\zeta_2}\,\partial_{12}^\alpha - 2\,\partial_{\zeta_1}^\alpha\,\partial_{\zeta_2}\cdot\partial_1 + 2\,\partial_{\zeta_2}^\alpha\,\partial_{\zeta_1}\cdot\partial_2\right]\right) \times\left[1+\partial\!\!\!/_{\zeta_2}\right]\Psi_1\left(x \mp i\,\sqrt{\frac{\alpha'}{2}}\,\xi\,,\,\zeta_1 \mp i\sqrt{2\,\alpha'}\,\partial_2\right)\Phi_2\left(x \pm i\sqrt{\frac{\alpha'}{2}}\,\xi\,,\,\zeta_2 \pm i\sqrt{2\alpha'}\,\partial_1\right)\Big|_{\zeta_i=0}\,.$$

Off-Shell Cubic Vertices

(AS, Taronna, 2010)

One can **complete uniquely** the cubic string vertices. The result is **gauge invariant up the Fronsdal (or compensator) equations.** New ingredient:

$$\mathcal{H}_{ij} = (1 + \partial_{\xi_i} \cdot \partial_{\xi_j})\, i\,\mathcal{D}_j - \frac{1}{2}\, p_j \cdot \partial_{\xi_i}\, \partial_{\xi_j} \cdot \partial_{\xi_j}$$

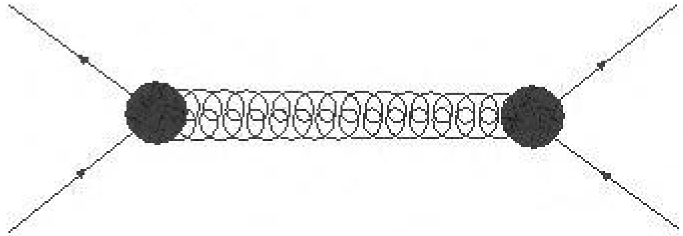
where

$$i\,\mathcal{D}_i = p_i \cdot \partial_{\xi_i} - \frac{1}{2}\, p_i \cdot \xi_i\, \partial_{\xi_i} \cdot \partial_{\xi_i}$$

$$\mathcal{A}_{\pm} = \exp \pm \left\{ \sqrt{\frac{\alpha'}{2}} \Big[(\partial_{\xi_1} \cdot \partial_{\xi_2} + 1)(\partial_{\xi_3} \cdot p_{12}) + (\partial_{\xi_2} \cdot \partial_{\xi_3} + 1)(\partial_{\xi_1} \cdot p_{23}) + (\partial_{\xi_3} \cdot \partial_{\xi_1} + 1)(\partial_{\xi_2} \cdot p_{31}) \Big] \right\}$$
$$\times\, \phi_1(p_1; \xi_1)\, \phi_2(p_2; \xi_2)\, \phi_3(p_3; \xi_3) \Big|_{\xi_i = 0} \equiv e^{\pm \Gamma}\, \phi_1(p_1; \xi_1)\, \phi_2(p_2; \xi_2)\, \phi_3(p_3; \xi_3) \Big|_{\xi_i = 0}$$

$$\mathcal{A}^{\pm\,(tot)} = e^{\pm\Gamma} \left[1 + \left(\frac{\alpha'}{2}\right) (\mathcal{H}_{12}\mathcal{H}_{13} + \mathcal{H}_{21}\mathcal{H}_{23} + \mathcal{H}_{31}\mathcal{H}_{32}) \pm \left(\frac{\alpha'}{2}\right)^{\frac{3}{2}} \times \right.$$
$$\left. (\,:\mathcal{H}_{21}\mathcal{H}_{32}\mathcal{H}_{13}: - :\mathcal{H}_{12}\mathcal{H}_{31}\mathcal{H}_{23}:\,) \right] \varphi_1(p_1, \xi_1)\, \varphi_2(p_2, \xi_2)\, \varphi_3(p_3, \xi_3) \Big|_{\xi_i = 0}$$

Exchanges & Coupling Functions



- **Current-exchange formula:** (Fronsdal, 1978; Francia, Mourad, AS, 2007)

$$\sum_n \frac{1}{n!\,2^{2n}(3-\frac{d}{2}-s)_n}\;\langle J^{[n]}\;,\;J^{[n]}\rangle\,,$$

- **Scalar currents & "coupling function"** (Berends, Burgers, van Dam, 1986) (Bekaert, Joung, Mourad, 2009)

$$J(x,u)=\Phi^\star(x+iu)\,\Phi(x-iu)\,,\qquad a(z)=\sum_r \frac{z^r}{r!}\,a_r$$

- **Bekaert-Joung-Mourad amplitude (D=4):**

$$\mathcal{A}^{(s)}\;=\;-\frac{1}{\alpha' s}\left[a\left(\frac{\alpha'}{4}(u-t)+\frac{\alpha'}{2}\sqrt{-ut}\right)+a\left(\frac{\alpha'}{4}(u-t)-\frac{\alpha'}{2}\sqrt{-ut}\right)-a_0\right]\times\phi_1(p_1)\,\phi_2(p_2)\,\phi_3(p_3,)\,\phi_4(p_4)$$

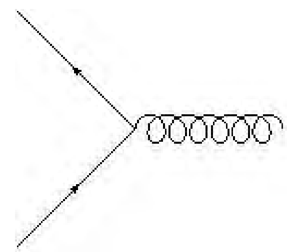
4-point Functions and Beyond

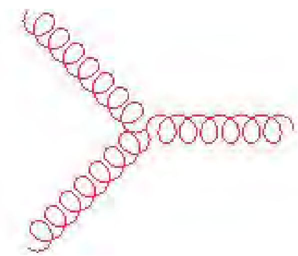
(Taronna, 2011)

- **(Limiting) 3-pt functions:**

$$\mathcal{A}_{\pm} = \exp\left\{\sqrt{\frac{\alpha'}{2}}\Big[(\partial_{\xi_1}\cdot\partial_{\xi_2}+1)\partial_{\xi_3}\cdot p_{12} + (\partial_{\xi_2}\cdot\partial_{\xi_3}+1)\partial_{\xi_1}\cdot p_{23} + (\partial_{\xi_3}\cdot\partial_{\xi_1}+1)\partial_{\xi_2}\cdot p_{31}\Big]\right\} \times \phi_1(p_1;\xi_1)\,\phi_2(p_2;\xi_2)\,\phi_3(p_3;\xi_3)\Big|_{\xi_i=0}$$

- What is the meaning of Γ?





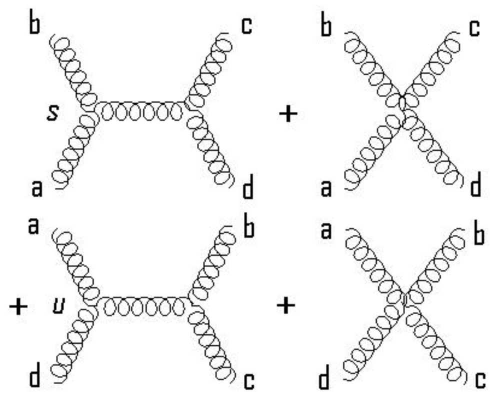
- **Cubic vertex:** tensor products of YM and scalar vertices
- These quantities are **gauge invariant**, up to the DFP conditions

- What are the analogues of the two basic vertices for N>3 pts?

[**"Lego bricks"** for S-matrix amplitudes]

4-point Functions and Beyond

(Taronna, 2011)



- **Quartic YM vertex:**

  "conterterm" for linearized gauge symmetry

- **Actually:**

  gauge symmetry requires "planarly dual" pairs

- **YM amplitudes:**

$$A_\mu^a\, A_\nu^b\, A_\rho^c\, A_\sigma^d \left(\frac{\alpha^{\,s}_{\mu\nu\rho\sigma}}{s} + \beta^{\,s}_{\mu\nu\rho\sigma} + \frac{\alpha^{\,u}_{\sigma\mu\nu\rho}}{u} + \beta^{\,u}_{\sigma\mu\nu\rho} \right) \left[tr\left(T^aT^bT^cT^d\right) + tr\left(T^dT^cT^bT^a\right) \right] + \ldots$$

- **Gauge invariant HS amplitudes (open-string-like):**

$$\phi^a_{\mu_1\ldots\mu_k}\, \phi^b_{\nu_1\ldots\nu_k}\, \phi^c_{\rho_1\ldots\nu_k}\, \phi^d_{\sigma_1\ldots\sigma_k} \left(\frac{\alpha^{\,s}}{s} + \beta^{\,s} + \frac{\alpha^{\,u}}{u} + \beta^{\,u} \right)^k (su)^{k-1} \left[tr\left(T^aT^bT^cT^d\right) + tr\left(T^dT^cT^bT^a\right) \right] + \ldots$$

- **Weinberg's 1964 argument bypassed:** lowest exchanged spin = 2s-1
- → **Non-local** Lagrangian couplings
- **Closed-string-like amplitudes:** striking differences already for s=2 !

Outlook

- **Free HS Fields:**
  - Constraints, compensators and curvatures
  - (String Theory ($\alpha' \rightarrow \infty$) : triplets)

- **Interacting HS Fields:**
  - External currents and vDVZ (dis)continuity
  - Cubic interactions and (conserved) currents

- **(Old) Frontier crossed:** (class of) 4-point amplitudes
  - Massless flat limits vs locality

Beyond String Theory?