

# Higher-Spin Fermionic Gauge Fields & Their Electromagnetic Coupling

**Rakibur Rahman**

**Université Libre de Bruxelles, Belgium**



**April 18, 2012**

**ESI Workshop on Higher Spin Gravity**

**Erwin Schrödinger Institute, Vienna**



**M. Henneaux**



**G. Lucena Gómez**

- G. Lucena Gómez, M. Henneaux and RR, [arXiv:1204.XXXX](#) [[hep-th](#)] (to appear shortly).
- G. Barnich and M. Henneaux, *Phys. Lett. B* **311**, 123 (1993) [[hep-th/9304057](#)].
- R. R. Metsaev, *Nucl. Phys. B* **859**, 13 (2012) [[arXiv:0712.3526](#) [[hep-th](#)]].
- A. Sagnotti and M. Taronna, *Nucl. Phys. B* **842**, 299 (2011) [[arXiv:1006.5242](#) [[hep-th](#)]].

# Motivations

- We will consider the coupling of an arbitrary-spin massless fermion to a  $U(1)$  gauge field, in **flat spacetime with  $D \geq 4$** . Such a study is important in that fermionic fields are required by supersymmetry. This fills a gap in the higher-spin literature.
- **No-go theorems** prohibit, in flat space, minimal coupling to gravity for  $s \geq 5/2$ , and to EM for  $s \geq 3/2$ . These particles may still interact through gravitational and EM multipoles.
- Metsaev's **light-cone formulation** restricts the possible number of derivatives in generic higher-spin cubic vertices.
- Sagnotti-Taronna used the tensionless limit of **string theory** to present generating function for off-shell trilinear vertices.

- **BRST cohomological methods** could independently reconfirm, rederive and check these results.
- Search for consistent interactions in a gauge theory becomes very **systematic** as one as one takes the cohomological approach.
- Any non-trivial consistent interaction **cannot go unnoticed**.
- **Full off-shell vertices** are natural output.
- Whether a given vertex calls for **deformation of the gauge transformations and the gauge algebra** is known by construction, and the deformations are given explicitly.
- **Higher-order consistency** of vertices can be checked easily.

# Results

- Cohomological proof of **no minimal EM coupling** for  $s \geq 3/2$ .
- Reconfirmation of Metsaev's **restriction on the number of derivatives** in a cubic  $1$ - $s$ - $s$  vertex, with  $s = n+1/2$ . There are Only three allowed values:  $2n-1$ ,  $2n$ , and  $2n+1$ .
- Explicit construction of **off-shell cubic vertices** for arbitrary  $s = n+1/2$ , and presenting them in a very neat form .
  1. **Non-abelian  $(2n-1)$ -derivative vertex containing the  $(n-1)$ -curl of the field.**
  2. Abelian  $2n$ -derivative vertex, for  $D \geq 5$ , involving  $n$ -curl (curvature tensor).
  3. Abelian  $(2n+1)$ -derivative vertex of Born-Infeld type (3-curvature term).
- Explicit **matching** with known results for lower spins.
- Generic **obstruction** for the non-abelian cubic vertices.

# Outline

- **EM coupling of massless spin 3/2**: simple but nontrivial. We start with free theory and perform **cohomological reformulation** of the gauge system. We employ the **BRST deformation scheme** to construct consistent parity-preserving off-shell cubic vertices.
- We generalize to **arbitrary spin,  $s = n + 1/2$** , coupled to EM. There appear restrictions on the gauge parameter and the field. These actually make easy the search for consistent interactions!
- Comparative study of the vertices with known results.
- Second-order deformations & issues with locality.
- Concluding remarks.

# Massless Rarita-Schwinger Field Coupled to Electromagnetism

# Step 0: Free Gauge Theory

- The free theory contains a photon  $A_\mu$  and a massless spin-3/2 Rarita-Schwinger field  $\psi_\mu$ , described by the action:

$$S^{(0)}[A_\mu, \psi_\mu] = \int d^D x \left[ -\frac{1}{4} F_{\mu\nu}^2 - i \bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho \right]$$

- It enjoys two abelian the gauge invariances:

$$\delta_\lambda A_\mu = \partial_\mu \lambda, \quad \delta_\varepsilon \psi_\mu = \partial_\mu \varepsilon.$$

- **Bosonic gauge parameter:  $\lambda$** , fermionic gauge parameter:  $\varepsilon$
- Curvature for the fermionic field:  $\Psi_{\mu\nu} \equiv \partial_\mu \psi_\nu - \partial_\nu \psi_\mu$



# Step 1: Introduce Ghosts

- For each gauge parameter, we **introduce a ghost field**, with the same algebraic symmetries but opposite Grassmann parity:
  - **Grassmann-odd bosonic ghost:  $C$**
  - Grassmann-even fermionic ghost:  $\xi$
- The original fields and ghosts are collectively called fields:

$$\Phi^A = \{A_\mu, C, \psi_\mu, \xi\}$$

- Introduce the grading: **pure ghost number,  $pgh$** , which is
  - 1 for the ghost fields
  - 0 for the original fields

## Step 2: Introduce Antifields

- One introduces, for each field and ghost, an antifield  $\Phi_A^*$ , with the same algebraic symmetries but opposite Grassmann parity. Each antifield has 0 pure ghost number:  $pgh(\Phi_A^*)=0$ .

$$\Phi_A^* = \{A^{*\mu}, C^*, \bar{\psi}^{*\mu}, \bar{\xi}^*\}$$

- Introduce the grading: antighost number,  $agh$ , which is 0 for the fields and non-zero for the antifields:

$$agh(\Phi_A^*) = pgh(\Phi^A) + 1$$

## Step 3: Define Antibracket

- On the space of fields and antifields, one defines an odd symplectic structure, called **the antibracket**:

$$(X, Y) \equiv \frac{\delta^R X}{\delta \Phi^A} \frac{\delta^L Y}{\delta \Phi_A^*} - \frac{\delta^R X}{\delta \Phi_A^*} \frac{\delta^L Y}{\delta \Phi^A}.$$

- Here  $R$  and  $L$  respectively mean right and left derivatives.
- The antibracket satisfies **graded Jacobi identity**.

## Step 4: Construct Master Action

- The master action  $S_0$  is an extension of the original action; it includes terms involving ghosts and antifields.

$$S_0 = \int d^D x \left[ -\frac{1}{4} F_{\mu\nu}^2 - i\bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho \right. \\ \left. + A^{*\mu} \partial_\mu C - (\bar{\psi}^{*\mu} \partial_\mu \xi - \partial_\mu \bar{\xi} \psi^{*\mu}) \right]$$

- Because of Noether identities, it solves the master equation:

$$(S_0, S_0) = 0$$

- The antifields appear as sources for the “gauge” variations, with gauge parameters replaced by corresponding ghosts.

## Step 5: BRST Differential

- $S_0$  is the generator of the BRST differential  $s$  of the free theory

$$sX = (S_0, X)$$

- Then the free master equation means:  $S_0$  is BRST-closed.
- Graded Jacobi identity of the antibracket gives:

$$s^2 = 0$$

- The free master action  $S_0$  is in the cohomology of  $s$ , in the local functionals of the fields, antifields and their derivatives. Locality calls for a finite number of derivatives.

- The BRST differential decomposes into two differentials:

$$s = \Gamma + \Delta$$

- $\Delta$  is the Koszul-Tate differential. It implements the equations of motion by acting only on the antifields. It decreases the *agh* by one unit while keeping unchanged the *pgh*.
- $\Gamma$  is the longitudinal derivative along the gauge orbits. It acts only on the original fields to produce the gauge transformations. It increases the *pgh* by one unit without modifying the *agh*.
- They obey:  $\Gamma^2 = \Delta^2 = 0, \Gamma \Delta + \Delta \Gamma = 0$ .
- All  $\Gamma, \Delta, s$  increase the ghost number, *gh*, by one unit, where

$$gh = pgh - agh$$

## Step 6: Properties of $\Phi^A$ & $\Phi^*_A$

$Z$	$\Gamma(Z)$	$\Delta(Z)$	$pgh(Z)$	$agh(Z)$	$gh(Z)$	$\epsilon(Z)$
$A_\mu$	$\partial_\mu C$	$0$	$0$	$0$	$0$	$0$
$C$	$0$	$0$	$1$	$0$	$1$	$1$
$A^{*\mu}$	$0$	$-\partial_\nu F^{\mu\nu}$	$0$	$1$	$-1$	$1$
$C^*$	$0$	$-\partial_\mu A^{*\mu}$	$0$	$2$	$-2$	$0$
$\psi_\mu$	$\partial_\mu \xi$	$0$	$0$	$0$	$0$	$1$
$\xi$	$0$	$0$	$1$	$0$	$1$	$0$
$\bar{\psi}^{*\mu}$	$0$	$-\frac{i}{2} \bar{\Psi}_{\alpha\beta} \gamma^{\alpha\beta\mu}$	$0$	$1$	$-1$	$0$
$\bar{\xi}^*$	$0$	$\partial_\mu \bar{\psi}^{*\mu}$	$0$	$2$	$-2$	$1$

## An Aside: BRST Deformation Scheme

- The solution of the master equation incorporates compactly all consistency conditions pertaining to the gauge transformations.
- Any consistent deformation of the theory corresponds to:

$$S = S_0 + gS_1 + g^2S_2 + O(g^3)$$

where  $S$  solves the deformed master equation:  $(S, S) = 0$ .

- Coupling constant expansion gives, up to  $O(g^2)$ :

$$(S_0, S_0) = 0,$$

$$(S_0, S_1) = 0,$$

$$(S_1, S_1) = -2(S_0, S_2).$$



- The first equation is fulfilled by assumption.
- The second equation says  $S_1$  is BRST-closed:

$$sS_1 = 0$$

- First order non-trivial consistent local deformations:  $S_1 = \int a$  are in one-to-one correspondence with elements of  $H^0(s/d)$  – the cohomology of the free BRST differential  $s$ , modulo total derivative  $d$ , at ghost number 0. One has the cocycle condition:

$$sa + db = 0$$

- A cubic deformation with  $0$  ghost number cannot have  $agh > 2$ .

Thus one can expand  $a$  in antighost number:

$$a = a_0 + a_1 + a_2, \quad agh(a_i) = i$$

- $a_0$  is the deformation of the Lagrangian.  $a_1$  and  $a_2$  encode information about the deformations of the gauge transformations and the gauge algebra respectively.
- Then the cocycle condition reduces, by  $s = \Gamma + \Delta$ , to a cascade

$$\Gamma a_2 = 0,$$

$$\Delta a_2 + \Gamma a_1 + db_1 = 0,$$

$$\Delta a_1 + \Gamma a_0 + db_0 = 0.$$

- The cubic vertex will deform the gauge algebra if and only if  $\mathbf{a}_2$  is in the cohomology of  $\Gamma$ .
- Otherwise, one can always choose  $\mathbf{a}_2 = \mathbf{0}$  and  $\mathbf{a}_1 = \Gamma$ -closed. In this case, if  $\mathbf{a}_1$  is in the cohomology of  $\Gamma$ , the vertex deforms the gauge transformations.
- If this is also not the case, we can take  $\mathbf{a}_1 = \mathbf{0}$ , so that the vertex is abelian, i.e.  $\mathbf{a}_0$  is in the cohomology of  $\Gamma$  modulo  $\mathbf{d}$ .
- The cohomology of  $\Delta$  is also relevant in that the Lagrangian deformation  $\mathbf{a}_0$  is  $\Delta$ -closed, whereas trivial interactions are given by  $\Delta$ -exact terms.

## Step 7: Cohomology of $\Gamma$

Cohomology of  $\Gamma$  isomorphic to the space of functions of:

- the undifferentiated ghosts  $\{C, \xi\}$ ,
  - the antifields  $\{A^{*\mu}, C^*, \bar{\psi}^{*\mu}, \bar{\xi}^*\}$  and their derivatives,
  - the curvatures  $\{F_{\mu\nu}, \Psi_{\mu\nu}\}$  and their derivatives.
- These are nothing but “gauge-invariant” objects, that themselves are not “gauge variation” of something else.
- Note: Fronsdal tensor is already included in this list.

## Step 8: Non-Abelian Vertices

- Recall that  $a_2$  must be Grassmann even, satisfying:

$$\Gamma a_2 = 0, \quad gh(a_2) = 0, \quad agh(a_2) = 2 = pgh(a_2)$$

- The most general **parity-even** Lorentz scalar solution is:

$$a_2 = -g_0 C \left( \bar{\xi}^* \xi + \bar{\xi} \xi^* \right) - g_1 C^* \bar{\xi} \xi$$

- It is a linear combination of two independent terms: **one that contains  $C$ , another that contains  $C^*$** . The former one potentially gives rise to minimal coupling, while the latter could produce dipole interactions (look at the cascade and count derivatives).

- Each of the terms can be lifted to an  $a_1$ :

$$a_1 = g_0 \left[ \bar{\psi}^{*\mu} (\psi_\mu C + A_\mu \xi) + \text{h.c.} \right] \\ + g_1 A^{*\mu} (\bar{\psi}_\mu \xi - \bar{\xi} \psi_\mu) + \tilde{a}_1$$

- The ambiguity is in the cohomology of  $\Gamma$ :

$$\Gamma \tilde{a}_1 = 0$$

- The  $\Delta$  variation of none of the unambiguous pieces is  $\Gamma$ -exact modulo  $d$ . The  $\Delta$  variation of the ambiguity must kill, modulo  $d$ , the non-trivial part, so that  $\Delta a_1$  could be  $\Gamma$ -exact modulo  $d$ :

$$\Delta a_1 + \Gamma a_0 + db_0 = 0$$

- Any element of the cohomology of  $\Gamma$  at antighost number 1 contains at least 1 derivative, so that such a cancellation is not possible for the would-be minimal coupling, simply because the ambiguity contains too many derivatives.
- Thus minimal coupling is ruled out, and we must set  $g_0 = 0$ .
- For the would-be dipole interaction, one has

$$\Delta a_1 = -\Gamma(g_1 \bar{\psi}_\mu F^{\mu\nu} \psi_\nu) - \frac{1}{2} g_1 F^{\mu\nu} (\bar{\Psi}_{\mu\nu} \xi - \bar{\xi} \Psi_{\mu\nu}) + \Delta \tilde{a}_1 + d(\dots)$$

- To see that this can be lifted to an  $\mathbf{a}_0$ , we use the identity

$$\eta^{\mu\nu|\alpha\beta} = -\frac{1}{2} \gamma^{\mu\nu\alpha\beta} + \frac{1}{2} \gamma^{\mu\nu} \gamma^{\alpha\beta} - 2\gamma^{[\mu} \eta^{\nu][\alpha} \gamma^{\beta]}$$

- And also the Bianchi identity:  $\partial_{[\mu} F_{\nu\rho]} = 0$ , to arrive at

$$\begin{aligned} \Delta a_1 = & -\Gamma(g_1 \bar{\psi}_\mu F^{+\mu\nu} \psi_\nu) + \Delta \tilde{a}_1 + d(\dots) \\ & -\frac{1}{4} g_1 \left[ (\bar{\Psi} \not{F} - 4 \bar{\Psi}_{\mu\nu} \gamma^\mu F^{\nu\rho} \gamma_\rho) \xi - \bar{\xi} (\not{F} \Psi - 4 \gamma_\mu F^{\mu\alpha} \gamma^\beta \Psi_{\alpha\beta}) \right] \end{aligned}$$

- In view of all possible forms of the EoMs:

$$\{ \gamma^{\mu\nu} \Psi_{\mu\nu}, \gamma^\mu \Psi_{\mu\nu}, \not{\partial} \Psi_{\mu\nu}, \partial^\mu \Psi_{\mu\nu} \}, \{ \partial^\mu F_{\mu\nu}, \square F_{\mu\nu} \}$$

it is clear that the second line is  $\Delta$ -exact, and it can cancel the  $\Delta$  variation of the ambiguity, if the latter is chosen as:

$$\tilde{a}_1 = i g_1 \left[ \bar{\psi}^{*\mu} \gamma^\nu F_{\mu\nu} - \frac{1}{2(D-2)} \bar{\psi}^* \not{F} \right] \xi + \text{h.c.}$$



- This leaves us with the non-abelian Lagrangian deformation:

$$a_0 = g_1 \bar{\psi}_\mu F^{+\mu\nu} \psi_\nu$$

- This is a 1-derivative Pauli term that corresponds to  $g = 2$ .
- The same appears in  $N = 2$  SUGRA, where the dimensionful coupling constant  $g_1$  is simply the inverse Planck mass.
- To proceed, we note that we have exhausted all possible  $a_2$ . Any other possible vertex will not deform the gauge algebra.

## Step 9: Gauge-Symmetry-Preserving Vertices

- If a vertex comes from  $a_1$  or  $a_0$  itself, one can always write:

$$a_0 = j^\mu A_\mu, \quad \Gamma j^\mu = 0, \quad \partial_\mu j^\mu = \Delta M, 0$$

- Therefore, the most generic form of the vertex is:

$$a_0 = \left( \bar{\Psi}_{\mu\nu} X^{\mu\nu\alpha\beta\lambda} \Psi_{\alpha\beta} \right) A_\lambda$$

- Any derivative contained in  $X$  must have one of the 5 indices.
- It is not difficult to see if  $X$  contains more than 1 derivatives,  $a_0$  is  $\Delta$ -exact modulo  $d$  (trivial). **At most 3 derivatives in  $a_0$ .**

- There is a non-trivial 3-derivative vertex, corresponding to

$$X^{\mu\nu\alpha\beta\lambda} = \frac{1}{2} \eta^{\mu\nu|\alpha\beta} \overleftrightarrow{\partial}^\lambda,$$

- Other possibilities differ by trivial terms.
- Up to  $\Delta$ -exact modulo  $d$  terms, this is an abelian 3-curvature term (Born-Infeld type):

$$a_0 \sim \bar{\Psi}_{\mu\alpha} \Psi^{\alpha\nu} F^\mu{}_\nu$$

- A 2-derivative vertex can follow from two possibilities:

$$X^{\mu\nu\alpha\beta\lambda} = \eta^{\mu\nu|\alpha\beta}\gamma^\lambda, \gamma^{\mu\nu\alpha\beta\lambda}$$

- But they differ by  $\Delta$ -exact terms, thanks to the identities:

$$\eta^{\mu\nu|\alpha\beta} = -\frac{1}{2}\gamma^{\mu\nu\alpha\beta} + \frac{1}{2}\gamma^{\mu\nu}\gamma^{\alpha\beta} - 2\gamma^{[\mu}\eta^{\nu][\alpha}\gamma^{\beta]}$$

$$\frac{1}{2}\gamma^{\mu\nu\alpha\beta}\gamma^\lambda + \frac{1}{2}\gamma^\lambda\gamma^{\mu\nu\alpha\beta} = \gamma^{\mu\nu\alpha\beta\lambda}$$

- For  $D > 4$ , we have an abelian 2-derivative vertex, that is gauge invariant up to a total derivative:

$$a_0 = \left( \bar{\Psi}_{\mu\nu}\gamma^{\mu\nu\alpha\beta\lambda}\Psi_{\alpha\beta} \right) A_\lambda$$

# Arbitrary Spin: $s = n + 1/2$

- The set of fields and antifields are:

$$\{A_\mu, C, \psi_{\mu_1 \dots \mu_n}, \xi_{\mu_1 \dots \mu_{n-1}}\}$$

$$\{A^{*\mu}, C^*, \bar{\psi}^{*\mu_1 \dots \mu_n}, \bar{\xi}^{*\mu_1 \dots \mu_{n-1}}\}$$

- The ghost field  $C$  is Grassmann odd.
- Grassmann-even rank- $(n-1)$  fermionic ghost field  $\xi_{\mu_1 \dots \mu_{n-1}}$  is  $\gamma$ -traceless. The original fermion is triply  $\gamma$ -traceless:

$$\xi_{\mu_1 \dots \mu_{n-2}} = 0, \quad \psi^{\mu_2}_{\mu_2 \mu_3 \dots \mu_n} = 0$$

- The  $n$ -curl curvature  $\Psi_{\mu_1 \nu_1 | \mu_2 \nu_2 | \dots | \mu_n \nu_n}$  obeys Bianchi identity and EoMs identical to the spin-3/2 curvature.

- Cohomology of  $\Gamma$  is isomorphic to the space of functions of
  - The curvatures  $\{F_{\mu\nu}, \Psi_{\mu_1\nu_1|\mu_2\nu_2|\dots|\mu_n\nu_n}\}$ , and their derivatives.
  - The antifields  $\{A^{*\mu}, C^*, \bar{\psi}^{*\mu_1\dots\mu_n}, \bar{\xi}^{*\mu_1\dots\mu_{n-1}}\}$ , and their derivatives.
  - The undifferentiated ghosts  $\{C, \xi_{\mu_1\dots\mu_{n-1}}\}$ , and the  $\gamma$ -traceless part of all possible curls of the spinorial ghost  $\{\xi_{\mu_1\nu_1|\dots|\mu_m\nu_m\|\nu_{m+1}\dots\nu_{n-1}}^{(m)}, m \leq n-1\}$ .
  - The Fronsdal tensor  $\mathcal{S}_{\mu_1\dots\mu_n}$ , and its symmetrized derivatives.
- A derivative of  $0, 1, \dots, (n-2)$  curls of the fermionic ghost is in the cohomology of  $\Gamma$ , but that of  $(n-1)$  curl is  $\Gamma$ -exact.

$$\partial_\alpha \xi_{\mu_1\nu_1|\dots|\mu_{n-1}\nu_{n-1}}^{(n-1)} = \Gamma \psi_{\mu_1\nu_1|\dots|\mu_{n-1}\nu_{n-1}\|\alpha}^{(n-1)}$$

- The list of candidates of  $a_2$ , for cross-coupling, is:
  1. A set containing  $C$ ,  $i$ -th curl of the fermionic ghost and  $i$ -th curl of its antifield.  $i=0,1,\dots,n-1$ .
  2. A set containing  $C^*$ ,  $i$ -th curl of the fermionic ghost and  $i$ -th curl of its Dirac conjugate.  $i=0,1,\dots,n-1$ .
- The second kind cannot be lifted to  $a_1$  unless  $i = n-1$ .
- For the first kind, all can be lifted to  $a_1$ .  $i = 0$  corresponds to **minimal coupling: ruled out** like in  $s = 3/2$ . Other possibilities cannot also be lifted to  $a_0$ , because of different natures of the unambiguous piece and the ambiguity in  $a_1$ .
- The rest of the story is like in spin  $3/2$ .  $(n-1)$ -curl of the fermion appears in non-abelian vertex, and  $n$ -curl in the others.

# Comparative Study of Vertices

- Comparison of ours with Sagnotti-Taronna off-shell vertices reveal that they differ by  $\Delta$ -exact modulo  $d$  terms. Off-shell calculation has been carried out for the  $1--3/2--3/2$  vertices.
- The Sagnotti-Taronna vertices, written in the most naïve way, contains many terms. For them, it not straightforward at all to see that the 2-derivative vertex vanishes for  $D = 4$ .
- Our number of derivative count matches with Metsaev.
- Our off-shell vertices have a neat form for all spin.
- In the transverse-traceless gauge, our vertices also reduce to known results in the literature, in particular to Sagnotti-Taronna for higher spins.



## Second-Order Deformation

- Consistent 2nd-order deformation requires  $(S_1, S_1)$  be  $s$ -exact:

$$(S_1, S_1) = -2sS_2 = -2\Delta S_2 - 2\Gamma S_2.$$

- For abelian vertices, this antibracket is zero, so the first-order deformations always go unobstructed. Non-abelian vertices, however, are more interesting in this respect.

- If the consistency condition holds, the  $\Gamma$  variation of the antibracket at zero antifields must be  $\Delta$ -exact.

$$\Gamma [(S_1, S_1)]_{\Phi_A^*=0} = \Gamma \Delta M = -\Delta (\Gamma M)$$

$$[S_2]_{c_\alpha^*=0} = -\frac{1}{2}M$$

- For our non-abelian vertices, we see easily

$$[(S_1, S_1)]_{\Phi_A^*=0} = 2 \left( \int a_0, \int a_1 \right)$$

- Straightforward computation for spin 3/2 gives

$$[(S_1, S_1)]_{\Phi_A^*=0} = 4\partial_\nu \left( \bar{\psi}^{[\mu} \psi^{\nu]} + \frac{1}{2} \bar{\psi}_\alpha \gamma^{\mu\nu\alpha\beta} \psi_\beta \right) (\bar{\psi}_\mu \xi - \bar{\xi} \psi_\mu) \\ + \left\{ i\bar{\psi}_\mu F^{+\mu\nu} \left[ 2\gamma^\rho F_{\nu\rho} - \frac{1}{(D-2)} \gamma_\nu F \right] \xi + \text{h.c.} \right\}$$

- Its  $\Gamma$  variation is clearly not  $\Delta$ -exact. So the **non-abelian vertex is obstructed** beyond the cubic order.
- The proof is very similar for arbitrary spin.

Notice that the non-abelian  $1-3/2-3/2$  vertex is precisely the Pauli term appearing in  $N = 2$  SUGRA. The theory, however, contains additional degrees of freedom, namely graviton, on top of a complex massless spin  $3/2$  and a  $U(1)$  field.

It is this new DOF that renders the vertex unobstructed, while keeping locality intact. If one decouples gravity by sending Planck mass to infinity, the Pauli term vanishes because the dimensionful coupling constant is nothing but the inverse Planck mass. One could integrate out the **massless** graviton to obtain a system of spin- $3/2$  and spin-1 fields only. The resulting theory contains the Pauli term, but is necessarily **non-local**.

Thus, higher-order consistency of the non-abelian vertex is possible either by forgoing locality or by adding a new dynamical field (graviton).

# Remarks & Future Perspectives

- Gravitational coupling of fermions.
- Mixed Symmetry fields.
- Similarities with bosonic  $1-s-s$  and  $2-s-s$  results by Boulanger!
- Chargeless massless scaling limit of massive theory in 4D.
- Comparison with BCFW results in 4D.
- Hint of non-locality at the quartic level.
- Construction of vertices in AdS spaces, and compare with the the results of Joung-Lopez-Taronna.