3d higher spin black holes III: ∞

Eric Perlmutter (UCLA)
April 10, 2012

ESI Workshop on Higher Spin Gravity

Based on: hep-th/1006.4788 [Ammon, Gutperle, Kraus, EP]
hep-th/1008.2567 [Kraus, EP]

Outline

1. (Part of) 3d Vasiliev gravity as a Chern-Simons theory
2. Introduction to hs[λ] and extended chiral symmetry, $W_\infty[\lambda]$
3. Making a hs[λ] black hole
4. AdS_3/CFT_2 vector model duality
   - Comparison to free CFT with $W_\infty[\lambda]$ symmetry
5. Open questions

3d Vasiliev gravity

- Degrees of freedom:
  - Metric (s=2), plus infinite tower of massless higher spin fields, s = 3,4,…
  - # matter multiplets (e.g. scalar fields); masses fixed by symmetry
- Dynamics fixed by higher spin gauge invariance (~Diff + more)
- Variables:
  - Spacetime coordinates (x^μ)
  - Bosonic spinors $(\tilde{y}_\alpha, \tilde{z}_\alpha), \quad \alpha = 1,2$
  - A pair of Clifford algebras comprised of $(k, \rho; \psi_{1,2})$

[Prokushkin, Vasiliev]

- "Master field" content:

$$W = \underbrace{W_\mu(\tilde{y}, \tilde{z}; x)}_{\text{Higher spin fields}} dx^\mu, S = \underbrace{S_\alpha(\tilde{y}, \tilde{z}; x)}_{\text{Realizes internal higher spin symmetry}} d\tilde{z}^\alpha, B = \underbrace{B(\tilde{y}, \tilde{z}; x)}_{\text{Matter fields}}$$

3d Vasiliev gravity

- Higher spin "oscillator" algebra:

$$[\tilde{y}_\alpha, \tilde{y}_\beta]_\star = 2i\epsilon_{\alpha\beta}(1+\nu k)$$

with multiplication by Moyal (star) product; similarly for $\tilde{z}$

- ν has many roles:
  - Appears in higher spin algebra ("deformation parameter")
  - Parameterizes AdS vacua
  - Fixes scalar mass
- Spin-s generator ~ $\tilde{y}^{2s-2}$
  - e.g. SL(2) subalgebra: $S_{\alpha\beta} = \tilde{y}_{(\alpha}\tilde{y}_{\beta)}$
- Expand master fields in oscillators, e.g.

$$W_\mu = W_\mu^{(2)}(x)\tilde{y}\tilde{y} + W_\mu^{(3)}(x)\tilde{y}\tilde{y}\tilde{y}\tilde{y} + \ldots$$

3d Vasiliev gravity

- Master field equations satisfied for:

$$B = \nu$$

$$S \propto \tilde{z}$$

$$dW(\tilde{y}) + W(\tilde{y}) \wedge \star W(\tilde{y}) = 0$$

- Simplest solution is AdS:

$$W = \omega^{\alpha\beta}\tilde{y}_\alpha\tilde{y}_\beta + \psi_1 e^{\alpha\beta}\tilde{y}_\alpha\tilde{y}_\beta$$

- But any flat W will do → higher spin backgrounds
- Re-write W in terms of gauge fields:

$$W = \mathcal{P}_+ A + \mathcal{P}_- \overline{A}, \quad \text{where} \quad \mathcal{P}_\pm = \frac{1 \pm \psi_1}{2}$$

- Then

$$dA + A \wedge \star A = 0$$

$$d\overline{A} + \overline{A} \wedge \star \overline{A} = 0$$

3d Vasiliev gravity

- What is the Lie algebra?
- Define generators:

$$V^s_m \equiv \prod_{i=1}^{2s-2} \tilde{y}_{\alpha_i}\Big|_{\mathrm{Sym}}, \quad \text{where} \quad 2m = N_1 - N_2$$

e.g.

$$V^2_0 \propto \tilde{y}_1\tilde{y}_2 + \tilde{y}_2\tilde{y}_1$$

$$V^3_1 \propto \tilde{y}_1\tilde{y}_1\tilde{y}_1\tilde{y}_2\Big|_{\mathrm{Sym}}$$

- The {V} generate the higher spin algebra hs[λ], where

$$\lambda = \frac{1+\nu}{2}$$

- Conclusion: the gauge sector of Vasiliev theory can be written as two copies of hs[λ] Chern-Simons theory.
- e.g. AdS now looks like

$$A = e^\rho V^2_1 dx^+ + V^2_0 d\rho$$

$$\overline{A} = -e^\rho V^2_{-1} dx^- - V^2_0 d\rho$$

simply generalizing SL(2) construction.

Vasiliev theory

3D	4D
Gauge higher spin d.o.f.	All d.o.f. propagate
One pair of oscillators $(\tilde{y}; \tilde{z})$	Two pairs of oscillators $(y, \overline{y}; z, \overline{z})$
Deformed oscillator algebra	Only undeformed oscillators
Arbitrary numbers of scalars	Cannot add matter multiplets
N(spins) = finite or infinite	N(spins) = infinite
Field equations fixed	Scalar self-interaction ambiguity
Easier?	Harder?

Introduction to hs[λ]

[Pope, Romans, Shen]

- Commutation relations

$$[V_m^s, V_n^t] = \sum_{u=2,4,6,\ldots}^{s+t-|s-t|-1} g_u^{st}(m,n;\lambda) V_{m+n}^{s+t-u}$$

- Structure constants are known
- At λ=N, an ideal forms: recover SL(N)
- Exactly one SL(2) subalgebra; no other SL(N) subalgebras for generic λ
- Low spin examples: $[V_2^3, V_0^2] = 2V_2^3$

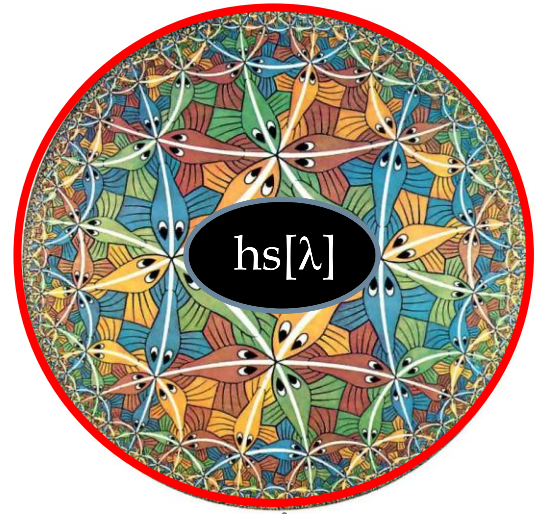
$$[V_2^3, V_{-2}^3] = 8V_0^4 - \frac{4}{5}(\lambda^2 - 4)V_0^2$$

- Underlying associative "lone star product":

$$V_m^s \star V_n^t \equiv \frac{1}{2} \sum_{u=1,2,3,\ldots}^{s+t-|s-t|-1} g_u^{st}(m,n;\lambda) V_{m+n}^{s+t-u}$$

- Identity is V_0^1

hs[λ] and asymptotic symmetry



Virasoro (W_2)

W_N

$W_\infty[\lambda]$

Generalized Brown-Henneaux boundary conditions give extended conformal algebras

In reverse: bulk Lie algebras are "wedge subalgebras" of boundary conformal algebras (vacuum invariance)

[Campoleoni, Fredenhagen, Pfenninger, Theisen;
Gaberdiel, Hartman;
Henneaux, Rey]

$W_\infty[\lambda]$

- $W_\infty[\lambda]$ is highly nonlinear algebra; structure constants now known in closed form

[Campoleoni, Fredenhagen, Pfenninger]

- Currents J_s, with mode expansions $J_s = \sum_{n\in Z} \frac{(J_s)_n}{z^{n+s}}$
- Schematically,

$$J_3 J_3 \sim J_4 + J_2 + \frac{1}{c}(J_2)^2 + c$$

In a Virasoro primary basis, nonlinearity increases with spin.

- To recover hs[λ]:
  1. Restrict to wedge modes, |n|<s: eliminates central terms
  2. Take large c: eliminates nonlinear terms
  - Analogous to SL(2) embedding in Virasoro:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}n(n^2-1)\delta_{m,-n}$$

Taking stock

- The gauge sector of 3d Vasiliev theory is (two copies of) a hs[λ] Chern-Simons theory.
- The asymptotic symmetry algebra of AdS hs[λ] gravity is $W_\infty[\lambda]$
  - CFTs with $W_\infty[\lambda]$ symmetry live on the boundary of AdS.

- Goal: to write down a black hole solution of hs[λ] gravity with nonzero higher spin charges, and compute its partition function.
  - A Cardy formula for higher spin black holes
  - Later, we will "count" the entropy microscopically in a simple theory with $W_\infty[\lambda]$ symmetry: free bosons

Making higher spin black holes

- Recall the SL(3) black hole connection with spin-3 chemical potential, μ:

$$a = \left(L_1 - \frac{2\pi}{k}\mathcal{L}L_{-1} - \frac{\pi}{2k}\mathcal{W}W_{-2}\right)dx^+ + \mu\left(W_2 - \frac{4\pi\mathcal{L}}{k}W_0 + \frac{4\pi^2\mathcal{L}^2}{k^2}W_{-2} + \frac{4\pi\mathcal{W}}{k}L_{-1}\right)dx^-$$

- Manifestly a flat connection: $$a_- = 2\mu\left[(a_+)^2 - \frac{1}{3}\mathrm{Tr}(a_+)^2\right]$$

- Suggests general method for constructing higher spin black hole connections with spin-s potential, μ_s, in *any* bulk CS theory with SL(2) subalgebra:

$$\boxed{\begin{aligned} a_+ &= a_+^{BTZ} + (\text{higher spin charges}) \\ a_- &\sim \sum_s \mu_s\left[(a_+)^{s-1} - \text{trace}\right] \end{aligned}}$$

- Metric will look like black hole (e.g. have a horizon) in some gauge… but is it smooth?

Making higher spin black holes (smooth)

- Enforce BTZ holonomy constraint. This determines which charges you need, and their functional dependence on $\{\tau, \mu_s\}$.
  - With $\omega = 2\pi(\tau a_+ - \bar{\tau} a_-)$, solve

$$\boxed{\mathrm{Tr}(\omega^n) = \mathrm{Tr}(\omega^n_{BTZ})}$$

An Algorithm

1. Deform BTZ solution by adding chemical potential(s), $\{\mu_s\}$, and some number of higher spin charges while maintaining flatness.

$$a_+ = a_+^{BTZ} + (\text{higher spin charges})$$
$$a_- \sim \sum_s \mu_s \left[(a_+)^{s-1} - \text{trace}\right]$$

2. Determine charges as a function of $\{\tau, \mu_s\}$ by enforcing the BTZ holonomy constraint: the black hole will now be smooth.

$$\mathrm{Tr}(\omega^n) = \mathrm{Tr}(\omega^n_{BTZ})$$

Making a hs[λ] black hole

- Simplest case: turn on spin-3 chemical potential
- Step 1: Write down the solution:

$$a_+ = V_1^2 - \frac{2\pi\mathcal{L}}{k}V_{-1}^2 - N(\lambda)\frac{\pi\mathcal{W}}{2k}V_{-2}^3 + J$$
$$a_- = \mu N(\lambda)\left(a_+ \star a_+ - \frac{2\pi\mathcal{L}}{3k}(\lambda^2-1)\right)$$

Recall:
$V_{\pm1}^2 \sim L_{\pm1}$
$V_{\pm2}^3 \sim W_{\pm2}$
$V_0^2 \sim L_0$

where

$$J = J_4 V_{-3}^4 + J_5 V_{-4}^5 + \ldots$$

and

$$A = e^{-\rho V_0^2}(a+d)e^{\rho V_0^2}$$

- Black hole is a saddle point contribution to the CFT partition function

$$Z(\tau,\alpha;\overline{\tau},\overline{\alpha}) = \mathrm{Tr}\left[e^{4\pi^2 i(\tau\mathcal{L}+\alpha\mathcal{W}-\overline{\tau}\overline{\mathcal{L}}-\overline{\alpha}\overline{\mathcal{W}})}\right]$$

- As in SL(3), take $\alpha = \overline{\tau}\mu, \quad \overline{\alpha} = \tau\overline{\mu}$

hs[λ] vs. SL(3)

$$a_+ = V^2_1 - \frac{2\pi\mathcal{L}}{k}V^2_{-1} - N(\lambda)\frac{\pi\mathcal{W}}{2k}V^3_{-2} + J_4V^4_{-3} + \ldots$$

$$a_- = \mu N(\lambda)\left(a_+ \star a_+ - \frac{2\pi\mathcal{L}}{3k}(\lambda^2-1)\right)$$

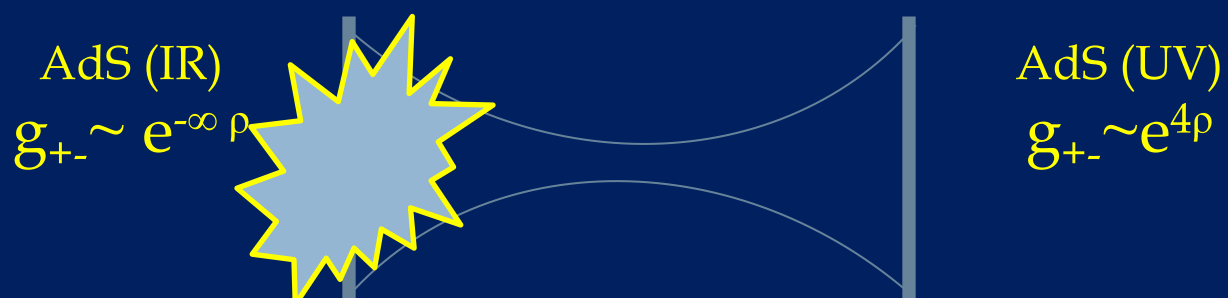
- Novel infinities:
  - N(holonomy equations)
  - N(higher spin charges): $\langle\mathcal{L}\rangle_\alpha \sim \langle\mathcal{L}\rangle + \alpha^2\langle\mathcal{L}\mathcal{W}\mathcal{W}\rangle + \ldots$

$$\langle\mathcal{W}\rangle_\alpha \sim \alpha\langle\mathcal{W}\mathcal{W}\rangle + \alpha^3\langle\mathcal{W}\mathcal{W}\mathcal{W}\mathcal{W}\rangle + \ldots$$

$$\langle J_4\rangle_\alpha \sim \alpha^2\langle J_4\mathcal{W}\mathcal{W}\rangle + \alpha^4\langle J_4\mathcal{W}\mathcal{W}\mathcal{W}\mathcal{W}\rangle \ldots$$

  - Non-perturbative curvature: in wormhole gauge,

hs[λ]:



Making a hs[λ] black hole (smooth)

- Step 2: Solve holonomy equations:

$$\mathrm{Tr}(\omega^n) = \mathrm{Tr}(\omega^n_{BTZ}), \quad n = 2, 3, \ldots, \infty$$

- Work perturbatively in α:

$$\mathcal{L} = \mathcal{L}^{(0)} + \alpha^2 \mathcal{L}^{(2)} + \ldots$$
$$\mathcal{W} = \alpha \mathcal{W}^{(1)} + \alpha^3 \mathcal{W}^{(3)} + \ldots$$
$$J_4 = \alpha^2 J_4^{(2)} + \alpha^4 J_4^{(4)} + \ldots$$

- Solution through O(α^8):

$$\ln Z(\tau,\alpha) = \frac{i\pi k}{2\tau}\Bigg[1 - \frac{4}{3}\frac{\alpha^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\alpha^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4 - 85\lambda^2 + 377}{(\lambda^2-4)^2}\frac{\alpha^6}{\tau^{12}}$$
$$+ \frac{32000}{81}\frac{20\lambda^6 - 600\lambda^4 + 6387\lambda^2 - 23357}{(\lambda^2-4)^3}\frac{\alpha^8}{\tau^{16}}\Bigg] + \ldots$$

- Entropy and integrable charges $(\mathcal{L}, \mathcal{W})$ follow by differentiation, all charges (J) also fixed

hs[λ] black hole: comments

- Higher spin, but no scalar, "hair"
- Reproduces SL(3) result at λ=3
- Compare to partition function of U(1)-charged BTZ black hole:

$$\ln Z(\tau, \alpha) = \frac{i\pi k}{2\tau} - \frac{2\pi i \alpha^2}{\tau}$$

[Kraus, Larsen]

- Grand canonical partition function of $W_\infty[\lambda]$ CFT deformed by spin-3 chemical potential

---

- Holography: Reproduce this from CFT?
  - At T→∞, modular transformation maps to vacuum OPE structure.

[Gaberdiel, Hartman, Jin]

- What CFTs have $W_\infty[\lambda]$ symmetry? W_N minimal models in 't Hooft limit *

(*we think)

AdS3/CFT2: W_N coset duality

- Consider coset model $$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}}$$

[Gaberdiel, Gopakumar]

Take 't Hooft limit: $N, k \to \infty, \quad \lambda \equiv \frac{N}{k+N} \quad \text{fixed}$

- Central charge scales like N
- Dual to 3d Vasiliev gravity with pair of complex scalars:

$$\begin{aligned} \phi_\pm \quad &\Leftrightarrow \quad \mathcal{O}_\pm \\ m^2 = \lambda^2 - 1 \quad &\Leftrightarrow \quad \Delta_{\mathcal{O}_\pm} = 1 \pm \lambda \end{aligned}$$

- Coset believed to have $W_\infty[\lambda]$ symmetry in 't Hooft limit
- Substantial evidence:
  - Partition functions [Gaberdiel, Gopakumar, Hartman, Raju]
  - $W_\infty[\lambda]$ symmetry [Ahn]
  - 3-pt correlators [Chang, Yin; Ammon, Kraus, EP]

Z at λ =1 from free CFT

- A simpler realization of $W_\infty[1]$: free, complex, singlet bosons

$$T = -\partial\overline{\phi}^i \partial\phi_i$$

[Bakas, Kiritsis]

$$\mathcal{W} \propto \partial^2\overline{\phi}^i \partial\phi_i - \partial\overline{\phi}^i \partial^2\phi_i$$

- Compute Z non-perturbatively:

$$\ln Z(\tau,\alpha) = -\frac{3ik}{2\pi\tau}\int_0^\infty dx \left[\ln\left(1 - e^{-x+\frac{ia\alpha}{\tau^2}x^2}\right) + \ln\left(1 - e^{-x-\frac{ia\alpha}{\tau^2}x^2}\right)\right]$$

where $a = \sqrt{\frac{5}{3\pi^2}}$

- Perturbative expansion matches bulk result at λ=1
- Note: zero radius of convergence

Open questions

- Interesting effects from multiple potentials
- Scalar in hs[λ] black hole background
  - Wave equation known, in principle, at given order in α

[Ammon, Kraus, EP]

- Subleading contributions to Z
- Better understanding of holonomy-integrability relationship
- D=4 black holes