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1110.5918 Taronna-EJ : Massless

1203.6578 Lopez-Taronna-EJ : Massive & Partially-massless

Cubic interactions of massive and (partially-)massless HS fields in (A)dS

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HS Interactions

- **Massless HS**
  - Gauge Invariance
  - No-go Weinberg'64 Aragon-Deser ...
  - Yes-go Berends-Burgers-van Dam Fradkin-Vasiliev Metsaev ...
  - Vasiliev's Equations

- **Massive HS**
  - Reproduce Fierz sys.
  - No-go Fierz-Pauli Velo-Zwanziger ...
  - Yes-go Federbush Argyres-Nappi ...
  - String Theory

Systematic approach to
massless *and* ***massive*** *HS interactions*
in AdS and flat background

Noether Procedure

$$S_{\mathrm{YM}} = \int \partial^2 A^2 + \boldsymbol{g} \int \partial A^3 + \boldsymbol{g}^2 \int A^4$$

$$g_2 \int \partial^3 A^3 \to F^3$$

$$S_{\mathrm{GR}} = \int \partial^2 h^2 + \boldsymbol{G}^{\frac{1}{2}} \int \partial^2 h^3 + \boldsymbol{G} \int \partial^2 h^4 + \cdots$$

$$g_2 \int \partial^4 h^3 \to \sqrt{-g}\, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$

$$g_3 \int \partial^6 h^3 \to \sqrt{-g}\, R^{\mu}{}_{\nu} R^{\nu}{}_{\rho} R^{\rho}{}_{\mu}$$

$$\boldsymbol{S} = \boldsymbol{S^{(2)}} + \boldsymbol{S^{(3)}} + \boldsymbol{S^{(4)}} + \cdots$$

$$\delta \varphi = \delta^{(0)} \varphi + \delta^{(1)} \varphi + \delta^{(2)} \varphi + \cdots$$

Gauge Invariance

$$\delta S = 0 \Rightarrow \begin{cases} \delta^{(0)} S^{(2)} = 0 \\ \delta^{(0)} S^{(3)} + \delta^{(1)} S^{(2)} = 0 \\ \vdots \end{cases}$$

≈ 0

Cubic Interactions

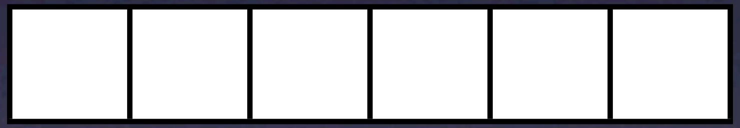
- **Massless HS**
  - Gauge Invariance
- **Massive HS**
  - Reproduce Fierz sys.

- Correct Propagation of DoF
- Light-Cone(LC) Gauge Metsaev 2006
  - Complete list of couplings
  - Massive & Massless , Cubic , Flat

HS in (A)dS

- Only **Transverse & Traceless (TT)** part
  - LC ⊂ TT ⊂ “Full”

  - Key to “Full” Manvelyan-Mkrtchyan-Ruehl Taronna-Sagnotti
  - On-Shell Info (n-pt fns , ...)
- Only Symmetric Bosons



- Only Parity-Invariant Interactions
- Any Dimensions $d \geq 4$
- Metric-like formulation

Massless Cubic Vertices in Frame-like : Vasiliev 2011

OUTLINE

- Massless HS in Flat-Space (warm-up ex.)
- Ambient-Space Formalism for (A)dS
    - Free HS fields
    - Interactions
- Cubic Interactions
    - Massive & Massless in (A)dS
    - Partially-Massless in dS

Warm-up Ex

Massless HS in Flat Space

- *Free Action* $S^{(2)} = -\int \varphi^{\mu\cdots} \Box \varphi_{\mu\cdots} + \cdots$
  - **Field Eqs** $\Box \varphi + \cdots \approx 0$
  - Constrained Fronsdal
  - Unconstrained Francia-Mourad-Sagnotti
    Buchbinder-Galajinsky-Krykhtin

divergence, trace, auxiliary fields

- *Cubic Action* $S^{(3)} = \int c\,(\partial^{\cdots}\varphi^{\cdots})(\partial^{\cdots}\varphi^{\cdots})(\partial^{\cdots}\varphi_{\cdots})$
- *Noether Procedure* $\delta^{(0)}\varphi_{\mu\nu\rho\cdots} = \partial_{(\mu}\varepsilon_{\nu\rho\cdots)}$

$$\delta^{(0)}S^{(3)} = \int c\,(\partial^{\cdots}\varepsilon^{\cdots})\,(\partial^{\cdots}\varphi^{\cdots})\,(\partial^{\cdots}[\Box\varphi_{\cdots} + \cdots])$$

Warm-up Ex

Massless HS in Flat Space

- *Focus TT part*

$$S^{(3)} = \int \; c \, (\partial^{\cdots} \varphi^{\cdots})(\partial^{\cdots} \varphi^{\cdots})(\partial^{\cdots} \varphi_{\ldots})$$

$$= \quad \textit{TT in } \varphi \quad + \quad \textit{Others}$$

$$\delta^{(0)} S^{(3)} = \int \; c \, (\partial^{\cdots} \cdots) \, (\partial^{\cdots} \varphi^{\cdots}) \, (\partial^{\cdots} [\Box \varphi_{\ldots} + \cdots])$$

$$= \quad \textit{TT in } \varphi \; \& \; \varepsilon \approx 0 \quad + \quad \textit{Others} \approx 0$$

- *Noether Procedure* ➜ $\delta^{(0)} \left(S^{(3)} \right)_{TT} \Big|_{TT} \approx 0$

Warm-up Ex

Massless HS in Flat Space

- *Cubic Action* Taronna-EJ

$$y_1 := \partial_{u_1}\cdot\partial_{x_2} \quad y_2 := \partial_{u_2}\cdot\partial_{x_3} \quad y_3 := \partial_{u_3}\cdot\partial_{x_1}$$
$$z_1 := \partial_{u_2}\cdot\partial_{u_3} \quad z_2 := \partial_{u_3}\cdot\partial_{u_1} \quad z_3 := \partial_{u_1}\cdot\partial_{u_2}$$

$$S^{(3)} \sim \int C(\partial_{u_i}\cdot\partial_{x_j}, \partial_{u_i}\cdot\partial_{u_j})\, \varphi(x_1,u_1)\,\varphi(x_2,u_2)\,\varphi(x_3,u_3)\,\Big|_{\substack{x_i=x\\ u_i=0}}$$

Divergent

$$\partial_u\cdot\partial_x\,\varphi(x,u) = \frac{1}{(s-1)!}\,\partial^{\mu_s}\varphi_{\mu_s\,\mu_1\cdots\mu_{s-1}}(x)\,u^{\mu_1}\,\cdots\,u^{\mu_{s-1}}$$

Trace

$$\partial_u^2\,\varphi(x,u) = \frac{1}{(s-2)!}\,\varphi'_{\mu_1\cdots\mu_{s-2}}(x)\,u^{\mu_1}\,\cdots\,u^{\mu_{s-2}}$$

$$\varphi(x,u) = \frac{1}{s!}\,\varphi_{\mu_1\cdots\mu_s}(x)\,u^{\mu_1}\,\cdots\,u^{\mu_s}$$

Warm-up Ex

Massless HS in Flat Space

$$y_1 := \partial_{u_1}\cdot\partial_{x_2} \quad y_2 := \partial_{u_2}\cdot\partial_{x_3} \quad y_3 := \partial_{u_3}\cdot\partial_{x_1}$$
$$z_1 := \partial_{u_2}\cdot\partial_{u_3} \quad z_2 := \partial_{u_3}\cdot\partial_{u_1} \quad z_3 := \partial_{u_1}\cdot\partial_{u_2}$$

- *Cubic Action* Taronna-EJ

$$S^{(3)} \sim \int C(\,y_i\;,\;z_i\,)\,\varphi(x_1,u_1)\,\varphi(x_2,u_2)\,\varphi(x_3,u_3)\,\Big|_{\substack{x_i=x\\u_i=0}}$$

$$\delta^{(0)}S^{(3)} \sim \int (y_3\,\partial_{z_2} - y_2\,\partial_{z_3})\,C(y,z)\,\varepsilon(x_1,u_1)\,\varphi(x_2,u_2)\,\varphi(x_3,u_3)\,\Big|_{\substack{x_i=x\\u_i=0}}$$

$$\boldsymbol{(y_3\,\partial_{z_2} - y_2\,\partial_{z_3})\,C(y,z) = 0}$$

Gradient

$$u\cdot\partial_x\,\varepsilon(x,u) = \frac{1}{s!}\,\partial_{(\mu_1}\varepsilon_{\mu_2\cdots\mu_s)}(x)\,u^{\mu_1}\;\cdots\;u^{\mu_s}$$

Warm-up Ex

Massless HS in Flat Space

$y_1 := \partial_{u_1} \cdot \partial_{x_2} \quad y_2 := \partial_{u_2} \cdot \partial_{x_3} \quad y_3 := \partial_{u_3} \cdot \partial_{x_1}$

$z_1 := \partial_{u_2} \cdot \partial_{u_3} \quad z_2 := \partial_{u_3} \cdot \partial_{u_1} \quad z_3 := \partial_{u_1} \cdot \partial_{u_2}$

- *Cubic Action* Taronna-EJ

$$S^{(3)} \sim \int C(y,z)\,\varphi(x_1,u_1)\,\varphi(x_2,u_2)\,\varphi(x_3,u_3)\,\Big|_{\substack{x_i=x\\ u_i=0}}$$

Gauge invariance

$$(y_i\,\partial_{z_{i+1}} - y_{i+1}\,\partial_{z_i})\,C(y,z) = 0$$

General solution

$$C = \mathcal{K}(\,y_1\,,\,y_2\,,\,y_3\,,\,g\,)$$

$g := y_1\,z_1 + y_2\,z_2 + y_3\,z_3$

- **Generating fn of all consistent couplings**
- **Go to LC → Metsaev's results**
- **Relax "~" → Full vertices**

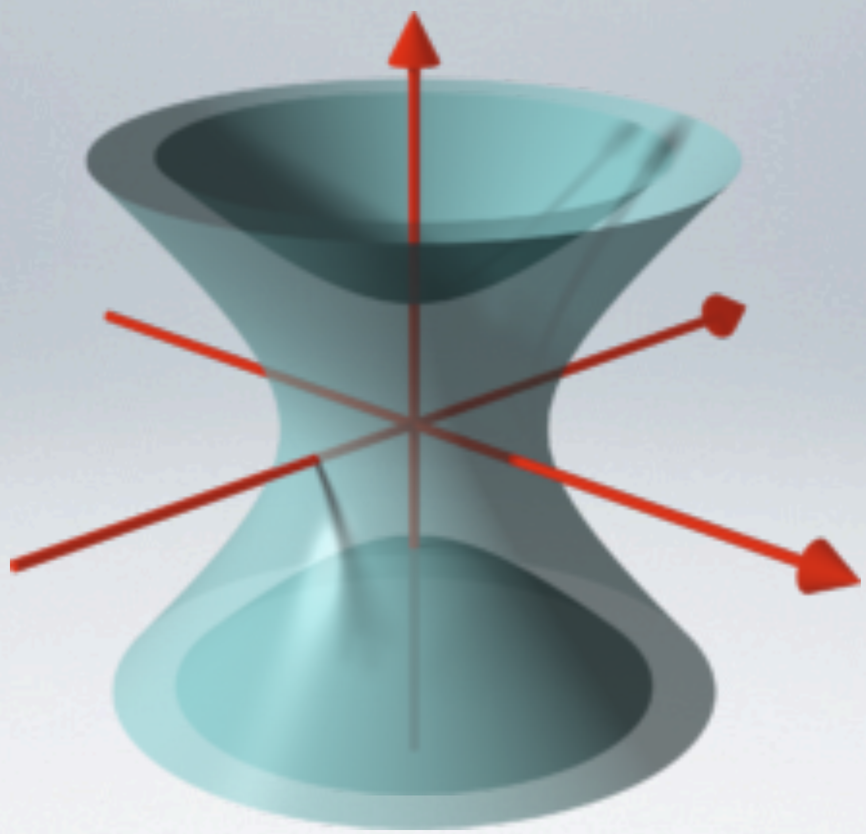
Generalization to (A)dS

$$\partial_x \quad \Rightarrow \quad \nabla_x$$?

AMBIENT SPACE

Fronsdal Metsaev Biswas-Siegel ...

$\mathbb{R}^{1,d}$



de Sitter

$X^2 = L^2$

Anti de Sitter

$X^2 = -L^2$

AMBIENT SPACE

Fronsdal Metsaev Biswas-Siegel ...

- Ambient-Space HS Fields $\Phi(X,U) = \boldsymbol{R^{\Delta_h}\,\varphi(x,u)}$
  - Homogeneity $\boxed{X\cdot\partial_X\,\Phi = \Delta_h\,\Phi}$
  - Tangentiality $\boxed{X\cdot\partial_U\,\Phi = 0}$

$$X = (R, x)$$
$$U = (v, u)$$

- **MASSLESS** equation in **Ambient Space**
  - **massive** & (**partially-**)**massless** in **(A)dS** depending on Δ_h
  - Gauge symmetry $\boxed{\delta^{(0)}\,\Phi = U\cdot\partial_X\,E}$

AMBIENT SPACE

- Consistency of Gauge sym. with Tangentiality & Homogeneity

Lopez-Taronna-EJ

$$\delta^{(0)}\,\Phi = U\cdot\partial_X\,E \quad \begin{cases} X\cdot\partial_U\,\Phi = 0 \\ (X\cdot\partial_X - U\cdot\partial_U + \mu + 2)\,\Phi = 0 \\ (X\cdot\partial_X - U\cdot\partial_U + \mu)\,E = 0 \end{cases}$$

- Massless : $\mu = 0$ $\quad X\cdot\partial_U\,E = 0$
- Massive : $\mu \notin \{0,\cdots,s-1\}$ $\quad E = 0$
- Partially-massless : $\mu \in \{1,\cdots,s-1\}$ $\quad E = (U\cdot\partial_X)^{\mu}\,\Omega$, $\quad X\cdot\partial_U\,\Omega = 0$

AMBIENT SPACE

- Gauge Symmetry $\quad (X \cdot \partial_X - U \cdot \partial_U + \mu + 2)\, \Phi = 0$
  - Massless $\quad$ (A)dS : $\mu = 0$

    $$\boxed{\delta^{(0)} \Phi = U \cdot \partial_X\, E}$$

  - Massive $\quad$ (A)dS : $\mu \notin \{0, \cdots, s-1\}$

    (no gauge sym.)

  - Partially-massless $\quad$ dS : $\mu \in \{1, \cdots, s-1\}$

    $$\boxed{\delta^{(0)} \Phi = (U \cdot \partial_X)^{\mu+1}\, \Omega}$$

How to deal with the **ACTION** ?

Radial dependence of the Lagrangian

$$\mathsf{L}^{(2)}_{(s,\mu)} = R^{2(s-\mu-3)}\,\mathcal{L}^{(2)}_{(s,\mu)}$$

$$\mathsf{L}^{(2)}_{(s,\mu)} = \Phi \cdot \Box\,\Phi$$

$$\mathcal{L}^{(2)}_{(s,\mu)} = \varphi \cdot (\Box_{(A)dS} + m_\mu^2)\,\varphi$$

$$\mathsf{S}^{(2)}_{(s,\mu)} = \left(\int_0^\infty dR\, R^{d+2(s-\mu-3)}\right) S^{(2)}_{(s,\mu)}$$

$\infty_{s,\mu}$

HS Cubic Interactions in (A)dS

- How to deal with the R-dependence?
  - Remove it by introducing $\delta\big(\sqrt{\epsilon\, X^2} - L\big)$
- Then, what are the differences w.r.t. the (A)dS intrinsic way?
  - We still work with ∂_X not ∇_x
- Only simplifications? No subtleties?
  - A subtlety arises in "TOTAL DERIVATIVES"

$Y_1 = \partial_{U_1} \cdot \partial_{X_2}$ $Y_2 = \partial_{U_2} \cdot \partial_{X_3}$ $Y_3 = \partial_{U_3} \cdot \partial_{X_1}$
$Z_1 = \partial_{U_2} \cdot \partial_{U_3}$ $Z_2 = \partial_{U_3} \cdot \partial_{U_1}$ $Z_3 = \partial_{U_1} \cdot \partial_{U_2}$

- Cubic Action in (A)dS

$$S^{(3)} \sim \int d^{d+1}X \; \delta\big(\sqrt{\epsilon\, X^2} - L\big) \; C(Y_i, Z_i) \times \\ \times \Phi(X_1, U_1)\, \Phi(X_2, U_2)\, \Phi(X_3, U_3) \,\Big|_{\substack{X_i = X \\ U_i = 0}}$$

- Massless Gauge Symmetry

$$\boldsymbol{\delta^{(0)}} S^{(3)} \sim \int d^{d+1}X \; \delta\big(\sqrt{\epsilon\, X^2} - L\big) \; \big[\, C(Y_i, Z_i), \boldsymbol{U_1 \cdot \partial_{X_1}} \,\big] \times \\ \times \boldsymbol{E(X_1, U_1)}\, \Phi(X_2, U_2)\, \Phi(X_3, U_3) \,\Big|_{\substack{X_i = X \\ U_i = 0}}$$

- Partially-massless Gauge Symmetry

$$\boldsymbol{\delta^{(0)}} S^{(3)} \sim \int d^{d+1}X \; \delta\big(\sqrt{\epsilon\, X^2} - L\big) \; \big[\, C(Y_i, Z_i), \boldsymbol{(U_1 \cdot \partial_{X_1})^{r+1}} \,\big] \times \\ \times \boldsymbol{\Omega(X_1, U_1)}\, \Phi(X_2, U_2)\, \Phi(X_3, U_3) \,\Big|_{\substack{X_i = X \\ U_i = 0}}$$

$$Y_1 = \partial_{U_1}\cdot\partial_{X_2} \quad Y_2 = \partial_{U_2}\cdot\partial_{X_3} \quad Y_3 = \partial_{U_3}\cdot\partial_{X_1}$$
$$Z_1 = \partial_{U_2}\cdot\partial_{U_3} \quad Z_2 = \partial_{U_3}\cdot\partial_{U_1} \quad Z_3 = \partial_{U_1}\cdot\partial_{U_2}$$

- Cubic Action in (A)dS

$$S^{(3)} \sim \int d^{d+1}X\ \delta\big(\sqrt{\epsilon\, X^2} - L\big)\ C(Y_i, Z_i) \times$$
$$\times\, \Phi(X_1,U_1)\,\Phi(X_2,U_2)\,\Phi(X_3,U_3)\,\Big|_{\substack{X_i=X\\ U_i=0}}$$

- Massless Gauge Symmetry

$$\big[\,C(Y,Z)\,,\,U_1\cdot\partial_{X_1}\,\big] = (Y_3\,\partial_{Z_2} - Y_2\,\partial_{Z_3})\,C(Y,Z)$$
$$+\ (\text{total derivatives})$$

- Partially-massless Gauge Symmetry

$$\big[\,C(Y,Z)\,,\,(U_1\cdot\partial_{X_1})^{r+1}\big] = (Y_3\,\partial_{Z_2} - Y_2\,\partial_{Z_3})^{r+1}\,C(Y,Z)$$
$$+\ (\text{total derivatives})$$

- Total Derivatives

$$\delta(\sqrt{X^2} - L)\ \partial_X \cdot (\,-\,)\big|_{X_i = X} = -\frac{1}{L}\,\delta'(\sqrt{X^2} - L)\ X \cdot (\,-\,)\big|_{X_i = X}$$

$$X_i \cdot \partial_{X_i} \qquad X_i \cdot \partial_{U_i}$$

  - Homogeneity and Tangentiality
  - Move them to the right, and Act them on fields
    - Use commutation relations with Y_i and Z_i

- Cubic Action in (A)dS

$$Y_1 = \partial_{U_1}\cdot\partial_{X_2} \quad Y_2 = \partial_{U_2}\cdot\partial_{X_3} \quad Y_3 = \partial_{U_3}\cdot\partial_{X_1}$$
$$Z_1 = \partial_{U_2}\cdot\partial_{U_3} \quad Z_2 = \partial_{U_3}\cdot\partial_{U_1} \quad Z_3 = \partial_{U_1}\cdot\partial_{U_2}$$

$$S^{(3)} \sim \int d^{d+1}X\ \delta\big(\sqrt{\epsilon\, X^2} - L\big)\ C(Y_i, Z_i) \times$$
$$\times\, \Phi(X_1, U_1)\, \Phi(X_2, U_2)\, \Phi(X_3, U_3)\,\Big|_{\substack{X_i = X \\ U_i = 0}}$$

- Massless Gauge Symmetry

$$\Big[Y_2\partial_{Z_3} - Y_3\partial_{Z_2} + \tfrac{\hat\delta}{L}\,\big(Y_2\partial_{Y_2} - Y_3\partial_{Y_3} - \tfrac{\mu_2-\mu_3}{2}\big)\,\partial_{Y_1}\Big]\,C(Y,Z) = 0$$

$$\delta^{(n)}\big(\sqrt{\epsilon\, X^2} - L\big) \equiv \delta\big(\sqrt{\epsilon\, X^2} - L\big)\,(\epsilon\,\hat\delta)^n$$

- Partially-massless Gauge Symmetry

$$\sum_{l_1+l_2+l_3=r_1+1} \binom{r_1+1}{l_1\, l_2\, l_2} \Big(Y_3\,\partial_{Y_3} - Y_2\,\partial_{Y_2} - Z_3\,\partial_{Z_3} + \tfrac{1}{2}\,(\mu_2-\mu_3)\Big)_{l_1}$$
$$\times\ \hat\delta^{l_1}\, Y_3^{l_2}\,\Big(-Y_2 + \hat\delta\, Z_3\,\partial_{Y_1}\Big)^{l_3}\ \partial_{Y_1}^{l_1}\,\partial_{Z_2}^{l_2}\,\partial_{Z_3}^{l_3}\,C(Y,Z) = 0$$

- How to solve

$$\sum_{l_1+l_2+l_3=r_1+1} \binom{r_1+1}{l_1\, l_2\, l_2} \Big(Y_3\, \partial_{Y_3} - Y_2\, \partial_{Y_2} - Z_3\, \partial_{Z_3} + \tfrac{1}{2}\, (\mu_2 - \mu_3)\Big)_{l_1}$$

$$\times\, \hat{\delta}^{l_1}\, Y_3^{l_2} \Big(-Y_2 + \hat{\delta}\, Z_3\, \partial_{Y_1}\Big)^{l_3}\, \partial_{Y_1}^{l_1}\, \partial_{Z_2}^{l_2}\, \partial_{Z_3}^{l_3}\, C(Y,Z) = 0$$

- Straightforward for given spins: $s_1 - s_2 - s_3$
  - Expand the coupling in $Y_1^{m_1}\, Y_2^{m_2}\, Y_3^{m_3}\, Z_1^{n_1}\, Z_2^{n_2}\, Z_3^{n_3}$ with arbitrary coeff.
  - System of linear equations for those coeff.

$$\overbrace{\begin{bmatrix} \cdot & \cdots & \cdot \\ \vdots & \ddots & \vdots \\ \cdot & \cdots & \cdot \end{bmatrix}}^{N} M \qquad N\,,\ M \sim\ s_1\, s_2\, s_3$$

  - Numerical methods Mathematica

Ex) 4-4-2 Couplings

- Spin 4 : first partially-massless point (μ=1)
- Spin 2 : massless (μ=0)

$$
\begin{aligned}
C_1 &= Y_1^4\,Y_2^4\,Y_3^2 - 12\,\hat\delta^2\,Y_1^2\,Y_2^2\,(Y_1\,Z_1+Y_2\,Z_2)^2 + 48\,\hat\delta^3\,Y_1\,Y_2\,(Y_1\,Z_1+Y_2\,Z_2)\,Z_3\,(2\,Y_1\,Z_1+2\,Y_2\,Z_2+Y_3\,Z_3)\\
&\quad -\,24\,\hat\delta^4\,Z_3^2\left[6\,Y_1^2\,Z_1^2+6\,Y_2^2\,Z_2^2+4\,Y_2\,Y_3\,Z_2\,Z_3+Y_3^2\,Z_3^2+2\,Y_1\,Z_1\,(7\,Y_2\,Z_2+2\,Y_3\,Z_3)\right]+96\,\hat\delta^5\,Z_1\,Z_2\,Z_3^3\,,\\
C_2 &= Y_1^3\,Y_2^3\,Y_3^2\,Z_3 - 3\,\hat\delta\,Y_1^2\,Y_2^2\,(Y_1\,Z_1+Y_2\,Z_2)^2 + 12\,\hat\delta^2\,Y_1\,Y_2\,(Y_1\,Z_1+Y_2\,Z_2)\,Z_3\,(2\,Y_1\,Z_1+2\,Y_2\,Z_2+Y_3\,Z_3)\\
&\quad -\,6\,\hat\delta^3\,Z_3^2\left[6\,Y_1^2\,Z_1^2+6\,Y_2^2\,Z_2^2+4\,Y_2\,Y_3\,Z_2\,Z_3+Y_3^2\,Z_3^2+2\,Y_1\,Z_1\,(7\,Y_2\,Z_2+2\,Y_3\,Z_3)\right]+24\,\hat\delta^4\,Z_1\,Z_2\,Z_3^3\,,\\
C_3 &= Y_1^3\,Y_2^3\,Y_3\,(Y_1\,Z_1+Y_2\,Z_2)+\hat\delta\,Y_1^2Y_2^2\left(6\,Y_1^2\,Z_1^2+11\,Y_1\,Y_2\,Z_1\,Z_2+6\,Y_2^2\,Z_2^2\right)\\
&\quad -\,18\,\hat\delta^2\,Y_1\,Y_2\,(Y_1\,Z_1+Y_2\,Z_2)\,Z_3\,(2\,Y_1\,Z_1+2\,Y_2\,Z_2+Y_3\,Z_3)\\
&\quad +\,6\,\hat\delta^3\,Z_3^2\left[6\,Y_1^2\,Z_1^2+2\,Y_2\,Z_2\,(3\,Y_2\,Z_2+Y_3\,Z_3)+Y_1\,Z_1\,(15\,Y_2\,Z_2+2\,Y_3\,Z_3)\right]-12\,\hat\delta^4\,Z_1\,Z_2\,Z_3^3\,,\\
C_4 &= -\,Y_1^2\,Y_2^2\left(Y_1^2\,Z_1^2+2\,Y_1\,Y_2\,Z_1\,Z_2+Y_2^2\,Z_2^2-Y_3^2\,Z_3^2\right)\\
&\quad +\,4\,\hat\delta\,Y_1\,Y_2\,(Y_1\,Z_1+Y_2\,Z_2)\,Z_3\,(2\,Y_1\,Z_1+2\,Y_2\,Z_2+Y_3\,Z_3)\\
&\quad -\,2\,\hat\delta^2\,Z_3^2\left[6\,Y_1^2\,Z_1^2+6\,Y_2^2\,Z_2^2+4\,Y_2\,Y_3\,Z_2\,Z_3+Y_3^2\,Z_3^2+2\,Y_1\,Z_1\,(7\,Y_2\,Z_2+2\,Y_3\,Z_3)\right]+8\,\hat\delta^3\,Z_1\,Z_2\,Z_3^3\,,\\
C_5 &= Y_1^2\,Y_2^2\,(Y_1\,Z_1+Y_2\,Z_2)\,(Y_1\,Z_1+Y_2\,Z_2+Y_3\,Z_3)\\
&\quad -\,\hat\delta\,Y_1\,Y_2\,Z_3\left[6\,Y_1^2\,Z_1^2+2\,Y_2\,Z_2\,(3\,Y_2\,Z_2+2\,Y_3\,Z_3)+Y_1\,Z_1\,(13\,Y_2\,Z_2+4\,Y_3\,Z_3)\right]\\
&\quad +\,2\,\hat\delta^2\,Z_3^2\left[3\,Y_1^2\,Z_1^2+Y_2\,Z_2\,(3\,Y_2\,Z_2+Y_3\,Z_3)+Y_1\,Z_1\,(8\,Y_2\,Z_2+Y_3\,Z_3)\right]-2\,\hat\delta^3\,Z_1\,Z_2\,Z_3^3\,,\\
C_6 &= Y_1\,Y_2\,Z_3\,(Y_1\,Z_1+Y_2\,Z_2+Y_3\,Z_3)^2\\
&\quad -\,\hat\delta\,Z_3^2\left[3\,Y_1^2\,Z_1^2+3\,Y_2^2\,Z_2^2+4\,Y_2\,Y_3\,Z_2\,Z_3+Y_3^2\,Z_3^2+4\,Y_1\,Z_1\,(2\,Y_2\,Z_2+Y_3\,Z_3)\right]\\
&\quad +\,4\,\hat\delta^2\,Z_1\,Z_2\,Z_3^3\,.
\end{aligned}
$$

- Can we simplify the results?

- Can we get a general formula valid for any s_1-s_2-s_3 ?

Generating function of all massive & massless cubic interactions

- Recall the massless flat results

$$C = \mathcal{K}(\, y_1\,,\, y_2\,,\, y_3\,,\, g\,)$$

- Is it possible to write the (A)dS interactions in the same way?

$$C = \mathcal{K}(\,?\,,\,?\,,\,?\,,\,?\,)$$

- [PROBLEM]
  COUPLINGS ARE **NOT** HOMOGENEOUS IN THE NUMBER OF DERIVATIVES
- DIFFERENT DERIVATIVE PARTS ARE SIZED BY POWERS OF $\hat{\delta}$ (OR $1/L$)

EX: MINIMUM-DERIVATIVE 4-4-4 COUPLING

$$(Y_1\,Z_1+Y_2\,Z_2+Y_3\,Z_3)^4-12\,\hat{\delta}\,Z_1\,Z_2\,Z_3\,(Y_1\,Z_1+Y_2\,Z_2+Y_3\,Z_3)^2+12\,\hat{\delta}^2\,Z_1^2\,Z_2^2\,Z_3^2$$

$$\delta^{(n)}\Big(\sqrt{\epsilon\,X^2}-L\Big)\equiv\delta\Big(\sqrt{\epsilon\,X^2}-L\Big)\,(\epsilon\,\hat{\delta})^n$$

- BUILDING BLOCKS WITH

  TOTAL DERIVATIVES !

Cubic Interactions of MASSIVE & MASSLESS HS

- **3 massive** $C = \mathcal{K}(\,Y_1\,,\,Y_2\,,\,Y_3\,,\,Z_1\,,\,Z_2\,,Z_3\,)$
- **2 massive** ($\mu_1 = \mu_2$) **& 1 massless** $C = \mathcal{K}(\,\tilde{Y}_1\,,\,\tilde{Y}_2\,,\,\tilde{Y}_3\,,\,Z_1\,,\,\tilde{G}\,)$
- **2 massive** ($\mu_1 \neq \mu_2$) **& 1 massless** $C = \mathcal{K}(\,Y_2\,,Y_3\,,\;Z_1\,,\,\tilde{H}_2\,,\tilde{H}_3\,)$
- **1 massive & 2 massless** $C = \mathcal{K}(\,Y_3\,,\,\tilde{H}_1\,,\tilde{H}_2\,,\tilde{H}_3\,)$
- **3 massless** $C = \mathcal{K}(\,\tilde{Y}_1\,,\tilde{Y}_2\,,\tilde{Y}_3\,,\tilde{G}\,)$

$$\tilde{Y}_i = Y_i + \alpha_i\,\partial_X \cdot \partial_{U_i}$$

$$\tilde{G} = \textstyle\sum_i\,(Y_i + \beta_i\,\partial_X \cdot \partial_{U_i})\,Z_i$$

$$\alpha_1 = \alpha\,,\qquad \alpha_2 = -\frac{1}{\alpha+1}\,,\qquad \alpha_3 = -\frac{\alpha+1}{\alpha}$$

$$\beta_1 = \beta\,,\qquad \beta_2 = -\frac{\beta+1}{\alpha+1}\,,\qquad \beta_3 = -\frac{\alpha-\beta}{\alpha}$$

$$\tilde{H}_i = Y_{i+1}\,(Y_{i-1} - \partial_X \cdot \partial_{U_{i-1}}) - \tfrac{1}{2}\,\partial_X \cdot (\partial_{X_i} - \partial_{X_{i+1}} - \partial_{X_{i-1}})\,Z_i$$

Ex: minimum-derivative 4-4-4 coupling

$$(Y_1\,Z_1 + Y_2\,Z_2 + Y_3\,Z_3)^4 - 12\,\hat{\delta}\,Z_1\,Z_2\,Z_3\,(Y_1\,Z_1 + Y_2\,Z_2 + Y_3\,Z_3)^2 + 12\,\hat{\delta}^2\,Z_1^2\,Z_2^2\,Z_3^2 \;=\; \tilde{G}^{\,4}$$

Cubic Interactions of MASSIVE & MASSLESS HS

- 3 massive $\qquad C = \mathcal{K}(\,Y_1\,,\,Y_2\,,\,Y_3\,,\,Z_1\,,\,Z_2\,,Z_3\,)$
- 2 massive ($\mu_1 = \mu_2$) & 1 massless $\qquad C = \mathcal{K}(\,\tilde{Y}_1\,,\,\tilde{Y}_2\,,\,\tilde{Y}_3\,,\,Z_1\,,\,\tilde{G}\,)$
- 2 massive ($\mu_1 \neq \mu_2$) & 1 massless $\qquad C = \mathcal{K}(\,Y_2\,,Y_3\,,\;Z_1\,,\,\tilde{H}_2\,,\tilde{H}_3\,)$
- 1 massive & 2 massless $\qquad C = \mathcal{K}(\,Y_3\,,\,\tilde{H}_1\,,\tilde{H}_2\,,\tilde{H}_3\,)$
- 3 massless $\qquad C = \mathcal{K}(\,\tilde{Y}_1\,,\tilde{Y}_2\,,\tilde{Y}_3\,,\tilde{G}\,)$

$$\tilde{Y}_i = Y_i + \alpha_i\,\partial_X \cdot \partial_{U_i}$$

$$\tilde{G} = \textstyle\sum_i\,(Y_i + \beta_i\,\partial_X \cdot \partial_{U_i})\,Z_i$$

$$\alpha_1 = \alpha\,, \qquad \alpha_2 = -\tfrac{1}{\alpha+1}\,, \qquad \alpha_3 = -\tfrac{\alpha+1}{\alpha}$$

$$\beta_1 = \beta\,, \qquad \beta_2 = -\tfrac{\beta+1}{\alpha+1}\,, \qquad \beta_3 = -\tfrac{\alpha-\beta}{\alpha}$$

$$\tilde{H}_i = Y_{i+1}\,(Y_{i-1} - \partial_X \cdot \partial_{U_{i-1}}) - \tfrac{1}{2}\,\partial_X \cdot (\partial_{X_i} - \partial_{X_{i+1}} - \partial_{X_{i-1}})\,Z_i$$

flat limit ➜ Metsaev's results

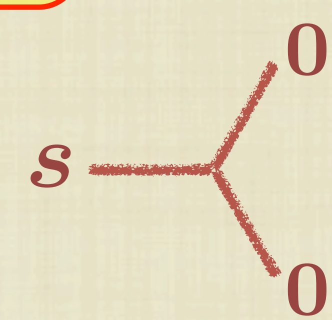
- “Highest-derivative” type interactions of **partially-massless** HS

$$C = \mathcal{K}(\,\tilde{Y}_1\,,\,\tilde{Y}_2\,,\tilde{Y}_3\,)$$

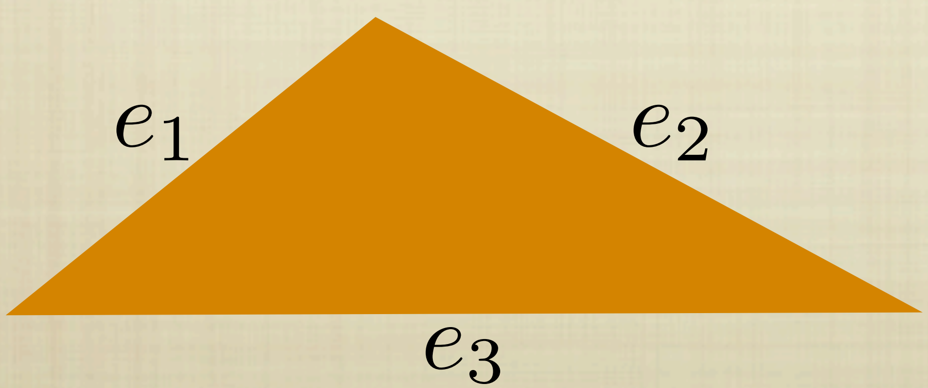
- This type of interactions exists if and only if

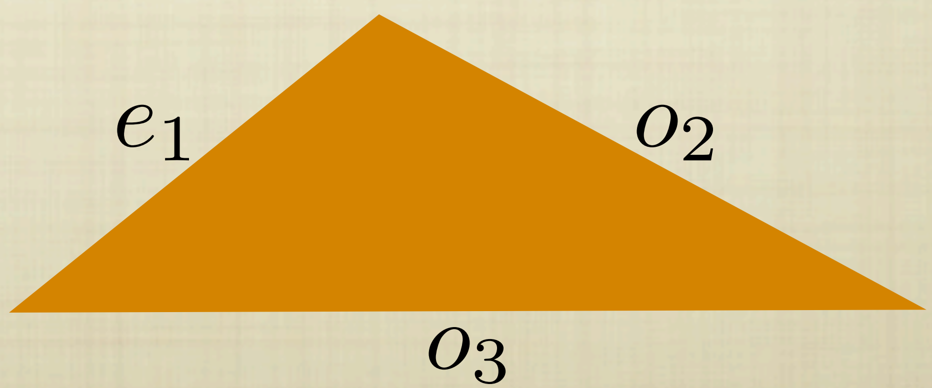
$$\mu_1 - |\mu_2 - \mu_3| \in 2\,\mathbb{N}$$

- Restriction on couplings of **PM** HS to **two scalars**



- Three **PM** HS : triangular inequality





Summary & Outlook

- PDE for consistent interactions in (A)dS
  - Massive & Massless : ***generating function***
  - Partially-massless : for given spins
- **[** Stückelberg formulation & Massless limit **]**
- Fermions and Mixed-symmetry
- Deformations of HS Gauge Algebra
- AdS/CFT computations