Families of exact solutions to Vasiliev's 4D equations with spherical, cylindrical and biaxial symmetry

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C.I. , P. Sundell – JHEP 12 (2011) 084
arXiv:1107.1217 [hep-th]

ESI, Vienna, April 19 2012

Why higher spins?

1. Crucial issue in Field Theory

2. Key role in String Theory

- Strings beyond low-energy SUGRA
- HSGT as symmetric phase of String Theory?

3. HS/CFT correspondences

Summary

- The 4D Vasiliev equations

  - HS algebras as extensions of $\mathfrak{so}(3,2)$
  - Unfolded formulation

- Solving the equations

  - Gauge function method and separation of variables in twistor space

- Exact solutions

  - Spherically-symmetric solution.
  - Weyl 0-form and deformed oscillators.
  - Construction of some HS invariants. Singularities?
  - Cylindrically and biaxisymmetric solutions.

- Conclusions and Outlook

The Vasiliev Equations

Interactions? Consistent!, in presence of:

- Infinitely many fields
- Cosmological constant $\Lambda \neq 0$
- Higher-derivative vertices

Consistent non-linear equations for **all spins** (all symm tensors): $\phi_{\mu_1 \ldots \mu_s}$

- Diff invariant
- so(5;ℂ)-invariant natural vacuum solutions (S^4, H_4, (A)dS_4, $H_{(3,2)}$)
- Infinite-dimensional (tangent-space) algebra
- Correct free field limit → Fronsdal or Francia-Sagnotti eqs
- Arguments for uniqueness

Focus on D = 4 bosonic model
[NO Chan-Paton-like internal symmetry]

The Vasiliev Equations

- ∞-dim. extension of AdS-gravity with gauge fields valued in HS tangent-space algebra $\mathfrak{hs}(3,2) \subset \mathcal{U}(\mathfrak{so}(3,2))/\mathcal{I}(D)$

$\mathfrak{so}(3,2)$: $[M_{ab}, M_{cd}]_\star = 4i\eta_{[c|[b}M_{a]|d]}\ , \quad [M_{ab}, P_c]_\star = 2i\eta_{c[b}P_{a]}\ , \quad [P_a, P_b]_\star = i\lambda^2 M_{ab}$

Oscillator realization:

$$M_{ab} = -\frac{1}{8}\left[(\sigma_{ab})^{\alpha\beta} y_\alpha y_\beta + (\bar{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}\right] \ , \qquad P_a = \frac{\lambda}{4}(\sigma_a)^{\alpha\dot{\beta}} y_\alpha \bar{y}_{\dot{\beta}}$$

$$\mathfrak{sp}(4,\mathbb{R}) \text{ quartet } \leftarrow Y_{\underline{\alpha}} = (y_\alpha, \bar{y}_{\dot{\alpha}})\ , \qquad [Y_{\underline{\alpha}}, Y_{\underline{\beta}}]_\star = 2iC_{\underline{\alpha\beta}} = 2i\begin{pmatrix} \varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}$$

- Weyl-ordered star-product (implements operator product on symbols):

$$F(Y) \star G(Y) = \int_{\mathcal{R}} \frac{d^4U d^4V}{(2\pi)^4} e^{iV^{\underline{\alpha}} U_{\underline{\alpha}}} F(Y+U)\, G(Y+V)$$

Generators of $\mathfrak{hs}(3,2)$:
(symm. and TRACELESS!)

$$T_s \sim y_{\alpha_1} \dots y_{\alpha_n} \bar{y}_{\dot{\alpha}_1} \dots \bar{y}_{\dot{\alpha}_m}\ , \quad \frac{n+m}{2} + 1 = s$$

The Vasiliev Equations

- Antiautomorphism τ:

$$\tau(X\star Y) \;=\; \tau(Y)\star\tau(X)\,, \qquad \tau(y_\alpha) = i y_\alpha\,,\; \tau(\bar{y}_{\dot\alpha}) = i\bar{y}_{\dot\alpha}$$

- Automorphisms π, $\bar{\pi}$:

$$\pi(X\star Y) \;=\; \pi(X)\star\pi(Y)\,, \qquad \pi(y_\alpha) = -y_\alpha\,,\; \pi(\bar{y}_{\dot\alpha}) = \bar{y}_{\dot\alpha}\,,$$
$$\bar\pi(X\star Y) \;=\; \bar\pi(X)\star\bar\pi(Y)\,, \qquad \bar\pi(y_\alpha) = y_\alpha\,,\; \bar\pi(\bar{y}_{\dot\alpha}) = -\bar{y}_{\dot\alpha}$$

- Nonminimal bosonic HS algebra (s=1,2,...) : $\{X(y,\bar{y}):\ \pi\bar\pi(X) = X\}$

- Minimal bosonic HS algebra (s=2,4,...): $\{X(y,\bar{y}):\ \tau(X) = -X\}$

$$\boxed{[T_{s_1}\,,T_{s_2}] = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \ldots + T_{|s_1-s_2|+2}}$$

- Maximal finite (bosonic) subalgebra → $\mathfrak{so}(3,2)$
- Even spins close, minimal infinite extension → minimal HS algebra $\mathfrak{ho}(3,2)$
- Spin 2 always appears

The Vasiliev Equations

- Gauge field $\in \mathrm{Adj}(\mathfrak{ho}(3,2))$ (*master 1-form*):

$$A_\mu(x|y,\bar{y}) = \sum_{n+m=2\mathrm{mod}4}^{\infty} \frac{i}{2n!m!} dx^\mu A_\mu{}^{\alpha_1\ldots\alpha_n\dot{\alpha}_1\ldots\dot{\alpha}_m}(x) y_{\alpha_1}\cdots y_{\alpha_n}\bar{y}_{\dot{\alpha}_1}\cdots\bar{y}_{\dot{\alpha}_m}$$

(every spin-s sector contains all one-form connections that are necessary for a frame-like formulation of HS dynamics (finitely many))

But:
- **Massless UIRs** with all spins in AdS include **a scalar**!
- "Unfolded" eq.ns require a "twisted adjoint" *master 0-form* (contains a scalar, Weyl, HS Weyl and derivatives)

$$T(X)(\Phi) = [X,\Phi]_{\star,\pi} \equiv X\star\Phi - \Phi\star\pi(X)$$

Nonminimal bos. twisted adj. (s=0,1,2,...) : $\Phi(y,\bar{y}) \;:\; \pi\bar{\pi}(\Phi) \;=\; \Phi$

Minimal bos. twisted adj. (s=0,2,4,...) : $\Phi(y,\bar{y}) \;:\; \tau(\Phi) \;=\; \bar{\pi}(\Phi)$

The Vasiliev Equations

- Introduce a *master 0-form* (contains a scalar, Weyl, HS Weyl and derivatives)

$$\Phi(x|y,\bar{y}) = \sum_{|n-m|=0 \mathrm{mod} 4}^{\infty} \frac{1}{n!m!} \Phi^{\alpha_1 \dots \alpha_n \dot{\alpha}_1 \dots \dot{\alpha}_m}(x)\, y_{\alpha_1} \dots y_{\alpha_n} \bar{y}_{\dot{\alpha}_1} \dots \bar{y}_{\dot{\alpha}_m}$$

N.B.: spin-s sector → infinite-dimensional
(upon constraints, all on-shell-nontrivial covariant derivatives of the physical fields, *i.e.*, all the local dof encoded in the 0-form at a point)

- Unfolding s=2: Ricci = 0 ↔ Riemann = Weyl [**tracelessness ⇒ dynamics** !]
[Bianchi ⇒ infinite chain of ids. $\Phi^{\alpha(n)\dot{\alpha}(m)} \sim (\nabla^{\alpha\dot{\alpha}})^m \Phi^{\alpha(2s)}$]

Unfolded free HS eqs:

$$F_1(x|y,\bar{y}) = \frac{i}{4}\left[e_0^{\alpha\dot{\gamma}} \wedge e_0{}^{\beta}{}_{\dot{\gamma}} \frac{\partial}{\partial y^\alpha}\frac{\partial}{\partial y^\beta}\Phi(x|y,0) + e_0^{\gamma\dot{\alpha}} \wedge e_0{}_{\gamma}{}^{\dot{\beta}} \frac{\partial}{\partial \bar{y}^{\dot{\alpha}}}\frac{\partial}{\partial \bar{y}^{\dot{\beta}}}\Phi(x|0,\bar{y})\right]$$

$$\mathcal{D}_0\Phi(x|y,\bar{y}) \equiv d\Phi + [\omega_0,\Phi]_\star + \{e_0,\Phi\}_\star = 0$$

(Vasiliev '89)

- Manifest HS-covariance
- Consistency ($d^2 = 0$) ⇒ gauge invariance (FDA)
- **NOTE:** covariant constancy conditions, **but** infinitely many fields
  + trace constraints ⇒ **DYNAMICS**

The Vasiliev Equations

- NC extension, x →(x,Z):

$$Z_{\underline{\alpha}} = (z_\alpha, -\bar{z}_{\dot\alpha}) \ , \qquad [Z_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = -2iC_{\underline{\alpha\beta}} \ , \quad [Y_{\underline{\alpha}}, Z_{\underline{\beta}}]_\star = 0$$

- Extended star-product, normal-ordering (wrt $A^+ = (Y-Z)/2i$, $A^- = (Y+Z)/2$)

$$\widehat{F}(Y,Z) \star \widehat{G}(Y,Z) = \int_{\mathcal{R}} \frac{d^4U d^4V}{(2\pi)^4} e^{iV^{\underline{\alpha}}U_{\underline{\alpha}}} \widehat{F}(Y+U, Z+U)\, \widehat{G}(Y+V, Z-V)$$

- Fields live on correspondence space, locally $\mathcal{X}$ x $\mathcal{Y}$ x $\mathcal{Z}$:

$$d \to \widehat{d} = d + d_Z = dx^\mu \frac{\partial}{\partial x^\mu} + dz^\alpha \frac{\partial}{\partial z^\alpha} + d\bar{z}^{\dot\alpha} \frac{\partial}{\partial \bar{z}^{\dot\alpha}}$$

$$A(x|Y) \to \hat{A}(x|Z,Y) \equiv (dx^\mu \hat{A}_\mu + dz^\alpha \hat{A}_\alpha + d\bar{z}^{\dot\alpha} \hat{A}_{\dot\alpha})(x|Z,Y) \ , \quad A_\mu(x|Y) = \hat{A}_\mu\big|_{Z=0}$$

$$\Phi(x|Y) \to \hat{\Phi}(x|Z,Y) \ , \quad \Phi(x|Y) = \hat{\Phi}(x|Z,Y)\big|_{Z=0}$$

- π automorphism becomes inner, generated by the inner kleinian κ

$$\begin{array}{llll} \pi(\widehat{f}) = \kappa \star \widehat{f} \star \kappa \ , & \kappa = (-1)_\star^{\widehat{n}} \ , & \hat{n} = a^{+\alpha} \star a^-_\alpha & \kappa \star \kappa = 1 \\ \bar\pi(\widehat{f}) = \bar\kappa \star \widehat{f} \star \bar\kappa \ , & \bar\kappa = (-1)_\star^{\widehat{\bar{n}}} \ , & \hat{\bar{n}} = \bar{a}^{+\alpha} \star \bar{a}^-_\alpha & \bar\kappa \star \bar\kappa = 1 \end{array}$$

$$\pi(\widehat{f}(y,\bar{y};z,\bar{z})) = \widehat{f}(-y,\bar{y};-z,\bar{z}) \ , \quad \bar\pi(\widehat{f}(y,\bar{y};z,\bar{z})) = \widehat{f}(y,-\bar{y};z,-\bar{z})$$

The Vasiliev Equations

- In normal-ordering: $\kappa = e^{iy^\alpha z_\alpha}\ , \qquad \bar\kappa = e^{i\bar y^\alpha \bar z_\alpha}$

$$\kappa = \kappa_y \star \kappa_z\ , \quad \bar\kappa = \bar\kappa_{\bar y} \star \bar\kappa_{\bar z}\ , \quad \kappa_y \star \kappa_y = 1 \quad idem\ \kappa_z,\ \bar\kappa_{\bar y} \text{ and } \bar\kappa_{\bar z}$$

$$\kappa_y = 2\pi\delta^{(2)}(y) = 2\pi\delta(y_1)\delta(y_2)$$

Unfolded full eqs:

(Vasiliev '90)

$$\boxed{\begin{array}{l}\hat F \equiv \hat d\hat A + \hat A \star \hat A = \dfrac{i}{4}(dz^\alpha \wedge dz_\alpha\, \hat{\mathcal{B}} \star \hat\Phi \star \kappa + d\bar z^{\dot\alpha} \wedge d\bar z_{\dot\alpha} \hat{\bar{\mathcal{B}}} \star \hat\Phi \star \bar\kappa) \\ \hat{\mathcal{D}}\hat\Phi \equiv \hat d\hat\Phi + \hat A \star \hat\Phi - \hat\Phi \star \bar\pi(\hat A) = 0\end{array}}$$

Local sym: $\delta\hat A = \hat D\hat\epsilon\ , \quad \delta\hat\Phi = -[\hat\epsilon, \hat\Phi]_\pi$

$$\hat F_{\mu\nu} = \hat F_{\alpha\mu} = \hat F_{\dot\alpha\mu} = \hat F_{\alpha\dot\alpha} = 0\ ,$$

$$\hat F_{\alpha\beta} = -\frac{i}{2}\epsilon_{\alpha\beta}\hat{\mathcal{B}} \star \hat\Phi \star \kappa\ ,$$

$$\hat F_{\dot\alpha\dot\beta} = -\frac{i}{2}\epsilon_{\dot\alpha\dot\beta}\hat{\bar{\mathcal{B}}} \star \hat\Phi \star \bar\kappa\ ,$$

$$\hat{\mathcal{D}}_\mu\hat\Phi = \hat{\mathcal{D}}_\alpha\hat\Phi = \hat{\mathcal{D}}_{\dot\alpha}\hat\Phi = 0$$

Solving for Z-contractions yields consistent nonlinear corrections as an expansion in Φ.

For spacetime components, project on spacetime manifold $\mathcal{X}$

{Z=0} →
$$\boxed{\hat F_{\mu\nu}(x|A,\Phi;Y)\Big|_{Z=0} = 0\ , \quad (\hat{\mathcal{D}}_\mu\hat\Phi)(x|\Phi;Y)\Big|_{Z=0} = 0}$$

Black Holes and Higher Spins

- Crucial to look into the non-perturbative sector of the theory, may shed some light on peculiarities of HS physics and prompts to study global issues in HS gravity (boundary conditions, asymptotic charges, global dof in $\mathcal{Z}\ldots$). Very likely new tools, and HS geometry adapted to HS symmetries, have to be developed.

- HS Gravity does not admit a consistent truncation to spin 2. No obvious embedding of gravitational bhs.

- Characterization of bhs rests on geodesic motion, but relativistic interval $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ is NOT HS-invariant . What is to be called a "higher-spin black hole"?

- Do non-local interactions & HS gauge symmetries smooth out singularities? (already from ST we are used to higher-derivative stringy correction affecting the nature of singularities)

Exact Solutions: gauge function method

- Full eqns:

$$\begin{aligned} \widehat{F}_{\mu\nu} &= \widehat{F}_{\mu\alpha} = \widehat{F}_{\mu\dot{\alpha}} = 0 \,, \qquad \widehat{D}_\mu \widehat{\Phi} = 0 \,, \\ \left[\widehat{S}'_\alpha, \widehat{S}'_\beta\right]_\star &= -2i\epsilon_{\alpha\beta}(1 - \mathcal{B} \star \widehat{\Phi}' \star \kappa) \,, \\ \left[\widehat{S}'_{\dot{\alpha}}, \widehat{S}'_{\dot{\beta}}\right]_\star &= -2i\epsilon_{\dot{\alpha}\dot{\beta}}(1 - \bar{\mathcal{B}} \star \widehat{\Phi}' \star \bar{\kappa}) \\ \left[\widehat{S}'_\alpha, \widehat{S}'_{\dot{\beta}}\right]_\star &= 0 \,, \\ \widehat{S}'_\alpha \star \widehat{\Phi}' &+ \widehat{\Phi}' \star \pi(\widehat{S}'_\alpha) = 0 \,, \\ \widehat{S}'_{\dot{\alpha}} \star \widehat{\Phi}' &+ \widehat{\Phi}' \star \bar{\pi}(\widehat{S}'_{\dot{\alpha}}) = 0 \end{aligned}$$

$$\hat{S}_\alpha = z_\alpha - 2i\hat{A}_\alpha$$

- Project on $\mathcal{Z}$! (base ↔ fiber evolution)
  Locally give x-dep. via gauge functions (spacetime ~ pure gauge!)

$$\hat{A}_\mu = \hat{L}^{-1} \star \partial_\mu \hat{L} \,, \quad \hat{S}_\alpha = \hat{L}^{-1} \star (\hat{S}'_\alpha) \star \hat{L} \,, \quad \hat{\Phi} = \hat{L}^{-1} \star \hat{\Phi}' \star \pi(\hat{L})$$

$$\hat{L} = \hat{L}(x|Z,Y) \,, \; \hat{L}(0|Z,Y) = 1 \qquad \hat{S}'_\alpha = \hat{S}_\alpha(0|Z,Y) \,, \quad \hat{\Phi}' = \hat{\Phi}(0|Z,Y)$$

- Z-eq.^{ns} can be solved exactly: 1) imposing symmetries on primed fields
  2) via projectors

- "Dress" with x-dependence. Lorentz tensors are coefficients of:

$$\widehat{W}_\mu := \widehat{A}_\mu - \widehat{K}_\mu \,, \quad \widehat{K}_\mu := \frac{1}{4i}\omega_\mu^{\alpha\beta}\widehat{M}_{\alpha\beta} - \text{h.c.} \,, \quad \widehat{M}_{\alpha\beta} := y_\alpha y_\beta - z_\alpha z_\beta + \widehat{S}_{(\alpha} \star \widehat{S}_{\beta)}$$

AdS$_4$ Vacuum Solution

- AdS$_4$ vacuum sol.:

$$\boxed{\widehat{\Phi} = 0\,, \quad \widehat{S}_\alpha = \widehat{S}^{(0)}_\alpha = z_\alpha\,, \quad \widehat{S}_{\dot\alpha} = \widehat{S}^{(0)}_{\dot\alpha} = \bar z_{\dot\alpha}\,, \quad \widehat{A}_\mu = \Omega^{(0)}_\mu = L^{-1}\star\partial_\mu L}$$

The gauge function
$(h = \sqrt{1-\lambda^2 x^2})$

$$L(x;y,\bar y) = e_\star^{i\lambda\tilde x^\mu(x)\delta^a_\mu P_a} = \frac{2h}{1+h}\exp\left[\frac{i\lambda x^{\alpha\dot\alpha}y_\alpha\bar y_{\dot\alpha}}{1+h}\right]$$

gives AdS$_4$ connection

$$\Omega^{(0)}_\mu = -i\left(\frac{1}{2}\omega^{ab}_{(0)}M_{ab} + e^a_{(0)}P_a\right) = \frac{1}{4i}\left(\omega^{\alpha\beta}_{(0)}y_\alpha y_\beta + \bar\omega^{\dot\alpha\dot\beta}_{(0)}\bar y_{\dot\alpha}\bar y_{\dot\beta} + 2e^{\alpha\dot\beta}_{(0)}y_\alpha\bar y_{\dot\beta}\right)$$

$$e_{(0)}{}^{\alpha\dot\alpha} = -\frac{\lambda(\sigma^a)^{\alpha\dot\alpha}dx_a}{h^2}\,, \qquad \omega_{(0)}{}^{\alpha\beta} = -\frac{\lambda^2(\sigma^{ab})^{\alpha\beta}dx_a x_b}{h^2}$$

leading to AdS$_4$ metric in stereographic coords.: $\displaystyle ds^2_{(0)} = \frac{4dx^2}{(1-\lambda^2x^2)^2}$

- Global symmetries:

$$\begin{aligned} \delta S^{(0)}_\alpha &= [z_\alpha, \widehat\epsilon]_\star = 0 \quad \Rightarrow \quad \widehat\epsilon = \epsilon^{(0)}(x|Y) \\ \delta\Omega^{(0)}_\mu &= D^{(0)}_\mu\epsilon^{(0)}(x|Y) = 0 \end{aligned}$$

Y^2-sector: $\displaystyle \epsilon^{(0)} = -i\left(\frac{1}{2}\kappa^{ab}M_{ab} + v^aP_a\right)$ $\longrightarrow$

$$\begin{aligned} \delta e^a_{(0)} &= 0 \quad \Rightarrow \quad \nabla^{(0)}_a v_b = \kappa_{ab} \\ \delta\omega^{ab}_{(0)} &= 0 \quad \Rightarrow \quad \nabla^{(0)}_a\kappa_{bc} = g^{(0)}_{ac}v_b - g^{(0)}_{ab}v_c \end{aligned}$$

Local properties of 4D black holes

- Bh Weyl tensor is of Petrov-type D, ((anti-)selfdual part) has 2 principal spinors :

$$\Phi_{\alpha\beta\gamma\delta} = \nu(x)\, u^+_{(\alpha} u^-_{\beta} u^+_{\gamma} u^-_{\delta)} , \qquad u^{+\alpha} u^-_{\alpha} = 1$$

- Local characterization of 4D bhs: sol.ns of Einstein's eqs. in vacuum (flat or AdS) such that their Weyl tensor's principal spinors are collinear with those of the Killing 2-form of an asymptotically *timelike* KVF, $\kappa_{\mu\nu} = \nabla_\mu v_\nu$ *(Mars, '99; Didenko-Matveev-Vasiliev, '08-'09)* ,

$$\Phi_{\alpha\beta\gamma\delta} \sim \frac{M}{(\varkappa^2)^{5/2}} \varkappa_{(\alpha\beta} \varkappa_{\gamma\delta)} , \qquad \varkappa^2 := \frac{1}{2} \varkappa^{\alpha\beta} \varkappa_{\alpha\beta}$$

- Bh metric admits Kerr-Schild form (linearizes field eqs.):

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \frac{2M}{H} k_\mu k_\nu , \quad g^{\mu\nu} = g^{\mu\nu}_{(0)} - \frac{2M}{H} k^\mu k^\nu , \qquad k^\mu k_\mu = 0 = k^\mu D_\mu k_\nu$$

- A generic bh is completely determined by a chosen background global symmetry parameter $Y^{\underline{\alpha}} K_{\underline{\alpha\beta}} Y^{\underline{\beta}}$
(Didenko-Matveev-Vasiliev, '09)

$$K_{\underline{\alpha\beta}} = \begin{pmatrix} \varkappa_{\alpha\beta} & v_{\alpha\dot\beta} \\ \bar v_{\dot\alpha\beta} & \bar\varkappa_{\dot\alpha\dot\beta} \end{pmatrix} , \qquad D_0 K_{\underline{\alpha\beta}} = 0$$

Properties of bh encoded in algebraic conditions: $K^2 = -1$ → static:

$$K_{\underline{\alpha}}{}^{\underline{\beta}} K_{\underline{\beta}}{}^{\underline{\gamma}} = -\delta_{\underline{\alpha}}{}^{\underline{\gamma}} \quad \Leftrightarrow \quad \left\{ \begin{array}{rcl} \varkappa^2 + v^2 & = & 1 \\ \varkappa^2 & = & \bar\varkappa^2 \\ \varkappa_\alpha{}^\beta v_\beta{}^{\dot\gamma} + v_\alpha{}^{\dot\beta} \bar\varkappa_{\dot\beta}{}^{\dot\gamma} & = & 0 \end{array} \right. \longrightarrow \quad v_{[\mu} \nabla_\nu v_{\rho]} = 0$$

HS black-hole-like Ansatz

- Weyl zero-form $\widehat{\Phi} = \widehat{L}^{-1} \star \widehat{\Phi}' \star \pi(\widehat{L})$: reduces eqs. to linearized on AdS

$$\partial_\mu \widehat{\Phi} + [\widehat{A}_\mu, \widehat{\Phi}]_\pi = 0 \quad \rightarrow \quad \partial_\mu \Phi + [\Omega_\mu^{(0)}, \Phi]_\pi = 0$$

with

$$\widehat{L}(x|Y,Z) = L(x|Y) \star \widetilde{L}(x|Z) \ , \quad \pi(\widetilde{L}) = \widetilde{L} \ ; \qquad \widehat{\Phi}' = \Phi'(Y)$$

- Link with global sym parameters: to any HS global sym parameter $\varepsilon_{(0)}(x|Y)$ ($D^{(0)}\epsilon^{(0)} = 0$) is associated a solution $\epsilon^{(0)} \star \kappa_y$ of the linearized Weyl 0-form eqn.

$$\partial_\mu(\epsilon^{(0)} \star \kappa_y) + [\Omega_\mu^{(0)}, (\epsilon^{(0)} \star \kappa_y)]_\pi = (D^{(0)}\epsilon^{(0)}) \star \kappa_y = 0$$

$$\Phi(x|Y) = \epsilon^{(0)}(x|Y) \star \kappa_y = L^{-1} \star \epsilon'_{(0)}(Y) \star L \star \kappa_y \quad \Rightarrow \quad \Phi'(Y) = \epsilon'_{(0)}(Y) \star \kappa_y$$

- Bh determined by a chosen AdS KVF $K_{\underline{\alpha\beta}}(x)$ → by a rigid $K'_{\underline{\alpha\beta}} \in \mathfrak{sp}(4,\mathbb{C})$. Generalize to a HS global sym parameter *(Didenko-Vasiliev '09)*

$$\epsilon_0(x|Y) = f(Y^{\underline{\alpha}} K_{\underline{\alpha\beta}}(x) Y^{\underline{\beta}}) \ , \quad \Rightarrow \quad \epsilon'_0(Y) = f(Y^{\underline{\alpha}} K'_{\underline{\alpha\beta}} Y^{\underline{\beta}}) \ ,$$

- "Static" → $K'_{\underline{\alpha}}{}^{\underline{\beta}} K'_{\underline{\beta}}{}^{\underline{\gamma}} = -\delta_{\underline{\alpha}}{}^{\underline{\gamma}} \quad \Rightarrow \quad K'_{\underline{\alpha\beta}} \sim (\Gamma_{AB})_{\underline{\alpha\beta}} \ , \quad M_{AB} = -\frac{1}{8} Y^{\underline{\alpha}} (\Gamma_{AB})_{\underline{\alpha\beta}} Y^{\underline{\beta}}$

  Up to $\mathfrak{sp}(4,\mathbb{R})$ rotations this selects $\quad Y^{\underline{\alpha}} K'_{\underline{\alpha\beta}} Y^{\underline{\beta}} \sim E, J, iB, iP$

- Spherical symmetry → $\quad Y^{\underline{\alpha}} K'_{\underline{\alpha\beta}} Y^{\underline{\beta}} \sim E$

HS black-hole-like Ansatz

- Which f(E)? Choose a projector (enforces Kerr-Schild property in gauge fields):

$$\mathcal{P}_1(x|Y) := 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K_{\underline{\alpha\beta}}(x)Y^{\underline{\beta}}}\ , \qquad \mathcal{P}_1 \star \mathcal{P}_1 = \mathcal{P}_1\ , \qquad \left(K_{\underline{\alpha}}{}^{\underline{\beta}}K_{\underline{\beta}}{}^{\underline{\gamma}} = -\delta_{\underline{\alpha}}{}^{\underline{\gamma}}\right)$$

$\Longrightarrow$ Solve the full twisted adjoint eq. with $\Phi(x|Y) \propto \mathcal{P}_1 * \kappa_y$, and

$$\Phi(x) = iM\mathcal{P}_1 \star \kappa_y = \frac{4M}{\sqrt{\varkappa^2}} \exp\left[\frac{1}{2}\left(y^\alpha \varkappa^{-1}_{\alpha\beta} y^\beta + \bar{y}^{\dot\alpha}\bar{\varkappa}^{-1}_{\dot\alpha\dot\beta}\bar{y}^{\dot\beta} + 2iy^\alpha \bar{y}^{\dot\alpha}\varkappa^{-1}_{\alpha\beta}v^{\beta}{}_{\dot\alpha}\right)\right]$$

containing the spin-s generalized type-D Weyl 0-form components:

(Didenko-Vasiliev, '09)

$$\Phi_{\alpha(2s)} = \frac{4M}{2^s\,(-\varkappa^2)^{s+1/2}}\,\varkappa_{(\alpha_1\alpha_2}\ldots\varkappa_{\alpha_{2s-1}\alpha_{2s})}$$

M = real deformation parameter

- Using the gauge function:

$$P_1(Y) = 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K'_{\underline{\alpha\beta}}Y^{\underline{\beta}}} = 4e^{-y^\alpha\sigma^0_{\alpha\dot\alpha}\bar{y}^{\dot\alpha}} = 4e^{-4E} \;\rightarrow\; L^{-1}\star P_1(Y)\star L = 4e^{-\frac{1}{2}Y^{\underline{\alpha}}K_{\underline{\alpha\beta}}(x)Y^{\underline{\beta}}}$$

In AdS_4 spherical coords. (t,r,θ,φ) $[ds^2 = (1+r^2)\,dt^2 + (1+r^2)^{-1}\,dr^2 + r^2\,d\Omega^2]$

$$K'_{\underline{\alpha\beta}} = (\Gamma_0)_{\underline{\alpha\beta}} = \begin{pmatrix} 0 & u^+_\alpha \bar{u}^+_{\dot\beta} + u^-_\alpha \bar{u}^-_{\dot\beta} \\ \bar{u}^+_{\dot\alpha}u^+_\beta + \bar{u}^-_{\dot\alpha}u^-_\beta & 0 \end{pmatrix} \longrightarrow K_{\underline{\alpha\beta}} = \begin{pmatrix} 2r\tilde{u}^+_{(\alpha}\tilde{u}^-_{\beta)} & \sqrt{1+r^2}(\tilde{u}^+_\alpha\tilde{\bar{u}}^+_{\dot\beta} + \tilde{u}^-_\alpha\tilde{\bar{u}}^-_{\dot\beta}) \\ \sqrt{1+r^2}(\tilde{\bar{u}}^+_{\dot\alpha}\tilde{u}^+_\beta + \tilde{\bar{u}}^-_{\dot\alpha}\tilde{u}^-_\beta) & 2r\tilde{\bar{u}}^+_{(\dot\alpha}\tilde{\bar{u}}^-_{\dot\beta)} \end{pmatrix}$$

$$u^{+\alpha}u^-_\alpha = 1 = \tilde{u}^{+\alpha}(x)\tilde{u}^-_\alpha(x)$$

$$\Phi_{\alpha(2s)} = \frac{4M}{r^{s+1}}\,\tilde{u}^+_{(\alpha_1}\tilde{u}^-_{\alpha_2}\ldots\tilde{u}^+_{\alpha_{2s-1}}\tilde{u}^-_{\alpha_{2s})}$$

Spherically symmetric type-D solutions

- Solution based on scalar singleton ground-state projector!, i.e. …

$$\begin{aligned} E\star e^{-4E} &= e^{-4E}\star E = \frac{1}{2}e^{-4E}\ , \\ L^-_r\star e^{-4E} &= 0 = e^{-4E}\star L^+_r\ , \\ M_{rs}\star e^{-4E} &= 0 \end{aligned}$$

$\Longrightarrow$ $$4e^{-4E} \simeq |1/2;0\rangle\langle 1/2;0| \in \mathcal{D}_0\otimes\mathcal{D}_0^*$$

(C.I., P. Sundell '08)

- (More) general spherically symm. type-D ansatz: include *all* projectors on scalar (super)singleton modes (all $\mathfrak{so}(3)$-invariant excitations of 4 exp(-4E)) and their negative-energy counterparts.

$$P_n \sim \begin{cases} a^{\dagger\, i_1}\dots a^{\dagger\, i_n}\star|1/2;0\rangle\langle 1/2;0|\star a_{i_1}\dots a_{i_n}\ , & n>0 \\ a_{i_1}\dots a_{i_{|n|}}\star|-1/2;0\rangle\langle -1/2;0|\star a^{\dagger\, i_1}\dots a^{\dagger\, i_{|n|}}\ , & n<0 \end{cases}$$

$$P_n\star P_m = \delta_{nm}P_n$$

$$\begin{aligned} a_1 &= \frac{1}{2}(y_1+i\bar y_{\dot 2})\ , \quad a^{\dagger\,1} = \frac{1}{2}(\bar y_{\dot 1}-iy_2)\ , \\ a_2 &= \frac{1}{2}(-y_2+i\bar y^{\dot 1})\ , \quad a^{\dagger\,2} = \frac{1}{2}(-\bar y_{\dot 2}-iy_1) \end{aligned}$$

$$[a_i, a^{\dagger\, j}]_\star = \delta_i^j$$

$\Rightarrow$ Φ'(Y) = any f(Y) diagonalizable on such basis of projectors $*\ \kappa_y$:

$$\Phi'(Y) = \sum_{n=\pm1,\pm2,\dots}\nu_n\, P_n(Y)\star\kappa_y$$

Spherically symmetric type-D solutions

- Weyl-ordered (integral) realization (ε := sign(n), n= ±1,±2,…):

$$P_n(E) \;=\; 4(-1)^{n-\frac{1+\varepsilon}{2}}\, e^{-4E}\, L^{(1)}_{n-1}(8E) \;=\; 2(-1)^{n-\frac{1+\varepsilon}{2}} \oint_{C(\varepsilon)} \frac{ds}{2\pi i} \left(\frac{s+1}{s-1}\right)^{n} e^{-4sE}$$

- Weyl 0-form: $$\Phi(x|Y) \;=\; \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{ds}{2\pi i}\left(\frac{s+1}{s-1}\right)^{n} \underbrace{L^{-1}(x) \star e^{-4sE} \star L(x) \star \kappa_y}$$

Type-D
Weyl 0-form
generating f :

$$\bar{y}=0: \quad \frac{1}{s\sqrt{\varkappa^2}} \exp\left(\frac{1}{2s}\, y^{\alpha} \varkappa^{-1}_{\alpha\beta} y^{\beta}\right) \;, \quad \varkappa^{-1}_{\alpha\beta} = -\frac{\varkappa_{\alpha\beta}}{\varkappa^2} \;, \quad \varkappa^2 = -r^2$$

$$C^{(n)}_{\alpha(2s)} \;\sim\; \frac{i^{n-1}\mu_n}{r^{s+1}}\, (\tilde{u}^{+}\tilde{u}^{-})^{s}_{\alpha(2s)}$$

- ➢ Deformation parameter is real for scalar singleton, imaginary for spinor singl.
  → generalized electric/magnetic charge (or mass/NUT).
  E/m duality connects Type A/B models?
- ➢ Spacetime coords. enter as parameter of a limit representation of a delta function . $$\widehat{\Phi}_1 \;\overset{r\to 0}{\longrightarrow}\; \widehat{\Phi}'_1 \;=\; \nu_1 \kappa_{y-i\sigma_0\bar{y}} \;=\; 2\pi\nu_1\left[\delta^2(y-i\sigma_0\bar{y})\right]$$

- Residual symmetry → stabilizer of E $\Rightarrow$ $\mathfrak{h} = \mathfrak{so}(2)_{\mathrm{E}} \oplus \mathfrak{so}(3)_{\mathrm{M_{ij}}}$.

$$\delta\Phi(x|Y) \;=\; -[\epsilon(x|Y),\Phi(x|Y)]_{\star,\pi} \;=\; 0 \;\Leftrightarrow\; [\epsilon'(Y), e^{-4sE}]_{\star} \;=\; 0$$

Internal Z-Space Solution

- Ansatz for internal eqs., separation of Y and Z variables, absorb Y-dep. in $P_n(Y)$:

$$\widehat{S}'_\alpha = z_\alpha - 2i \sum_{n=0}^{\infty} P_n(Y) \star A^n_\alpha(z) \ , \qquad \bar{\widehat{S}}'_{\dot\alpha} = \bar z_{\dot\alpha} - 2i \sum_{n=0}^{\infty} P_n(Y) \star \bar A^n_{\dot\alpha}(\bar z)$$

Reduced deformed oscillators: $\Sigma^n_\alpha = z_\alpha - 2iA^n_\alpha \ , \quad \bar\Sigma^n_{\dot\alpha} = \bar z_{\dot\alpha} - 2i\bar A^n_{\dot\alpha}$

- ✓ Orthogonality of projectors ⇒ eqs. for different n split;
- ✓ Projectors only Y-dep. ⇒ spectators, out of commutators;
- ✓ ν_n = cost and $\pi(\Sigma) = -\Sigma$ solve $\{S',\Phi'\}_\pi = 0$;
- ✓ Holomorphicity in z of S' solves $[S', \overline{S}'] = 0$

- Left with the deformed oscillator problem :

$$\begin{aligned} \left[\Sigma^n_\alpha, \Sigma^n_\beta\right]_\star &= -2i\epsilon_{\alpha\beta}(1 - \mathcal{B}_n\nu_n\kappa_z) \ , \\ \left[\bar\Sigma^n_{\dot\alpha}, \bar\Sigma^n_{\dot\beta}\right]_\star &= -2i\epsilon_{\dot\alpha\dot\beta}(1 - \bar{\mathcal{B}}_n\bar\nu_n\bar\kappa_{\bar z}) \end{aligned}$$

Can solve by a general method *(Prokushkin-Vasiliev '98, Sezgin-Sundell '05)* for regular deformation terms. Use a limit representation of $\kappa_z \sim \delta^2(z)$ or first go to normal-ordering where κ_z = gaussian .

Solution for Z-space deformed oscillators

- Introduce basis spinors $u^{\pm}{}_{\alpha}$ (a priori non-collinear with $\tilde{u}^{\pm}{}_{\alpha}(x)$) :

$$z^{\pm} := u^{\pm\alpha} z_{\alpha} \;, \quad w_z := z^{+} z^{-} \;, \quad [z^{-}, z^{+}]_{\star} = -2i$$

- Solve $[\Sigma_n^{+}, \Sigma_n^{-}]_{\star} \;=\; -1 + \mathcal{B}_n \nu_n \kappa_z$ w/ Laplace-like transform:

$$\Sigma_n^{\pm} \;=\; 4z^{\pm} \int_{-1}^{1} \frac{dt}{(t+1)^2}\, f_{\sigma_n}^{n\pm}(t)\, e^{i\sigma_n \frac{t-1}{t+1} w_z}$$

and using the limit representation $\lim\limits_{\varepsilon\to 0} \frac{1}{\varepsilon} e^{-i\frac{\sigma}{\varepsilon} z^{+} z^{-}} \;=\; \sigma\, [\kappa_z]^{\mathrm{Weyl}}$.

Leads to manageable algebraic eqns for $f^{n}{}_{\pm}(t)$. Can either solve symmetrically, $f^{n}{}_{+} = f^{n}{}_{-}$, or asymmetrically (gauge freedom on S).
Study <u>sym case</u>: particular, ν-dependent solution

$$f_{\sigma_n}^{n\pm}(t) \;=\; \delta(t-1) \;-\; \frac{\sigma_n \mathcal{B}_n \nu_n}{4}\, {}_1F_1\left[\frac{1}{2}; 2; \frac{\sigma_n \mathcal{B}_n \nu_n}{2} \log \frac{1}{t^2}\right]$$

- Also: a general way of solving the homogeneous ($\nu_n = 0$) eq. is the projector solution: $X^2 = 1 \;\rightarrow\; X = 1 - 2P \;, \quad P^2 = P$

Solution for Z-space deformed oscillators

- Internal Z-space connection:

$$
\begin{aligned}
A^n_\pm &= A^{n\,(reg)}_\pm + A^{n\,(proj)}_\pm \\
A^{n\,(reg)}_\pm &= \frac{i\sigma_n \mathcal{B}_n \nu_n}{2} z^\pm \int_{-1}^{1} \frac{dt}{(t+1)^2}\, e^{i\sigma_n \frac{t-1}{t+1} w_z} \left[{}_1F_1\left(1/2; 2; \frac{\nu_n}{2} \log\frac{1}{t^2}\right)\right] \\
A^{n\,(proj)}_\pm &= -iz^\pm \sum_{k=0}^{\infty} (-1)^k \theta_k L_k[\nu_n] P_k(z) \ , \quad P_k(z) = \frac{(z^+ z^-)^k}{k!}\, e^{-z^+ z^-} \ , \\
L_k[\nu] &= \int_{-1}^{1} dt\, t^k f^n_\pm(t) \longrightarrow 1 \ \text{ as } \ \nu_n \longrightarrow 0 \ , \quad \theta_k \ = \ 0, 1
\end{aligned}
$$

Sol.ns depend on two infinite sets of parameters:

- continuous parameters ν_n → Φ-moduli;
- discrete parameters θ_k → S-moduli, a “landscape” of vacua.

- Divergent deformed oscillators (t = -1) but S(x|Y,Z) only singular in r = 0 ! Pushed out of integration domain by star-product with $\mathcal{P}_n$(x|Y). For n=1:

$$
\widehat{S}^\pm = \tilde{z}^\pm + 8\,\mathcal{P}_1(x|Y)\,\tilde{a}^\pm \int_{-1}^{1} \frac{dt}{(t+1+i\sigma_n r(t-1))^2}\, j_1^\pm(t)\, e^{\frac{i\sigma_n (t-1)}{t+1+i\sigma r(t-1)}\,\tilde{a}^+\tilde{a}^-}
$$

$$
\tilde{a}^\pm := \tilde{u}^{\alpha\pm} a_\alpha \ , \qquad a_\alpha \ = \ z_\alpha + i(\varkappa_\alpha{}^\beta y_\beta + v_\alpha{}^{\dot\beta} \bar{y}_{\dot\beta}) \ , \quad z_\alpha \star \mathcal{P}_1 = a_\alpha \mathcal{P}_1 \ , \quad [a_\alpha, \, a_\beta]_\star \ = \ -2i\epsilon_{\alpha\beta}
$$

Gauge fields

- Internal Z-space connection: $\widehat{W}_\mu = \Omega^{(0)}_\mu + \widetilde{L}^{-1}_{(K)} \star \partial_\mu \widetilde{L}_{(K)} - \widehat{K}_{(K)\mu}$

$$\widehat{K}_\mu(Z,Y|x) = \frac{1}{4i}\left(\omega^{++}_\mu \widehat{M}^{--} + \omega^{--}_\mu \widehat{M}^{++} - 2\omega^{-+}_\mu \widehat{M}^{+-}\right) - \text{h.c.}\,,$$

with

$$\left[\widehat{M}^{++}\right]^{\widehat{N}_+} = \tilde{y}^+\tilde{y}^+ + \sum_{n=\pm1,\pm2,\ldots} 8(-1)^{n-\frac{1+\epsilon}{2}} \oint_{C(\epsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \mathcal{P}_1(\eta E^L)\,(\mathcal{F}-\mathcal{G})\,\tilde{\breve{a}}^+\tilde{\breve{a}}^+\,,$$

$$\left[\widehat{M}^{--}\right]^{\widehat{N}_+} = \tilde{y}^-\tilde{y}^- + \sum_{n=\pm1,\pm2,\ldots} 8(-1)^{n-\frac{1+\epsilon}{2}} \oint_{C(\epsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \mathcal{P}_1(\eta E^L)\,(\mathcal{F}-\mathcal{G}')\,\tilde{\breve{a}}^-\tilde{\breve{a}}^-\,,$$

$$\left[\widehat{M}^{+-}\right]^{\widehat{N}_+} = \tilde{y}^+\tilde{y}^- - \sum_{n=\pm1,\pm2,\ldots} 8(-1)^{n-\frac{1+\epsilon}{2}} \oint_{C(\epsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \mathcal{P}_1(\eta E^L)$$

$$\times\left[(\mathcal{Q}+\mathcal{F})\tilde{\breve{a}}^+\tilde{\breve{a}}^- - \eta r(\mathcal{P}+\mathcal{R})\right]\,,$$

Gauge fields

- Parametric integrals (functions of number operator $\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-}$):

$$\mathcal{F}(\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-};\eta;r) = \int_{-1}^{1} dt\, j^{n}(t)\, \frac{t+1}{\chi^{3}}\, e^{\frac{i\sigma_n(t-1)}{\chi}\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-}}\,, \tag{4.32}$$

$$\mathcal{G}(\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-};\eta;r) = \int_{-1}^{1} dt\, j^{n}(t) \int_{-1}^{1} dt'\, j^{n}(t')\, \frac{(t'-1)(1+\sigma_n)+(t-1)(1-\sigma_n)+2}{\tilde{\chi}^{3}}\, e^{\frac{i\sigma_n(\tilde{t}-1)}{\tilde{\chi}}\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-}}\,, \tag{4.33}$$

$$\mathcal{G}'(\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-};\eta;r) = \int_{-1}^{1} dt\, j^{n}(t) \int_{-1}^{1} dt'\, j^{n}(t')\, \frac{(t-1)(1+\sigma_n)+(t'-1)(1-\sigma_n)+2}{\tilde{\chi}^{3}}\, e^{\frac{i\sigma_n(\tilde{t}-1)}{\tilde{\chi}}\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-}}\,, \tag{4.34}$$

$$\mathcal{P}(\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-};\eta;r) = \int_{-1}^{1} dt\, j^{n}(t)\, \frac{e^{\frac{i\sigma_n(t-1)}{\chi}\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-}}}{\chi^{2}}\,, \tag{4.35}$$

$$\mathcal{Q}(\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-};\eta;r) = \int_{-1}^{1} dt\, j^{n}(t) \int_{-1}^{1} dt'\, j^{n}(t')\, \frac{\tilde{t}+1}{\tilde{\chi}^{3}} e^{\frac{i\sigma_n\tilde{t}}{\tilde{\chi}}\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-}}\,, \tag{4.36}$$

$$\mathcal{R}(\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-};\eta;r) = \int_{-1}^{1} dt\, j^{n}(t) \int_{-1}^{1} dt'\, j^{n}(t')\, \frac{e^{\frac{i\sigma_n(\tilde{t}-1)}{\tilde{\chi}}\breve{\tilde{a}}^{+}\breve{\tilde{a}}^{-}}}{\tilde{\chi}^{2}}\,. \tag{4.37}$$

where $\chi := t+1+i\sigma_n\eta r(t-1)$ and $\tilde{\chi} := \tilde{t}+1+i\sigma_n\eta r(\tilde{t}-1)$.

Deformation parameters and asymptotic charges

- Building solutions on more than one projector opens up interesting possibilities.

➢ Every singleton-state projector contains a tower of fields of all spins → can change basis and diagonalize on spin (and not occupation number)

$$\mathcal{C}(x|y) \;=\; \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i}\left(\frac{\eta+1}{\eta-1}\right)^n \frac{1}{\eta\sqrt{\varkappa^2}}\,\exp\left(\frac{1}{2\eta}\,y^\alpha \varkappa^{-1}_{\alpha\beta} y^\beta\right)$$

$$\mathcal{C}(x|y) \;=\; \sum_{s=0}^{\infty} \frac{1}{(2s)!} C_{\alpha(2s)}(x)\, y^{\alpha(2s)}\;, \qquad C_{\alpha(2s)} \;\sim\; \frac{\mathcal{M}_s}{r^{s+1}}\,(\tilde{u}^+\tilde{u}^-)^s_{\alpha(2s)}$$

"HS asymptotic charge", $f(\nu_n)$: $\quad \mathcal{M}_s \sim \sum_n \nu_n \tilde{\mathcal{N}}_n \oint_{C(\epsilon)} \frac{d\eta}{2\pi i \eta^{s+1}}\left(\frac{\eta+1}{\eta-1}\right)^n$

(Can we choose ν_n such that $\mathcal{M}_s \;\sim\; \delta_{s,k}$, switching off all spins except one?)

➢ Possible to turn on an angular dependence in the Weyl tensor singularity via specific choices of deformation parameters (e.g. $\nu_n = q^n$, exchanging sum and integral) → Kerr-like HS black-hole?

Reading off asymptotic charges

- Having found the gauge-fields generating functions, one may try to read off asymptotic charges from the sources of field strengths for r → ∞, i.e. analyzing the asymptotics of the gauge field eq.

$$\nabla\widehat{W} + \widehat{W}\star\widehat{W} + \tfrac{1}{4i}\left(r^{\alpha\beta}\widehat{M}_{\alpha\beta} + \bar{r}^{\dot{\alpha}\dot{\beta}}\widehat{\overline{M}}_{\dot{\alpha}\dot{\beta}}\right) \;=\; 0$$

$$r^{\alpha\beta} \;:=\; d\omega^{\alpha\beta} + \omega^{\alpha\gamma}\omega^{\beta}{}_{\gamma}\;, \qquad \nabla\widehat{W} \;=\; d\widehat{W} + \tfrac{1}{4i}\left[\omega^{\alpha\beta}\widehat{M}^{(0)}_{\alpha\beta} + \bar{\omega}^{\dot{\alpha}\dot{\beta}}\widehat{\overline{M}}^{(0)}_{\dot{\alpha}\dot{\beta}}\;,\;\widehat{W}\right]_{\star}$$

after moving to the standard gauge of perturbation theory and reducing to spacetime submanifold {Z=0}.

- Possible mixing between different orders in $\mathcal{M}_s$ due to s-dependent r-behaviour of spin-s component fields

$$(\nabla_{(0)}W + \{e_{(0)},W\}_{\star})_{\alpha(2s)} \;\sim\; e_{(0)}\wedge e_{(0)}\,\partial^{(y)}_{\alpha(2s)}\Phi + \text{h.o.t.} \;=\; \frac{\widehat{\mathcal{M}}_s}{r^{s+1}}(u^+u^-)^s_{\alpha(2s)}$$

leads to possible asymptotic charge redefinition

$$\widehat{\mathcal{M}}_s \;=\; \mathcal{M}_s + O(\mathcal{M}_s^2)$$

Twistor gauge and asymptotic charges

- To compare solutions in x-space, need to bring them in "universal" twistor gauge via some extended HS gauge transformation $G_{(v)}^{(K)}(x|Y,Z)$.

$$\widehat{v}^{\alpha}(x|Y,Z)\widehat{A}_{\alpha}(x|Y,Z) \;=\; \widehat{f}(x|Y,Z)\,, \qquad \frac{\partial}{\partial\nu_n}\widehat{v}^{\alpha} \;=\; \frac{\partial}{\partial K_{\underline{\alpha\beta}}}\widehat{v}^{\alpha} \;=\; 0 \;=\; \frac{\partial}{\partial\nu_n}\widehat{f} \;=\; \frac{\partial}{\partial K_{\underline{\alpha\beta}}}\widehat{f}$$

with residual gauge symmetries → $\mathfrak{ho}(3,2)$, e.g., standard choice $v^{\alpha} = z^{\alpha}$.

- Our solutions are in some twistor gauge but NOT in universal twistor gauge (v^{α} depends explicitly on K). Can be brought to twistor gauge, e.g., the standard gauge of perturbative analysis, order by order in ν_n.

- The action of G^{v}_{K} on solutions will redefine the HS asymptotic charges, too!

$$\begin{aligned} &\widehat{\Phi}_{(v)} \;=\; (\widehat{G}^{(K)}_{(v)})^{-1} \star \widehat{\Phi}_{(K)} \star \pi(\widehat{G}^{(K)}_{(v)}) \\ \longrightarrow\quad &\mathcal{M}_s\Big|_{(v)} \;=\; \mathcal{M}_s\Big|_{(K)} + \sum_{s,'s''} \mathcal{M}_{s'}\Big|_{(K)}\mathcal{M}_{s''}\Big|_{(K)} f_s^{s's''} + \dots \end{aligned}$$

- Finally, $\mathfrak{ho}(3,2)$ asymptotic symmetries (possibly enhanced to current algebra of free fields) will act $\mathcal{M}_s$. Invariants $\mathcal{O}(\mathcal{M}_s)$?

HS Invariants

- Define HS observables, gauge invariant off-shell. Weyl-curvature invariants:

$$
\begin{aligned}
\mathcal{C}^{\pm}_{2p} &= \mathcal{N}_{\pm}\widehat{Tr}_{\pm}[\mathcal{C}_{2p}]\ , \qquad \mathcal{C}_{2p} = [\widehat{\Phi}\star\pi(\widehat{\Phi})]^{\star p} \\
\widehat{Tr}_{+}[f(Y,Z)] &= \int \frac{d^4Y\,d^4Z}{(2\pi)^4} f(Y,Z)\ , \qquad \widehat{Tr}_{-}[f(Y,Z)] = \widehat{Tr}_{+}[f(Y,Z)\star\kappa\bar{\kappa}]
\end{aligned}
$$

- Ciclicity:

$$\widehat{Tr}_{\pm}[f(Y,Z)\star g(Y,Z)] = \widehat{Tr}_{\pm}[g(\pm Y,\pm Z)\star f(Y,Z)]$$

- Conserved on the field equations:

$$\widehat{D}_{\mu}\widehat{\Phi} = 0 \ \Rightarrow\ \partial_{\mu}(\widehat{\Phi}\star\kappa)^{\star q} = -[\widehat{A}_{\mu},(\widehat{\Phi}\star\kappa)^{\star q}]_{\star}$$

Ciclicity + A_{μ} even function of oscilllators

$$d\,\widehat{Tr}_{\pm}[\mathcal{C}^{\pm}_{2p}] = 0$$

$$
\begin{aligned}
\mathcal{C}^{[0]}_{k} &= \widehat{\mathrm{Tr}}_{+}\left[(\widehat{\Phi}\star\pi(\widehat{\Phi}))^{\star k}\star\kappa\bar{\kappa}\right] \\
\mathcal{I}(\sigma,k,\bar{k};\lambda,\bar{\lambda}) &= \widehat{\mathrm{Tr}}_{+}\left[(\widehat{\kappa}\widehat{\bar{\kappa}})^{\star\sigma}\star\exp_{\star}(\lambda^{\alpha}\widehat{S}_{\alpha}+\bar{\lambda}^{\dot{\alpha}}\widehat{\bar{S}}_{\dot{\alpha}})\star(\widehat{\Phi}\star\widehat{\kappa})^{\star k}\star(\widehat{\Phi}\star\widehat{\bar{\kappa}})^{\star\bar{k}}\right]
\end{aligned}
$$

Singularity?

- Radial dependence of individual spin-s Weyl tensor $\sim r^{-s-1}$. However, HS-invariants for finitely many projectors are finite!

$$Tr_+ \left[(\widehat{\Phi} \star \pi(\widehat{\Phi}))^N \star \kappa\bar{\kappa} \right] = -4 \sum_{n=\pm1,\pm2,\ldots} |n|(-1)^{(N+1)n} \mu_n^{2N}$$

Note: invariants are also (formally) insensitive to changes of ordering!
Can the singularity be only an artefact of basis choice for function of operators?
(crucial with non-polynomial f(operators))

- Examine master-fields in r = 0:

$$\Phi(r=0) = L^{-1}|_{r=0} \star P_1(E) \star L|_{r=0} \star \kappa_y = P_1(E) \star \kappa_y \sim \delta^2(y - i\sigma^0\bar{y})$$

[L (r = 0) = f(E)]

$\Rightarrow$ Weyl tensors generating function ~ $\delta^2(y)$
→ a regular function ($\exp(-2N_y)$) in normal ordering!

Cylindrically-symmetric solutions

- Condition $K'_{\underline{\alpha}}{}^{\underline{\beta}} K'_{\underline{\beta}}{}^{\underline{\gamma}} = -\delta_{\underline{\alpha}}{}^{\underline{\gamma}}$ solved by any $Y^{\underline{\alpha}} K'_{\underline{\alpha\beta}} Y^{\underline{\beta}} \sim E, J, iB, iP$

➔ Solutions with $\mathfrak{so}(2,1)_{\mathfrak{h}} \oplus \mathfrak{so}(2)_{\mathrm{YK'Y}}$ symmetry ($\mathfrak{h}$ = stabilizer of YK'Y).

- In particular, for K' = Γ_{12}, $P_1(Y) := 4e^{-\frac{1}{2}Y^{\underline{\alpha}} K'_{\underline{\alpha\beta}} Y^{\underline{\beta}}} = 4e^{-4J}$

  Again a ground state of a 2D Fock-space (a non-compact ultra-short irrep, singleton-like but with roles of E and J exchanged, E < J instead of E > J).

  [Systematic procedure to extract creation/annihilation operators, based on 3 possible inequivalent embeddings of a complexified Heisenberg algebra in the complexified Cartan subalgebra of $\mathfrak{so}$ (3,2)]

- Same steps yield $\Phi(x|Y) = \sum_n \nu_n \mathcal{N}_n \oint_{C(\varepsilon)} \frac{ds}{2\pi i} \left(\frac{s+1}{s-1}\right)^n \underbrace{L^{-1}(x) \star e^{-4sJ} \star L(x) \star \kappa_y}$

$$\bar{y} = 0 : \quad \frac{1}{s\sqrt{\varkappa^2}} \exp\left(\frac{1}{2s}\, y^\alpha \varkappa^{-1}_{\alpha\beta} y^\beta\right) , \quad \varkappa^{-1}_{\alpha\beta} = -\frac{\varkappa_{\alpha\beta}}{\varkappa^2} , \quad \varkappa^2 = 1 + r^2 \sin^2\theta$$

$$C^{(n)}_{\alpha(2s)} \sim \frac{i^{n+s+1}\mu_n}{(1 + r^2\sin^2\theta)^{(s+1)/2}} \, (\tilde{u}^+ \tilde{u}^-)^s_{\alpha(2s)}$$

Conclusions & Outlook

- Found a general class of (almost) type-D solutions, with various symmetries:
  - spherical, HS generalization of Schwarzschild bh
  - cylindrical, HS counterpart of GR Melvin solution (regular everywhere)
  - biaxial (building blocks of the previous two, “almost type-D”)

  and other ones whose physical interpretation and GR analogues are yet to be found.

- Singular? Not obvious, not at the level of invariants nor master-fields.
  1) A closed 2-form charge could detect singularities
  2) Divergent curvature invariants with infinitely many excitations

  A HS-invariant characterization of bhs is yet to be found.

- Must gain a better understanding of HS invariants and evaluate more of them.
  [HS “metrics” $G_{\mu_1 \dots \mu_s} = \widehat{Tr}_+ \left[\widehat{\kappa}\widehat{\bar{\kappa}} \star \widehat{E}_{(\mu_1} \star \cdots \star \widehat{E}_{\mu_s)} \right] \,, \quad \widehat{E}_\mu = \frac{1}{2}(1-\pi)\widehat{W}_\mu$]

- Multi-body solutions? [Preliminary analysis of consistency of a 2-body problem by evaluating 0-form invariants for $\Phi(x) + \Phi(y)$. Cross terms fall off as $V((1+r^2)^{-1/2}; n)$. Hierarchy of excitations ?]

- Thermodynamics in invariants? Horizon? Trapped surfaces?...

Conclusions & Outlook

- Study the boundary duals of such solutions. Many interesting questions:
  - What are the dual configurations in U(N)/O(N) vector models?
  - Hawking-Page phase transitions? *(Shenker-Yin '11 → No uncharged bhs in Type A minimal model)*
  - Are spacetime boundary conditions (partly) encoded in (Y,Z)-space behaviour? [Distiction small/large gauge transformation and superselection sectors]

- Role of Z-space in non-perturbative sector of the theory . In particular, "Z-space vacua", topologically non-trivial flat Z-connections.

- Solutions mixing AdS massless particle state + soliton-like state. [Particles alone are inconsistent as solutions of the full eqs., backreaction forces addition of non-perturbative states]