Black holes: From GR to HS

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Plan

- **Introduction**

  Unfolded dynamics.

- **GR black holes in $d = 4, 5$**

  Algebraic facts. Unfolding formulation.

- **Generalization to higher-spins**

  Strategy. Exact solution in $d = 4$. Symmetries.

- **Conclusion**

Unfolding of pure gravity

Einstein equations

$$R_{ab} = 0 \quad \Leftrightarrow \quad R_{ab,cd} = C_{ab,cd}$$ **Riemann=Weyl**

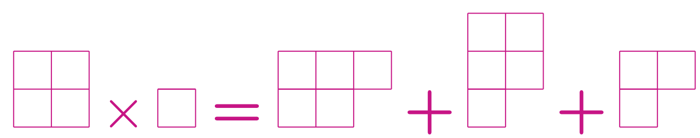
Cartan equations

$$d\omega_{ab} + \omega_a{}^c \wedge \omega_{cb} = \mathbf{C_{ac,bd}} e^c \wedge e^d \,,$$

$$De_a \equiv de_a + \omega_a{}^b \wedge e_b = 0 \,.$$

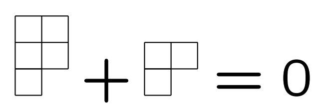
Unfolding..

$$DC_{ab,cd} = C_{ab,cd|f} e^f :$$

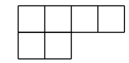


Bianchi identities:

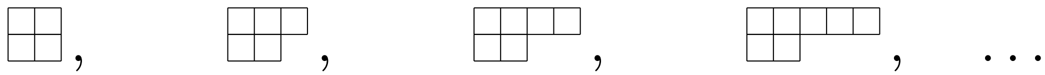
$$e^b \wedge e^d DC_{ab,cd} = 0 \quad \Rightarrow$$



$$\mathbf{DC_{ab,cd} = (2C_{abf,cd} + C_{abc,df} + C_{abd,cf})e^f}$$

$$DC_{abc,de} = C_{abc,de|f}e^f \quad \textbf{Bianchi identities} \quad \Rightarrow \quad C_{abc,de|f} \sim$$ 

Gravity unfolded module $C_{\dots,\dots}$:



Unfolded equations

$$DC_{a_1\dots a_{k+2},b_1b_2} = ((k+2)C_{a_1\dots a_{k+2}c,b_1b_2} + C_{a_1\dots a_{k+2}b_1,b_2c} + C_{a_1\dots a_{k+2}b_2,b_1c})e^c$$

Second Bianchi identity

$$[D,D] \sim C_{ab,cd} \quad \Rightarrow \quad \text{nonlinear corrections}$$

$$DC_{a_1a_2a_3,b_1b_2} = (3C_{a_1a_2a_3c,b_1b_2} + C_{a_1a_2a_3b_1,b_2c} + C_{a_1a_2a_3b_2,b_1c} + (\textbf{Weyl})^2)e^c$$

schematically

$$\mathrm{DC} = \mathcal{F}_{\mathrm{a}}(\mathrm{C})\mathrm{e}^{\mathrm{a}}$$

solved up to $O(C^2)$ in $d=4$, M.A. Vasiliev, '89

Black holes in GR.

- At least two isometries (any d)

- $d = 4$ Weyl tensor is of Petrov type D

$$C^{\pm}_{ab,cd} \sim (\Phi^{\pm}\Phi^{\pm})_{ab,cd}\,, \qquad \Phi_{ab} = -\Phi_{ba}\,.$$

- $d = 3$ BTZ black hole is completely determined by an AdS_3 single isometry ξ

$$BTZ = AdS_3/\exp t\xi$$

  Type of BH is classified by inequivalent ξ with respect to AdS_3 adjoint action

$$\xi \to M\xi M^{-1}$$

- Hidden symmetries (Killing-Yano tensor Φ_{ab})

$$D\Phi_{ab} = v_a \mathbf{e}_b - v_b \mathbf{e}_a \,, \qquad \Phi_{ab} = -\Phi_{ba}$$

General decomposition:

$$D\Phi_{ab} = \Phi_{ab|c}\mathbf{e}^c \quad \Phi_{ab|c} \rightarrow \yng(1,1) \times \yng(1) = \yng(2,1) + \yng(1,1,1) + \yng(1)$$

Absence of $\yng(2,1)$ and $\yng(1,1,1)$ is a manifestation of the **hidden symmetry**

Φ_{ab} entails (on-shell): v_a – Killing vector, $K_{(ab)} = \Phi_a{}^c\Phi_{cb}$ – Killing tensor, $K_{ab}v^b$ –Killing vector.

Φ_{ab} in AdS (Minkowski) space-time

AdS :

$$d\omega_{ab} + \omega_a{}^c \wedge \omega_{cb} = \Lambda \mathbf{e}_a \wedge \mathbf{e}_b \,,$$

$$d\mathbf{e}_a + \omega_a{}^b \wedge \mathbf{e}_b = 0$$

embedding - zero curvature representation

$$W_{AB} = (\omega_{ab}, \sqrt{\Lambda}\mathbf{e}_a) \quad \Rightarrow \quad dW_{AB} + W_A{}^C \wedge W_{CB} = 0$$

Global symmetries

$$\delta W_{AB} = D_0 \xi_{AB} \equiv d\xi_{AB} + W_A{}^C \xi_{CB} + W_B{}^C \xi_{AC} = 0 \,, \qquad \boldsymbol{\xi_{AB} \to (\Phi_{ab}, v_a)}$$

$$D v_a = -\Lambda \Phi_{ab} \mathbf{e}^b$$

$$D \Phi_{ab} = v_a \mathbf{e}_b - v_b \mathbf{e}_a$$

ξ_{AB} – AdS **global symmetry parameter**

AdS Black holes from AdS global symmetry parameter

- $d = 3$ BTZ black hole from a single AdS_3 isometry factorization

  (M. Henneaux+BTZ, '93)

- **d=4** $AdS - Kerr$ from an AdS isometry μ-deformation

  (V.D, A.S. Matveev, M.A. Vasiliev) **d=5** **(V.D)**

$$Dv_a = -\Lambda\Phi_{ab}\mathbf{e}^b + \mathcal{F}(\mu, \mathbf{\Phi_{ab}}, \mathbf{e_a})$$

$$D\Phi_{ab} = v_a\mathbf{e}_b - v_b\mathbf{e}_a$$

Integrating flow $\frac{\partial}{\partial\mu}$ links AdS to BH

$$g^{BH}_{mn} = g^{AdS}_{mn} + \mathbf{f_{mn}}(\xi_{\mathbf{AB}})$$

BH mass m and angular momenta a_i are encoded in Casimir invariants $Tr(\xi^n_{AB})$. $\sharp(m, a_i)$**=rank O(d-1,2)**

d=4 Kerr-NUT Black hole

Spinor form for AdS_4 equations:

$$DV_{\alpha\dot\alpha} = \frac{1}{2}\mathbf{e}^{\gamma}{}_{\dot\alpha}\Phi_{\gamma\alpha} + \frac{1}{2}\mathbf{e}_{\alpha}{}^{\dot\gamma}\bar\Phi_{\dot\alpha\dot\gamma}$$

$$D\Phi_{\alpha\alpha} = \lambda^2\mathbf{e}_{\alpha}{}^{\dot\gamma}V_{\alpha\dot\gamma}\,, \qquad D\bar\Phi_{\dot\alpha\dot\alpha} = \lambda^2\mathbf{e}^{\gamma}{}_{\dot\alpha}V_{\gamma\dot\alpha}\,.$$

1. AdS_4 covariant form

$$K_{AB} = K_{BA} = \begin{pmatrix} \lambda^{-1}\Phi_{\alpha\beta} & V_{\alpha\dot\beta} \\ V_{\beta\dot\alpha} & \lambda^{-1}\bar\Phi_{\dot\alpha\dot\beta} \end{pmatrix}, \qquad \Omega_{AB} = \Omega_{BA} = \begin{pmatrix} \Omega_{\alpha\beta} & -\lambda\mathbf{e}_{\alpha\dot\beta} \\ -\lambda\mathbf{e}_{\beta\dot\alpha} & \bar\Omega_{\dot\alpha\dot\beta} \end{pmatrix} =$$

$$D_0 K_{AB} = 0\,, \qquad D_0^2 \sim R_{0AB} = d\Omega_{AB} + \frac{1}{2}\Omega_A{}^C\wedge\Omega_{CB} = 0\,.$$

K_{AB} – AdS_4 **global symmetry parameter**

2. Two first integrals

related to two AdS_4 invariants (Casimir operators)

$$C_2 = \frac{1}{4}K_{AB}K^{AB} = I_1\,, \qquad C_4 = \frac{1}{4}\mathrm{Tr}K^4 = I_1^2 + \lambda^2 I_2$$

Deformation of $AdS_4 \rightarrow$ black hole unfolded system

(Keep the same form of the unfolded equations)

$$\mathcal{D}V_{\alpha\dot{\alpha}} = \frac{1}{2}\rho\, \mathbf{e}^{\gamma}{}_{\dot{\alpha}}\Phi_{\gamma\alpha} + \frac{1}{2}\bar{\rho}\, \mathbf{e}_{\alpha}{}^{\dot{\gamma}}\bar{\Phi}_{\dot{\alpha}\dot{\gamma}}\,,$$

$$\mathcal{D}\Phi_{\alpha\alpha} = \mathbf{e}_{\alpha}{}^{\dot{\gamma}}V_{\alpha\dot{\gamma}}\,,$$

$$\mathcal{D}\bar{\Phi}_{\dot{\alpha}\dot{\alpha}} = \mathbf{e}^{\gamma}{}_{\dot{\alpha}}V_{\gamma\dot{\alpha}}\,.$$

Unlike the AdS_4 case with $\rho = -\lambda^2$ we assume ρ to be arbitrary

Bianchi identities: $\mathcal{D}^2 \sim \mathbf{R}\,, \qquad \mathcal{D}\mathbf{R} = 0$

fix ρ uniquely in the form

$$\rho(G,\bar{G}) = \mathcal{M}G^3 - \lambda^2 - q\,\bar{G}G^3\,, \qquad G = \frac{1}{\sqrt{\det \Phi_{\alpha\beta}}}$$

Integrating flow and solution space

The flow $\frac{\partial}{\partial \chi}$, where $\chi = (\mathcal{M}, q)$

$$[d, \frac{\partial}{\partial \chi}] = 0$$

allows one to express BH fields in terms of AdS_4 global symmetry parameter K_{AB}. This identifies the solution space

- **Generic K_{AB}, $\mathcal{M}$-complex** –

  Carter-Plebanski class of metrics. Parameters: $Re\mathcal{M}$, $Im\mathcal{M}$, C_2, C_4, q, Λ

- **Kerr-Newman, $C_2 > 0$, $\mathcal{M} > 0$**

  Parameters: $M, a(C_4), q$

- $K_A^2{}^B = -\delta_A{}^B$ – Schwarzschild ($V^2 < 0$), Taub-NUT ($V^2 > 0$)

$d = 5$ black holes

metric (Hawking-Hunter)

$$ds^2 = -\frac{\Delta}{\rho^2}\Big(dt - \frac{a\sin^2\theta}{\Xi_a}d\phi - \frac{b\cos^2\theta}{\Xi_b}d\psi\Big)^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\Big(adt - \frac{r^2+a^2}{\Xi_a}d\phi\Big)^2 + \frac{\Delta_\theta \cos^2\theta}{\rho^2}\Big(bdt - \frac{r^2+b^2}{\Xi_b}d\psi\Big)^2 +$$

$$+\frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{(1-\Lambda r^2)}{r^2\rho^2}\Big(abdt - \frac{b(r^2+a^2)\sin^2\theta}{\Xi_a}d\phi - \frac{a(r^2+b^2)\cos^2\theta}{\Xi_b}d\psi\Big)^2$$

where

$$\Delta = \frac{1}{r^2}(r^2+a^2)(r^2+b^2)(1-\Lambda r^2) - 2M\,, \qquad \Delta_\theta = 1 + \Lambda a^2\cos^2\theta + \Lambda b^2 \sin^2\theta$$

$$\rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta\,, \qquad \Xi_a = 1 + \Lambda a^2\,, \qquad \Xi_b = 1 + \Lambda b^2$$

Horizon:

$$\Delta(r_+) = 0$$

Black hole unfolded system

AdS_5 unfolded equations

$$Dv_{\alpha\beta} = -\frac{\Lambda}{8}(\Phi_\alpha{}^\gamma \mathbf{e}_{\gamma\beta} - \Phi_\beta{}^\gamma \mathbf{e}_{\gamma\alpha}), \quad D\Phi_{\alpha\beta} = \frac{1}{2}(v_\alpha{}^\gamma \mathbf{e}_{\gamma\beta} + v_\beta{}^\gamma \mathbf{e}_{\gamma\alpha}) \quad \text{consistency} \quad D^2 \sim R_{ads}$$

μ-deformation $\rightarrow$

$$Dv_{\alpha\beta} = -\frac{\Lambda}{8}(\Phi_\alpha{}^\gamma \mathbf{e}_{\gamma\beta} - \Phi_\beta{}^\gamma \mathbf{e}_{\gamma\alpha}) + \frac{\mu}{H}(\mathbf{\Phi}_\alpha^{-1\gamma} \mathbf{e}_{\gamma\beta} - \mathbf{\Phi}_\beta^{-1\gamma} \mathbf{e}_{\gamma\alpha}),$$

$$D\Phi_{\alpha\beta} = \frac{1}{2}(v_\alpha{}^\gamma \mathbf{e}_{\gamma\beta} + v_\beta{}^\gamma \mathbf{e}_{\gamma\alpha}), \qquad H = \sqrt{\det \Phi_{\alpha\beta}}$$

$$D^2 \sim R_{ads} + \mathbf{C_{BH}}$$

$$C_{\alpha\beta\gamma\delta} = -\frac{32\mu}{H^3}((\Phi^{-1})_{\alpha\beta}(\Phi^{-1})_{\gamma\delta} + (\Phi^{-1})_{\alpha\gamma}(\Phi^{-1})_{\beta\delta} + (\Phi^{-1})_{\alpha\delta}(\Phi^{-1})_{\beta\gamma})$$

Black holes

$$v_{\alpha\beta} = v^0_{\alpha\beta} = const\,, \quad \Phi_{\alpha\beta} = \frac{1}{2}(v_\alpha{}^\gamma x_{\gamma\beta} + v_\beta{}^\gamma x_{\gamma\alpha}) + \Phi^0_{\alpha\beta}\,, \quad \Phi^0_{\alpha\beta} = const$$

Type	**Killing vector**	**Lorentz generator** $\Phi^0_{\alpha\beta}$	P^2	I_1	I_2
Kerr	$\frac{\partial}{\partial t}$	$a\Gamma^{xy}_{\alpha\beta} + b\Gamma^{zu}_{\alpha\beta}$	-1	$b^2 + a^2$	$2ab$
light-like Kerr	$\frac{\partial}{\partial t} + \frac{\partial}{\partial x}$	$a\Gamma^{xy}_{\alpha\beta} + b\Gamma^{zu}_{\alpha\beta}$	0	a^2	$2ab$
tachyonic Kerr	$\frac{\partial}{\partial x}$	$a\Gamma^{ty}_{\alpha\beta} + b\Gamma^{zu}_{\alpha\beta}$	$+1$	$a^2 - b^2$	$2ab$

Classification of Kerr-Schild solutions on 5d Minkowski space according to its Poincare invariants

$$\mathbf{g_{mn}} = \eta_{\mathbf{mn}} + \frac{2\mu}{H}\mathbf{k_m k_n}$$

Projectors and Kerr-Schild vectors

Projectors:

$$\Pi^{\pm}_{\alpha\beta} = \frac{1}{2}(\epsilon_{\alpha\beta} \pm X_{\alpha\beta})$$

$$X_{\alpha\beta} = X_{\beta\alpha}\,, \quad \mathbf{X_\alpha{}^\gamma X_\gamma{}^\beta = \delta_\alpha{}^\beta} \Rightarrow$$

$$\Pi^{\pm}_{\alpha}{}^{\gamma}\Pi^{\pm}_{\gamma\beta} = \Pi^{\pm}_{\alpha\beta}\,, \quad \Pi^{\pm}_{\alpha}{}^{\gamma}\Pi^{\mp}_{\gamma\beta} = 0\,, \quad \Pi^{+}_{\alpha\beta} = -\Pi^{-}_{\beta\alpha}\,.$$

Light-like vectors:

$$v^{+}_{\alpha\beta} = \Pi^{+}_{\alpha\gamma}\Pi^{+}_{\beta\delta}v^{\gamma\delta}\,, \quad v^{-}_{\alpha\beta} = \Pi^{-}_{\alpha\gamma}\Pi^{-}_{\beta\delta}v^{\gamma\delta} \Rightarrow \quad \mathbf{v^{+}v^{+} = v^{-}v^{-} = 0}$$

Specify $X_{\alpha\beta}$:

$$\mathbf{X}_{\alpha\beta} = \frac{1}{2r}(\mathbf{\Phi}_{\alpha\beta} + \mathbf{H}\mathbf{\Phi}^{-1}_{\alpha\beta})\,, \quad \mathbf{r^2} = \frac{1}{2}(\mathbf{H} - \mathbf{Q})$$

Kerr-Schild vectors :

$$k_{\alpha\beta} = \frac{v^{+}_{\alpha\beta}}{v^{+}v^{-}}\,, \qquad n_{\alpha\beta} = \frac{v^{-}_{\alpha\beta}}{v^{+}v^{-}}\,, \qquad \mathbf{v^{+}v^{-}} = \frac{1}{4}\mathbf{v}^{+}_{\alpha\beta}\mathbf{v}^{-\alpha\beta}$$

$$k_a v^a = n_a v^a = 1\,, \qquad k^a k_a = n^a n_a = 0$$

geodetic condition:

$$\mathbf{k^a D_a k_b = n^a D_a n_b = 0}$$

Kerr-Schild vectors and massless fields

$$\phi_{a_1 \ldots a_s} = \frac{1}{H} k_{a_1} \ldots k_{a_s}$$

$$\Box \phi_{a_1 \ldots a_s} - s D_b D_{(a_1} \phi_{a_2 \ldots a_s)}{}^b = -\frac{\Lambda}{2}(\mathbf{s}-\mathbf{1})(\mathbf{s}+\mathbf{2})\phi_{a_1 \ldots a_s}$$

Black holes

$$g_{mn} = g^{0}_{mn} + \frac{2\mu}{H} k_m k_n$$

Towards higher-spin BH

- **d=4 – Static BPS HS black hole** (V.D, M.A. Vasiliev, 2009)

- **d=4 – D-type class of solutions** (C. Iazeolla, P. Sundell, 2011)

- **d=3 HS asymptotic symmetries** (M. Henneaux, S-J. Rey, $\oplus$ A. Campoleoni, S. Fredenhagen, S. Pfenninger, S. Theisen, 2010)

- **d=3 – Static $sl(3) \oplus sl(3)$ black hole** (M. Gutperle, P. Kraus, 2011)

  $sl(N) \oplus sl(N)$, $hs(\lambda) \oplus hs(\lambda)$ – great deal of interest

GR black holes $\rightarrow$ SUGRA black holes $\rightarrow$ HS black holes **???**

Obstacles:

1. HS does not have decoupled spin-2 sector $\rightarrow$ all higher spins involved in the equations of motion.

2. The interval $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ is not gauge invariant quantity in higher spin algebra.

Perturbative analysis available

HS theory $\rightarrow$ 0-th order vacua AdS $\rightarrow$ 1-st order free field Fronsdal equations $\rightarrow$ 2-nd order interactions $\rightarrow \ldots$

Program for HS black holes

vacuum AdS_4 $\rightarrow$ black hole massless fields $\phi_{\mu_1 \ldots \mu_s}(x)$ $\rightarrow$ HS corrections $\rightarrow \ldots$

Kerr-Schild fields from free HS theory

- **Free HS equations**

HS field strengths $C(y,\bar{y}|x) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} C_{\alpha(n),\dot{\alpha}(m)} y^{\alpha} \dots y^{\alpha} \bar{y}^{\dot{\alpha}} \dots \bar{y}^{\dot{\alpha}}$

HS potentials $w(y,\bar{y}|x) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} w_{\alpha(n),\dot{\alpha}(m)} y^{\alpha} \dots y^{\alpha} \bar{y}^{\dot{\alpha}} \dots \bar{y}^{\dot{\alpha}}$

Equations of motion:

$$\tilde{\mathcal{D}}_0 \mathbf{C} \equiv \mathbf{dC} - \mathbf{w_0} \star \mathbf{C} + \mathbf{C} \star \tilde{\mathbf{w}}_0 = 0 \qquad \leftarrow \textbf{twisted-adjoint}$$

$$\mathcal{D}_0 \mathbf{w} \equiv \mathbf{dw} - [\mathbf{w_0}, \mathbf{w}]_\star = \mathbf{R_1(C)} \qquad \leftarrow \textbf{adjoint}$$

$\tilde{f}(y,\bar{y}) = f(-y,\bar{y}) \leftarrow$ **twist operator**

$\mathbf{w_0(y,\bar{y}|x)}$ $-$ AdS_4 vacuum connection

matter fields: scalar $s=0 \rightarrow C(x)$, fermion $s=1/2 \rightarrow C_\alpha(x) \oplus \bar{C}_{\dot{\alpha}}(x)$

HS fields: potentials $\rightarrow \omega_{\alpha(s-1),\dot{\alpha}(s-1)}$, strengths$\rightarrow$ $C_{\alpha(2s)} \oplus \bar{C}_{\dot{\alpha}(2s)}$

- **Star-product operation**

  Let $Y_A = (y_\alpha, \bar{y}_{\dot{\alpha}})$ be commuting variables.

$$\mathbf{(f \star g)(Y) = \int f(Y + U)g(Y + V)e^{U_A V^A} dUdV} \longrightarrow$$

  associative algebra with

$$\mathbf{[Y_A, Y_B]_\star = -2\epsilon_{AB}}$$

- **AdS_4 vacuum**

Introduce 1-form $w_0 \in o(3,2) \sim sp(4)$

$$w_0 = -\frac{1}{8}(\omega_{\alpha\alpha} y^\alpha y^\alpha + \bar{\omega}_{\dot\alpha\dot\alpha}\bar{y}^{\dot\alpha}\bar{y}^{\dot\alpha} - 2\lambda \mathbf{e}_{\alpha\dot\alpha} y^\alpha \bar{y}^{\dot\alpha})\,, \qquad dw_0 - w_0 \star \wedge w_0 = 0$$

Equiv. to

$$d\omega_{\alpha\alpha} + \frac{1}{2}\omega_\alpha{}^\gamma \wedge \omega_{\gamma\alpha} = \frac{\lambda^2}{2}\mathbf{e}_{\alpha\dot\gamma} \wedge \mathbf{e}_\alpha{}^{\dot\gamma} \quad \rightarrow \quad$$ AdS_4 **Riemann tensor**

$$d\mathbf{e}_{\alpha\dot\alpha} + \frac{1}{2}\omega_\alpha{}^\gamma \mathbf{e}_{\gamma\dot\alpha} + \frac{1}{2}\bar{\omega}_{\dot\alpha}{}^{\dot\gamma} h_{\alpha\dot\gamma} = 0 \quad \rightarrow \quad$$ **zero torsion**

Kerr-Schild HS solution

- **Fix AdS_4 global symmetry parameter $K_{AB} = K_{BA}$**

$$D_0 K_{AB} = 0 \quad \Rightarrow$$

$$\mathcal{D}_0 K_{AB} Y^A Y^B \equiv dK_{AB} Y^A Y^B - [w_0, K_{AB} Y^A Y^B]_\star = 0$$

any function $f(K_{AB} Y^A Y^B)$ is a HS global symmetry parameter

$$\mathcal{D}_0 f(K_{AB} Y^A Y^B) = 0$$

- **Solving for linearized $C(y, \bar{y}|x)$ in the curvature sector**

$$dC - w_0 \star C + C \star \tilde{w}_0 = 0\,, \qquad \mathbf{C = 2\pi f(\frac{1}{2}K_{AB}Y^AY^B) \star \delta^{(2)}(y)}$$

$$K_{AB} = K_{BA} = \begin{pmatrix} \lambda^{-1}\Phi_{\alpha\beta} & V_{\alpha\dot{\beta}} \\ V_{\beta\dot{\alpha}} & \lambda^{-1}\bar{\Phi}_{\dot{\alpha}\dot{\beta}} \end{pmatrix}$$

$$\mathbf{C(y, \bar{y}|x) = \int d^2u f(\frac{1}{2}\Phi_{\alpha\beta}u^\alpha u^\beta + V_{\alpha\dot{\alpha}}u^\alpha\bar{y}^{\dot{\alpha}} + \frac{1}{2}\bar{\Phi}_{\dot{\alpha}\dot{\beta}}\bar{y}^{\dot{\alpha}}\bar{y}^{\dot{\beta}})} \exp(\mathbf{iu_\alpha y^\alpha})$$

- **Reality condition**

$$(C(y,\bar{y}|x))^{\dagger} = C(-y,\bar{y}|x)$$

Generic $f(\frac{1}{2}K_{AB}Y^{A}Y^{B})$ **does not meet reality condition for** C

- **Choose** f **in the form**

$$f = \mathbf{M} \exp \frac{1}{2} K_{AB} Y^{A} Y^{B}$$

- $C^{\dagger} = \tilde{C} \qquad \Rightarrow$

$$\mathbf{K_A{}^C K_C{}^B} = -\delta_{\mathbf{A}}{}^{\mathbf{B}}$$

$V_{\alpha\dot{\alpha}}$ **– time-like**, $\quad M = \bar{M} \quad \Rightarrow$ **Schwarzschild**

$V_{\alpha\dot{\alpha}}$ **– space-like**, $\quad M = -\bar{M} \quad \Rightarrow$ **Taub-NUT**

Higher-spin curvatures:

$$C = \frac{M}{r} \exp\left(\frac{1}{2}\Phi^{-1}_{\alpha\alpha} y^\alpha y^\alpha + \frac{1}{2}\bar{\Phi}^{-1}_{\dot\alpha\dot\alpha}\bar{y}^{\dot\alpha}\bar{y}^{\dot\alpha} - i\Phi^{-1}_{\alpha\gamma} v^\gamma{}_{\dot\alpha} y^\alpha \bar{y}^{\dot\alpha}\right)$$

$$\mathbf{C}_{\alpha(\mathbf{2n})} = \frac{M}{r}(\mathbf{\Phi}^{-\mathbf{1}}_{\alpha\alpha})^{\mathbf{n}}, \qquad \bar{\mathbf{C}}_{\dot\alpha(\mathbf{2n})} = \frac{M}{r}(\bar{\mathbf{\Phi}}^{-\mathbf{1}}_{\dot\alpha\dot\alpha})^{\mathbf{n}}$$

Connections:

$$\phi_{\mu_1\ldots\mu_n} = \frac{M}{r}\mathbf{k}_{\mu_1}\ldots\mathbf{k}_{\mu_n} \quad \rightarrow \textbf{BH Fronsdal fields}$$

- **Nonlinear HS equations**

$$w(y,\bar{y}|x) \to W(y,\bar{y},z,\bar{z}|x)\,, \qquad C(y,\bar{y}|x) \to B(y,\bar{y},z,\bar{z}|x)$$

Pure gauge compensator 1-form $\to$ $\quad S(Z,Y|x) = S_\alpha dz^\alpha + \bar{S}_{\dot\alpha} dz^{\dot\alpha}$

Introducing $A = d + W + S$, HS nonlinear equations take the form

$$A \star \wedge A = \mathcal{R}(B, v, \bar{v})\,, \qquad [B, A]_\star = 0$$

Component form (bosonic eqs.) $\to$

$$dW - W \star \wedge W = 0\,, \qquad dB - W \star B + B \star \tilde{W} = 0\,,$$

$$dS_\alpha - [W, S_\alpha]_\star = 0\,, \quad d\bar{S}_{\dot\alpha} - [W, \bar{S}_{\dot\alpha}]_\star = 0\,,$$

$$S_\alpha \star S^\alpha = 2(1 + B \star v)\,, \quad \bar{S}_{\dot\alpha} \star \bar{S}^{\dot\alpha} = 2(1 + B \star \bar{v})\,, \quad [S_\alpha, \bar{S}_{\dot\alpha}]_\star = 0\,,$$

$$B \star \tilde{S}_\alpha + S_\alpha \star B = 0\,, \quad B \star \tilde{\bar{S}}_{\dot\alpha} + \bar{S}_{\dot\alpha} \star B = 0\,,$$

Dynamical potentials and field strengths: $W(Y,Z|x)_{|Z=0}\,, B(Y,Z|x)_{|Z=0}$

New ingredients

- (Y, Z) **star-product:** Let $Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$ and $Z_A = (z_\alpha, \bar{z}_{\dot\alpha})$ be commuting variables.

$$(\mathbf{f} \star \mathbf{g})(\mathbf{Y}, \mathbf{Z}) = \int \mathbf{f}(\mathbf{Y} + \mathbf{s}, \mathbf{Z} + \mathbf{s})\mathbf{g}(\mathbf{Y} + \mathbf{t}, \mathbf{Z} - \mathbf{t})\mathbf{e}^{\mathbf{s}_\mathbf{A}\mathbf{t}^\mathbf{A}}\mathbf{dsdt} \longrightarrow$$

associative algebra with

$$[\mathbf{Z_A}, \mathbf{Z_B}]_\star = -[\mathbf{Y_A}, \mathbf{Y_B}]_\star = 2\epsilon_{\mathbf{AB}}\,, \qquad [\mathbf{Y_A}, \mathbf{Z_B}]_\star = \mathbf{0}$$

- **Klein operators**

$$\mathbf{v} = \exp\left(\mathbf{z}_\alpha \mathbf{y}^\alpha\right), \quad \bar{\mathbf{v}} = \exp\left(\bar{\mathbf{z}}_{\dot\alpha} \bar{\mathbf{y}}^{\dot\alpha}\right)$$

$$\mathbf{v} \star \mathbf{v} = \bar{\mathbf{v}} \star \bar{\mathbf{v}} = \mathbf{1}\,, \qquad \mathbf{v} \star \mathbf{f}(\mathbf{y}, \mathbf{z}) = \mathbf{f}(-\mathbf{y}, -\mathbf{z}) \star \mathbf{v}\,, \qquad \bar{\mathbf{v}} \star \mathbf{f}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) = \mathbf{f}(-\bar{\mathbf{y}}, -\bar{\mathbf{z}})$$

Solving nonlinear HS equations

Main idea: Function $F_K = \exp\left(\frac{1}{2}K_{AB}Y^AY^B\right)$ generates invariant subspace in the star-product algebra and provides suitable ansatz for solving nonlinear HS equations

Properties of F_K

1. $F_K \star F_K = F_K\,, \quad Y_{-A} \star F_K = F_K \star Y_{+A} = 0\,,$ $Y_{\pm A} = \Pi_{\pm A}{}^C Y_C = \frac{1}{2}(\delta_A{}^B \pm K_A{}^B)Y_B$ $\leftarrow$ **Fock vacuum projector**
2. $\mathcal{D}_0 F_K = 0$ $\quad\leftarrow$ **by definition**
3. **Generates subalgebra of the form** $F_K\phi(a|x)$, where $a_A = Z_A + K_A{}^B Y_B$

$$(F_K\phi_1(a|x)) \star (F_K\phi_2(a|z)) = F_K(\phi_1(a|x) * \phi_2(a|x))$$

$*$ - **is Fock induced associative star-product operation on the space of a_A - oscillators**

* - properties

1. **associativity** $\rightarrow$ $(\phi_1 * \phi_2) * \phi_3 = \phi_1 * (\phi_2 * \phi_3)$

$$[a_A, a_B]_* = 2\epsilon_{AB}$$

2. **Admits Klein operators of the form**

$$K = \frac{1}{r} \exp\left(\frac{1}{2}\Phi^{-1}_{\alpha\alpha} a^\alpha a^\alpha\right), \qquad \bar{K} = \frac{1}{r} \exp\left(\frac{1}{2}\bar{\Phi}^{-1}_{\dot\alpha\dot\alpha} \bar{a}^{\dot\alpha} \bar{a}^{\dot\alpha}\right)$$

$$K * K = \bar{K} * \bar{K} = 1\,, \quad \{K, a_\alpha\}_* = \{\bar{K}, \bar{a}_{\dot\alpha}\}_* = 0$$

3. **Differential** $\rightarrow$ $\mathcal{Q} = \hat{d} - \frac{1}{2} dK^{AB} \frac{\partial^2}{\partial a^A \partial a^B}$

$$\mathcal{Q}(f(a|x) * g(a|x)) = \mathcal{Q}f(a|x) * g(a|x) + f(a|x) * \mathcal{Q}g(a|x)\,,$$

$$\mathcal{Q}^2 = 0\,, \qquad \mathcal{Q}a_A = 0\,, \qquad \mathcal{Q}K = 0$$

The Ansatz

$$B = MF_K \star \delta(y)\,,$$

$$S_\alpha = z_\alpha + F_K \sigma_\alpha(a|x)\,, \qquad \bar{S}_{\dot\alpha} = \bar{z}_{\dot\alpha} + F_K \bar{\sigma}_{\dot\alpha}(\bar{a}|x)\,,$$

$$W = w_0(y,\bar{y}|x) + F_K(\omega(a|x) + \bar{\omega}(\bar{a}|x))\,,$$ w_0 **is the** AdS_4 **connection**

HS equations reduce to "$3d$ massive equations" :

$$[s_\alpha, s_\beta]_* = 2\epsilon_{\alpha\beta}(1 + M\cdot K)\,,$$

$$\mathcal{Q}s_\alpha - [\omega, s_\alpha]_* = 0\,,$$

$$\mathcal{Q}\omega - \omega * \wedge\omega = 0\,,$$

where $s_\alpha \equiv a_\alpha + \sigma_\alpha(a|x)$ - the so called deformed oscillators (Wigner).

Note: $3d$ **HS equations around the vacuum** $B_0 = \nu = const$ **were considered by Prokushkin and Vasiliev and were shown to provide massive field dynamics with the mass scale** ν

Exact solution

$$S_\alpha = z_\alpha + MF_K \frac{a^+_\alpha}{r} \int_0^1 \mathrm{d}t \, \exp\left(\frac{t}{2}\kappa^{-1}_{\beta\beta} a^\beta a^\beta\right),$$

$$\bar{S}_{\dot\alpha} = \bar{z}_{\dot\alpha} + MF_K \frac{\bar{a}^+_{\dot\alpha}}{r} \int_0^1 \mathrm{d}t \, \exp\left(\frac{t}{2}\bar\kappa^{-1}_{\dot\beta\dot\beta} \bar{a}^{\dot\beta} \bar{a}^{\dot\beta}\right),$$

$$B = \frac{M}{r} \exp\left(\frac{1}{2}\kappa^{-1}_{\alpha\beta} y^\alpha y^\beta + \frac{1}{2}\bar\kappa^{-1}_{\dot\alpha\dot\beta} \bar{y}^{\dot\alpha} \bar{y}^{\dot\beta} - \kappa^{-1}_{\alpha\gamma} v^\gamma{}_{\dot\alpha} y^\alpha \bar{y}^{\dot\alpha}\right),$$

$$W = W_0 + \left(\frac{M}{8r} F_K d\tau^{\alpha\beta} \pi^{+\alpha}_\beta a_\alpha a_\alpha \int_0^1 dt(1-t) \exp\left(\frac{t}{2}\kappa^{-1}_{\beta\beta} a^\beta a^\beta\right) + F_K f_0 + c.c.\right),$$

Symmetries

Let $\epsilon(Z, Y|x)$ be a global symmetry parameter $\rightarrow$

$$B \star \epsilon - \tilde{\epsilon} \star B = 0\,, \quad [S, \epsilon]_\star = 0\,, \quad d\epsilon - [W, \epsilon]_\star = 0 \quad \Rightarrow$$

$$[F_K, \epsilon]_\star = 0$$

$$\epsilon(Y|x) = \sum_{m,n=1}^{\infty} f_{0A(m),B(n)}(x) \underbrace{Y_+^A \star \ldots \star Y_+^A}_{m} \star \underbrace{Y_-^B \star \ldots \star Y_-^B}_{n} + c_0(x) =: f(Y_-, Y_+) : + c_0, \quad \mathcal{D}_0 f = 0$$

Max. finite dimensional subalgebra: $T^{AB} = Y_+^{(A} Y_-^{B)}$, $T = Y_{-A} Y_+^A \quad \rightarrow su(2) \oplus u(1)$

More (Supersymmetry)! Global SUSY is a quarter of the $\mathcal{N} = 2$ SUSY with two supergenerators $\mathcal{Q}_A^\alpha$ of the AdS_4 vacuum.

Vacuum AdS_4 symmetry algebra $osp(2,4)$ is broken giving BPS HS black hole

Conclusion

- It is demonstrated that unfolded formulation allows to describe GR black holes in terms of AdS global symmetry parameter in coordinate invariant way.

- The construction admits natural generalization to higher-spins, resulting in HS Schwarzschild and Taub-NUT exact solutions.

Open problems

- What is black about HS black hole? (**Horizons, singularities, entropy, temperature**)

- Black rings, are they within reach of the AdS global symmetry parameter?