Higher Spin Gravity
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Topics in asymptotically flat gravity in 3 and 4 dimensions

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Overview

3d AdS gravity in BMS gauge

3d flat gravity as a modified Penrose limit

G. B., A. Gomberoff, and H. Gonzalez, "The flat limit of three dimensional asymptotically anti-de Sitter spacetimes." to appear.

"Higher spin like" infinite-dimensional extension of 3d flat gravity

G. B., A. Garbarz, G. Giribet, and M. Leston, "A Chern-Simons action for the Virasoro algebra." in preparation.

4d flat gravity, null infinity: symmetries & charges

G. B., C. Troessaert,

"Symmetries of asymptotically flat 4 dimensional spacetimes at null infinity revisited," *Phys. Rev. Lett.* **105** (2010) 111103, 0909.2617.

"Aspects of the BMS/CFT correspondence," *JHEP* **05** (2010) 062, 1001.1541.

"BMS charge algebra," *JHEP* **1112** (2011) 105, 1106.0213.

G. B., P.-H. Lambert, "A note on the Newman-Unti group," 1102.0589.

AdS3 FG gauge

Fefferman-Graham gauge

$$g_{\mu\nu} = \begin{pmatrix} \frac{l^2}{r^2} & 0 \\ 0 & g_{AB} \end{pmatrix} \qquad \Lambda = -\frac{1}{l^2}$$

2d metric $\quad g_{AB} = r^2 \bar{\gamma}_{AB}(x^C) + O(1) \qquad r \quad t, \phi$

$\bar{\gamma}_{AB} = \eta_{AB}$ flat metric on the cylinder $\quad \eta_{AB} = -d\tau^2 + d\phi^2, \quad \tau = \frac{t}{l}$

existence of general solution

integration "constants"

$$\Xi_{++} = \Xi_{++}(x^+), \quad \Xi_{--} = \Xi_{--}(x^-)$$

$$x^{\pm} = \tau \pm \phi$$

$$g_{AB}dx^A dx^B = -(r^2 + \frac{l^4}{r^2}\Xi_{++}\Xi_{--})dx^+dx^- + l^2\Xi_{++}(dx^+)^2 + l^2\Xi_{--}(dx^-)^2,$$

BTZ black hole $\quad \Xi_{\pm\pm} = 2G(M \pm \frac{J}{l})$ $\quad$ AdS3 space $\quad M = -\frac{1}{8G}, J = 0$

$l \to \infty$ $\quad$ cannot be taken naively in these coordinates

AdS3 BMS gauge

conformal boundary for flat case: null infinity

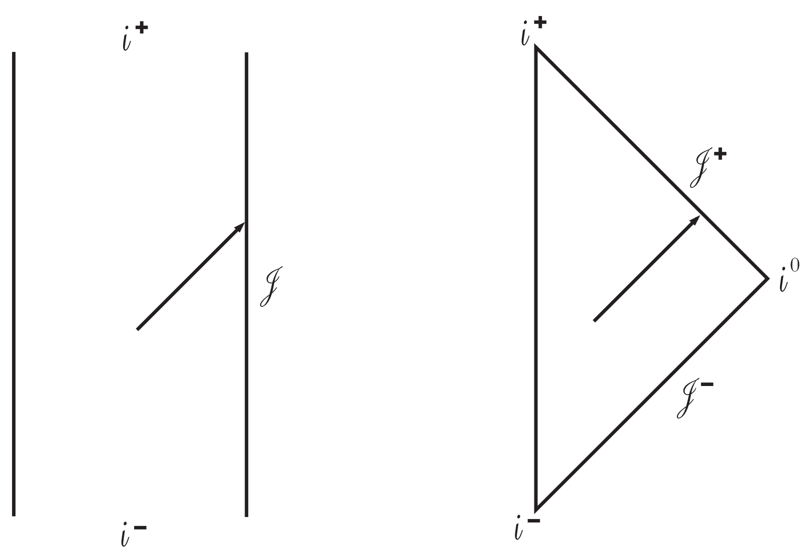
BMS gauge
$$ds^2 = e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}dudr + r^2(d\phi - U du)^2$$

same gauge for asymptotically flat and AdS3 spacetimes

AdS3
$$ds^2 = -(\frac{r^2}{l^2}+1)du^2 - 2dudr + r^2 d\phi^2$$
$$t = u + l\arctan\frac{r}{l}$$

Minkowski
$$ds^2 = -du^2 - 2dudr + r^2 d\phi^2$$
$$t = u + r$$

fall-offs
$$\frac{V}{r} = -\frac{r^2}{l^2} + O(1), \quad \beta = O(r^{-1}), \quad U = O(r^{-2})$$



Asymptotic symmetries

asymptotic symmetries

$$\mathcal{L}_\xi g_{rr} = 0 = \mathcal{L}_\xi g_{r\phi}, \quad \mathcal{L}_\xi g_{\phi\phi} = 0,$$
$$\mathcal{L}_\xi g_{ur} = O(r^{-1}), \quad \mathcal{L}_\xi g_{u\phi} = O(1), \quad \mathcal{L}_\xi g_{uu} = O(1)$$

general (exact) solution

$$\xi^u = f, \quad \xi^\phi = Y - \partial_\phi f \int_r^\infty dr'\, r'^{-2} e^{2\beta}, \quad \xi^r = -r(\partial_\phi \xi^\phi - U \partial_\phi f),$$
$$f = \frac{l}{2}(Y^+ + Y^-), \quad Y = \frac{1}{2}(Y^+ - Y^-)$$
$$Y^\pm = Y^\pm(x^\pm) \qquad x^\pm = \frac{u}{l} \pm \phi$$

modified bracket linear representation of conformal algebra in bulk spacetime

$$[\xi_{Y_1}, \xi_{Y_2}]^\mu_M \equiv [\xi_{Y_1}, \xi_{Y_2}]^\mu - \delta^g_{\xi_{Y_1}} \xi^\mu_{Y_2} + \delta^g_{\xi_{Y_2}} \xi^\mu_{Y_1} = \xi^\mu_{[Y_1, Y_2]}.$$

$$l^+_m = \xi^\mu_{e^{imx^+},0} \partial_\mu, \quad l^-_m = \xi^\mu_{0,e^{imx^-}} \partial_\mu$$

$$i[l^\pm_m, l^\pm_n]_M = (m-n) l^\pm_{m+n}, \quad [l^\pm_m, l^\mp_n]_M = 0$$

AdS3 Solution space and conformal properties

general solution to EOM

$$ds^2 = \left(-\frac{r^2}{l^2} + \mathcal{M}\right) du^2 - 2dudr + 2\mathcal{N} dud\phi + r^2 d\phi^2$$

$$\mathcal{M}(u,\phi) = 2(\Xi_{++} + \Xi_{--}), \quad \mathcal{N}(u,\phi) = l(\Xi_{++} - \Xi_{--})$$

$$\Xi_{\pm\pm} = \Xi_{\pm\pm}(x^{\pm})$$

asymptotic symmetries transform solutions into solutions

$$g_{\mu\nu} = g_{\mu\nu}(x, \Xi),$$
$$\mathcal{L}_\xi g_{\mu\nu} = g_{\mu\nu}(x - \delta\Xi)$$

conformal transformation properties

$$-\delta_Y \Xi_{\pm\pm} = Y^{\pm}\partial_{\pm}\Xi_{\pm\pm} + 2\partial_{\pm}Y^{\pm}\Xi_{\pm\pm} - \frac{1}{2}\partial^3_{\pm}Y^{\pm}$$

AdS3 Charge algebra

surface charge generators

$$Q_Y = \frac{1}{16\pi G}\int_0^{2\pi} d\phi\,\left[f(\mathcal{M}+1)+2Y\mathcal{N}\right]$$

$$= \frac{l}{8\pi G}\int_0^{2\pi} d\phi\Big[Y^+(\Xi_{++}+\frac{1}{4})+Y^-(\Xi_{--}+\frac{1}{4})\Big].$$

Dirac bracket algebra $\{Q_{Y_1}, Q_{Y_2}\} = \delta_{Y_1} Q_{Y_2}$

modes

$$L^+_m = Q_{e^{imx^+},0}, \quad L^-_m = Q_{0,e^{imx^-}}$$

$$i\{L^\pm_m, L^\pm_n\} = (m-n)L^\pm_{m+n} + \frac{c^\pm}{12}m(m^2-1)\delta^0_{m+n}, \quad \{L^\pm_m, L^\mp_n\} = 0,$$

$$c^\pm = \frac{3l}{2G}$$

Charge fields on the plane

conventional normalization: $$T_{\pm\pm}(x^{\pm}) = -\frac{l}{4G}\Xi_{\pm\pm}$$

$$-\delta_Y T_{\pm\pm} = Y^{\pm}\partial_{\pm}T_{\pm\pm} + 2\partial_{\pm}Y^{\pm}T_{\pm\pm} + \frac{c^{\pm}}{12}\partial_{\pm}^3 Y^{\pm},$$

$$Q_Y = -\frac{1}{2\pi}\int_0^{2\pi} d\phi\Big[Y^{+}(T_{++} - \frac{c^{+}}{24}) + Y^{-}(T_{--} - \frac{c^{-}}{24})\Big],$$

$$L_m^{\pm} = -\frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{imx^{\pm}}(T_{\pm\pm} - \frac{c^{\pm}}{24}), \quad T_{\pm\pm} = -\sum_m L_m^{\pm}e^{-imx^{\pm}} + \frac{c^{\pm}}{24}$$

mapping to the plane:

$$t = i\tau \qquad z = e^{ix^{+}}, \quad \bar{z} = e^{ix^{-}} \qquad T_{++}(x^{+}) - \frac{c^{+}}{24} = -z^2 T(z)$$

$$L_m = \frac{1}{2\pi i}\oint_{|z|=1} dz\, z^{m+1}T, \quad T(z) = \sum_m L_m z^{-m-2}$$

$$L_m(w) = \frac{1}{2\pi i}\oint_{|z|=1} dz\,(z-w)^{m+1}T(z),$$

$$i\{L_m(w), L_n(z)\}|_{w=z} = (m-n)L_{m+n}(z) + \frac{c}{12}m(m^2-1)\delta^0_{m+n}$$

Penrose limit — Generalities

action $$S[g;G,l]=\frac{1}{16\pi G}\int d^3x\sqrt{|g|}(R+\frac{2}{l^2})$$

scaling
$$S[g^{(\lambda)};G^{(\lambda)},l^{(\lambda)}]=S[g;G,l]$$
$$g^{(\lambda)}=\lambda^{-2}g,\quad G^{(\lambda)}=\lambda^{-1}G,\quad l^{(\lambda)}=\lambda^{-1}l$$

most general solution $\quad g(\lambda):\Xi_{\pm\pm}(\lambda)\iff L^{\pm}_{m}(\lambda)$

$g^{(\lambda)}(\lambda)$ solution with $l^{(\lambda)}$

$$g=\lim_{\lambda\to 0}\lambda^{-2}g(\lambda)$$ flat space solution

Penrose rescaling of coordinates $\quad (u,r,\phi)\to(\lambda^2u,r,\lambda\phi)$

$$\lambda^{-2}ds^2_\lambda=-\lambda^2\left[\frac{r^2}{l^2}-\mathcal{M}(\lambda^2u,\lambda\phi)\right]du^2-2dudr+2\lambda\mathcal{N}(\lambda^2u,\lambda\phi)dud\phi+r^2d\phi^2$$

limit : null orbifold $\quad ds^2=-2dudr+r^2d\phi^2$

Penrose limit modified scaling

alternative scaling $(u, r, \phi) \to (\lambda u, \lambda r, \phi)$

$$\lambda^{-2} ds^2_\lambda = \Big[-\frac{\lambda^2 r^2}{l^2} + \mathcal{M}(\lambda u, \phi)\Big] du^2 - 2dudr + 2\lambda^{-1}\mathcal{N}(\lambda u, \phi) du d\phi + r^2 d\phi^2$$

$$\mathcal{M}(\lambda u, \phi) = -1 + 8G \sum_m \left(\frac{L^+_m(\lambda) e^{-im\frac{\lambda u}{l}} + L^-_{-m}(\lambda) e^{im\frac{\lambda u}{l}}}{l} \right) e^{-im\phi},$$

$$\lambda^{-1}\mathcal{N}(\lambda u, \phi) = \frac{4G}{\lambda} \sum_m \left(L^+_m(\lambda) e^{-im\frac{\lambda u}{l}} - L^-_{-m}(\lambda) e^{im\frac{\lambda u}{l}} \right) e^{-im\phi}$$

well defined limit if

$$L^+_m(\lambda) = \frac{1}{2} l P_m + \lambda L'^+_m(0) + O(\lambda^2),$$
$$L^-_m(\lambda) = \frac{1}{2} l P_{-m} + \lambda L'^-_m(0) + O(\lambda^2).$$

limiting metric

$$\lim_{\lambda \to 0} \lambda^{-2} ds^2_\lambda = \Theta(\phi) du^2 - 2dudr + 2\Big[\Xi(\phi) + \frac{u}{2}\partial_\phi \Theta(\phi)\Big] du d\phi + r^2 d\phi^2.$$

3d flat

Charge algebra contraction

appropriate combination for the limit

$$P_m = \frac{1}{l}(L^+_m + L^-_{-m}), \ J_m = L^+_m - L^-_{-m},$$

$$i\{J_m, J_n\} = (m-n)J_{m+n} + \frac{c^+ - c^-}{12} m(m^2-1)\delta^0_{m+n},$$

$$i\{J_m, P_n\} = (m-n)P_{m+n} + \frac{c^+ + c^-}{12\ell} m(m^2-1)\delta^0_{m+n},$$

$$i\{P_m, P_n\} = \frac{1}{l^2}\left((m-n)J_{m+n} + \frac{c^+ - c^-}{12} m(m^2-1)\delta^0_{m+n}\right).$$

Virasoro algebra contracts to

$$\mathfrak{bms}_3 \cup \mathfrak{iso}(2,1)$$

$$i\{J_m, J_n\} = (m-n)J_{m+n} + \frac{c_1}{12} m(m^2-1)\delta^0_{m+n},$$

$$i\{J_m, P_n\} = (m-n)P_{m+n} + \frac{c_2}{12} m(m^2-1)\delta^0_{m+n},$$

$$i\{P_m, P_n\} = 0,$$

$$c_1 = 0, \quad c_2 = \frac{3}{G}$$

relation to AdS_3 similar to contraction between $\mathfrak{so}(2,2) \to \mathfrak{iso}(2,1)$

Virasoro factor: centrally non extended superrotations

Charge fields

normalized fields $$P_{++}(\phi) = -\frac{1}{8G}\Theta, J_{++}(\phi) = -\frac{1}{4G}\Xi$$

$$-\delta P_{++} = Y\partial_\phi P_{++} + 2\partial_\phi Y P_{++} + \frac{c_2}{12}\partial_\phi^3 Y,$$

$$-\delta J_{++} = Y\partial_\phi J_{++} + 2\partial_\phi Y J_{++} + T\partial_\phi P_{++} + 2\partial_\phi T P_{++} + \frac{c_2}{12}\partial_\phi^3 T,$$

$$Q_{T,Y} = -\frac{1}{2\pi}\int_0^{2\pi} d\phi\, [T(P_{++} - \frac{c_2}{24}) + YJ_{++}],$$

Minkowski background

$$z = e^{i\phi}$$

$$P_m = \frac{1}{2\pi i}\oint_{|z|=1} dz\, z^{m+1}P, \quad J_m = \frac{1}{2\pi i}\oint_{|z|=1} dz\, z^{m+1}J,$$

$$P(z) = \sum_m P_m z^{-m-2}, \quad J(z) = \sum_m J_m z^{-m-2}.$$

$$i\{J(z), J(w)\} = \partial_w J(w)\delta(z-w) + 2J(w)\partial_w\delta(z-w)$$

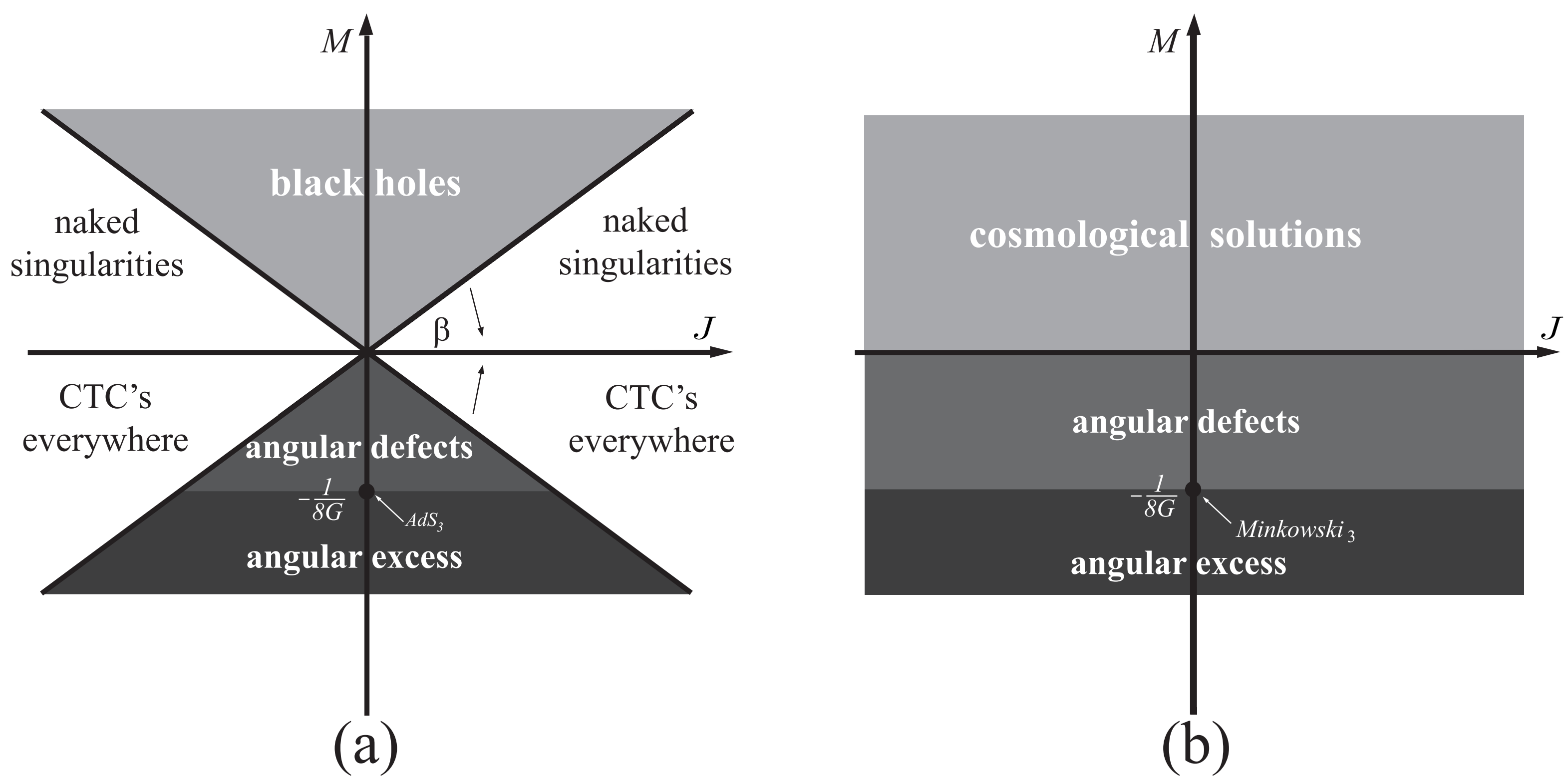
$$i\{J(z), P(w)\} = \partial_w P(w)\delta(z-w) + 2P(w)\partial_w\delta(z-w) + \frac{c_2}{12}\partial_w^3\delta(z-w)$$

$$i\{P(z), P(w)\} = 0$$

Zero modes

zero mode solutions in both cases

$$ds^2 = \left([-\tfrac{r^2}{l^2}] - 1 + 8GM\right) du^2 - 2dudr + 8GJdud\phi + r^2 d\phi^2$$



$$\tan\beta = \frac{1}{l}$$

3d flat CS extension   Chern-Simons for Virasoro

3d flat gravity, Chern-Simons formulation $\mathfrak{iso}(2,1)$

invariant metric $< P_a, J_b >= \eta_{ab}, < P_a, P_b >= 0 =< J_a, J_b >$

always exists for $\mathfrak{g} \ltimes \mathfrak{g}^*$

$$[e_a, e_b] = f^c_{ab} e_c, \quad [e_a, e^{*b}] = -f^b_{ac} e^{*c}, \quad [e^{*a} e^{*b}] = 0$$

$$< e_a, e^{*b} >= \delta^b_a, \quad < e_a, e_b >= 0 =< e^{*a}, e^{*b} >$$

Virasoro

$$[l_m, l_n] = (m-n) l_{m+n} + \tfrac{c}{12} \delta^0_{m+n} m(m-1)(m+1) Z, \quad [l_m, Z] = 0,$$

$$[l_m, l^{*n}] = (n-2m) l^{*n-m}, \quad [l_m, Z^*] = -\frac{c}{12} m(m-1)(m+1) l^{*-m},$$

$$[Z, l^{*n}] = 0 = [Z, Z^*], \quad [l^{*m}, l^{*n}] = 0 = [l^{*m}, Z^*]$$

CS $S[A] = \kappa \int \frac{1}{2} < A, dA + \frac{2}{3} A^2 >$ $A = A^A_\mu T_A \, dx^\mu, T_A = (l_m, Z, l^{*m}, Z^*)$

co-adjoint vs adjoint

extension of 3d flat gravity $l_{\pm 1}, l_0, l^{*\pm 1}, l^{*0}$

$\mathfrak{bms}_3$

3d flat CS extension

Deformations

$iso(2,1)$ cosmological constant $[P_a, P_b] = \frac{1}{l^2}\epsilon^c_{ab}P_c$

$\longrightarrow$ $\mathfrak{so}(2,2)$ same inner product

extended theory $[l^{*m}, l^{*n}] = k^{mnl} l_l$

invariance of metric implies completely skew, Jacobi implies invariant under co-adjoint action

no such tensor, AdS deformation does not survive extension

$iso(2,1)$ "exotic deformation"

survives on its own related to $H^3(\mathfrak{witt}, \mathbb{R}) \neq 0$

$$S^{ex} = \int \frac{1}{3!}\beta_{mnk} B^m B^n B^k$$

$$\beta_{mnk} = \beta_{[mnk]} = \delta^0_{m+n+k}\left(mn(m-n) + nk(n-k) + mk(k-m)\right)$$

to be studied further: asymptotics, boundary theory, solutions, 1-loop effects ...

Asymptotically flat spacetimes

BMS ansatz

$$g^{\mu\nu} = \begin{pmatrix} 0 & -e^{-2\beta} & 0 \\ -e^{-2\beta} & -\frac{V}{r}e^{-2\beta} & -U^B e^{-2\beta} \\ 0 & -U^A e^{-2\beta} & g^{AB} \end{pmatrix}$$

$$u \qquad r \qquad x^A = \begin{cases} \theta, \phi \\ \zeta, \bar{\zeta} \end{cases} \qquad \zeta = \cot\frac{\theta}{2} e^{i\phi}$$

Minkowski $\quad u = t - r$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

$$g_{AB} dx^A dx^B = r^2 \bar{\gamma}_{AB} dx^A dx^B + O(r)$$

Sachs: unit sphere $\quad \bar{\gamma}_{AB} = e^{2\varphi}\, {}_0\gamma_{AB} \quad {}_0\gamma_{AB} dx^A dx^B = d\theta^2 + \sin^2\theta d\phi^2$

Riemann sphere $\quad \zeta = e^{i\phi}\cot\frac{\theta}{2}, \quad \bar{\gamma}_{AB} dx^A dx^B = e^{2\widetilde{\varphi}} d\zeta d\bar{\zeta}$

$$d\theta^2 + \sin^2\theta d\phi^2 = P^{-2} d\zeta d\bar{\zeta}, \quad P(\zeta,\bar{\zeta}) = \frac{1}{2}(1+\zeta\bar{\zeta}), \qquad \tilde{\varphi} = \varphi - \ln P$$

determinant condition $\quad \det g_{AB} = \frac{r^4}{4} e^{4\widetilde{\varphi}}$

fall-off conditions $\quad \beta = O(r^{-2}), \quad U^A = O(r^{-2}), \quad V/r = -\frac{1}{2}\bar{R} + O(r^{-1})$

Asymptotic symmetries

asymptotic symmetries $\mathcal{L}_\xi g_{rr} = 0, \quad \mathcal{L}_\xi g_{rA} = 0, \quad \mathcal{L}_\xi g_{AB} g^{AB} = 0,$

$$\mathcal{L}_\xi g_{ur} = O(r^{-2}), \quad \mathcal{L}_\xi g_{uA} = O(1), \quad \mathcal{L}_\xi g_{AB} = O(r), \quad \mathcal{L}_\xi g_{uu} = O(r^{-1})$$

general solution

$$\begin{cases} \xi^u = f, & \dot{f} = f\dot{\varphi} + \frac{1}{2}\psi \iff f = e^{\varphi}\big[T + \frac{1}{2}\int_0^u du' e^{-\varphi}\psi\big], \\ \xi^A = Y^A + I^A, \quad I^A = -f_{,B}\int_r^\infty dr'(e^{2\beta}g^{AB}), \\ \xi^r = -\frac{1}{2}r(\bar{D}_A\xi^A - f_{,B}U^B + 2f\partial_u\varphi), & \psi = \bar{D}_A Y^A \end{cases}$$

$Y^A = Y^A(x^B)$ conformal Killing vectors of the sphere

$T = T(x^B)$ generators for supertranslations

spacetime vectors with modified bracket form linear representation of $\mathfrak{bms}_4$ algebra

$$[(Y_1, T_1), (Y_2, T_2)] = (\widehat{Y}, \widehat{T})$$

$$\begin{aligned} \widehat{Y}^A &= Y_1^B \partial_B Y_2^A - Y_1^B \partial_B Y_2^A, \\ \widehat{T} &= Y_1^A \partial_A T_2 - Y_2^A \partial_A T_1 + \frac{1}{2}(T_1 \partial_A Y_2^A - T_2 \partial_A Y_1^A) \end{aligned}$$

Sachs 1962

standard GR choice: restrict to globally well-defined transformations

$SL(2,\mathbb{C})/\mathbb{Z}_2 \simeq SO(3,1)$

Y^A generators of Lorentz algebra

New proposal

CFT choice : allow for meromorphic functions on the Riemann sphere

solution to conformal Killing equation $Y^{\zeta} = Y^{\zeta}(\zeta), \quad Y^{\bar{\zeta}} = Y^{\bar{\zeta}}(\bar{\zeta})$

generators

$$l_n = -\zeta^{n+1}\frac{\partial}{\partial \zeta}, \quad \bar{l}_n = -\bar{\zeta}^{n+1}\frac{\partial}{\partial \bar{\zeta}}, \quad n \in \mathbb{Z}$$

superrotations

$$T_{m,n} = \zeta^m \bar{\zeta}^n, \quad m, n \in \mathbb{Z}$$

supertranslations

commutation relations

$$[l_m, l_n] = (m-n)l_{m+n}, \quad [\bar{l}_m, \bar{l}_n] = (m-n)\bar{l}_{m+n}, \quad [l_m, \bar{l}_n] = 0,$$

$$[l_l, T_{m,n}] = (\frac{l+1}{2} - m)T_{m+l,n}, \quad [\bar{l}_l, T_{m,n}] = (\frac{l+1}{2} - n)T_{m,n+l}.$$

Poincaré subalgebra $l_{-1},\, l_0,\, l_1, \quad \bar{l}_{-1},\, \bar{l}_0,\, \bar{l}_1, \quad T_{0,0},\, T_{1,0},\, T_{0,1},\, T_{1,1},$

solution space

$Re(\Psi_2^0)(u,\zeta,\bar\zeta)$

u dependence fixed through evolution equation
integration "constants

$\Psi_1^0(u,\zeta,\bar\zeta)$

free data

$\Psi_5^0(u,r,\zeta,\bar\zeta) = O(r^{-5})$

plays no role asymptotically

$\sigma^0(u,\zeta,\bar\zeta)$

free u dependence

$\dot\sigma^0(u,\zeta,\bar\zeta)$

news tensor

unit sphere

$$\eth\eta^s = P^{1-s}\bar\partial(P^s\eta^s), \qquad \bar\eth\eta^s = P^{1+s}\partial(P^{-s}\eta^s)$$

$$\mathcal{Y} = P^{-1}\bar{Y}$$

$$-\delta_{\mathcal{Y},\bar{\mathcal{Y}}}\eta^{s,w} = [\mathcal{Y}\eth + \bar{\mathcal{Y}}\bar\eth + \frac{s-w}{2}\eth\mathcal{Y} - \frac{s+w}{2}\bar\eth\bar{\mathcal{Y}}]\eta^{s,w}$$

Conformal properties

bms4 transformations

$$-\delta_S\sigma^0 = [f\partial_u + \mathcal{Y}\eth + \bar{\mathcal{Y}}\bar{\eth} + \frac{3}{2}\eth\mathcal{Y} - \frac{1}{2}\bar{\eth}\bar{\mathcal{Y}}]\sigma^0 - \eth^2 f\,,$$
$$-\delta_S\dot{\sigma}^0 = [f\partial_u + \mathcal{Y}\eth + \bar{\mathcal{Y}}\bar{\eth} + 2\eth\mathcal{Y}]\dot{\sigma}^0 - \frac{1}{2}\eth^2\psi\,,$$
$$-\delta_S\Psi^0_2 = [f\partial_u + \mathcal{Y}\eth + \bar{\mathcal{Y}}\bar{\eth} + \frac{3}{2}\eth\mathcal{Y} + \frac{3}{2}\bar{\eth}\bar{\mathcal{Y}}]\Psi^0_2 - 2\eth f\eth\dot{\bar{\sigma}}^0,$$
$$-\delta_S\Psi^0_1 = [f\partial_u + \mathcal{Y}\eth + \bar{\mathcal{Y}}\bar{\eth} + 2\eth\mathcal{Y} + \bar{\eth}\bar{\mathcal{Y}}]\Psi^0_1 + 3\eth f\Psi^0_2\,.$$

$$f = T + \frac{1}{2}u\psi \qquad\qquad \psi = \eth\mathcal{Y} + \bar{\eth}\bar{\mathcal{Y}}$$

Interpretation and consequences: work in progress

field dependent Schwarzian derivative:
Lie algebra $\longrightarrow$ Lie algebroid

Charge algebra

asymptotic charge : non integrable due to the news

$$\not{\delta} Q_\xi[\delta\mathcal{X},\mathcal{X}] = \delta\left(Q_s[\mathcal{X}]\right) + \Theta_s[\delta\mathcal{X},\mathcal{X}]\,,$$

$$Q_s[\mathcal{X}] = -\frac{1}{8\pi G}\int d^2\Omega\Big[\big(f(\Psi_2^0+\sigma^0\dot{\bar\sigma}^0)+\mathcal{Y}(\Psi_1^0+\sigma^0\eth\bar\sigma^0+\frac{1}{2}\eth(\sigma^0\bar\sigma^0))\big)+\text{c.c.}\Big],$$

$$\Theta_s[\delta\mathcal{X},\mathcal{X}] = \frac{1}{8\pi G}\int d^2\Omega\, f\,[\dot{\bar\sigma}^0\delta\sigma^0+\text{c.c.}]$$

Proposal : “Dirac” bracket

$$\{Q_{s_1},Q_{s_2}\}^*[\mathcal{X}] = (-\delta_{s_2})Q_{s_1}[\mathcal{X}] + \Theta_{s_2}[-\delta_{s_1}\mathcal{X},\mathcal{X}]\,.$$

Proposition : if one can integrate by parts $\int d^2\Omega\,\eth\eta^{-1} = 0 = \int d^2\Omega\,\bar\eth\eta^1,$

$$\{Q_{s_1},Q_{s_2}\}^* = Q_{[s_1,s_2]} + K_{s_1,s_2}\,,$$

$$K_{s_1,s_2}[\mathcal{X}] = \frac{1}{8\pi G}\int d^2\Omega\,\Big[\big(\frac{1}{2}\bar\sigma^0 f_1\eth^2\psi_2 - (1\leftrightarrow 2)\big)+\text{c.c.}\Big]$$

generalized cocycle condition $K_{[s_1,s_2],s_3} - \delta_{s_3}K_{s_1,s_2} + \text{cyclic }(1,2,3) = 0.$

Charges for Kerr black hole

supertranslations :

$$Q_{T_{m,n},0}[\mathcal{X}^{Kerr}] = \frac{2M}{G} I_{m,n}, \quad I_{m,n} = \frac{1}{4\pi} \int d^2\Omega \frac{1}{1+\zeta\bar{\zeta}} \zeta^m \bar{\zeta}^n .$$

$$I_{m,n} = \delta^m_n I(m) \qquad I(m) = \frac{1}{4} \int_{-1}^{1} d\mu \frac{(1+\mu)^m}{(1-\mu)^{m-1}}$$

$$Q_{T=1,Y=0}[\mathcal{X}^{Kerr}] = \frac{M}{G} ,$$

divergences for proper supertranslations !

superrotations :

$$Q_{0,l_m}[\mathcal{X}^{Kerr}] = -\delta^m_0 \frac{iaM}{2G} .$$

$$\partial_\phi = -i(l_0 - \bar{l}_0) \qquad Q_{T=0,Y^\phi=1,Y^\theta=0}[\mathcal{X}^{Kerr}] = -\frac{Ma}{G}$$

Central charges for Kerr black hole

central charges :

$$K_{(0,l_m),(0,l_n)}[\mathcal{X}^{Kerr}] = 0 = K_{(0,\bar{l}_m),(0,\bar{l}_n)}[\mathcal{X}^{Kerr}] = K_{(0,l_m),(0,\bar{l}_n)}[\mathcal{X}^{Kerr}],$$

$$K_{(0,l_l),(T_{m,n},0)}[\mathcal{X}^{Kerr}] = \frac{a\,l(l-1)(l+1)}{16G} J_{m+l,n}\,,$$

$$K_{(0,\bar{l}_l),(T_{m,n},0)}[\mathcal{X}^{Kerr}] = \frac{a\,l(l-1)(l+1)}{16G} J_{m,n+l}\,,$$

$$J_{m,n} = \delta^m_n J(m) \qquad J(m) = 2\int_{-1}^{1} d\mu\, \frac{(1+\mu)^{m-\frac{3}{2}}}{(1-\mu)^{m+\frac{1}{2}}},$$

form in-line with extremal Kerr/CFT correspondence,
but divergences !

problem: one cannot integrate by parts if there are poles

$$\int_M d^2x\, \bar{\partial}(\frac{1}{z} f(z)) = \left\{ \begin{array}{c} \int d^2x_M\, \bar{\partial}(\frac{1}{z}) f(z) \\ \oint_{\partial M} dz\, \frac{f(z)}{z} \end{array} \right\} = \pi f(0)$$

there are no poles for $\mathfrak{bms}_4^{glob}$ but then $K_{s_1,s_2}[\chi] = 0$

way out (i) define the analog of charge fields to regularize the divergences in the charges

$$\frac{1}{z-w} \to \frac{\sqrt{(1+\zeta\bar{\zeta})(1+\eta\bar{\eta})}}{\zeta-\eta}$$

Green's function for $\eth$

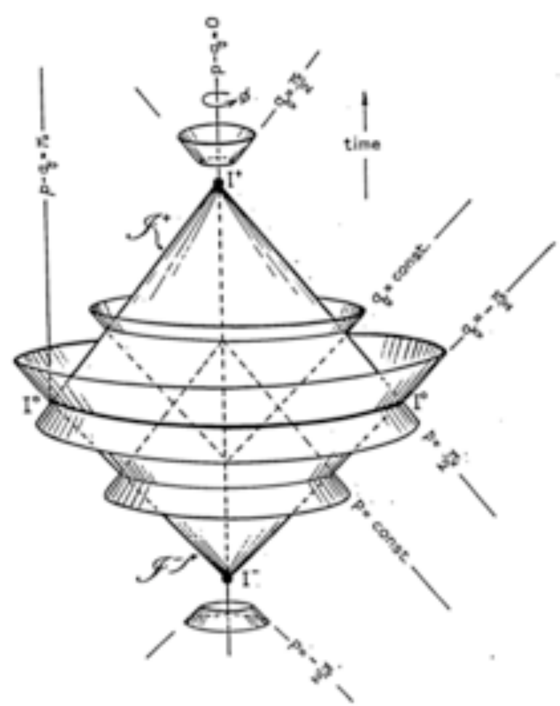
(ii) take correctly into account the boundary contributions to correct the central charges

Conclusions and perspectives

4d gravity is dual to an extended conformal field theory

to be done: particles as UIRREPS for $\mathfrak{bms}_4$

scattering theory between $\mathscr{I}^-$ and $\mathscr{I}^+$



Penrose, Les Houches 1963

Conclusions and perspectives

angular momentum problem in GR:

> This analogy goes even deeper. For example, there is no single, natural Lorentz subalgebra of the Poincarè Lie algebra. Similarly, there is no single, natural $\mathcal{L}/\mathcal{S}$ subalgebra of $\mathcal{L}$. However, it is possible to realize the Lorentz Lie algebra as a subalgebra of the Poincaré Lie algebra. For example, fix a point of Minkowski space, and consider the collection of all Killing fields which vanish at that point. These form a subalgebra of Poincarè, isomorphic with Lorentz. Of course, this subalgebra is not "natural", because its determination requires the choice of a point of Minkowski space.

Geroch, Asymptotic structure of spacetime, 1977

Lorentz = Poincaré /translation — 4 conditions needed to fix rotations

Lorentz = bms4(old)/supertranslations — infinite # conditions needed to fix rotations

bms4(new)/supertranslations = Virasoro — infinite # conditions needed to fix infinite # of superrotations

Lie algebroids

Lie algebroid

$$\begin{array}{ccc} A & \xrightarrow{\rho_A} & TM \\ & \searrow \quad \swarrow & \\ & M & \end{array}$$

base space M vector bundles A and TM $[\cdot,\cdot]_A$ Lie bracket on $\Gamma[A]$

bundle map, "anchor" $\rho_A : A \to TM$ Lie algebra homomorphism + Leibniz rule

$$[\alpha, f\beta]_A = f[\alpha,\beta]_A + (\rho_A(\alpha)f)\beta \qquad \alpha,\beta \in \Gamma[A],\ f \in C^\infty(M)$$

local coordinates $M : \phi^i$ $\quad A \ni f = f^\alpha(\phi)e_\alpha \quad \rho_A(f) = f^\alpha R^i_\alpha \frac{\partial}{\partial \phi^i} = \delta_f$

$$[f_1, f_2]_A = \left(C^\gamma_{\alpha\beta}(\phi)(f_1^\alpha, f_2^\beta) + \delta_{f_1} f_2^\gamma - \delta_{f_2} f_1^\gamma\right)e_\gamma$$

Gauge algebroid   Irreducible gauge theories

gauge theories M $\phi^i_s(x)$ solutions to underdetermined EL $\frac{\delta L}{\delta \phi^i} \approx 0$

TM $\delta\phi^i_s(x)$ linearized solutions

$R^{\dagger i}_\alpha$ generating set of irreducible Noether operators

$$R^{+i}_\alpha[\frac{\delta L}{\delta \phi^i}] = 0$$

determines   structure functions and algebra A field dependent gauge parameters on M

quantitative control on functional aspects:
jet-spaces & variational bicomplex

GR: fields are Riemannian metrics satisfying Einstein's equation

gauge parameters are metric dependent vector fields

bracket: derived bracket from antibracket in BV description

isotropy Lie algebra : dynamical Killing vectors
at a particular solution

asymptotic context: physically meaningful sub-Lie algebroids of the gauge algebroid for GR reduce to action Lie algebroids involving the Virasoro algebras

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