Generating formulation for HS gauge fields

(Howe duality, BRST, and many faces of HS dynamics)

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Plan

Review part:

- AdS spacetime symmetry algebra and its «particle» representations
- Various field-theoretical realizations

Generating formulation:

- Algebraic tools: auxiliary variables & Howe duality (standard and twisted)

- Ambient space formulation: fields, BRST operator, and constraints
- Physical interpretation: Casimir operator, spins and masses

- Generating formulation: fields, BRST operator, and constraints
- Example: Maxwell theory
- Cohomological analysis: a relation to the unfolded formulation

AdS spacetime symmetry algebra and its «particle» representations

(non)-unitary elementary particles ≡ highest weight irreps

Anti-de Sitter spacetime AdS: $o(d-1,2) = T^- \oplus T^0 \oplus T^+$

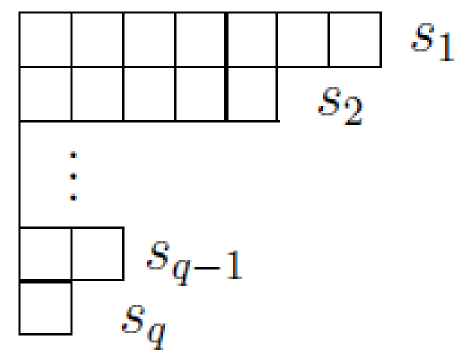
energy

spins

Maximal compact subalgebra $T^0 = o(2) \oplus o(d-1) \subset o(d-1,2)$

Creation and annihilation operators forming $T^\pm$ act on the vacuum space which is an irrep of the maximal compact subalgebra.

o(d-2) Young diagram



Finite-dimensional irreps:

- All traces = 0
- Young symmetry

Spins $\mathbf{s} = (s_1, \ldots, s_q)\ , \quad q = \left[\dfrac{d-1}{2}\right]$

Energy $E_0 \in \mathbb{R}$

sym d = 2,3,4
mix d ≥ 5

Unitarity region: $E_0 \geqslant E_0(\mathbf{s}) = s - p + d - 2$

Metsaev ('95)

Non-unitary region: non-unitary massless,
non-unitary partially-massless

singular vectors

Brink-Metsaev-Vasiliev (BMV) decomposition for unitary massless fields

$$o(d-2) \subset o(d-1)$$

Massless Minkowski field

Massless anti-de Sitter field

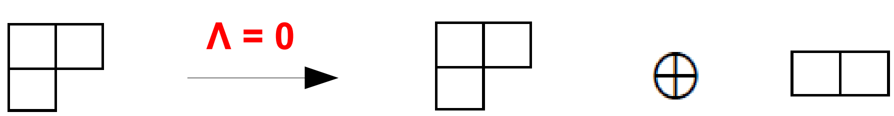
cosmological constant

Λ = 0 : massless AdS field = a number of flat massless fields

Boulanger Iazeolla Sundell ('08)
Alkalaev Grigoriev ('09)

AdS hook | flat hook | flat graviton

«Hook» field :



Field-theoretical realizations

Metric - like formulation for HS fields: $\phi^{(s_1,\ldots,s_q)}$ $\ydiagram{4,2}$

- Young symmetry
- Trace conditions

Fronsdal ('78), Labastida ('88), Sagnotti Francia ('02)

Minkowski fields

$$\delta\phi^{(s_1,\ldots,s_q)} = \sum_{n=1}^{q} \partial\xi^{(s_1,\ldots,s_n-1,\ldots,s_q)}$$

Fronsdal ('79), Metsaev ('95)

AdS fields massless n=1,...,q

$$\delta_n\phi^{(s_1,\ldots,s_q)} = \nabla\xi^{(s_1,\ldots,s_n-1,\ldots,s_q)}$$

«hook» field: (2,1)

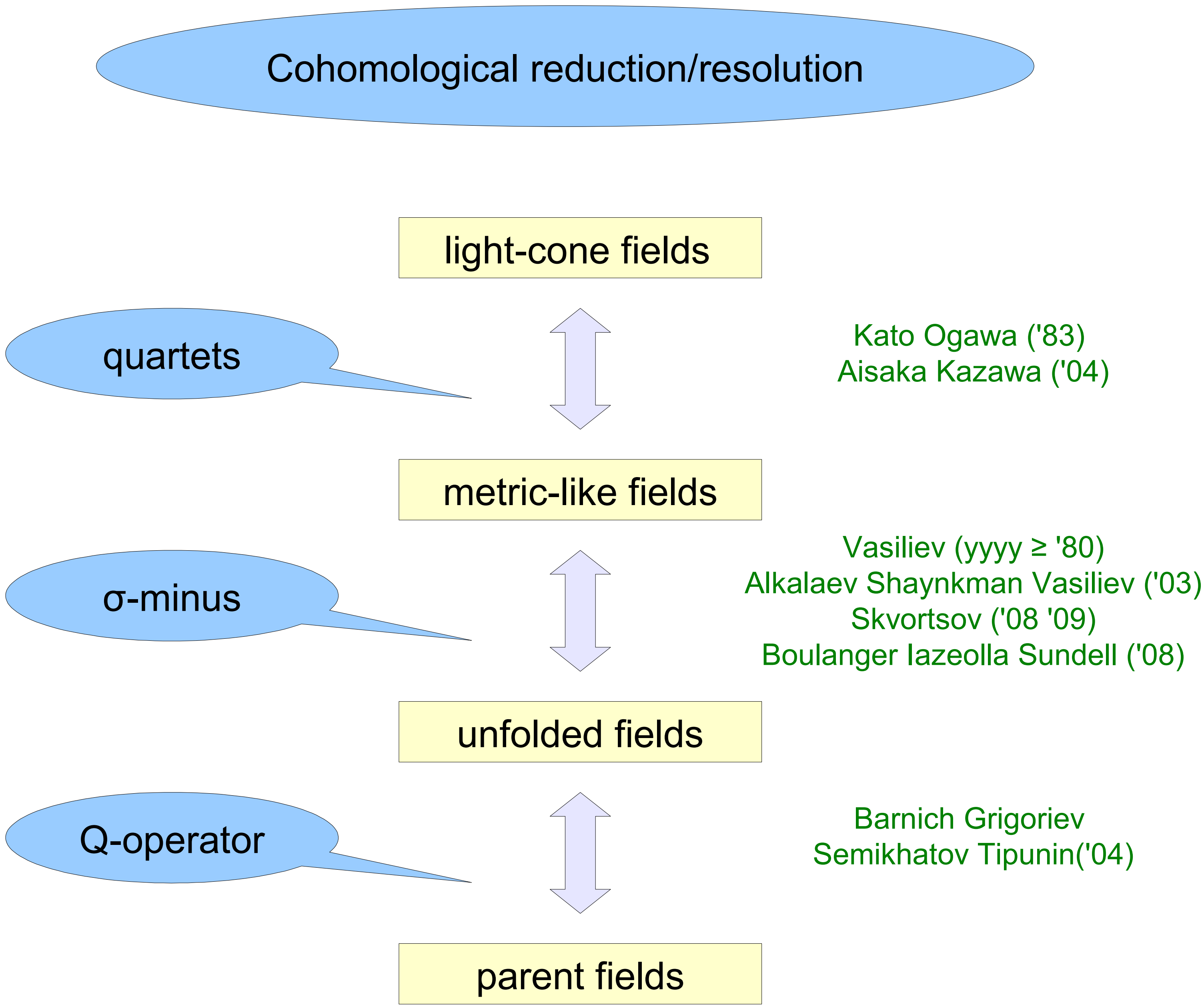
unitary theory	non-unitary theory
$\delta_1 \ydiagram{2,1} = \nabla \ydiagram{1,1}$	$\delta_2 \ydiagram{2,1} = \nabla \ydiagram{2}$

AdS fields partially-massless n=1,...,q

depth t

Deser, Nepomechie (84), Deser Waldron ('01) Skvortsov ('09)

$$\delta_{n,t}\phi^{(s_1,\ldots,s_q)} = (\nabla)^t\xi^{(s_1,\ldots,s_n-t-1,\ldots,s_q)}$$



Cohomological reduction/resolution
light-cone fields
quartets
Kato Ogawa ('83)
Aisaka Kazawa ('04)
metric-like fields
σ-minus
Vasiliev (yyyy ≥ '80)
Alkalaev Shaynkman Vasiliev ('03)
Skvortsov ('08 '09)
Boulanger Iazeolla Sundell ('08)
unfolded fields
Q-operator
Barnich Grigoriev
Semikhatov Tipunin('04)
parent fields

Algebraic tools: auxiliary variables & Howe duality

Auxiliary variables

Two types of indices

$$A_I^A$$

running $A = 0, ..., d$ and $I = 0, ..., n-1$

Polynomials $\Phi = \Phi(A)$

Expansion coefficients are covariant tensors

$$\Phi(A) = \sum \Phi_{A_1 \,...\, A_{m_0}; \,......\, ; C_1 \,...\, C_{m_{n-1}}} A_0^{A_1} \cdots A_0^{A_{m_0}} \;\cdots\; A_{n-1}^{C_1} \cdots A_{n-1}^{C_{m_{n-1}}}$$

Howe dual pair o(d-1,2) - sp(2n)

Orthogonal algebra o(d-1,2)

rotations

$$J^{AB} = A_I^A \frac{\partial}{\partial A_{BI}} - A_I^B \frac{\partial}{\partial A_{AI}}$$

Symplectic algebra sp(2n)

$$T_{IJ} = A_I^A A_{AJ}\,, \qquad T_I{}^J = \frac{1}{2}\{A_I^A, \frac{\partial}{\partial A_J^A}\}\,, \qquad T^{IJ} = \frac{\partial}{\partial A_I^A}\frac{\partial}{\partial A_{AJ}}\,.$$

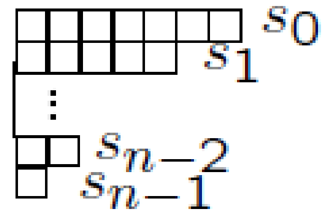
Trace creation

Young symmetrizer

Trace annihilation

Note that the algebras commute to each other $[J, T] = 0$!

Finite-dimensional irrep of o(d-1,2) algebra



$$\Uparrow$$

Howe duality

$$\Downarrow$$

Highest weight conditions of sp(2n) algebra

spin weights

$$T_I{}^I \phi = (s_I + \frac{d+1}{2})\, \phi$$

$$T^{IJ}\phi = 0\,, \qquad T_I{}^J\phi = 0 \qquad I < J$$

zeroth traces

Young symmetry

Two types of constraints: from sp(2n) to sp(2n-2)

Take a distinguished direction along A_0^A so that from now on we consider

variables A_0^A and $A_i^A,\ i = 1, ..., n-1$ separately.

Then we identify stability subalgebra preserving the direction:

$$sp(2n-2) \subset sp(2n)$$

Introduce new notation:

$$N_i^{\ j} \equiv T_i^{\ j} = A_i^A \frac{\partial}{\partial A_j^A} \quad i \neq j \qquad N_i = N_i^{\ i} \equiv T_i^{\ i} - \frac{d+1}{2} = A_i^A \frac{\partial}{\partial A_i^A}$$

These form gl(n-1) subalgebra. There are also

$$T_{ij} = A_i^A A_{jA} \qquad\qquad T^{ij} = \frac{\partial}{\partial A_i^A} \frac{\partial}{\partial A_{jA}}$$

These complete the above set of elements to sp(2n-2) algebra.

There are two different realizations of sp(2n) generators involving

A_0^A and/or $\partial / \partial A_0^A$

Realization on the space of polynomials in A_i^A with coefficients in functions on R^{d+1}. In this case

$$A_0^A = X^A \qquad \text{and} \qquad \frac{\partial}{\partial A_0^A} = \frac{\partial}{\partial X^A}$$

where X^A are Cartesian coordinates in R^{d+1}

Generators that involve X^A and/or $\partial/\partial X^A$ are denoted by

$$\mathcal{S}_i^\dagger = A_i^A \frac{\partial}{\partial X^A}, \qquad \bar{\mathcal{S}}^i = X^A \frac{\partial}{\partial A_i^A},$$

$$\mathcal{S}^i = \frac{\partial}{\partial A_i^A} \frac{\partial}{\partial X_A}, \qquad \Box_X = \frac{\partial}{\partial X^A} \frac{\partial}{\partial X_A}.$$

This is the standard realization.

There exists a realization on the space of polynomilas in A_i^A with coefficients in formal power series in variables Y^A such that

$$A_0^A = Y'^A = Y^A + V^A\,, \qquad \frac{\partial}{\partial A_0^A} = \frac{\partial}{\partial Y^A}$$

where V^A is some o(d-1,2) vector

$$V^A V_A = -1$$

compensator

Respective sp(2n) generators are realized by inhomogeneous differential operators. In this case:

$$S_i^\dagger = A_i^A \frac{\partial}{\partial Y^A}\,, \qquad \bar{S}^i = (Y^A + V^A)\frac{\partial}{\partial A_i^A}\,,$$

$$S^i = \frac{\partial}{\partial A_i^A}\frac{\partial}{\partial Y_A}\,, \qquad \Box_Y = \frac{\partial}{\partial Y^A}\frac{\partial}{\partial Y_A}\,.$$

This is the twisted realization. It is inequivalent to the standard one (V=0) because in such a change Y' = Y+V is ill-defined in the space of formal power series.

Ambient space formulation

Fronsdal ('79), Metsaev ('95)

Ambient space functions

hyperboloid

ambient space

$$AdS_d \quad \subset \quad \mathbb{R}^{d-1,2}$$

Functions on the ambient space

$$\phi = \phi(X, A)$$

- $X^A, \; A = 0, ..., d$ - $R^{d-1,2}$ spacetime coordinates
- $A_i^A, \; i = 1, ..., n-1$ target space variables (standard auxiliary variables)

To describe fields on the hyperboloid some constraints and gauge equivalence are required. They are given by certain sp(2n) generators or their higher powers.

Constraints and gauge symmetries

- General off-shell constraints

Young sym

spins

trace

$$T^{ij}\phi = 0\,, \qquad N_i{}^j\phi = 0 \quad i<j\,, \qquad N_i\phi = s_i\phi$$

- The radial dependence is fixed by

$$h\phi = 0\,, \qquad h = N_X - w\,, \qquad N_X = X^A\frac{\partial}{\partial X^A}$$

In proper coordiinates the condition is solved by $\phi = \phi_0(x,A)\,r^w$

- The equations of motion

wave equation

$$\Box_X\phi = 0\,, \qquad S^i\phi = 0$$

Lorentz gauge

- Tangent constraints

$$\bar{S}^{\hat{\alpha}}\phi = 0\,, \qquad \hat{\alpha} = p+1,\,\ldots,n-1$$

- Extra (polynomial) constraints

depth

$$(\bar{S}^p)^t\phi = 0\,, \qquad t = 1,2,\ldots,t_{\max}\,, \qquad t_{\max} = s_p - s_{p+1}$$

Gauge symmetry:

Let us fix integer number $p \leqslant n-1$ and ntroduce gauge parameters

$$\chi^\alpha = \chi^\alpha(X, A) \qquad \alpha = 1, \ldots, p$$

A gauge parameter version of the above constraints reads

$$N_i{}^j\chi^\alpha + \delta_i^\alpha \delta_\beta^j \chi^\beta = 0 \quad i < j \qquad N_i\chi^\alpha + \delta_i^\alpha\chi^\alpha - s_i\chi^\alpha = 0$$

$$(N_X - w - 1)\chi^\alpha = 0$$

no Young symmetry for gauge parameters

A gauge equivalence is defined by

$$\phi \sim \phi + \mathcal{S}_\alpha^\dagger \chi^\alpha \qquad \text{or} \qquad \delta_\chi \phi = \mathcal{S}_\alpha^\dagger \chi^\alpha$$

The consistency of this gauge transformation law is guaranteed by sp(2n) algebra.

Functions on the ambient space are extended by Grassmann odd variables

$$\Psi = \Psi(X, A|b)$$

where b^α are ghosts, $\mathrm{gh}\, b_\alpha = -1$. BRST extended constraints:

$$(N_i{}^j + B_i{}^j)\Psi = 0 \quad i < j \qquad (N_i + B_i)\chi = s_i \Psi \qquad (N_X - B - w)\Psi = 0$$

where

$$B_i{}^j = \delta_i^\alpha\, \delta_\beta^j\, b_\alpha \frac{\partial}{\partial b_\beta}\,, \qquad B_\alpha = b_\alpha \frac{\partial}{\partial b_\alpha}\,, \qquad B = \sum_{\alpha=1}^{p} B_\alpha$$

Physical fields are ghost-number-zero elements

$$\Psi^{(0)} = \phi(X, A)$$

Gauge parameters are ghost-number-one elements

$$\Psi^{(-1)} = \chi(X, A|\, b) = \chi^\alpha(X, A) b_\alpha$$

Now, the gauge symmetry is given by

no ghost cubic terms

$$\delta\phi = Q_p \chi\,, \qquad Q_p = \mathcal{S}_\alpha^\dagger \frac{\partial}{\partial b_\alpha}\,, \qquad \mathcal{S}_\alpha^\dagger = A_\alpha^A \frac{\partial}{\partial X^A}$$

where Q_p is BRST operator, $\mathrm{gh}\, Q_p = 1$

Comments:

- Equations of motion + off-shell constraints + gauge equivalence form an algebra generated by sp(2n) elements. In the massless unitary case this algebra is a parabolic subalgebra of sp(2n).

- Consistency of the constraints and gauge equivalence requires

$$w = s_p - p - t$$

-- massless and partially-massless fields.

- Relaxing polynomial constraint $(\bar{S}^p)^t\phi = 0$ makes w arbitrary

– massive fields. In this case the gauge symmetry can be shown to be purely algebraic so that there are no gauge fields at all.

Physical interpretation of parameters

- Spins $s_1 \geq s_2 \geq \ldots \geq s_{n-1}$
- Radial weight w, integer parameter p, depth t

To see which representation we are dealing with let $\Phi(X, A)$: represents an equivalence class of field configurations modulo the gauge equivalence relation generated by the introduced earlier BRST operator, i.e.

$$\Phi \sim \Phi + Q_p \chi \qquad \text{with} \qquad \chi = b_\alpha \chi^\alpha$$

To this end we explicitly evaluate the value of the quadratic Casimir operator

$$C_2 = -\frac{1}{2} J_{AB} J^{AB}, \qquad J_{AB} = L_{AB} + M_{AB}$$

where

$$L_{AB} = X_A \frac{\partial}{\partial X^B} - X_B \frac{\partial}{\partial X^A}$$

$$M_{AB} = \sum_{i=1}^{n-1} \left(A_{Ai} \frac{\partial}{\partial A_i^B} - A_{Bi} \frac{\partial}{\partial A_i^A} \right)$$

A direct calculation shows

$$C_2\Phi = \Big(w(w+d-1) + \sum_{l=1}^{n-1} s_l(s_l - 2l + d - 1)\Big)\Phi - 2\sum_{l=1}^{n-1} \mathcal{S}_l^\dagger \bar{\mathcal{S}}^l \Phi$$

Last term: summands $\mathcal{S}_l^\dagger \bar{\mathcal{S}}^l$ with $l > p$ vanish because of the constraints.

The remaining summands can be rewritten as

$$\sum_{\alpha=1}^{p} \mathcal{S}_\alpha^\dagger \bar{\mathcal{S}}^\alpha \Phi = \mathcal{Q}_p \chi\,, \qquad \chi = b_\alpha \bar{\mathcal{S}}^\alpha \Phi\,.$$

It is easy to see that the gauge parameter χ satisfies all the necessary constraints provided Φ does. So the value of the second Casimir operator in the $\mathcal{Q}_p$ - cohomology at zeroth degree is given by

$$C_2\Phi = \Big(w(d-1+w) + \sum_{l=1}^{n-1} s_l(s_l - 2l + d - 1)\Big)\Phi$$

The standard expression gives

$$E_0(E_0 - d + 1) = w(w + d - 1)$$

Fronsdal ('79), Metsaev ('95), Deser Waldron ('01), Skvortsov ('09)

we find

$$E_0 = w + d - 1 = (\text{special } w = s_p - p - t) = s_p - p - t + d - 1$$

Generating BRST formulation

Space of functions

Functions on the hyperboloid

$$\Phi = \Phi(x, Y, A|\, b, \theta)$$

- $x^m,\ m = 0, ..., d-1$ intrinsic AdS coordinates
- $Y^A, A_i^A,\ i = 1, ..., n-1$ target space variables (twisted Howe realization)
- $b^\alpha,\ \alpha = 1, ..., p \le n-1$ target space ghosts, $gh\, b_\alpha = -1$
- $\theta^m,\ m = 0, ..., d-1$ spacetimes ghosts, $gh\, \theta^m = 1$

Covariant background derivative

De Rham differential

BRST operator

$$\nabla = d + \frac{1}{2}\theta^m \omega_m^{AB} J_{AB}\,, \qquad d = \theta^m \frac{\partial}{\partial x^m}$$

$$\nabla^2 = 0\,, \quad \mathrm{gh}\,\nabla = 1$$

AdS background connection ω_m^{AB} satisfies

zero-curvature condition

$$d\omega^{AB} + \omega^A{}_C \wedge \omega^{CB} = 0$$

Here $V^A = const$. Basis differential forms dx^m are replaced with extra Grassmann odd ghost variables $\theta^m,\ m = 0, ..., d-1$ because ∇ is interpreted as a part of BRST operator.

Operator Q_p and off-shell constraints

BRST operator

$$Q_p = S^{\dagger}_{\alpha}\frac{\partial}{\partial b_{\alpha}}\,, \qquad Q_p^2 = 0\,, \qquad \mathrm{gh}\, Q_p = 1$$

Off-shell constraints along with Q_p form some algebra

1) Massless and massive fields: constraints are linear in sp(2n) generators

2) Partially-massless fields: some of constraints are polynomial in sp(2n) generators. The respective polynomial order is proportional to the depth of partial masslessness.

important remark:

Anti-de Sitter algebra acts in the Q_p cohomology:

$$[J^{AB}, Q_p] = 0$$

Total BRST operator: the generating formulaltion

BRST operator

$$\Omega = \nabla + Q_p \,, \qquad \Omega^2 = 0 \,,$$

$$\nabla^2 = Q_p^2 = 0 \,, \qquad [Q_p, \nabla] = 0$$

Plus appropriate off-shell (BRST extended) constraints.

BRST operator is a sum of the term associated to the spacetime isometry algebra and the term associated to the Howe dual symplectic algebra

Part of algebraic constraints can be consistently relaxed =
infinitely reducible string-like system

Field content and equations of motion

Ghost-number-zero field: $$\Phi^{(0)} = \phi_0 + \phi_1 + \ldots + \phi_p$$

where

$$\phi_k = \phi^{\alpha_1 \ldots \alpha_k}_{a_1 \ldots a_k}(x, Y, A) b_{\alpha_1} \ldots b_{\alpha_k} \theta^{a_1} \ldots \theta^{a_k}$$

Ghost-number-one gauge parameters $$\xi^{(-1)} = \xi_1 + \xi_2 + \ldots + \xi_p$$

where

$$\xi_k = \xi^{\alpha_1 \ldots \alpha_k}_{i_1 \ldots i_{k-1}}(x, A, Y) b_{\alpha_1} \ldots b_{\alpha_k} \theta^{i_1} \ldots \theta^{i_{k-1}}$$

(Fields and gauge parameters are identified as differential forms!)

Equations and symmetries for $\Omega = \nabla + Q_p$ are

$$\Omega \Phi^{(0)} = 0 \qquad \delta \Phi^{(0)} = \Omega \xi^{(-1)} \qquad \delta \xi^{(-1)} = \Omega \xi^{(-2)}$$

In components

$$\boxed{\nabla \phi_k + S^\dagger_\alpha \frac{\partial}{\partial b_\alpha} \phi_{k+1} = 0\,, \qquad k = 0, \ldots, p}$$

Algebraic constraints

$$T^{ij}\Psi = 0, \qquad (N_i{}^j + B_i{}^j)\Psi = 0 \quad i < j, \qquad (N_i + B_i)\Psi = s_i\Psi$$

and $$\Box_Y \Psi = 0\,, \qquad S^i\Psi = 0$$

and $$h\Psi = 0\,, \qquad h = N_Y - B - w\,,$$

$$\bar{S}^{\hat{\alpha}}\Psi = 0\,, \qquad \hat{\alpha} = p+1,\,\ldots\,,n-1$$

Here we recall that

$$N_Y = (Y^A + V^A)\frac{\partial}{\partial Y^A} \qquad \text{and} \qquad \bar{S}^i = (Y^A + V^A)\frac{\partial}{\partial A_i^A}$$

For special values $w = s_p - p - t$ one additionaly imposes

$$(\bar{S}^p)^t\Psi = 0$$

Generating theory on Minkowski space

Functions on Minkowski space

$$\phi = \phi(x, y, a| b, \theta)$$

- $x^m, \; m = 0, ..., d-1$ - Cartesian coordinates
- y^a, a_i^a - auxiliary variables, $i = 1, ...n-1 \; a = 0, ..., d-1$
- b^i - target space ghosts, $i = 1, ...n-1 \quad gh\, b_i = -1$
- θ^a - spacetime ghosts $a = 0, ..., d-1 \quad gh\, \theta^a = 1$

BRST operator

$$\Omega = \nabla + Q \,, \qquad \Omega^2 = 0$$

where

$$\nabla = \theta^a \left(\frac{\partial}{\partial x^a} - \frac{\partial}{\partial y^a}\right) \qquad Q = S_i^\dagger \frac{\partial}{\partial b^i} \qquad S_i^\dagger = a_i^a \frac{\partial}{\partial y^a}$$

covariant derivative in Cartseian coordiantes

p = n-1

Field content and equations of motion

Ghost-number-zero field: $$\phi^{(0)} = \phi_0 + \phi_1 + \ldots + \phi_{n-1}$$

where $$\phi_p = \phi^{i_1 \ldots i_p}_{a_1 \ldots a_p}(x, y, a) b_{i_1} \ldots b_{i_p} \theta^{a_1} \ldots \theta^{a_p}$$

Ghost-number-one gauge parameters $$\xi^{(-1)} = \xi_1 + \xi_2 + \ldots + \xi_{n-1}$$

where $$\xi_p = \xi^{i_1 \ldots i_p}_{a_1 \ldots a_{p-1}}(x, a, y) b_{i_1} \ldots b_{i_p} \theta^{a_1} \ldots \theta^{a_{p-1}}$$

Equations and symmetries for $\Omega = \nabla + Q$ are

$$\Omega \Phi^{(0)} = 0 \qquad \delta \Phi^{(0)} = \Omega \xi^{(-1)} \qquad \delta \xi^{(-1)} = \Omega \xi^{(-2)}$$

In components

$$\nabla \phi_k + S_i^{\dagger} \frac{\partial}{\partial b_i} \phi_{k+1} = 0\,, \qquad k = 0, \ldots, n-1$$

Algebraic constraints

$$T^{IJ} \Psi = 0 \qquad (N_i{}^j + B_i{}^j) \Psi = 0 \quad i < j \qquad (N_i + B_i) \Psi = s_i \Psi$$

Dinamically equivalent theories

Theory $(\mathcal{H}, \Omega)$:

- $\mathcal{H}$ – representation space of Ω, $\Omega^2 = 0$;
- Equations of motion $\Omega\Phi = 0$, where $\Phi \in \mathcal{H}$.

$$\text{Triplet } \mathcal{H} = \mathcal{E} \oplus \mathcal{F} \oplus \mathcal{G}$$

- $\mathcal{E}$ – dynamical fields
- $\mathcal{F}$ – auxiliary fields
- $\mathcal{G}$ – Stueckelberg fields

Theory $(\mathcal{E}, \hat{\Omega})$:

- $\mathcal{E}$ – representation space of $\hat{\Omega}$, $\hat{\Omega}^2 = 0$;
- Equations of motion $\hat{\Omega}\Psi = 0$, where $\Psi \in \mathcal{E}$.

$$(\mathcal{H}, \Omega) \text{ equivalent } (\mathcal{E}, \hat{\Omega})$$

Homological reduction

Additional grading

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots$$

$$\Omega = \Omega_{-1} + \Omega_0 + \Omega_1 + \ldots$$

Definition:

Stueckelberg

$$\mathcal{E} \oplus \mathcal{G} = Ker\Omega_{-1}\,, \qquad \mathcal{G} = Im\Omega_{-1}\,, \qquad \mathcal{E} = \frac{Ker\Omega_{-1}}{Im\Omega_{-1}}$$

dynamical

String theory: light cone DoF, quartets

Unfolded formulation: BRST operator $\Omega_{-1} \cong \sigma_-$

Generating formulation: BRST operator $\Omega_{-1} \cong Q_p$

Q_p - cohomology

THEOREM: **The Q_p cohomology evaluated in the subspace singled out by off-shell constraints is non-empty only for**

$$H^k(Q_p, \mathcal{H}_{\text{on-shell}}) = \begin{cases} \text{Weyl module}\,, & k=0\,, \\ \quad 0\,, & k \neq 0, -p\,, \\ \text{Gauge module}\,, & k=-p\,. \end{cases}$$

- $gh = 0$ infinite-dimensional AdS Weyl module $\mathcal{M}_0$
  (0-form gauge invariant combinations of potentials)
- $gh = -p$ finite-dimensional AdS gauge module $\mathcal{M}_p$
  (p-form gauge potentials). For massive fields it is zero.

Boulanger Iazeolla Sundell ('08) Alkalaev Grigoriev ('09) Skvortsov ('09)

remark:

The form of AdS Weyl module for unitary massless mixed-symmetry confirms the Brink-Metsaev-Vasiliev decomposition

some elements

General (non-unitary) case:

$$\mathcal{M}_0 = \bigoplus_{k \geqslant 0} \bigoplus_{\{m\}_k} \mathcal{M}^{(k)}_{0,m,p} \,/\, \mathcal{Z}^{(k)}_{0,m,p}$$

Poincare Weyl modules

Conclusions

- Uniform and concise description of AdS fields: massless, partially-massless, and massive

- Using Howe duality makes the constraint structure of the theory manifest:

  AdS algebra o(d-1,2) – symplectic algebra sp(2n)

- Cohomological reductions and resolutions. In particular, the unfolded fields appear as the cohomology of the nilpotent operator built from some sp(2n) generator.

- Off-shell formulations (non-dynamical) of non-linear HS theories. Vasiliev ('05), Grigoriev ('06)

- The study of cubic HS vertices within the ambient approach. Joung Lopez and Taronna ('12)

- Our main believe: the approach is important for understanding the searched-for geometry underlying HS interactions.