Leaky Carrollian boundaries

Céline Zwikel (Perimeter Institute)

Outline

1. Introduction Leaky Carrollian boundaries

2. Asymptotically flat spacetimes in 4D

   Partial Bondi gauge & New gauge fixing

3. Null surfaces @ finite distance

Introduction Leaky Carrollian boundaries

Boundaries

: spacetimes with boundaries

symmetries ξ

gauge symm.

$$\boxed{\xi \overset{\text{Noether}}{\longleftrightarrow} Q_\xi}$$

on co-dim 2

"surface charge"

located on boundary

$\sim$ EM Gauss law

$\sim$ $Q_{\partial t} \rightarrow$ mass

* asymptotic
* finite boundary
black hole horizon

→ Covariant phase space method

[review Fiorucci's thesis 2112.07666]

→ Fall-offs of the fields closed to the boundary are crucial

$Q_\xi = 0$ → pure gauge

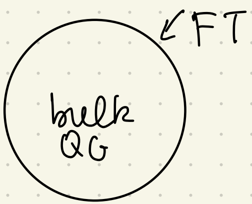
$Q_\xi \neq 0$ → physical, large

Charges form an algebra -> asymptotic symmetry algebra

↳ Charges label the states

↳ ASG organizes the phase space (EOM + fall offs)

↳ QG: states transforms under a representation of ASG



AdS_3/CFT_2 holographic dualities
[Brown - Henneaux]

How big this algebra can be? → Relaxation of the gauge

fall offs - relax enough solutions

- constrained well def.

3d'
[Pérez et al. '16]
[Campoleoni et al. 22']
[Geiller et al. 21] [Alessio et al.]
[Troessaert 13] [Ciambelli et al 21']
etc

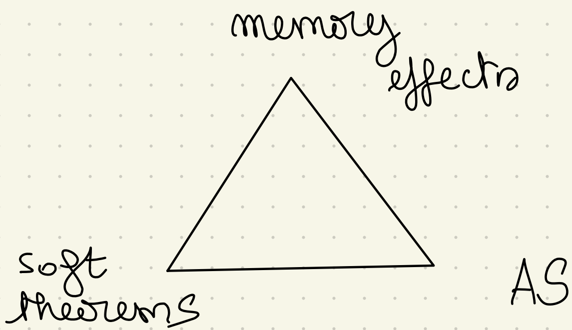
What is the relevant/fundamental group for gravity?

[Grumiller, Riegler '16]

[Freidel, Pranzetti 21'] [Ciambelli, Leigh 21']

[Adami et al 21']

For 4 dimensional asymptotically flat spacetimes: infrared triangle [Strominger et al]



Carrollian

Boundary is a null surface

Leaky $\sim$ Open system

$$\delta L = \text{EOM}\, \delta\text{field} + d\Theta(\delta\text{field})$$

$\Theta \neq 0$ (onshell)

$\longrightarrow$ Radiative dof eg 4D AF $\Theta = \frac{1}{2}\sqrt{q}\, C^{AB}\delta N_{AB}$

$\longrightarrow$ Unknown boundary dynamics

leaky cov. phase space (complete) is still work in progress

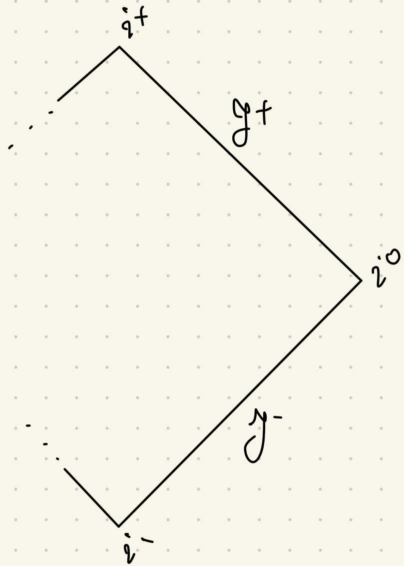
Today: 4d

* new gauge for AF [2205.11401 + WIP] with Marc Geiller

* finite surface [2110.04218] [2002.08396] with H. Adami, D. Grumiller, S. Jabbari, V. Taghiloo, H. Yavartanoo, S. Sadeghian

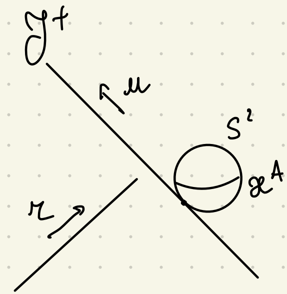
* prescription for renormalization of charges for leaky systems [2306.16451] with Robert McNees

Asymptotic flat spacetimes in 4 dimensions



Bondi coordinates (u, r, x^A) $A = 1,2$

approaches null $\mathcal{J}^+$ w/ null geod.



u = retarded time

x^A = celestial coord.

r = radial coord.

& conformal compactification

& no log term for this talk

& $\Lambda \neq 0$, not for this talk

Partial Bondi gauge

[2205.11401 & WIP]
w/ M. Geiller

null $= \partial_u$

$g^{uu} = 0$; $g^{uA} = 0$; no specification on r more than a param. along the null geod.

$\quad 1 \qquad\qquad 2$

$$ds^2 = e^{2\beta}\frac{V}{r}\,du^2 - 2e^{2\beta}\,du\,dr + g_{AB}\,(dx^A - U^A du)(dx^B - U^B du)$$

$g_{AB} = q_{AB}\, r^2 + C_{AB}\, r + \dots$; β, V, U^A arbitrary function of (u, r, x^A)

Newman-Unti gauge : r affine parameter $\quad \partial_r \beta = 0$

Bondi-Sachs gauge : r = areal distance $\quad \det g_{AB} = r^4 \det \underbrace{\mathring{q}_{AB}}_{S^2}$

Hierarchy of EOM

$$ds^2 = e^{2\beta}\frac{V}{r}\,du^2 - 2e^{2\beta}\,du\,dr + g_{AB}\,(dx^A - U^A du)(dx^B - U^B du)$$

$$g_{AB} = r^2 q_{AB} + r\left(C_{AB}^{TF} + \frac{1}{2}C\,q_{AB}\right) + \left(D_{AB}^{TF} + \frac{1}{2}D\,q_{AB}\right) + \frac{1}{r}\left(E_{AB}^{TF} + \frac{1}{2}E\,q_{AB}\right) + \dots$$

$E_{rr} = 0 \Rightarrow \quad \beta = \beta_0(u,\phi) + \frac{1}{r^2}(\ \dots\ ) + \dots$

$E_{rA} = 0 \Rightarrow \quad U^A = U_0^A(u,\phi) + \frac{1}{r}(\dots) + \frac{1}{r^2}(\dots) + \frac{1}{r^3}(P^A + \dots) + \mathcal{O}\left(\frac{1}{r^4}\right)$

$E_{ur} = 0 \Rightarrow \quad \frac{V}{r} = r(\dots) + (\dots) + \frac{1}{r}(M + \dots) + \dots$

$\rightarrow$ all the radial dep. is fixed ; the rest are evolution eq.

$E_{AB}^{TF} = 0 \Rightarrow \quad (\partial_u \ln\sqrt{q} - \partial_u)\,q_{AB} - (D_{(A}U^0_{B)})^{TF} = 0$; $\partial_u E_{AB}^{TF} = 0$ & all sub.

$E_{uu} = 0 \Rightarrow \quad \partial_u M = \dots$

$E_{uA} = 0 \Rightarrow \quad \partial_u P_A = \dots$

(Recommendation Newman - Penrose)

$$ds^2 = e^{2\beta}\frac{V}{r}\,du^2 - 2e^{2\beta}\,du\,dr + g_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

$$g_{AB} = r^2 q_{AB} + r\left(C^{TF}_{AB} + \frac{1}{2} C\, q_{AB}\right) + \left(D^{TF}_{AB} + \frac{1}{2} D\, q_{AB}\right) + \frac{1}{r}\left(E^{TF}_{AB} + \frac{1}{2} E\, q_{AB}\right) + \dots$$

$$E_{rr} = 0 \Rightarrow \beta = \beta_0(u,\phi) + \frac{1}{r^2}\left(\frac{1}{32}[CC] - 4D\right) + \frac{1}{r^3}(\dots)$$

$$E_{rA} = 0 \Rightarrow U^A = U^A_0(u,\phi) + \frac{1}{r}(\dots) + \frac{1}{r^2}(\dots) + \frac{1}{r^3}\left(P^A + \dots\right) + O\left(\frac{1}{r^4}\right)$$

$$E_{ur} = 0 \Rightarrow \frac{V}{r} = r(\dots) + (\dots) + \frac{1}{r}(M + \dots) + \dots$$

Solution space is
* kinematic data } arbitrary functions of (u, x^A)
* radiative data }
* constraints data } evolution is constrained

Notation $[CC] = C_{AB}C^{AB}$

Carroll structure

Intrinsic

$$\frac{ds^2}{r^2}\Big|_{r\to\infty} = q_{AB}\,(dx^A - U_0^A\,du)(dx^B - U_0^B\,du) = q_{ab} \qquad \text{Carroll metric}$$

$$\ell^a = e^{-2\beta_0}(\partial_u + U_0^A\partial_A)$$ null vector $\ell^a q_{ab} = 0$

θ_ℓ is free but shear is zero (due to EOM)

Ehresmann connect° $k = \partial_r \rightarrow \Theta(k) = \frac{2}{r} - \frac{1}{2r}C + \frac{1}{r^3}\left(D - \frac{1}{2}[CC]\right)$

$\nabla_k k = 2\partial_r\beta\,\partial_r$

shear(k) is free $\partial_r\gamma_{AB} \rightarrow C^{TF}_{AB}, \ldots$

BS: $\Theta(k) = \frac{2}{r}$

NU: $\nabla_k k = 2\partial_r\beta = 0$

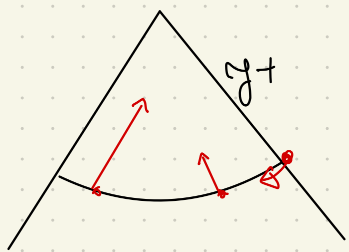
Asymptotic symmetries

ξ s.t $\quad g + \mathcal{L}_\xi g \in$ Partial Bondi gauge

Preserving $\quad g_{rr} = 0 \quad \Rightarrow \quad \xi^u = f(u, x^A)$

$\quad g_{rA} = 0 \quad \Rightarrow \quad \xi^A = Y^A(u, x^A) - \int_r^\infty dr' \, e^{2\beta} \gamma^{AB} \partial_B f$

$$= Y^A + \mathcal{O}\left(\frac{1}{r}\right)$$



$$g_{AB} = q_{AB} r^2 + \dots \quad \Rightarrow \quad \xi^r = r \, h(u, x^A) + \sum_{n=0}^{\infty} \frac{\xi^r_n}{r^n}$$

$$\xi^r = r h + \left(k - \frac{1}{2} \Delta f\right) + \frac{1}{r}\left(l - \frac{1}{2} D_A C^{AB} \partial_B f + \frac{1}{2} D^A (C \partial_A f)\right) + \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$\delta \ln \sqrt{q} = (2h + D_A Y^A)$$

$$\delta C = (\dots) C + 4k$$

$$\delta D = (\dots) D + 4l + kC - C^{AB}_{TF} D_A \partial_B f$$

↳ Changes analysis will show these are pure gauge

$$\mathrm{AKV} = \xi = f\,\partial_u + Y^A\,\partial_A + \Big(r\,h + k + \tfrac{1}{2}\Delta f + \tfrac{1}{r}\,\ell - \tfrac{1}{2} D_A C^{AB}\partial_B f + \tfrac{1}{2}\partial^A C\,\partial_A f\Big)$$

$+$ subLead.

Algebra : $\big[(f_1, Y_1, h_1, k_1, \ell_1), (f_2, Y_2, h_2, k_2, \ell_2)\big]_* = (f_{12}, Y_{12}, h_{12}, k_{12}, \ell_{12})$

$$f_{12} = f_1\,\partial_u f_2 + Y_1^A\,\partial_A f_2 - \delta_{\xi_1} f_2 - (1\leftrightarrow 2)$$

$$Y_{12}^A = f_1\,\partial_u Y_2^A + Y_1^B\,\partial_B Y_2^A - \delta_{\xi_1} Y_2^A - (1\leftrightarrow 2)$$

$$h_{12} = f_1\,\partial_u h_2 + Y_1^A\,\partial_A h_2 - \delta_{\xi_1} h_2 - (1\leftrightarrow 2)$$

$$k_{12} = f_1\,\partial_u k_2 + Y_1^A\,\partial_A k_2 - h_1 k_2 - \delta_{\xi_1} k_2 - (1\leftrightarrow 2)$$

$$\ell_{12} = f_1\,\partial_u \ell_2 + Y_1^A\,\partial_A \ell_2 - 2 h_1 \ell_2 - \delta_{\xi_1} \ell_2 - (1\leftrightarrow 2)$$

$$\boxed{\big(\mathrm{Diff}(\mathcal{J}^+) \oplus \mathbb{R}_h\big) \oplus \big(\mathbb{R}_k \oplus \mathbb{R}_\ell\big)}$$

Now changes...

Charges - covariant phase space formalism

$$\delta Q_\xi = \lim_{r \to \infty} \int_{S^2} k_\xi$$

$$\delta L = E^{\mu\nu} \delta g_{\mu\nu} + \partial_\mu \Theta^\mu (\delta g)$$

↳ symplectic potential

$$\omega(\delta_1 g, \delta_2 g) = \delta_1 \Theta(\delta_2 g) - \delta_2 \Theta(\delta_1 g)$$

$d\omega(\delta_\xi g, \delta g) = 0$ onshell $\to \omega(\delta_\xi g, \delta g) = k_\xi(\delta g)$

Ambiguities in definition of Θ (when imposing only bulk EOM)

$$\Theta \to \Theta + \delta \ell + dY$$

Assume $\partial_u q_{AB} = 0$; $\beta_0 = 0$; $U_0^A = 0$ $\Rightarrow$ $h = -\partial_u f$; $\partial_u Y^A = 0$; $\partial_u^2 f = 0$

Problem charge diverges $k^{ur} = (\dots)\, r + \text{finite} + \dots$

Solution - follow the prescription [2306.16451 w/ R. McNees]

choice of Y indep of ℓ s.t charges are finite

$$\Theta_{ren} = \Theta + dY + \delta \ell$$

$$Y^{ur} = -\int dr\, \Theta^u$$ "corner contribution of symplectic potential"

$$\Theta^r_{ren} = \Theta^r + \partial_u Y^{ru} = \Theta^r + \int dr\, \Theta^u + \delta(\dots)$$

$$\Theta^u_{ren} = \Theta^u + \partial_r Y^{ur} = 0 + \Theta(1/r^2)$$

$\rightarrow$ $\omega = dk$ $\Rightarrow$ $\omega^u = \partial_r k^{ur} + \partial_A k^{uA}$

$\Rightarrow$ $\partial_r \int_{S^2} k^{ur} = 0 + \text{sublead.}$ $\sim$ [Compère et al. '18]

In [2306.16451] we show the procedure in 2d & 3d where we then have symplectic charge.

[2107.01073] w/ C. Goeller & M. Geiller

For example in 3d Bondi-Weyl (equivalent of Partial Bondi gauge)

$$\Theta^u = \partial_r \left(\delta g_{\phi\phi} - \delta\beta \, g_{\phi\phi} \right)$$

interesting to not impose the fall-offs

→ Recover corner terms used to restore integrability & finiteness?

→ While it doesn't fix the finite ambiguity it might suggest one.

Comment on ℓ

* we were interested in having ω finite not Θ

* Θ_{ren} = finite + δ(exact)

  → FG & WFG hologr. renorm.

  → Bondi ∃ such terms but need corner dogr.

* Comparaison with other methods

• [Compere, Marolf '08] Y = corner symplectic potential Y_c

$$\left.\begin{aligned} \delta\ell &= EOM_{\partial M}\,\delta\phi_{\partial M} + d\,Y_c \\ Y &= -Y_c \end{aligned}\right\} \quad \Theta = EOM_{\partial M}\,\partial\phi_{\partial M}$$

• other prescription based on Noether charge [Freidel + 21']

$U_2^A = -\frac{1}{2} D_B C^{AB} + \frac{1}{2} \partial_B C$

$$\delta Q = \int_{S^2} \delta Q_Y + \delta Q_h + \delta Q_k + \delta Q_\ell + \delta Q_f$$

New Charges

$$\delta\!\!\!/ Q_Y = Y^A \delta \left[\sqrt{q} \left(2\mathcal{P}_A - \frac{3}{16} \partial_A (4D - [CC]) + C_{AB} U_2^B - C U_A^2 \right) \right],$$

$$\delta\!\!\!/ Q_h = h\delta \left[\sqrt{q} \left(\frac{3}{2} D + \frac{1}{4} C^2 - \frac{5}{8} [CC] \right) \right],$$

$$\delta\!\!\!/ Q_k = \frac{1}{2} k \left(\sqrt{q}\, C_{\mathrm{TF}}^{AB} \delta q_{AB} - C\, \delta \sqrt{q} \right),$$

$$\delta\!\!\!/ Q_\ell = -3\ell\delta\sqrt{q},$$

$$\begin{aligned} \delta\!\!\!/ Q_f = {} & 4f\delta(\sqrt{q}\,\mathcal{M}) - \frac{1}{2} f \sqrt{q}\, C_{AB}^{\mathrm{TF}} \delta N^{AB} - \frac{1}{4} f C \delta(\sqrt{q}\, R) \\ & + \sqrt{q}\, \delta q_{AB} \left[f \left(D^A U_2^B + \frac{1}{4} R C_{\mathrm{TF}}^{AB} + \frac{1}{8} \partial_u C C_{\mathrm{TF}}^{AB} + \frac{1}{8} C N^{AB} \right) + 2\partial^A f U_2^B + \frac{1}{4} \Delta f C_{\mathrm{TF}}^{AB} \right] \\ & + \delta\sqrt{q} \left[f \left(2\mathcal{M} - \frac{3}{4} \partial_u D - \frac{3}{16} \partial_u [CC] + \frac{1}{8} \partial_u C^2 - 2 D_A U_2^A \right) - 4 U_2^A \partial_A f - \frac{1}{4} C \Delta f \right] \end{aligned}$$

Reduces to previous analysis [Compère, Fiorucci, Ruzziconi '18] [Barnich, Troessaert '11]

Charges in conformal gauge

$q_{AB} = e^{\Phi}\, \mathring{q}_{AB}$; $\delta \mathring{q}_{AB} = 0$

* Charges associated to k is pure gauge $\rightarrow$ $C = 0$; $k \simeq 0$

* Change of slicing (field dep. redefinition of symm. generators)

$$\delta\!\!\!/ Q_Y = 2Y^A \delta\left[\sqrt{q}\left(\mathcal{P}_A + \partial_A \tilde{D} - \frac{1}{4}C_A\right)\right],$$

$$\delta\!\!\!/ Q_h = -4h\delta(\sqrt{q}\,\tilde{D}),$$

$$\delta\!\!\!/ Q_{\tilde{\ell}} = \tilde{\ell}\delta\sqrt{q},$$

$$\delta\!\!\!/ Q_f = 4f\delta(\sqrt{q}\,\mathcal{M}) - \frac{1}{2}f\sqrt{q}\,C^{AB}\delta\hat{N}_{AB} + (\mathrm{BT}),$$

$$\tilde{D} = -\frac{3}{8}D + \frac{5}{32}[CC]$$

$$C_A = \frac{1}{4}\partial_A [CC] + C_{AB} D_C C^{CB}$$

$$-3\ell = \tilde{\ell} - 2f\left(\mathcal{M} + \partial_u \tilde{D}\right) + \frac{1}{2}C^{AB} D_A \partial_B f.$$

* Integrable charges without radiation

* $(f, h, Y, \tilde{e})$ form an algebra without radiation

$$\tilde{\ell}_{12} = -2 f_1 \mathcal{J}^A \partial_A f_2 + Y_1^A \partial_A \tilde{\ell}_2 - 2 h_1 \tilde{\ell}_2 - (1 \leftrightarrow 2)$$

$$\mathcal{J}^A = \frac{1}{2} \nabla_B \hat{N}^{AB}$$

$\exists$ other choice s.t algebra even with radiation

$$-3\ell = \bar{\ell} - 2 f \partial_u \tilde{D} + \frac{1}{2} C^{AB} D_A \partial_B f$$

$$f = T(x^A) - u\, h(x^A)$$

$$\mapsto \boxed{\left(\mathrm{Diff}(S^1) \oplus \mathrm{Diff}(S^1) \ltimes \mathbb{R}_h\right) \ltimes \left(\mathbb{R}_T \oplus \mathbb{R}_\ell\right)}$$

Change algebra we used Koszul bracket (BT + "Mons")
To get rid of the field dep. in central charge. [Barnich, Fiorucci, Ruzziconi]

Asymptotic symmetry algebra

$f = T(x^A) - u\,h(x^A)$

× global BMS = Lorentz $\ltimes \mathbb{R}_T$ $\qquad \delta q_{AB} = 0 \quad (q_{AB} = \mathring{q}_{AB})$

[Bondi, van der Burg, Metzner ; Sachs '62]

× extended BMS = $\left((\mathrm{Diff}(S^1) \oplus \mathrm{Diff}(S^1)) \ltimes \mathbb{R}_T \right) \ltimes \mathbb{R}_h$

$\hookrightarrow$ Celestial holography

[Barnich, Troessaert '09]

$\det g_{AB} = r^4 \det \mathring{q}_{AB}$

× generalized BMS = $\mathrm{Diff}(S^2) \oplus \mathbb{R}_T$ $\qquad \delta\sqrt{q} = 0$

[Campiglia, Laddha '14]

× BMS Weyl = $(\mathrm{Diff}\,S \oplus \mathbb{R}_h) \ltimes \mathbb{R}_T$

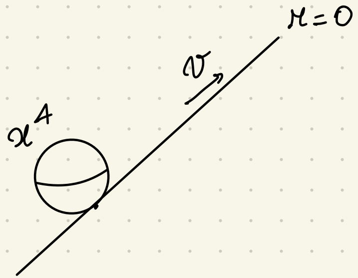
[Freidel, Oliveri, Pranzetti, Speziale '21]

$$\boxed{(\mathrm{Diff}(S^2) \ltimes \mathbb{R}_h) \ltimes (\mathbb{R}_T \oplus \mathbb{R}_h \oplus \mathbb{R}_e)}$$

$\longrightarrow$ Corresponds to the gauge fixing $\partial_r^3 (\det g_{AB}) = r^4 \det \mathring{q}_{AB}$

Finite surface

[2002.08346]
[2110.04218] & talk @ first Carroll workshop



Gaussian Null Coordinates

$$g_{rr}=0 \;;\; g_{rA}=0$$

$$ds^2 = e^{2\beta} V dv^2 + 2e^{2\beta} dv\, dr + g_{AB}(dx^A - U^A dv)(dx^B - U^B dv)$$

$$g_{AB} = q_{AB} + r\lambda_{AB} + r^2 \mu_{AB} + \dots$$

Solution space

$$E_{rr} = 0 \rightarrow \beta = \beta_0 + \left(\frac{\mu}{\lambda} - \frac{1}{4}\frac{[\lambda\lambda]}{\lambda}\right) r + O(r^2)$$

$$E_{rA} = 0 \rightarrow U^A = U^A_0 + P^A r + O(r^2)$$

$$E_{ur} = 0 \rightarrow V = 0 + (\dots) r + V_2 r^2 + O(r^3)$$ for the surface to be null

$$E^{TF}_{AB} = 0 \rightarrow \partial_u \lambda^{TF}_{AB} + \dots = \dots \;;\; E_{uA} = 0 \rightarrow \partial_u P_A = \dots$$

$$E_{uu} = 0 \rightarrow \partial_u \theta + \dots$$

Carroll fluid [Freidel + '22]

AS: $\xi = f(u,x^A) + h(u,x^A)\, r\,\partial_r + Y^A(u,x^A)\,\partial_A$ + subl.ead.

Charges: $$\delta Q_\xi = \frac{1}{16\pi G}\int d^{n-2}x \left(\sqrt{q}\, h + Y^A\, \delta P_A + T\,\delta A\right)$$

$$A = \sqrt{q}\ \underbrace{N^{AB}}_{\text{shear}\ \ell\ (\text{"}\partial_u q_{AB}\text{"})}\ \delta q_{AB} + \dots$$

$$\boxed{\mathrm{Diff}(\text{Null surface})_{f,Y} \ltimes \mathbb{R}_{h(u,x^A)}}$$

$\exists$ slicing s.t.

$\mathrm{Diff}(S^2) \oplus$ Heisenberg

Structure very different than AF

[for 3d see Adami et al '21]

Conclusion

* Partial Bondi gauge - solution space & symm.
* Charges computations $\rightarrow$ New gauge

solution space

Kinematic data: • boundary data $(q_{AB}, e^{\beta_0}, U_0^A)$ • C, D $\Big\}$ arbitrary u dep.

Radiative data: • C_{AB}^{TF}

Constraint data: • M, P_A, E_{AB}^{TF} & sublead. towers $\}$ EOM

* New prescription to obtain finite charges in open system
* Review on finite surface results

Futures directions

* towards a coord. indep. description
* Apply to a physical solution
  - → cosmological solution
  - → adding matter
* gluing @ i^0
* where are the new charges in holographic dictionary?
* $\Lambda \neq 0$
* how to fix the finite ambiguity in the charges?

Ευχαριστώ