Irreducible representations of the Poincaré group ($SO(1,d) \otimes_s T^{d+1}$) and $E_{11} \otimes_s l_1$

- Irreducible representations of the Poincaré group are the same as conformal representations living on $\mathcal{J}^+$
  (with Kevin Nguyen 2305.02884)

- The massless irreducible representation of $Ic(E_{11}) \otimes_s l_1$ contains only the degrees of freedom of maximal supergravity
  (1905.07324)

Irreducible representations of the Poincaré group

Wigner 1939, ...

For massless particles $p_\mu^2 = 0$ and we choose $p^{(0)+} = \frac{p^d + p^0}{\sqrt{2}} = 1$, $p^{(0)-} = 0 = p^{(0)i}$

This is preserved by

$$\tilde{\mathcal{L}} = \{\tilde{J}_{ij}, \tilde{J}_{i+}, \tilde{P}_\mu\} \qquad i,j = 1, \ldots, d-1$$

whose irreducible representation is

$$\tilde{J}_{ij}\,\psi_\sigma(p^{(0)}) = -(D_{ij})_\sigma{}^{\sigma'}\,\psi_{\sigma'}(p^{(0)}), \quad \tilde{J}_{i+}\,\psi_\sigma(p^{(0)}) = 0$$

(unitarity).

$$\tilde{P}_-\,\psi_\sigma(p^{(0)}) = i\,\psi_\sigma(p^{(0)}), \quad \tilde{P}_+\,\psi_\sigma(p^{(0)}) = 0 = \tilde{P}_i\,\psi_\sigma(p^{(0)})$$

The states in the representation of $SO(1,d)$ are defined by

$$\psi_\sigma(p) = e^{\varphi^i \tilde{J}_{-i}}\, e^{\phi J_{+-}}\, \psi_\sigma(p^{(0)}).$$

Acting with $\tilde{P}_\mu$ we find that

$$p^+ = e^\phi, \quad p^- = -\frac{1}{2}\varphi^i\varphi_i e^\phi, \quad p^i = -\varphi^i e^\phi$$

The d components of p^μ are parameterized by the d parameters φ^i and ϕ.

Under a $g \in SO(1,d)$ transformation

$$U(g)\, \psi_\sigma(p) = D(h_c^{-1})_\sigma{}^{\sigma'}\, \psi_{\sigma'}(p')$$

where $h_c \in \tilde{H}$ and is given by

$$g\, e^{x^i \hat{J}_{-i}}\, e^{\phi \hat{J}_{+-}} = e^{x^{i\prime} \hat{J}_{-i}}\, e^{\phi' J_{+-}}\, h_c$$

If we take

$$g = e^{-\frac{1}{2}\lambda^{\mu\nu} \hat{J}_{\mu\nu}} = e^{\lambda J_{+-} + a^i J_{-i} + k^i \hat{J}_{+}} \cdot e^{-\frac{1}{2}\omega^{ij} J_{ij}}$$

the parameters transform as

$$x^{i\prime} = x^i + a^i + w^i{}_j x^j - \lambda x^i + (x^j k_j) x^i - \frac{1}{2}(x^j x_j) k^i$$

$$\phi' = \phi + \lambda - (k^i x_i)$$

and

$$h_c = e^{-\frac{1}{2}\omega^{ij} J_{ij} - k^i x^j J_{ij} + e^{-\phi} k^i J_{+i}}$$

All irreducible representations of the Poincaré group are found in this way.

The states in Minkowski space are

$$\psi_\sigma(X) = \int \frac{d^d p}{2p^0 (2\pi)^d}\, e^{ip \cdot X}\, \psi_\sigma(p).$$

where X^μ are the usual coordinates.

Instead we choose the coordinates

$$(r, u, x^i)$$

where

$$X^\mu = \frac{1}{\sqrt{2}}(u, -u, 0) + \frac{r}{\sqrt{2}}(1+x^2, 1-x^2, 2x^i).$$

where upon the metric becomes

$$ds^2 = \eta_{\mu\nu}dX^\mu dX^\nu = -2\,du\,dr + 2r^2 dx^i \delta_{ij} dx^j$$

As $r \to \infty$ we find $\mathcal{I}^+$ with coordinates (u, x^i). We now find the states ψ on $\mathcal{I}^+$ by taking $r \to \infty$.

Now

$$e^{ip\cdot X} = e^{-i\omega u}\, e^{-ir\omega\left(x^i + \frac{\alpha^i}{\sqrt{2}}\right)^2}.$$

and

$$\frac{d^d p}{p^0} = -(\sqrt{2})^d \quad \omega^{d-1}\frac{d\omega}{\omega}\, d\alpha^i$$

where $\omega = e^\phi$

As $r \to \infty$ the second term in $e^{ip\cdot X}$ oscillates wildly and under the integral

$$x^i = -\frac{\alpha^i}{\sqrt{2}}$$

We find that

$$\psi_\sigma(u, x^i) \equiv \lim_{r\to\infty} r^{\frac{d-1}{2}} \psi_\sigma(X)$$

$$= \text{const} \int_0^\infty \frac{d\omega}{\omega}\, \omega^{\frac{d-1}{2}} e^{-iu\omega} e^{x^i P_i} e^{-\ln\omega P} \psi_\sigma(p^{(0)})$$

where $P_i = \sqrt{2}\,\hat{\tilde{J}}_{i-}$, $D = -\hat{\tilde{J}}_{+-}$

Up to a Fourier transform $(\omega \to u)$ $\mathcal{J}^+$ parameterizes the massless particle and vice-versa.

The massless particle is described by $\psi_\sigma(u, x^i)$ defined on $\mathcal{J}^+$ which obeys

$$J_{ij}\, \psi_\sigma(0,0) = -(D_{ij})_\sigma{}^{\sigma'} \psi_{\sigma'}(0,0)$$

$$D\, \psi_\sigma(0,0) = \left(\frac{d-1}{2}\right) \psi_\sigma(0,0)$$

and

$$K_i\, \psi_\sigma(0,0) = 0 = K \psi_\sigma(0,0) = B_i \psi_\sigma(0,0).$$

where $K_i = -\sqrt{2}\,\hat{\tilde{J}}_{i+}$, $K = \hat{\tilde{P}}_+$, $B_i = -\frac{1}{\sqrt{2}} \hat{\tilde{P}}_i$

$J_{ij} = \hat{\tilde{J}}_{ij}$

with

$$e^{\frac{u}{\sqrt{2}} H + x^i P_i}\, \psi_\sigma(0,0) = \psi_\sigma(u, x^i)$$

where

$$H = -\hat{\tilde{P}}_- \ , \quad P_i = -\sqrt{2}\,\hat{\tilde{J}}_{-i}$$

Hence we have an induced representation with subgroup.

$$H = \{ J_{ij}, K_i, K, B_i, D \}.$$

(note D now is the subgroup) and coset

H and P_i

Except for $\hat{P}_-$ translations $\hat{P}_i$ are swopped with the Lorentz transformations $\hat{J}_{-i}$

It looks like a representation of the conformal group. Indeed the Poincaré group is the conformal group of J^+.

Where are the covariant fields?

We take a non-unitary, reducible representati of Poincaré Ψ_n and embedd Ψ_σ in it

under $\mathcal{L}e$

$$J_{ij}\,\Psi_n(p^{(0)}) = -(\tilde{D}_{ij})_n{}^m\,\Psi_m(p^{(0)})$$

$$J_{+i}\,\Psi_n(p^{(0)}) = -(\tilde{D}_{+i})_n{}^m\,\Psi_m(p^{(0)}).$$

and define

$$\Psi_n(p) = e^{\alpha^i J_{-i}}\,e^{\phi J_{+-}}\,\Psi_n(p^{(0)})$$

Under the Lorentz group.

$$U(g)\,\Psi_n(p) = D(h_c^{-1})_n{}^m\,\Psi_m(p').$$

The covariant states are.

$$A_n(p) = \tilde{D}(e^{\alpha^i J_{-i}}\,e^{\phi J_{+-}})_n{}^m\,\Psi_m(p).$$

If spin 1 then $U(g)\,A_\mu = -\Lambda_\mu{}^\nu A_\nu$

- The embedding condition $p^\mu A_\mu(p) = 0$

$\sim\ \Psi_+(p) = 0$

- The gauge transformation $\delta A_\mu = p_\mu \Lambda$

$\sim\ \Psi_-(p) \equiv \Lambda$

Taking the Fourier transform to Minkowski space time and the $r \to \infty$ limit to get to $\mathcal{I}^+$ we find

$$\psi_i(u, x^i)\ , \quad \psi_-(u, x^i)$$

which transform into each other and the gauge transformation

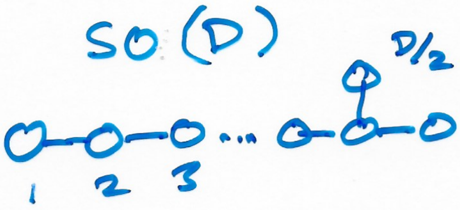
$$\delta\psi_i(u, x^i) = 0\ , \quad \delta\psi_-(u, x^i) = \Lambda(x^i; u)$$

Let us recaste the above in a more general way.

D even.

GL(D) $\qquad I_c \qquad$ SO(D)

$\xrightarrow[\text{involution}]{\text{invariant}}$



$K^a{}_b$ $\qquad J_{ab} = \eta_{ac}K^c{}_b - \eta_{bc}K^c{}_a$

The Cartan involution I_c acts as

$$I_c(R^\alpha) = -R^{-\alpha}$$

ie +ve ⟺ −ve roots $\qquad I_c(K^a{}_b) = -K^b{}_a$
$\cdot \eta\eta$

The vector or first fundamental representation (l_1) of $GL(D)$ is P_a.

$$[K^a{}_b, P_c] = -\delta^a_c P_b = -(D^a{}_b)_c{}^d P_d.$$

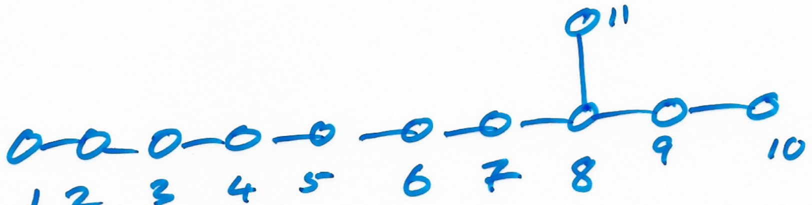
$$[K^a{}_b, K^c{}_d] = \delta^c_b K^a{}_d - \delta^a_d K^c{}_b.$$

Thus $I_c(GL(D)) \otimes_s l_1$ is $\{J_{ab}, P_c\}$ ie the Poincaré algebra.

$E_{11} \otimes_s l_1$ and its irreducible representatio

hep 1905.07324.

E_{11} is completely specified by its Dynkin diagram



Its generators at low levels using the decomposition to GL(11) are

$$\ldots, R_{a_1 \ldots a_8 b}, R_{a_1 \ldots a_6}, R_{a_1 a_2 a_3}, K^a{}_b, R^{a_1 a_2 a_3}, R^{a_1 \ldots a_6}, R^{a_1 \ldots a_8, b}, \ldots$$

$-$ve roots $\longleftrightarrow$ +ve roots

The Cartan involution acts as

$$I_c(R^\alpha) = - R_\alpha .$$

The invariant subalgebra is $R^\alpha - R_\alpha = S$

$I_c(S_\alpha) = S_\alpha$ contains

$$J_{ab} = K_{ab} - K_{ba}, \quad S^{a_1 a_2 a_3} = R^{a_1 \ldots a_3} - R_{a_1 \ldots a_3}$$

$$S^{a_1 \ldots a_6} = R^{a_1 \ldots a_6} - R_{a_1 \ldots a_6}, \ldots$$

Note $T^\alpha = R^\alpha + R_\alpha$ obeys $I_c(T^\alpha) = - T^\alpha$.

$$[S^\alpha, T^\beta] = g^{\alpha\beta}{}_\gamma T^\gamma$$

a

The vector (l_1) representation has

$$l_A = \{ P_a, \ Z^{a_1a_2}, \ Z^{a_1\dots a_5}, \ Z^{a_1\dots a_7,b}, \ Z^{a_1\dots a_8}, \dots \}$$

It contains all known ~~the~~ brane charge

The $E_{11} \otimes_s l_1$ algebra has the generato

R^α, R_α and l_A

$$[R^\alpha, l_A] = -(D^\alpha)_A{}^B \, l_B$$

$\uparrow$ vector rep.

Using the Wigner approach we can find the irreducible representation of $I_c(E_{11}) \otimes_s l_1$

For the massless representation we choose $p^+ = m$, all other p^μ's $= 0$ and all other charges zero ie $Z^{a_1a_2} = 0, \dots$

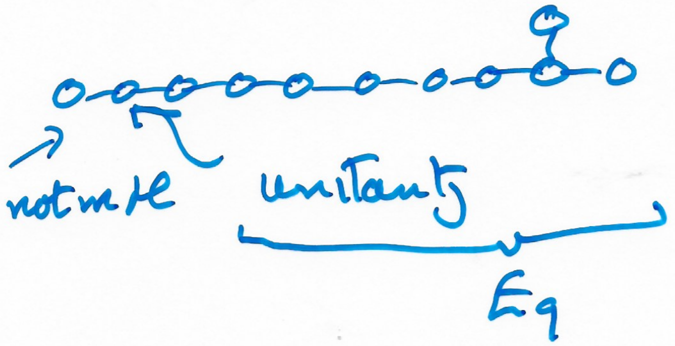
This choice is preserved by

$$\mathcal{H} = \{ J_{ij}, S_{i_1i_2i_3}, S_{i_1\dots i_6}, \dots \dots$$
$$J_{+i}, S_{+i_1i_2}, S_{+i_1\dots i_5}, \dots \}.$$

Unitarity requires J_{+i}, $S_{+i_1i_2}, \dots$ trivial

We are left with

$$I_c(E_9) = \{ J_{ij},\ S_{i_1 i_2 i_3},\ S_{i_1 \ldots i_6}, \ldots \}.$$



We must choose a representation of $I_c(E_9)$. Take the T's that is

$$\begin{tikzcd}
h_{ij}, \arrow[rrr, bend left, leftrightarrow] & A_{i_1 i_2 i_3}, \arrow[r, bend right, leftrightarrow] \arrow[rrr, bend right, leftrightarrow] & A_{i_1 \ldots i_6}, & A_{i_1 \ldots i_8, b}, & A_{i_1 \ldots i_9, i_1 i_2 i_3} , \ldots
\end{tikzcd}$$

This is highly reducible as there are $I_c(E_9)$ covariant conditions

$$A_{i_1 i_2 i_3} \sim \epsilon_{i_1 i_2 i_3}{}^{j_1 \ldots j_6} A_{j_1 \ldots j_6}$$

$$\sim \epsilon^{j_1 \ldots j_9} A_{j_1 \cdot j_9, i_1 i_2 i_3}$$

This leaves only

$$h_{ij},\ A_{i_1 i_2 i_3}$$

the bosonic degrees of freedom of maximal supergravity

- The irreducible representation depends on

$$\underset{e^i}{\underset{\wr}{P^{-i}}}, \quad \underset{e^\phi}{\underset{\wr}{P^{+-}}}, \quad P^{-i_1 i_2}, \quad P^{-+i}, \quad \ldots\ldots$$

or in space-time

$$\underset{x_i}{\underset{\shortparallel}{X_{-i}}}, \quad \underset{u}{\underset{\shortparallel}{X_{+-}}}, \quad X_{-i_1 i_2}, \quad X_{-+i}, \quad \ldots\ldots$$

- Can one take $z^{+i} \neq 0$ and find the string states or any of the other infinite degrees of freedom see later papers papers with Keith Glennon

- The non-linear equations of motion are given by the non-linear realisation of $E_{11} \otimes_s l_1$ with local subalgebra $I_c(E_{11})$. Explicitly carried out up to the level of the dual graviton. The agree precisely with those of maximal supergravity if we keep only the x^μ.

- Different decompositions of E_{11} lead to all the maximal supergravity theories. Also find all the gauged maximal supergravities. Unlike M theory this is a theory.

Duality Symmetries and Spacetime

E theory is the non-linear realization of the semi-direct product of the Kac-Moody algebra E_{11} with its vector representation

2001
2003.

* * *

- Contains the maximal supergravity theory in eleven dimensions. It mixes particles of different spins

$$h_a{}^b \leftrightarrow A_{a_1a_2a_3}$$

- Has an infinite number of duality symmetries

$$\begin{tikzcd} A_{a_1a_2a_3} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & A_{a_1\ldots a_6} \arrow[r, leftrightarrow] & A_{a_1\ldots a_9,b_1b_2b_3} \arrow[r, leftrightarrow] & {} \\ h_a{}^b \arrow[r, leftrightarrow] & h_{a_1\ldots a_8,b} \arrow[r, leftrightarrow] & h_{a_1\ldots a_9,b_1\ldots b_8,c} \arrow[r, leftrightarrow] & {} \end{tikzcd}$$

- Contains all the maximal supergravities and the gauged supergravities. Combines spacetime and internal symmetries.

- Unlike M theory it is a theory

The fields live in a space time which has coordinates beyond those usually used

$$x^{\mu}, \; x_{\mu_1\mu_2}, \; x_{\mu_1\ldots\mu_5}, \; \ldots$$

Kac-Moody Algebras.

Lie algebra $\rightsquigarrow$ Cartan subalgebra H_i and E_α, α roots

(su(2)) (H, E, F).

$\rightsquigarrow$ simple roots α_i and generators E_{α_i}, $F_{-\alpha_i}$, H_i $\rightsquigarrow$ Cartan matrix $A_{ij} = \frac{2(\alpha_i,\alpha_j)}{(\alpha_i,\alpha_i)}$

($A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$).

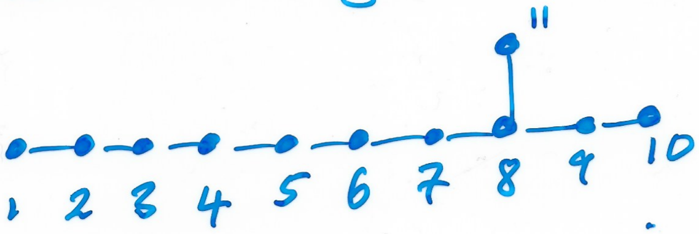
or Dykin diagram

(●)

In the 1950's Serre showed how subject to some relations one can go from Dykin diagram $\rightsquigarrow$ Lie algebra.

Kac and Moody (1969) just took more general Cartan matrices, no longer positive definite

E_{11} has the Dynkin diagram



We study by decomposing to known subalgebra

- delete node 11 $\rightsquigarrow$ $GL(11)$

[- delete node 3 $\rightsquigarrow$ $GL(3) \otimes E_8$].

Generators are classified by a level

$$\dots, \underset{-2}{R_{a_1 \dots a_6}}, \underset{-1}{R_{a_1a_2a_3}}, \underset{0}{K^a{}_b}, \underset{1}{R^{a_1a_2a_3}}, \underset{2}{R^{a_1 \dots a_6}}, \underset{3}{R^{a_1 \dots a_8, b}}, \dots$$

Algebra follows from level preservation plus Jacobi

$$[K^a{}_b, R^{c_1c_2c_3}] = 3\delta_b^{[c_1|}R^{a|c_2c_3]}$$

tensor of $GL(11)$

$$[R^{a_1a_2a_3}, R^{b_1b_2b_3}] = 2R^{a_1a_2a_3b_1b_2b_3}$$

The vector representation (l_1) of E_{11}

is the first fundamental representation $\left((\alpha^a, \lambda^{(1)}) = \delta^{a1} \right)$.

$$l_A = \{ \underset{\text{Point particle}}{P_a} ; \underset{\text{M2}}{Z^{a_1 a_2}} ; \underset{\text{M5}}{Z^{a_1 .. a_5}} ; \underset{\text{TaubNut}}{Z^{a_1 . a_7, a_8}} , Z^{a_1 . a_8} ; .. \}$$

The l_1 representation contains all known charges plus an infinite number of unknown branes or degrees of freedom

The semi-direct product of E_{11} with l_1 denote $E_{11} \otimes_s l_1$ has the algebra.

$$[R^\alpha, R^\beta] = f^{\alpha\beta}{}_\gamma \, R^\gamma \qquad \text{ie } E_{11}$$

$$[R^\alpha, l_A] = - (D^\alpha)_A{}^B \, l_B$$

↑ 1st fundamental

$$\text{ie} \quad [R^{a_1 a_2 a_3}, P_b] = 3 \delta_c^{[a_1} Z^{a_2 a_3]} , ..$$

The non-linear realisation of $E_{11} \otimes_s l_1$ with subalgebra $I_c(E_{11})$ is constructed from $g = g_l g_A$

$$g_A = e^{A_\alpha R^\alpha} = \dots e^{h_a{}^b K^a{}_b} e^{A_{a_1 a_2 a_3} R^{a_1 a_2 a_3}} \dots$$

$$g_l = e^{z^A l_A} = e^{x^a P_a} e^{x_{a_1 a_2} Z^{a_1 a_2}} \dots$$

We have the fields

$$\underset{\text{graviton}}{h_a{}^b}, \quad \underset{\text{3-form}}{A_{a_1 a_2 a_3}}, \quad \underset{\text{6-form}}{A_{a_1 \dots a_6}}, \quad \underset{\text{dual graviton.}}{h_{a_1 \dots a_8, b}}, \dots$$

which depend on

$$z^A = \{ \underset{\substack{\text{usual}\\ \text{space-time}}}{x^a}, x_{a_1 a_2}, x_{a_1 \dots a_5}, \dots \}$$

The dynamics is determined "uniquely" by the symmetry

$$g(z) \to \underset{\in E_{11}}{g_0} \, \underset{\in E_{11}}{g(z)} \, \underset{\in I_c(E_{11})}{h(z)}$$

can use

$$\mathcal{V} = g^{-1} dg \to h^{-1} \mathcal{V} h + h^{-1} dh.$$

We find an infinite number of duality relations

$$F_{a_1 \cdot a_4} = * \epsilon_{a_1 \cdot a_4}{}^{b_1 \cdot\cdot b_7} F_{b_1 \cdot\cdot b_7} \, .$$

$$\updownarrow$$

$$W_{a, b_1 b_2} = * \epsilon_{b_1 b_2}{}^{c_1 \cdot\cdot c_9} \underset{\partial_{c_1} h_{c_2 \cdot c_9, a} \, .}{\underset{\wr}{G_{c_1, c_2 \cdots c_9, a}}}$$

$$\vdots$$

These imply the 'usual'' field equations for the three form and the graviton which agree with those of eleven dimensional supergravity if we take the fields to depend only on x^μ.

- The construction of the irreducible representations of $Ic(E_{11})\otimes_s l_1$ is similar to those of Poincaré = $SO(1,d-1)\otimes_s T^d$. but now the "momentum" is from

$$P_a,\ Z^{a_1a_2},\ Z^{a_1\ldots a_5},\ \ldots$$

If we choose the massless particle ($P_- = m$) the little group is E_9 and the only physical states are the graviton and the three form.

[string states ?]

- If we delete node D

we find maximal supergravity in D dimensions.

- In E_{11} we have in D dimension the fields $A_{a_1\ldots a_{D-1},\circ} \sim F_{a_1\ldots a_D,\circ}$

$\sim$ action $\int d^Dx\, F^2_{a_1\ldots a_D} \sim F_{a_1\ldots a_D} = \Lambda^{\frac{1}{2}}\epsilon_{a_1\ldots a_D}$

$\sim$ action $\int d^Dx\, \Lambda$ and all the gauged maximal supergravities

Space-time and Duality symmetries

2023

Monopoles occur in spontaneously broken SU(2) Yang-Mills with adjoint Higgs. Solutions obey $B_i = D_i\phi$ and they have free parameters ie moduli. N monopoles has 4N moduli and one monopole has the moduli

$\hat{x}_i$ and z

(position)

large gauge transformation

$A_i \to A_i + D_i(\frac{z\phi}{v})$.

The slow motion of the monopoles comes by letting the moduli depend on time

$\hat{x}_i \to x_i(t)$, $z \to z(t)$.

and substituting into the action

$$\frac{M}{2}\int dt\left(\dot{\hat{x}}^i\dot{\hat{x}}_i + \frac{1}{v^2}\dot{z}\dot{z}\right)$$

Manton.

If we quantize we have.

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{ev\hbar^2}{8\pi}\frac{\partial^2}{\partial z^2}\right)\psi$$

Gibbons Manton

- The role of space time is to label events
- $\leadsto$ Monopoles move in a ***five*** dimensional space time (t, x_i, z)

- Maximally supersymmetric Yang-Mills theories have monopoles.

$$\{Q_\alpha^i, Q_\beta^j\} = 2(\gamma^a C)_{\alpha\beta} P_a + \epsilon^{ij}(\Phi + \gamma_5 G) C_{\alpha\beta}$$

The theory lives on the coset with group element

$$g = e^{x^\mu P_\mu + z\Phi + \lambda G} e^{\theta^{\alpha i} Q_{\alpha i}}$$

ie a space-time with $x^\mu, z, \lambda; \theta^{\alpha i}$

- Montenon-Olive duality says that we have a dual theory in which the monopoles are the elementary particles and the original particles are electrically charge solitons which can acquire magnetic charges due to large gauge transformations

original theory	x^μ.
original theory with electrically charge monopoles	x^μ, z.
dual theory with electric particles	x^μ, λ.
dualy symmetric formulation	x^μ, z, λ

Maximal supergravity theories

$$\frac{1}{2\kappa_D^2}\int d^Dx\,\sqrt{-g}\,(R - \dots e^{a\phi} F_{\mu_1\dots\mu_q}F^{\mu_1\dots\mu_q}\dots)$$

have brane solutions which have moduli which arise as large gauge transformations

$$ds^2 = N^{2\alpha}dx_i dx^i + N^{2\beta}dy^n dy_n$$

where
$$N = 1 + \frac{\text{constant}}{r^{D-p-3}}$$

and
$$F_{i_1\dots i_{p+1}n} = \text{constant}\,\partial_n N^{-1}\,\epsilon_{i_1\dots i_{p+1}}.$$

The theory has gauge symmetries

$$\delta A_{\mu_1\dots\mu_{p+1}} = (p+1)\,\partial_{[\mu_1}\Lambda_{\mu_2\dots\mu_{p+1}]}\,,\quad \delta h_{\mu\nu} = \xi^\lambda\partial_\lambda h_{\mu\nu} + \partial_\mu\xi^\lambda h_{\lambda\nu} + \partial_\nu\xi^\lambda h_{\mu\lambda}$$

The moduli appear by taking

$$\Lambda_{i_1\dots i_q} = N^{-1}B_{i_1\dots i_q}\,,\quad \xi^n = N^{-1}z^n$$
others vanish

Taking the moduli to depend on the brane world volume

$$B_{i_1\dots i_q} \to B_{i_1\dots i_q}(x^i)\,,\quad z^n \to z^n(x^i)$$

and substituting in the action we find the dynamics

$$\int d^dx\,(\partial_i z^q\partial^i z_q + G_{i_1\dots i_{q+1}}G^{i_1\dots i_{q+1}} + \dots)$$

where $G_{i_1\dots i_{q+1}} = \partial_{[i_1}B_{i_2\dots i_{q+1}]}$

Adawi, Cederwall, Gran, Nilsson, Razaznejad 1998

Maxfield Sethi 2017

- Just as the monopole carries electric charge the brane carries a charge due to its $B_{i_1 \dots i_q}$ moduli

- The motion of the brane is in a spacetime

$$\underbrace{x^i , z^n}_{x^\mu} , \quad B_{i_1 \dots i_q}$$

and when we quantize we have a wavefunction

$$\psi (x^\mu , B_{i_1 \dots i_q}).$$

- Now all moduli are large gauge transformations of a spontaneously broken symmetry. Taking account of all the possible moduli we find all of our usual spacetime (x^μ) and an infinite number of other coordinates.

Spacetime in E theory.

In E theory we have an infinite dimensional space time arising from the vector representation

field	$h_a{}^b$;	$A_{a_1a_2a_3}$;	$A_{a_1..a_6}$,	$h_{a_1..a_8,b}$, ...
brane charges	P_a ;	$Z^{a_1a_2}$;	$Z^{a_1..a_5}$;	$Z^{a_1..a_7,b}$, ...
gauge transformations	ξ^a ,	$\Lambda_{a_1a_2}$,	$\Lambda_{a_1..a_5}$,	$\Lambda_{a_1..a_7,b}$, ...
coordinates	x^a ,	$x_{a_1a_2}$,	$x_{a_1..a_5}$,	$x_{a_1..a_7,b}$, ..

The coordinates can be thought of as arising as moduli in the large gauge transformations

If we have just the particles of supergravity we do not need extra coordinates but we also lose duality symmetries

Conclusion

- Spacetime is a derived concept that should label events. The elementary particles in our familiar relativistic quantum field theories only require our usual coordinates x^μ

- Such theories often have solitonic solutions with moduli and these particles move in an enlarged spacetime.

- In theories of gravity our usual spacetime arises in this way.

- The maximal supergravity theories have an infinite number of branes which require an infinite spacetime. All these theories are encoded in E theory which has such a spacetime.

- Duality symmetries require an enlarged spacetime which provides a natural way to combine spacetime and internal symmetries.

- Spacetime should arise in large local symmetries which are spontaneously broken