Carroll stories

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Four Carroll stories

- Partition functions and thermodynamics
- Electric Carroll theories
- De Sitter and dark energy
- Carroll limits of black holes

Carroll and other limits

$$\frac{v}{c} \to 0 : \textit{Galilei limit}$$

$$\frac{v}{c} \to 1 : \textit{Ultra relativistic limit}$$

$$\frac{v}{c} \to \infty : \textit{Carroll limit}$$

Always compare c to some other velocity v in the system

A Carroll story on partition functions and thermodynamics

Massless relativistic particles

$$Z(T, V, v^i) = \frac{V}{h^3} \int d^3p \, e^{-\beta H + \beta v^i p_i}$$

$$H(p) = |\vec{p}|c$$

$$Z = \frac{8\pi V \gamma^4}{h^3 c^3 \beta^3} = \frac{8\pi V}{h^3 c^3 \beta^3 (1 - \frac{v^2}{c^2})^2}$$

Result convergent for $Re(\beta)Re(v - c) - Im(\beta)Im(v) < 0$.

Carroll limit of massless particles

$$Z = \frac{8\pi V\gamma^4}{h^3c^3\beta^3} = \frac{8\pi V}{h^3c^3\beta^3(1-\frac{v^2}{c^2})^2}$$

Timelike Carroll limit:

$$v = 0, c \to 0$$

Divergence: $Z \propto c^{-3}$

Carroll limit of massless particles

$$Z = \frac{8\pi V\gamma^4}{h^3c^3\beta^3} = \frac{8\pi V}{h^3c^3\beta^3(1-\frac{v^2}{c^2})^2}$$

Spacelike Carroll limit/regime:

$$v \to iv, \qquad v/c \to \infty$$

$$Z = \frac{8\pi Vc}{(h\beta)^3 v^4} \qquad \mathcal{E}+P = 0$$

$Re(\beta)Re(v-c) - Im(\beta)Im(v) < 0$

Imaginary chemical potentials

Partition function

- The strict Carroll limit is trivial or otherwise not well defined.
- It is better to consider the Carroll regime, in which c/v is very small (but non-zero).

Carroll QFT on S^1_R

- $S = \int d^2x\,(a^2\dot{\phi}^2 - b^2(\partial_x\phi)^2 - m^2\phi^2)$
- Relativistic: a=1/c and b=1
- Electric Carroll limit: $b \to 0$
- Magnetic Carroll limit: $a \to \infty$

Carroll QFT on S^1_R

- $S = \int d^2x\,(a^2\dot{\phi}^2 - b^2(\partial_x\phi)^2 - m^2\phi^2)$

- Energies: $\beta E_k = \sqrt{k^2x^2 + y^2} \quad k \in \mathbb{Z}$

$$x \equiv \frac{b\beta}{Ra} \qquad y \equiv \frac{\beta m}{a}$$

- Partition function $Z[x, y] = Tr[e^{-\beta H}]$

Carroll QFT on S^1_R

- Partition function $Z[x, y] = Tr[e^{-\beta H}]$

$$x \equiv \frac{b\beta}{Ra} \qquad y \equiv \frac{\beta m}{a}$$

- Electric limit: $x \to 0$, but then *R* drops out and partition function is not extensive.
- Magnetic limit: $x \to 0, y \to 0$. Z only depends on

$\frac{x}{y} = \frac{b}{Rm}$ fixed. But then β drops out.

- Conclusion: there is no thermodynamics in the strict Carroll limit.

- For flat space holography, this might be a virtue rather than a bug! In flat space, there does not exist thermal equilibrium either.

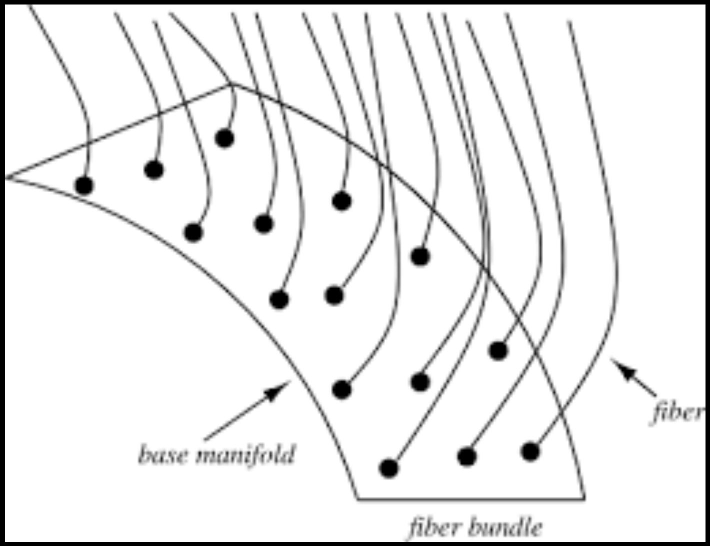
A Carroll story on

electric and magnetic

theories

Electric Carroll theories

- Electric theories are ultralocal in space and have non-trivial time dependence.
- A general class can be constructed as follows:



Analogy with fibre bundle: above each point *x* in the base M, we consider a QK system with Hamiltonian H_x.

$$\mathcal{L} = \int_M d^d x \sqrt{g}\, L_{QM}[\phi(x,t)]$$

Electric Carroll theories

$$\mathcal{L} = \int_M d^d x \sqrt{g}\, L_{QM}[\phi(x,t)]$$

Example: $S = \int dt\, \mathcal{L}$ with

$$L_{QM}[\phi(x,t)] = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$[H_x, H_y] = 0$$

Carroll invariant.

Electric Carroll theories

$$\mathcal{L} = \int_M d^d x \sqrt{g}\, L_{QM}[\phi(x,t)]$$

Conserved charges (~ supertranslations)

$$Q_a = \int_M d^d x \sqrt{g}\, a(x) H_x$$

Total Hamiltonian $H = \int_M d^d x \sqrt{g}\, H_x$. Ground state is product of all ground states at each *x*.

Energy eigenstates infinite degenerate.

Electric Carroll theories

Correlation functions of operators with vanishing one-point function in the ground state:

$$< 0|O_{x_1} \dots O_{x_n}|0 >$$

will vanish unless for each x_i there is at least another x_j with $x_i = x_j$.

Such correlators will produce a product of delta functions: ULTRALOCAL.

Magnetic Carroll theories

- Consider a d-dimensional Euclidean field theory $\mathcal{L}(\phi_a)$. Now construct a magnetic Carroll theory

$$S = \int dt d^d x \left(\chi^a \dot{\phi}_a - \mathcal{L}(\phi_a)\right)$$

The multipliers χ^a guarantee that the fields ϕ_a are time independent. Hamiltonian

$$H = \int d^d x \, \mathcal{L}$$

Correlators of ϕ_a-fields will be time-independent and so Carroll invariant and given by Euclidean correlator.

A Carroll story on

de Sitter and dark energy

De Sitter horizon

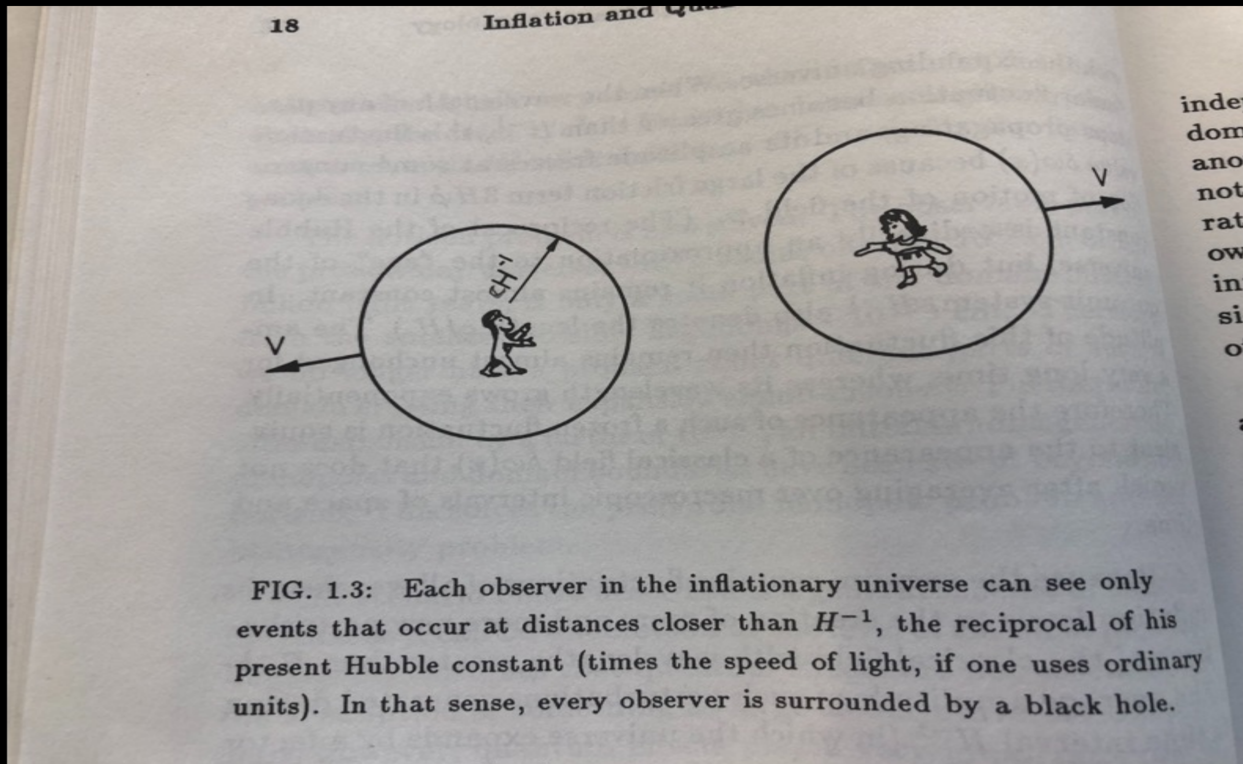
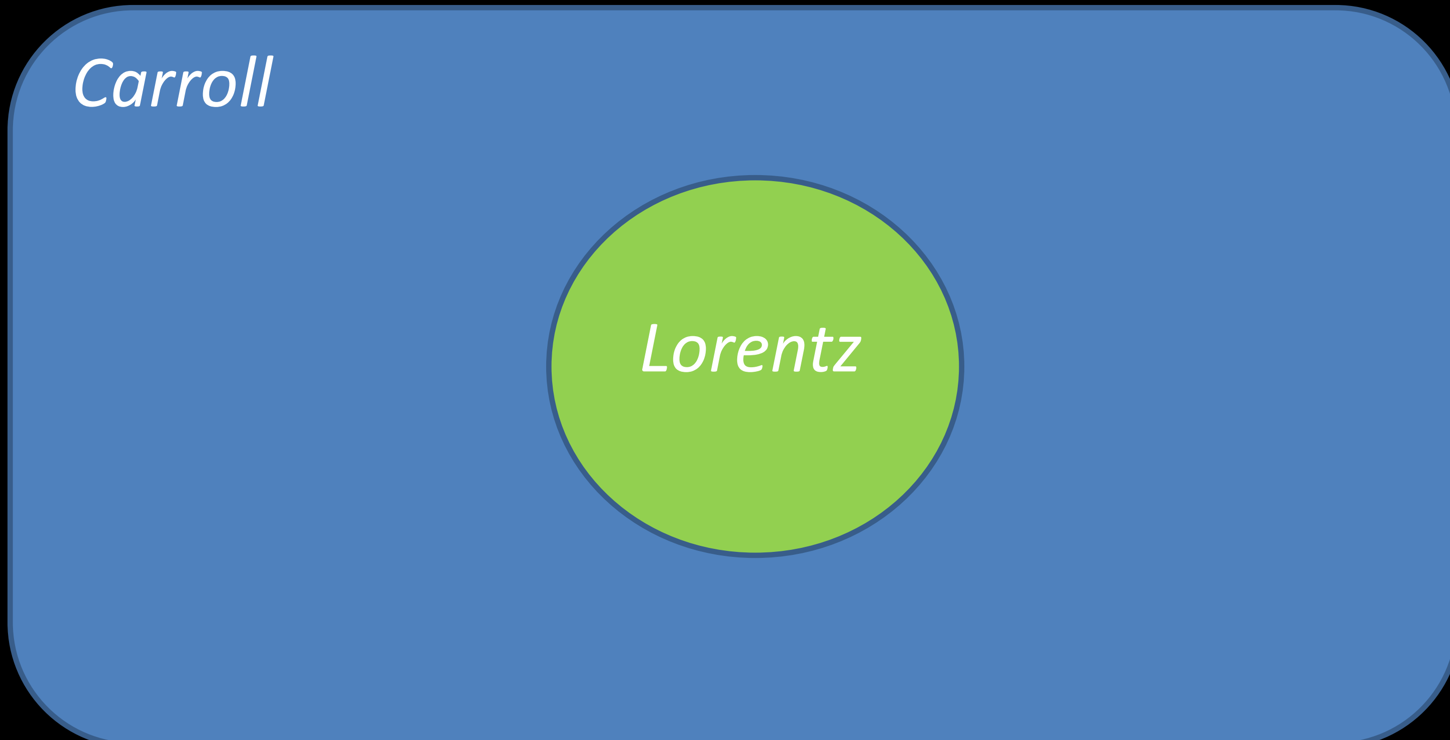


FIG. 1.3: Each observer in the inflationary universe can see only events that occur at distances closer than H^{-1}, the reciprocal of his present Hubble constant (times the speed of light, if one uses ordinary units). In that sense, every observer is surrounded by a black hole.

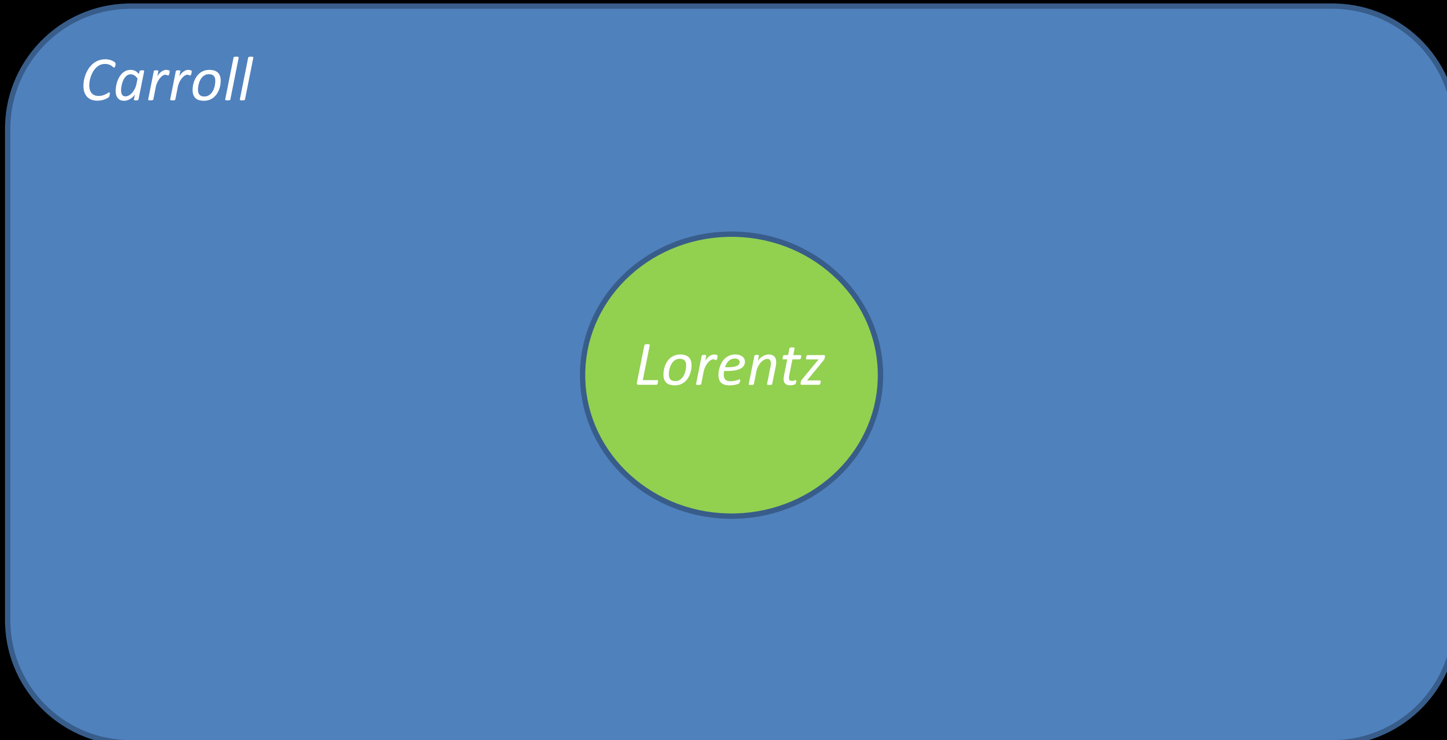
$$v = Hd \rightarrow \frac{v}{c} = \frac{d}{R_H}$$

Carroll universe



$$R_H = \frac{c}{H} \rightarrow 0$$

Carroll universe



$$R_H = \frac{c}{H} \rightarrow 0$$

Carroll universe

$$R_H = \frac{c}{H} \rightarrow 0$$

Carroll metric

- De Sitter metric in planar coordinates:

$$ds^2 = -c^2 dt^2 + e^{2Ht} d\vec{x}^2$$

- Carroll metric conformal to Euclidean flat space:

$$ds^2 = e^{2Ht} d\vec{x}^2$$

- Carroll limit is late time limit. dS/CFT. Euclidean theory on the boundary=magnetic Carroll theory.

Dark energy and Carroll particles

De Sitter requires dark energy with equation of state

$$P + \mathcal{E} = 0 \rightarrow w = -1$$

Microscopic interpretation: Carroll particles with imaginary chemical potentials v^i for the momenta p_i. They have $P + \mathcal{E} = 0$!

A Carroll story on black holes

Reissner-Nordstrom

- Electric and magnetic limits on black holes zoom in on the metric inside and outside the horizon.
- Charged RN black holes: introduce Maxwell

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Electric limit: $\mu_0 \to \infty$, ε_0 fixed

Magnetic limit: $\varepsilon_0 \to \infty$, μ_0 fixed

Electric limit

- Keep $E=Mc^2$ fixed, as well as Q and $G_C=G_N/c^2$. Take $\mu_0 \to \infty$ and set P=0. Define

$$a = 2EG_C = 2MG_N \qquad b = \frac{Q^2}{8\pi\varepsilon_0 E}$$

- Horizons: $r_\pm = \frac{a}{2c^2}(1 \pm \sqrt{1 - \frac{4c^2 b}{a}})$
- Carroll limit: $r_+ \to \infty, r_- \to b$. This is the region between inner and outer horizon.

Electric limit

- The geometry is some charged deformation of the Carroll-Kasner geometry. Carroll metric

$$h = \frac{a}{r}\left(1 - \frac{b}{r}\right) dt^2 + r^2 d\Omega^2$$

with $r \in (b, \infty)$ and

$$a = 2EG_C = 2MG_N \qquad b = \frac{Q^2}{8\pi\varepsilon_0 E}$$

Magnetic limit

- Keep $E=Mc^2$ fixed, as well as P and $G_C=G_N/c^4$. Take $\varepsilon_0 \rightarrow \infty$ and set Q=0. Define

$$R_S = 2EG_C = 2MG_N/c^2 \qquad R_P^2 = \frac{\mu_0 P^2 G_C}{4\pi}$$

- Horizons: $r_\pm = \frac{R_S}{2}(1 \pm \sqrt{1 - \frac{4R_P^2}{R_S^2}})$
- Carroll limit: horizons stay fixed.

Magnetic limit

- The Carroll metric

$$h = \left(1 - \frac{R_S}{r} + \frac{R_P^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

together with the magnetic field can be extended to a wormhole geometry, similar to Schwarzschild.

Conclusions

- There is no thermodynamics in the strict Carroll limit, but this is a good thing.
- Carroll symmetry not only relevant for flat space holography, but also for de Sitter holography
- The Carroll story continues…