


BMS$_3$ (Carrollian) field theories from a bound in the coupling of current-current deformations of CFT$_2$

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Conformal symmetries in D dimensions

Diffeomorphisms preserving flat spacetime, up to local scalings

$$ds^2 \to \Omega^2 ds^2$$

Conformal Killing eq. : $\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \lambda g_{\mu\nu}$

D > 2 : $J_{\mu\nu} \ , \ P_\mu \ ; \ D \ , \ K_\mu$

Isomorphic to $so(D,2) \ : \ \dfrac{(D+2)(D+1)}{2}$ **generators**

Conformal symmetries in 2D

Infinite-dimensional algebra :

Two copies of the Witt (or centerless Virasoro) algebra

Isomorphic to $\mathrm{Diff}(S^1)\oplus\mathrm{Diff}(S^1)$

$$[L_m, L_n] = (m-n)\,L_{m+n}$$

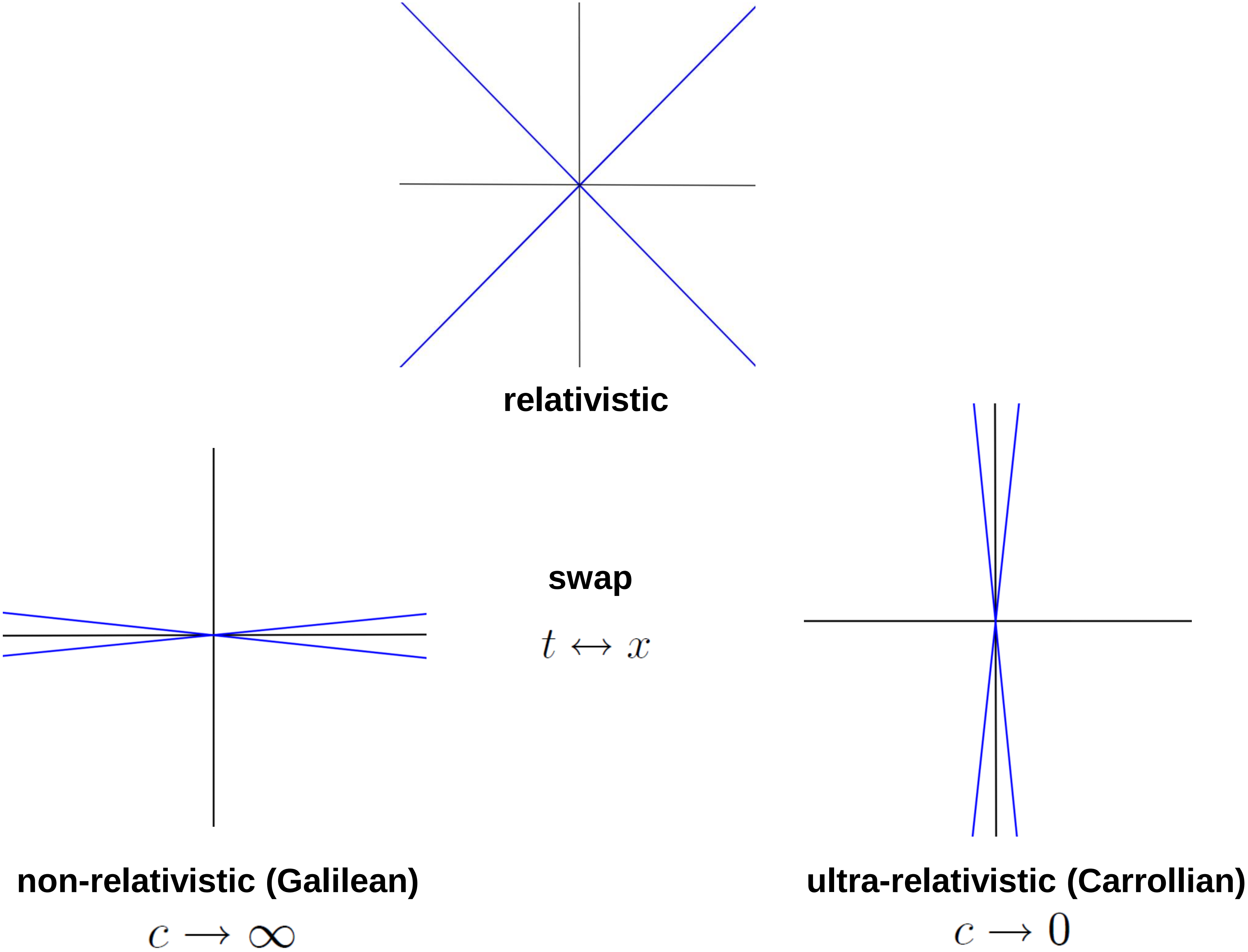
$$[\bar{L}_m, \bar{L}_n] = (m-n)\,\bar{L}_{m+n}$$

with $[L_m, \bar{L}_n] = 0$ and $m, n \in Z$

Ultra & non-relativistic limits

Another accident in 2D :

ultra & non-relativistic limits are isomorphic !


relativistic
swap
$t \leftrightarrow x$
non-relativistic (Galilean)
$c \longrightarrow \infty$
ultra-relativistic (Carrollian)
$c \longrightarrow 0$

Ultra & non-relativistic limits

Another accident in 2D :

ultra & non-relativistic limits are isomorphic !

Corresponding algebras are isomorphic $\mathrm{GCA}_2 \approx \mathrm{CCA}_2$

Additional curiosity :

ultra/non-relativistic algebras isomorphic to BMS3

[asymptotic symmetries of asymptotically flat spacetimes in 3D]

In sum : $$\mathrm{GCA}_2 \approx \mathrm{CCA}_2 \approx \mathrm{BMS}_3$$

BMS3 algebra

Semidirect sum of Witt algebra and supertranslations

$$[J_m, J_n] = (m-n)\, J_{m+n}$$

$$[J_m, P_n] = (m-n)\, P_{m+n}$$

$$\text{where } [P_m, P_n] = 0$$

Appears in different contexts, e.g., tensionless limit of strings

Admits unitary representations

Inonu-Wigner contractions

Conformal and BMS3 algebras are not isomorphic

$$P_m = \frac{1}{\ell}\left(L_m + \bar{L}_{-m}\right) \quad J_m = L_m - \bar{L}_{-m}$$

In the limit $\ell \to \infty$ **BMS3 is recovered**

$$P_m = \ell\left(L_m - \bar{L}_m\right),\; J_m = L_m + \bar{L}_m$$

In the limit $\ell \to 0$ **BMS3 is also recovered**

ℓ **: regarded as the inverse of the speed of light**

Outline

- **Introduction:**
  - **current-current ($j \cdot \bar{j}$) & $\sqrt{T\bar{T}}$ deformations of CFT_2's**
- **BMS_3 field theories from finite $j \cdot \bar{j}$ deformations**
  - **Coupling precisely fixed, up to a sign**
  - **2 inequivalent ones : electric-like and magnetic-like**
- **Finite $j \cdot \bar{j}$ deformations of the bosonic string**
  - **Electric-like deformation**

    [standard tensionless limit (Carrollian electric-type)]
  - **Magnetic-like deformation**

    [new action : nonstandard limits in the tension (zero or infinity !)]

Outline

- BMS_3 deformations from limiting cases of continuous exactly (integrably) marginal $j \cdot \bar{j}$ deformations
  - SO(1,1) automorphism of the currents
  - Bound in the coupling (def. parameter)
- Ending remarks
  - Beyond the bound: "Euclidean CFT_2's" [not thermal !]
  - Deformations & the Polyakov action

Introduction

- Continuous $j \cdot \bar{j}$ deformations

  Chaudhuri, Schwartz [PLB 1989]

- Deformations by “integrably marginal” operators [exactly marginal]
  - Lagrangian deformed by addition of op. of conf. dim. (1,1)
  - Continuous coupling (deformation parameter)
  - Preserves conformal symmetry

- CFT2’s with (left & right) Kac-Moody currents [(anti)-holomorphic)]

$$[J^a_m, J^b_n] = f^{ab}{}_c J^c_{m+n} + n g^{ab} \delta_{m+n,0} \qquad \text{same for } \bar{J}^a_m$$

Introduction

- **Continuous $j \cdot \bar{j}$ deformations**

Deformations of the form $g \, \tilde{c}_{IJ} \, J^I \bar{J}^J$

integrably marginal iff :

$J^I, \bar{J}^J$ stand for the subset of Abelian currents (Cartan subalgebra)

(Abelian currents preserved under the deformation)

- Valid to all orders in the def. parameter (finite g)

- holds for finite values in certain classes of CFT2

- **True for arbitrarily large g ?**
- **CFT2 moduli spaces with boundary in case of a bound in g ?**
- **Simple example (single free boson) yields a bound !**

Introduction

- Example taken from overlapping case of $j \cdot \bar{j}$ & $\sqrt{T\bar{T}}$ deformations of CFT2

Finite :

[Rodriguez, Tempo, Troncoso, arxiv:2106.09750 (JHEP)]

Continuous :

[Tempo, Troncoso, arxiv:2210.00059 (JHEP)]

- QFT2 $\sqrt{T\bar{T}}$ deformation from a Lagrangian flow eq. :

[Conti, Romano, Tateo [2206.03415] (JHEP)]

[Ferko, Sfondrini, Smith, Tartaglino-Mazzucchelli [2206.10515] (PRL)]

[Babaei-Aghbolagh, Babaei Velni, Yekta, Mohammadzadeh [2206.12677] (PRD)]

(also Hou [2208.05391] (JHEP))

Conformal algebra in 2D (continuum)

$$[L_m, L_n] = (m-n)\, L_{m+n}$$
$$\left[\bar{L}_m, \bar{L}_n\right] = (m-n)\, \bar{L}_{m+n}$$

with $\left[L_m, \bar{L}_n\right] = 0$ and $m, n \in Z$

Fourier modes :

$$L_m = \int d\phi \bar{T}\left(\phi\right) e^{-im\phi},\ \bar{L}_m = \int d\phi T\left(\phi\right) e^{im\phi}$$

Equivalently :

$$\left\{T\left(\varphi\right), T\left(\theta\right)\right\} = \left(2T\left(\varphi\right)\partial_\varphi + \partial_\varphi T\left(\varphi\right)\right)\delta\left(\varphi - \theta\right)\ ,$$
$$\left\{\bar{T}\left(\varphi\right), \bar{T}\left(\theta\right)\right\} = -\left(2\bar{T}\left(\varphi\right)\partial_\varphi + \partial_\varphi \bar{T}\left(\varphi\right)\right)\delta\left(\varphi - \theta\right)$$

with $\left\{T\left(\phi\right), \bar{T}\left(\phi'\right)\right\} = 0$, and $[\cdot,\cdot] = i\left\{\cdot,\cdot\right\}$

Introduction

- Example taken from overlapping case of $j \cdot \bar{j}$ & $\sqrt{T\bar{T}}$ deformations

  [Rodriguez, Tempo, Troncoso, arxiv:2106.09750 (JHEP)]
  [Tempo, Troncoso, arxiv:2210.00059 (JHEP)]

Brief review of $\sqrt{T\bar{T}}$ deformations

- BMS3 from a nonlinear map of the conformal algebra in 2D

$$H_{(\pm)} = T + \bar{T} \pm 2\sqrt{T\bar{T}} \quad ; \quad P = \bar{T} - T$$

- Not marginal, but still conformal defs. (ultrarelativistic CFTs !)
- Both sets $\{H_{(+)}, P\}$ and $\{H_{(-)}, P\}$ fulfill the BMS3 algebra
- [no limiting process involved !]

- Related by $H_{(+)} H_{(-)} = P^2$

[discrete nonlinear automorphism of BMS3]

Introduction

Brief review of $\sqrt{T\bar{T}}$ deformations

BMS3 generators from a limiting case :

- **Continuous nonlinear automorphism of the conformal algebra in 2D**

$$H_{(\alpha)} = \cosh(\alpha)\left(T + \bar{T}\right) + 2\sinh(\alpha)\sqrt{T\bar{T}}$$

Conformal algebra is preserved

- **Then rescaling** $\tilde{H}_{(\alpha)} := \dfrac{H_{(\alpha)}}{\cosh(\alpha)} = T + \bar{T} + 2\tanh(\alpha)\sqrt{T\bar{T}}$

when $\alpha \to \pm\infty$ $\qquad \tilde{H}_{(\pm\infty)} = H_{(\pm)}$

BMS3 supertranslation generators recovered

[def. with coupling $g = \pm 2$]

Introduction

The continuous deformation

$$I_{(\alpha)} = I_{(0)} + 2\tanh(\alpha) \int d\tilde{t} d\phi \sqrt{T\bar{T}}$$

generically preserves the conformal symmetry

unless, $\alpha \to \pm\infty$ **(** $g = \pm 2$ **)**

$$I_{(\pm)} = I_{(0)} \pm 2 \int d\tilde{t} d\phi \sqrt{T\bar{T}}$$

Marginal def. becomes nontrivial (2 different ultrarelativistic regimes)

[Conformal symmetry deforms to BMS3]

[Conformal Carrolian field theory $(c \to 0)$ **]** $\mathrm{GCA}_2 \approx \mathrm{CCA}_2 \approx \mathrm{BMS}_3$

Introduction

$\sqrt{T\bar{T}}$-deformed single free boson

$$I_{(0)}\left[\varphi\right] = -\frac{1}{2}\int d^2x\sqrt{-g}\partial_\mu\varphi\partial^\mu\varphi$$

$$T = j^2 \;\; ; \;\; \bar{T} = \bar{j}^2$$

Hence

$$\sqrt{T\bar{T}} = \pm J\bar{J}$$

Continuous $\sqrt{T\bar{T}}$ **deformation remains conformal**

Trivial: just a rescaling of the action : $I_{(\alpha)} = e^{\alpha}I_{(0)}$

Lagrangian flow $\sqrt{T\bar{T}}$ **-deformation for the free boson**

[Conti, Romano, Tateo [2206.03415] (JHEP)]

[Ferko, Sfondrini, Smith, Tartaglino-Mazzucchelli [2206.10515] (PRL)]

[Babaei-Aghbolagh, Babaei Velni, Yekta, Mohammadzadeh [2206.12677] (PRD)]

(also Hou [2208.05391] (JHEP))

Introduction

In sum: $\sqrt{T\bar{T}}$-deformed single free boson

$$I_{(0)}[\varphi] = -\frac{1}{2}\int d^2x\sqrt{-g}\partial_\mu\varphi\partial^\mu\varphi \qquad \sqrt{T\bar{T}} = j\bar{j}$$

- **Generic deformation : [Tempo, Troncoso, arxiv:2210.00059 (JHEP)]**
- **Bound in the coupling** $|g| \leq 2$
  - **If** $|g| < 2$ **, the conformal symmetry remains [trivial deformation]**
  - **when saturated (** $g = \pm 2$ **) one obtains**

$$I_{(+)} = \int dx^2\left(\pi\dot{\varphi} - \pi^2\right) \qquad I_{(-)} = \int dx^2\left(\pi\dot{\varphi} - \varphi'^2\right)$$

[Rodriguez, Tempo, Troncoso, arxiv:2106.09750 (JHEP)]

Carrollian limits $(c \to 0)$ **of electric & magnetic type**

[Henneaux, Salgado-Rebolledo, arxiv: 2109.06708 (JHEP)]

Next

- We aim to extend the result fom N + N abelian currents

- Some previous results along these lines in

  [Bagchi, Banerjee, Muraki, arxiv: 2205.05094 (JHEP)]

  [“infinite boosts" spanned by certain degenerate
  (non-invertible) linear transformations acting on the coordinates]

  [different approach here: agreement for some results on electric-like deformations]

Bosonic toroidal CFT$_2$'s

- **Starting point:**

N chiral & antichiral (holomorphic & antiholomorphic) abelian currents :

$$\{j^I(x), j^J(y)\} = -g^{IJ}\partial_x\delta(x-y)$$
$$\{\bar{j}^I(x), \bar{j}^J(y)\} = g^{IJ}\partial_x\delta(x-y)$$

Stress-energy tensor components :

$$\bar{T} = \bar{j}^2 \qquad T = j^2$$

Hereafter: $A \cdot B = g_{IJ}A^I B^J$ and $A^2 = g_{IJ}A^I A^J$

Mode expansion: $$j^I(\phi) = \frac{1}{2}\sum_n \bar{j}_n^I e^{-in\phi} \qquad \bar{j}^I(\phi) = \frac{1}{2}\sum_n j_n^I e^{in\phi}$$

$$T(\phi) = \frac{1}{2}\sum_n \bar{L}_n e^{-in\phi} \qquad \bar{T}(\phi) = \frac{1}{2}\sum_n L_n e^{in\phi}$$

- **Thus,** $$L_n = \frac{1}{2}\sum_k j_{n-k}\cdot j_k$$ **(similarly for** $\bar{L}_n$ **)**

Conformal algebra with currents

Thus, $$L_n = \frac{1}{2}\sum_k j_{n-k}\cdot j_k \qquad \bar{L}_k = \frac{1}{2}\sum_n \bar{j}_n \cdot \bar{j}_{k-n}$$

- **Algebra: 2 copies of the semidirect sum of Witt with currents**

$$[L_m, L_n] = (m-n)\, L_{m+n}$$

$$[L_n, j^I_m] = -m j^I_{m+n}$$

$$[j^I_n, j^J_m] = n g^{\mu\nu}\delta_{m+n,0}$$

$\bar{L}_m$ and $\bar{j}^I_m$ fulfill the same algebra

Here and afterwards: $[\cdot,\cdot] = i\,\{\cdot,\cdot\}$

Change of basis

Energy $H = \bar{T} + T$ **; Momentum density** $P = \bar{T} - T$

Also $k^I_{(\pm)} = \bar{j}^I \pm j^I$

- **Remarks :** Under parity $(\sigma \to -\sigma)$ j^I and $\bar{j}^I$ are swapped

  $k^I_{(+)} \to k^I_{(+)}$ (even), $k^I_{(-)} \to -k^I_{(-)}$ (odd)

- **Note that :** $$H = j^2 + \bar{j}^2 = \frac{1}{2}\left(k^2_{(+)} + k^2_{(-)}\right)$$

  $$P = j^2 - \bar{j}^2 = k_{(+)} \cdot k_{(-)}$$

- **In modes :**

  $$H_n = L_n + \bar{L}_{-n} \qquad P_n = L_n - \bar{L}_{-n}$$

  $$k^I_{(\pm)n} = j^I_n \pm \bar{j}^I_{-n}$$

Algebra in the energy-momentum basis

$$H_n = \frac{1}{4}\sum_m k_{(+)n-m}\cdot k_{(+)m} + \frac{1}{4}\sum_m k_{(-)n-m}\cdot k_{(-)m} \qquad P_n = \frac{1}{2}\sum_m k_{(+)n-m}\cdot k_{(-)m}$$

- In the base $H_n = L_n + \bar{L}_{-n} \quad P_n = L_n - \bar{L}_{-n} \quad k^I_{(\pm)n} = j^I_n \pm \bar{j}^I_{-n}$

The algebra reads (semidirect sum of conf. with currents)

$$[P_m, P_n] = (m-n)\, P_{m+n} \ ,$$

$$[P_m, H_n] = (m-n)\, H_{m+n} \ ,$$

$$[H_m, H_n] = (m-n)\, P_{m+n} \ ,$$

$$\left[k^I_{(+)m}, k^J_{(-)n}\right] = 2mg^{IJ}\delta_{m+n,0} \ ,$$

$$\left[k^I_{(\pm)m}, k^J_{(\pm)n}\right] = 0$$

$$\left[P, k^I_{(\pm)m}\right] = -mk^I_{(\pm)m+n}$$

$$\left[H_n, k^I_{(+)m}\right] = -mk^I_{(-)m+n}$$

$$\left[H_n, k^I_{(-)m}\right] = -mk^I_{(+)m+n}$$

BMS3 from finite $j \cdot \bar{j}$ deformations

Consider 2 inequivalent finite $j \cdot \bar{j}$ deformations of H, $H_{(+)}$ and $H_{(-)}$:

$$H_{(\pm)} = T + \bar{T} \pm 2j \cdot \bar{j}$$

equivalently:

$$H_{(\pm)} = k^2_{(\pm)}$$

In modes:

$$H_{(\pm)n} = L_n + \bar{L}_{-n} \pm \sum_k j_{n+k} \cdot \bar{j}_k$$

$$H_{(\pm)n} = \frac{1}{2} \sum_m k_{(\pm)n-m} \cdot k_{(\pm)m}$$

$H_{(+)}$: electric-like ; $H_{(-)}$: magnetic-like

[Energy density from square of vector or pseudovector]

BMS3 from finite $j \cdot \bar{j}$ deformations

Both sets: $\{H_{(+)}; P, k^I_{(+)}, k^I_{(-)}\}$ **(electric-like)**

and $\{H_{(-)}; P, k^I_{(+)}, k^I_{(-)}\}$ **(magnetic-like)**

yield to the BMS3 algebra

Algebra (both cases):

$$[P_m, P_n] = (m-n)\, P_{m+n} \ ,$$
$$[P_m, H_{(\pm)n}] = (m-n)\, H_{(\pm)m+n} \ ,$$
$$[H_{(\pm)m}, H_{(\pm)n}] = 0 \ ,$$

$$\left[k^I_{(+)m}, k^J_{(-)n}\right] = 2mg^{IJ}\delta_{m+n,0} \ ,$$

$$\left[k^I_{(\pm)m}, k^J_{(\pm)n}\right] = 0$$

$$\left[P, k^I_{(\pm)m}\right] = -mk^I_{(\pm)m+n} \ ,$$
$$\left[H_{(\pm)n}, k^I_{(\pm)m}\right] = 0 \ ,$$
$$\left[H_{(+)n}, k^I_{(-)m}\right] = -2mk^I_{(+)m+n} \ ,$$
$$\left[H_{(-)n}, k^I_{(+)m}\right] = -2mk^I_{(-)m+n} \ ,$$

BMS3 from finite $j \cdot \bar{j}$ deformations

Therefore : $$I_{CFT} = \int d^2x \left(\Pi\dot{\Phi} - H\right)$$ **[conformal gauge]**

Φ and Π collectively denote the fields and their momenta

Finite $j \cdot \bar{j}$ electric- & magnetic-like deformed actions

with $H_{(\pm)} = H \pm 2j \cdot \bar{j}$ **, are inv. under BMS3**

$$I^{(\pm)}_{BMS} = \int d^2x \left(\Pi\dot{\Phi} - H_{(\pm)}\right)$$

BMS3 from finite $j \cdot \bar{j}$ deformations

$$I_{CFT} = \int d^2x \left(\Pi\dot{\Phi} - H \right)$$

$$I^{(\pm)}_{BMS} = \int d^2x \left(\Pi\dot{\Phi} - H_{(\pm)} \right) \qquad H_{(\pm)} = H \pm 2j \cdot \bar{j}$$

No limiting process involved !

Advantage: quantization can be carried out from the same

rep. space of the original currents $j^I_n \, , \bar{j}^I_n$

Finite $j \cdot \bar{j}$ deformations: bosonic string

- **Polyakov action :**

$$I = -\frac{T}{2}\int d^2\sigma\,\sqrt{-h}h^{\alpha\beta}\partial_\alpha X\cdot\partial_\beta X$$

- **Hamiltonian action :**

$$I = \int d^2\sigma\,\left[\Pi\cdot\dot{X} - NH - N_\phi P\right]$$

- **Constraints & field eqs. :**

$$H = \frac{1}{2T}\left(\Pi^2 + T^2 X'^2\right)\ ,\qquad P = \Pi\cdot X'\ ,$$

$$\dot{X}^\mu = \frac{1}{T}N\Pi^\mu + N^\phi X^{\mu\prime}\ ,\qquad \dot{\Pi}_\mu = T\left(NX'_\mu\right)' + \left(N^\phi\Pi_\mu\right)'$$

Finite $j \cdot \bar{j}$ deformations: bosonic string

$$I = \int d^2\sigma \left[\Pi \cdot \dot{X} - NH - N_\phi P\right]$$

$$H = \frac{1}{2T}\left(\Pi^2 + T^2 X'^2\right) \ ,$$

$$P = \Pi \cdot X' \ ,$$

- **Currents :**

$$j^\mu = \frac{1}{2\sqrt{T}}\left(\Pi^\mu - T X^{\mu\prime}\right) \qquad \bar{j}^\mu = \frac{1}{2\sqrt{T}}\left(\Pi^\mu + T X^{\mu\prime}\right)$$

conserved on-shell

$$\dot{j}^\mu_\pm = \left[\left(N^\phi \pm N\right) j^\mu_\pm\right]'$$

- **Note that**

$$k^I_{(\pm)} = \bar{j}^I \pm j^I$$

$$k^\mu_{(+)} = \frac{1}{\sqrt{T}}\Pi^\mu$$

$$k^\mu_{(-)} = \sqrt{T} X^{\mu\prime}$$

Bosonic string: finite def. [electric-like]

$$H = \frac{1}{2T}\left(\Pi^2 + T^2 X'^2\right) \qquad j\cdot\bar{j} = \frac{1}{4T}\left(\Pi^2 - T^2 X'^2\right) \qquad k^{\mu}_{(+)} = \frac{1}{\sqrt{T}}\Pi^{\mu}$$

- **Hamiltonian :**

$$H_{(+)} = H + 2j\cdot\bar{j} = k^2_{(+)} = \frac{1}{T}\Pi^2$$

- **Electric-like action :**

$$I_{(+)} = \int d^2\sigma \left[\Pi\cdot\dot{X} - NH_{(+)} - N^{\phi}P\right]$$

$$I_{(+)} = \int d^2\sigma \left[\Pi\cdot\dot{X} - N_{(+)}\Pi^2 - N^{\phi}P\right]$$

with $N_{(+)} = NT^{-1}$ **[tension gauged away]**

Standard tensionless string

Recovered from the electric-like deformation

Bosonic string: tensionless limit

$$I_{(+)} = \int d^2\sigma \left[\Pi \cdot \dot{X} - N_{(+)}\Pi^2 - N^\phi P\right]$$

Standard tensionless string recovered from the electric-like def.

$$I = \int d^2\sigma \left[\Pi \cdot \dot{X} - NH - N_\phi P\right]$$

$$H = \frac{1}{2T}\left(\Pi^2 + T^2 X'^2\right) \ ,$$
$$P = \Pi \cdot X' \ ,$$

Rescaling lapse as $N = 2\tilde{N}_{(+)}T$, **and then** $T \to 0$

One obtains $I_{(+)}$ **with** $\tilde{N}_{(+)}$ **[compare with** $N_{(+)} = NT^{-1}$ **]**
[tension is gauged away in a different form]

- **Similar limit as that in of Henneaux & Salgado-Rebolledo:**
  [arxiv: 2109.06708 (JHEP)]

  Electric-like action (tensionless string) can also be seen to agree with the Carrolian limit $(c \ \to \ 0)$ **of electric-type**

[Electric-like] $j \cdot \bar{j}$ string def.: Lagrangian

- **Remarks about** $I_{(+)} = \int d^2\sigma \left[\Pi \cdot \dot{X} - N_{(+)}\Pi^2 - N^\phi \Pi \cdot X'\right]$

replacing $\Pi^\mu = \frac{1}{2N_{(+)}}(\dot{X}^\mu - N^\phi X'^\mu)$ **back into the Hamilt. action :**

$$
\begin{aligned}
I_{(+)} &= \int d^2\sigma \frac{1}{4N_{(+)}} \left[\dot{X} - N_\phi X'\right]^2 \\
&= \int d^2\sigma \, \mathscr{V}^\alpha \mathscr{V}^\beta \partial_\alpha X \cdot \partial_\beta X .
\end{aligned}
$$

- **Lagrangian action in terms of a vector density of weight ½**

$$
\mathscr{V}^\alpha = \frac{1}{2\sqrt{N_{(+)}}} \begin{pmatrix} 1 \\ -N_\phi \end{pmatrix}
$$

: Preserved under BMS3 diffs.

$$(\mathcal{L}_\xi \mathscr{V}^\alpha = 0)$$

Diffeomorphisms $\xi = \xi^\mu \partial_\mu$ close in the Lie bracket according to BMS$_3$

Bosonic string: fin. def. [magnetic-like]

$$H = \frac{1}{2T}\left(\Pi^2 + T^2 X'^2\right) \qquad j\cdot\bar{j} = \frac{1}{4T}\left(\Pi^2 - T^2 X'^2\right) \qquad k^{\mu}_{(-)} = \sqrt{T}X^{\mu\prime}$$

- **Hamiltonian :**

$$H_{(-)} = H - 2j\cdot\bar{j} = k^2_{(-)} = TX'^2$$

- **Magnetic-like action :**

$$I_{(-)} = \int d^2\sigma\left[\Pi\cdot\dot{X} - NH_{(-)} - N^{\phi}P\right]$$

$$I_{(-)} = \int d^2\sigma\left[\Pi\cdot\dot{X} - N_{(-)}X'^2 - N^{\sigma}\Pi\cdot X'\right]$$

with $N_{(-)} = NT$ **[tension gauged away again !]**

Magnetic-like deformation :

New action is also devoid of tension

Still relativistic, but with "inner Carrollian structure"

[Magnetic-like] $j \cdot \bar{j}$ string def.: remarks

$$I_{(-)} = \int d^2\sigma \left[\Pi \cdot \dot{X} - N_{(-)} X'^2 - N^\sigma \Pi \cdot X'\right]$$

Intrinsically Hamiltonian : Π **cannot be expressed in terms of** $\dot{X}$ **(nor** X'**)**

" Self-interacting null particle "

$$\dot{X}^\mu - N^\phi X^{\mu\prime} = 0$$

Field eqs :

$$\dot{\Pi}^\mu - 2\left(N_{(-)} X^{\mu\prime}\right)' - \left(N^\phi \Pi^\mu\right)' = 0$$

Choosing the gauge so that $N_{(-)}$ and N^σ are constants

$$X^\mu = X^\mu(\tilde{\sigma})$$

Describes a curve ! $\tilde{\sigma} = \sigma + N^\sigma \tau$

Constraints :

$$H_{(-)} = X'^2 = 0 \ ,$$

$$P = \Pi \cdot X' = 0 \ ,$$

Null curve: [not a geodesic in general]

$\tilde{\sigma}$ is an affine parameter

also :

$$\Pi^\mu = Y^\mu + 2N_{(-)} X^{\mu\prime\prime} \tau$$

with $Y \cdot X' = 0$ and $Y^\mu = Y^\mu(\tilde{\sigma})$

[Magnetic-like] $j \cdot \bar{j}$ string def. from limits

$$I_{(-)} = \int d^2\sigma \left[\Pi \cdot \dot{X} - N_{(-)} X'^2 - N^\sigma \Pi \cdot X'\right]$$

Recovered from a nonstandard limits in the tension

- **Different tensionless limit :**
- **Rescale** $X^\mu \to T^{-1} X^\mu, \; \Pi^\mu \to T \Pi^\mu$ **&** $N = 2T\tilde{N}_{(-)}$

when $T \to 0$**, one obtains** $I_{(-)}$ **[with** $\tilde{N}_{(-)}$ **]**

[compare with $N_{(-)} = NT$ **; tension gauged away differently]**

Field eqs. & BMS3 constraints recovered

Generic solution of the magnetic-like action also smoothly obtained !

Shares some similarity with Carrollian limit of "magnetic type" $(c \to 0)$

following the lines of [Henneaux & Salgado-Rebolledo, arxiv: 2109.06708 (JHEP)] plus with a suitable rescaling of the lapse

[Magnetic-like] $j \cdot \bar{j}$ string def. from limits

$$I_{(-)} = \int d^2\sigma \left[\Pi \cdot \dot{X} - N_{(-)} X'^2 - N^\sigma \Pi \cdot X' \right]$$

Another interesting limit :

Rescale only the lapse as $N = 2\mathcal{T}^{-1} \hat{N}_{(-)}$

[no rescaling of fields & momenta]

When $T \to \infty$ **, action** $I_{(-)}$ **is recovered with** $N_{(-)} \to \hat{N}_{(-)}$

Appealing possibility:

String length goes to zero -> null curve instead of a surface !

Also works well for field eqs. & constraints

However: generic solution not smoothly recovered

Further aspects in progress

Continuous integrably marginal $j \cdot \bar{j}$ deformations

Abelian currents algebra :

$$\left\{ j^{I}\left(x\right), j^{J}\left(y\right)\right\} = -g^{IJ}\partial_{x}\delta\left(x-y\right)$$

$$\left\{ \bar{j}^{I}\left(x\right), \bar{j}^{J}\left(y\right)\right\} = g^{IJ}\partial_{x}\delta\left(x-y\right)$$

with $g_{IJ} = \delta_{IJ}$ **admits an** $O\left(N,N\right)$ **automorphism**

$P = j^2 - \bar{j}^2$ **: clearly inv. under** $O\left(N,N\right)$ **[whole set of automorphisms]**

$H = j^2 + \bar{j}^2$ **: only inv. under** $O\left(N\right) \otimes O\left(N\right)$ **subset**

Deformations yielding spectral flow (changing H **) then go along**

$$\frac{O\left(N,N\right)}{O\left(N\right) \otimes O\left(N\right)}$$

For our purposes: relevant subset is $SO(1,1)$

Continuous integrably marginal $j \cdot \bar{j}$ deformations

Under $SO(1,1)$, currents transform according to :

$$j^{\mu}_{(\alpha)} = j^{\mu} \cosh\left(\frac{\alpha}{2}\right) + \bar{j}^{\mu} \sinh\left(\frac{\alpha}{2}\right)$$
$$\bar{j}^{\mu}_{(\alpha)} = \bar{j}^{\mu} \cosh\left(\frac{\alpha}{2}\right) + j^{\mu} \sinh\left(\frac{\alpha}{2}\right)$$

- **Hence,** $T_{(\alpha)} = j^2_{(\alpha)}$ and $\bar{T}_{(\alpha)} = \bar{j}^2_{(\alpha)}$ **also fulfill the conformal algebra**

- **Note that :**

$$T_{(\alpha)} = \cosh^2\left(\frac{\alpha}{2}\right) T + \sinh^2\left(\frac{\alpha}{2}\right) \bar{T} + \sinh(\alpha) j \cdot \bar{j}$$

$$\bar{T}_{(\alpha)} = \cosh^2\left(\frac{\alpha}{2}\right) \bar{T} + \sinh^2\left(\frac{\alpha}{2}\right) T + \sinh(\alpha) j \cdot \bar{j}$$

Automorphism induces a mixing of left & right sectors

Continuous integrably marginal $j \cdot \bar{j}$ deformations

In terms of
energy & momentum densities:

$$\bar{T}_{(\alpha)} = \frac{1}{2}\left(H_{(\alpha)} + P\right)$$

$$T_{(\alpha)} = \frac{1}{2}\left(H_{(\alpha)} - P\right)$$

$$P_{(\alpha)} = \bar{j}^2_{(\alpha)} - j^2_{(\alpha)} = P$$: **invariant [has to be]**

$$H_{(\alpha)} = j^2_{(\alpha)} + \bar{j}^2_{(\alpha)} = \cosh(\alpha) H + 2\sinh(\alpha) j \cdot \bar{j}$$

In the energy-momentum basis : $k^I_{(\alpha)(\pm)} = \bar{j}^I_{(\alpha)} \pm j^I_{(\alpha)}$

so that $$k^I_{(\alpha)(\pm)} = e^{\pm\alpha/2} k^I_{(\pm)}$$

The original algebra does not change : [trivial deformation]

[both sets $\{H_{(\alpha)}; P, k^I_{(\alpha)(+)}, k^I_{(-)}\}$ **&** $\{H; P, k^I_{(+)}, k^I_{(-)}\}$ **: same algebra]**

Int. marginal $j \cdot \bar{j}$ deformations: limiting cases

Useful to rescale $H_{(\alpha)} = \cosh(\alpha)\, H + 2\sinh(\alpha)\, j \cdot \bar{j}$

according to :
$$\tilde{\mathcal{H}}_{(\alpha)} := \frac{H_{(\alpha)}}{\cosh(\alpha)} = H + 2\tanh(\alpha)\, j \cdot \bar{j}$$

equivalently :
$$\tilde{\mathcal{H}}_{(\alpha)} = \frac{1}{2}\left(1+\tanh(\alpha)\right) k^2_{(+)} + \frac{1}{2}\left(1-\tanh(\alpha)\right) k^2_{(-)}$$

Relevant commutators :
$$\left[J_m, \tilde{\mathcal{H}}_{(\alpha)n}\right] = (m-n)\, \tilde{\mathcal{H}}_{(\alpha)m+n} \; ,$$
$$\left[\tilde{\mathcal{H}}_{(\alpha)m}, \tilde{\mathcal{H}}_{(\alpha)n}\right] = \cosh^{-2}(\alpha)\,(m-n)\, J_{m+n}$$
$$\left[\tilde{\mathcal{H}}_{(\alpha)n}, k^{\mu}_{(+)m}\right] = -\left(1-\tanh(\alpha)\right) m k^{\mu}_{(-)m+n}$$
$$\left[\tilde{\mathcal{H}}_{(\alpha)n}, k^{\mu}_{(-)m}\right] = -\left(1+\tanh(\alpha)\right) m k^{\mu}_{(+)m+n}$$

when $\alpha \to \pm\infty$ one recovers the BMS3 algebra with currents

Int. marginal $j \cdot \bar{j}$ deformations: limiting cases

$$\tilde{\mathcal{H}}_{(\alpha)} = \frac{1}{2}\left(1+\tanh(\alpha)\right) k_{(+)}^2 + \frac{1}{2}\left(1-\tanh(\alpha)\right) k_{(-)}^2$$

$$\left[J_m, \tilde{\mathcal{H}}_{(\alpha)n}\right] = (m-n)\,\tilde{\mathcal{H}}_{(\alpha)m+n} \ ,$$

$$\left[\tilde{\mathcal{H}}_{(\alpha)m}, \tilde{\mathcal{H}}_{(\alpha)n}\right] = \cosh^{-2}(\alpha)\,(m-n)\,J_{m+n}$$

$$\left[\tilde{\mathcal{H}}_{(\alpha)n}, k^{\mu}_{(+)m}\right] = -\left(1-\tanh(\alpha)\right) m k^{\mu}_{(-)m+n}$$

$$\left[\tilde{\mathcal{H}}_{(\alpha)n}, k^{\mu}_{(-)m}\right] = -\left(1+\tanh(\alpha)\right) m k^{\mu}_{(+)m+n}$$

when $\alpha \to \pm\infty$ **one recovers the BMS3 algebra with currents**

$$\left[P_m, P_n\right] = (m-n)\,P_{m+n} \ ,$$

$$\left[P_m, H_{(\pm)n}\right] = (m-n)\,H_{(\pm)m+n} \ ,$$

$$\left[H_{(\pm)m}, H_{(\pm)n}\right] = 0 \ ,$$

$$\left[k^I_{(+)m}, k^J_{(-)n}\right] = 2mg^{IJ}\delta_{m+n,0} \ ,$$

$$\left[k^I_{(\pm)m}, k^J_{(\pm)n}\right] = 0$$

$$\left[P, k^I_{(\pm)m}\right] = -mk^I_{(\pm)m+n} \ ,$$

$$\left[H_{(\pm)n}, k^I_{(\pm)m}\right] = 0 \ ,$$

$$\left[H_{(+)n}, k^I_{(-)m}\right] = -2mk^I_{(+)m+n} \ ,$$

$$\left[H_{(-)n}, k^I_{(+)m}\right] = -2mk^I_{(-)m+n} \ ,$$

electric- & magnetic-like generators recovered

$$\tilde{\mathcal{H}}_{(\pm\infty)} = H_{(\pm)} = H \pm 2j\cdot\bar{j} = k^2_{(\pm)}$$

Full conformal symmetry retained, in 2 alternative ultrarelativistic regimes

Ending remarks

- **Beyond the bound :** $|g| > 2$

After a suitable rescaling of $H_{(g)} = H + g\, j \cdot \bar{j}$

Euclidean version of the conformal algebra with currents :

$$\left[\tilde{\mathcal{H}}_{(g)m}, \tilde{\mathcal{H}}_{(g)n}\right] = -\,(m-n)\,J_{m+n}$$

[sign change at r.h.s.]

However, not a thermal version of the original (undeformed) CFT2

For a generic gauge choice (not conformal gauge) :

$$I_{(g)} = \int d^2\sigma \left[\Pi \cdot \dot{X} - N\tilde{\mathcal{H}}_{(g)} - N^{\phi} P\right]$$

[missing additional "i" in the action: not a thermal theory !]

[deformation does not implement the corresponding Wick rotation]

Ending remarks

- **Deformations & the Polyakov action**

$$I = -\frac{T}{2}\int d^2\sigma\ \sqrt{-h}h^{\alpha\beta}\partial_\alpha X\cdot\partial_\beta X$$

- **Deformed Hamiltonian action :**

$$I_{(g)} = \int d^2\sigma\ \left[\Pi\cdot\dot{X} - NH_{(g)} - N_\phi P\right]$$

$$H_{(g)} = H + g\ j\cdot\bar{j}$$

$$H = \frac{1}{2T}\left(\Pi^2 + T^2X'^2\right)\ ,$$

$$P = \Pi\cdot X'\ ,$$

$$j\cdot\bar{j} = \frac{1}{4T}\left(\Pi^2 - T^2X'^2\right)$$

$$I_{(g)} = \int d^2\sigma\ \left[\Pi\cdot\dot{X} - \frac{N}{2T}\left\{\left(1+\frac{g}{2}\right)\Pi^2 + \left(1-\frac{g}{2}\right)T^2X'^2\right\} - N_\phi\Pi\cdot X'\right]$$

Ending remarks

Deformed Hamiltonian action

$$I_{(g)} = \int d^2\sigma \left[\Pi \cdot \dot{X} - \frac{N}{2T}\left\{\left(1+\frac{g}{2}\right)\Pi^2 + \left(1-\frac{g}{2}\right)T^2 X'^2\right\} - N_\phi \Pi \cdot X'\right]$$

$$\Pi^\mu = \frac{T}{N}\left(1+\frac{g}{2}\right)^{-1}\left(\dot{X}^\mu - N_\phi X'^\mu\right)$$

Warning for $g=-2$ **:**
[not well-defined for magnetic-like BMS3]

Back into the Hamiltonian action : [Lagrangian action]

$$I_{(g)} = -\frac{T}{2}\left(1+\frac{g}{2}\right)^{-1}\int d^2\sigma\ N^{-1}\left[-\dot{X}^2 + 2N_\phi \dot{X}\cdot X' - \left\{N_\phi^2 - \left(1-\frac{g^2}{4}\right)N^2\right\}X'^2\right]$$

Inverse wordsheet metric & metric determinant :

$$h^{\alpha\beta} = \begin{pmatrix} -1 & N_\phi \\ N_\phi & -\left[N_\phi^2 - \left(1-\frac{g^2}{4}\right)N^2\right] \end{pmatrix} \qquad h = -\left(1-\frac{g^2}{4}\right)^{-1} N^{-2}$$

Ending remarks

$$I_{(g)} = -\frac{T}{2}\left(1+\frac{g}{2}\right)^{-1}\int d^2\sigma\ N^{-1}\left[-\dot{X}^2 + 2N_\phi \dot{X}\cdot X' - \left\{N_\phi^2 - \left(1-\frac{g^2}{4}\right)N^2\right\}X'^2\right]$$

$$h^{\alpha\beta} = \begin{pmatrix} -1 & N_\phi \\ N_\phi & -\left[N_\phi^2 - \left(1-\frac{g^2}{4}\right)N^2\right]\end{pmatrix} \qquad h = -\left(1-\frac{g^2}{4}\right)^{-1} N^{-2}$$

Three cases : sign(h)

$|g| < 2$ **Lorentzian metric :** $\sqrt{\left(1-\frac{g^2}{4}\right)}\sqrt{-h} = N^{-1}$

Same Polyakov action

$$I_{(g)} = -\frac{T_{(g)}}{2}\int d^2\sigma\ \sqrt{h}h^{\alpha\beta}\partial_\alpha X\cdot\partial_\beta X$$

[just rescaling tension]

$$T_{(g)} = T\sqrt{\frac{2-g}{2+g}} \qquad g = 2\tanh\alpha$$

equivalently $T_{(\alpha)} = Te^{-\alpha}$ **[parameter of the** $SO(1,1)$ **automorphism]**

Ending remarks

$$I_{(g)} = -\frac{T}{2}\left(1+\frac{g}{2}\right)^{-1}\int d^2\sigma\ N^{-1}\left[-\dot{X}^2 + 2N_\phi \dot{X}\cdot X' - \left\{N_\phi^2 - \left(1-\frac{g^2}{4}\right)N^2\right\}X'^2\right]$$

$$h^{\alpha\beta} = \begin{pmatrix} -1 & N_\phi \\ N_\phi & -\left[N_\phi^2 - \left(1-\frac{g^2}{4}\right)N^2\right] \end{pmatrix} \qquad h = -\left(1-\frac{g^2}{4}\right)^{-1} N^{-2}$$

Three cases : sign(h)

$|g| = 2$ **Degenerate metric :** $h = 0$

- **For** $g = 2$ **: tension can be gauged away**

  [electric-like deformation = tensionless string]

- **Note that the analysis is not valid for** $g = -2$ **!**

  [Magnetic-like deformation cannot be attained from this Lagrangian action]

Ending remarks

$$I_{(g)} = -\frac{T}{2}\left(1+\frac{g}{2}\right)^{-1}\int d^2\sigma\ N^{-1}\left[-\dot{X}^2 + 2N_\phi \dot{X}\cdot X' - \left\{N_\phi^2 - \left(1-\frac{g^2}{4}\right)N^2\right\}X'^2\right]$$

$$h^{\alpha\beta} = \begin{pmatrix} -1 & N_\phi \\ N_\phi & -\left[N_\phi^2 - \left(1-\frac{g^2}{4}\right)N^2\right]\end{pmatrix} \qquad h = -\left(1-\frac{g^2}{4}\right)^{-1} N^{-2}$$

Three cases :

$|g| > 2$ **Euclidean metric :** $\sqrt{\left(\frac{g^2}{4}-1\right)}\sqrt{h} = N^{-1}$

" Euclidean " Polyakov action :

$$I_{(g)} = -\frac{T^E_{(g)}}{2}\int d^2\sigma\ \sqrt{h}h^{\alpha\beta}\partial_\alpha X\cdot\partial_\beta X$$

[just rescaling tension]

$$T^E_{(g)} = T\sqrt{\frac{g-2}{g+2}}$$

Missing overall " i " : not a thermal theory
[deformation does not implement the Wick rotation]