Logarithmic Celestial Conformal Field Theory

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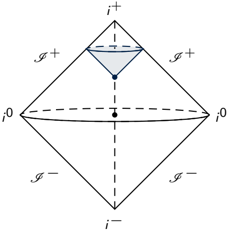
Flat space holography

How to formulate flat space holography?

- Correspondence between **gravity in asymptotically flat spacetimes** and a **lower-dimensional field theory** without gravity.
- **Bottom-up** approaches to build candidates for **holographic duals**.
- Strong constraints implied by the **Bondi-van der Burg-Metzner-Sachs (BMS) symmetries** on the putative dual theories. [Bondi-van der Burg-Metzner '62] [Sachs '62]



Carroll vs Celestial

- Two proposals for flat space holography have emerged:
  $\Longrightarrow$ **Carrollian holography**: the dual theory is a **3d Carrollian CFT** living at **null infinity** $\mathscr{I} \simeq \mathbb{R} \times S^2$.
  [Arcioni-Dappiaggi '03] [Dappiaggi-Moretti-Pinamonti '06] [Barnich-Compère '07] [Bagchi '10] [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '12] [Barnich-Gomberoff-Gonzalez '12] [Bagchi-Basu-Grumiller-Riegler '15] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Donnay-Fiorucci-Herfray-Ruzziconi '22] ...
  $\Longrightarrow$ **Celestial holography**: the dual theory is a **2d CFT** living on the **celestial sphere** S^2.
  [de Boer-Solodukhin '03] [He-Mitra-Strominger '15] [Kapec-Mitra-Raclariu-Strominger '16] [Cheung-de la Fuente-Sundrum '16] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17] [Donnay-Puhm-Strominger '18] [Stieberger-Taylor '18] [Pate-Raclariu-Strominger-Yuan '19] [Adamo-Mason-Sharma '21] ...
- These two proposals are related (massless scattering): [Donnay-Fiorucci-Herfray-Ruzziconi '22]

$$\underbrace{\langle \mathcal{O}_{\Delta_1}(z_1,\bar z_1)\dots\mathcal{O}_{\Delta_n}(z_n,\bar z_n)\rangle}_{\text{Celestial correlators on } S^2} = \prod_{i=1}^{n}\left(4\pi\, i^{\Delta_i+1}\Gamma[\Delta_i]\int_{-\infty}^{+\infty} du_i\, u_i^{-\Delta_i}\right)\underbrace{\langle \Phi(u_1,z_1,\bar z_1)\dots\Phi(u_n,z_n,\bar z_n)\rangle}_{\text{Carrollian correlators at } \mathscr{I}}$$

Conservation between $\mathscr{I}^-$ and $\mathscr{I}^+$ / Soft Theorems / Ward identities in CCFT

$$\Longleftrightarrow$$

(non-)Conservation along $\mathscr{I}$ / Flux-balance laws / Sourced Carrollian Ward identities

- Can use either of these approaches to formulate flat space holography.

Properties of CCFT

What is the nature of the dual field theory?

$\Longrightarrow$ Focus on the celestial approach to identify a relevant class of CFT.

List of (exotic) properties for the celestial CFT (CCFT):

- **Complex spectrum** including the principal series ($\Delta = 1 + i\lambda$, $\lambda \in \mathbb{R}$) [Pasterski-Shao '17]
  $\Longrightarrow$ Non-unitary (non-reflection positive) Euclidean CFT.
- **The central charge vanishes** ($c = 0$), at least at tree level:
  $\Longrightarrow$ Obtained from collinear and double-soft limit of amplitudes [Fotopoulos-Stieberger-Taylor-Zhu '19]
  $\Longrightarrow$ Obtained independently from an asymptotic symmetry analysis [Donnay-Ruzziconi '21]
- A non-unitary CFT might be non-trivial, even if $c = 0$, but there is still the **$c = 0$ catastrophe** [Gurarie '98]
- **Two $(2,0)$ operators** in the celestial CFT (celestial stress tensor + symplectic partner)
  [Pasterski-Shao '17] [Donnay-Puhm-Strominger '18] [Ball-Himwich-Narayanan-Pasterski-Strominger '19]

Smoking gun for a logarithmic CFT!

Definition of a log CFT

- A **log CFT** is defined through its **Jordan block structure**. [Gurarie '93]
- Logarithmic pair $(\mathcal{O}_h(z), \mathcal{O}_h^{\log}(z))$:

$$\delta_{\mathcal{Y}} \mathcal{O}_h = \mathcal{Y}\partial\mathcal{O}_h + h(\partial\mathcal{Y})\mathcal{O}_h \,,$$
$$\delta_{\mathcal{Y}} \mathcal{O}_h^{\log} = \mathcal{Y}\partial\mathcal{O}_h^{\log} + h(\partial\mathcal{Y})\mathcal{O}_h^{\log} + (\partial\mathcal{Y})\mathcal{O}_h \,.$$

- State-operator correspondence: $\mathcal{O}_h \leftrightarrow |\mathcal{O}_h\rangle$ and $\mathcal{O}_h^{\log} \leftrightarrow |\mathcal{O}_h^{\log}\rangle$

$$L_0 \begin{pmatrix} |\mathcal{O}_h^{\log}\rangle \\ |\mathcal{O}_h\rangle \end{pmatrix} = \begin{pmatrix} h & 1 \\ 0 & h \end{pmatrix} \begin{pmatrix} |\mathcal{O}_h^{\log}\rangle \\ |\mathcal{O}_h\rangle \end{pmatrix} .$$

- Ward identities imply logarithmic correlation functions ($b \in \mathbb{C}$, $\mu \in \mathbb{R}_0^+$):

$$\langle \mathcal{O}_h(z)\mathcal{O}_h(0)\rangle = 0 \,, \quad \langle \mathcal{O}_h^{\log}(z)\mathcal{O}_h(0)\rangle = \frac{b}{z^{2h}} \,, \quad \langle \mathcal{O}_h^{\log}(z)\mathcal{O}_h^{\log}(0)\rangle = -\frac{2b}{z^{2h}} \ln(\mu z) \,.$$

- μ is a physically irrelevant scale: $\mathcal{O}_h^{\log} \to \mathcal{O}_h^{\log} + \gamma\mathcal{O}_h$.
- If the stress tensor $T(z)$ has a logarithmic partner $t(z)$ ($h = 2$), then the central charge vanishes!
- **NB**: the construction can be trivially extended to the other chirality.

$c = 0$ catastrophe

- Log CFTs offer a possible **resolution of the $c = 0$ catastrophe**. [Gurarie '98] [Cardy '13]
- In a generic CFT, the OPE of some (chiral) primary with itself is given by

$$\mathcal{O}_h(z)\,\mathcal{O}_h(0) = \frac{a}{z^{2h}}\left(1 + \frac{2h}{c}\,z^2\,T(0) + \dots\right).$$

- In the limit $c \to 0$, the theory is ill defined, unless one of the three conditions is fulfilled:
  1. The normalization a vanishes for $c \to 0$.
  2. The conformal weight h vanishes for $c \to 0$.
  3. The $\dots$ terms contain another pole in c such that both poles cancel and the $c \to 0$ limit can be taken.
- The CCFT OPEs and spectrum being non-trivial, we must be in the third scenario:

$$\mathcal{O}_h(z)\,\mathcal{O}_h(0) = \frac{a}{z^{2h}}\left[1 + \frac{2h}{c}\,z^2\left(T(0) - M(0)\right) + \dots\right]$$

  such that $M(z) = T(z) + \mathcal{O}(c)$.

- Assume $c \sim \epsilon$ ($\epsilon \to 0$), and a family M_ϵ of $(2+\epsilon, 0)$ primaries such that $\lim_{\epsilon\to 0} M_\epsilon = T$.
- Log partner: $t_\epsilon(z) = \frac{T(z) - M_\epsilon(z)}{\epsilon}$, $t(z) = \lim_{\epsilon\to 0} t_\epsilon(z)$ [Cardy '13]

$$L_0|t_\epsilon\rangle = 2|t_\epsilon\rangle + |M_\epsilon\rangle \quad \Rightarrow \quad L_0|t\rangle = 2|t\rangle + |T\rangle\,.$$

Radiative phase space and symmetries

Is the CCFT a log CFT?

- Operators of the CCFT built out of the radiative phase space.
- $\mathscr{I}^+$ with coordinates $x^a = (u, z, \bar{z})$, conformal Carrollian structure in Bondi coordinates:

$$q_{ab} dx^a dx^b = 0 du^2 + 2 dz d\bar{z} \,, \qquad n^a \partial_a = \partial_u \,,$$

defined up to rescalings: $q_{ab} \sim \omega^2 q_{ab}$ and $n^a \sim \omega^{-1} n^a$. [Geroch '77] [Ashtekar '14] [Duval-Gibbons-Horvathy '14]
- BMS/Conformal Carroll symmetries: [Bondi-van der Burg-Metzner '62] [Sachs '62]

$$\xi = \left[\mathcal{T} + \frac{u}{2}(\partial \mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} \qquad \text{where} \qquad \partial \equiv \partial_z, \bar{\partial} \equiv \partial_{\bar{z}} \,.$$

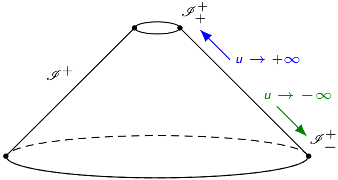
$\mathcal{T} = \mathcal{T}(z, \bar{z})$: supertranslation parameter, $(\mathcal{Y}(z), \bar{\mathcal{Y}}(\bar{z}))$: superrotation parameters [Barnich-Troessaert '10]
- Radiative phase space at $\mathscr{I}^+$: parametrized by the asymptotic shear $C_{zz}(u, z, \bar{z})$ ($C^*_{zz} = C_{\bar{z}\bar{z}}$) and the Bondi news tensor $N_{zz} = \partial_u C_{zz}$ (outgoing radiation).
- Transformation under BMS symmetries:

$$\delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})} C_{zz} = \left(\mathcal{T} + \frac{u}{2}(\partial \mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right) N_{zz} + \left(\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \frac{3}{2}\partial\mathcal{Y} - \frac{1}{2}\bar{\partial}\bar{\mathcal{Y}}\right) C_{zz} - 2\partial^2\mathcal{T} - u\partial^3\mathcal{Y} \,.$$

Falloffs in u

- Falloffs at $\mathscr{I}^+_\pm$ $(u \to \pm\infty)$, compatible with symmetries and contributions from loop diagrams: [Strominger '13] [Compère-Fiorucci-Ruzziconi '18] [Sahoo-Sen '18] [Campiglia-Peraza '20]

$$
\begin{aligned}
C_{zz}|_{\mathscr{I}^+_\pm} &= (u + C_\pm) N^{vac}_{zz} - 2\partial^2 C_\pm + O(u^{-1}) \,, \\
N_{zz}|_{\mathscr{I}^+_\pm} &= N^{vac}_{zz} + O(u^{-2}) \,.
\end{aligned}
$$



- $C_\pm(z,\bar{z})$ correspond to the values of the supertranslation field at $\mathscr{I}^+_\pm$ encoding the displacement memory effect: [Strominger-Zhiboedov '14]

$$
\delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})} C_\pm = \left(\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} - \frac{1}{2}\partial\mathcal{Y} - \frac{1}{2}\bar{\partial}\bar{\mathcal{Y}} \right) C_\pm + \mathcal{T} \,,
$$

- $N^{vac}_{zz}(z) = \frac{1}{2}(\partial\varphi)^2 - \partial^2\varphi$ is the Liouville stress tensor built out of the Liouville field $\varphi(z)$: [Compère-Long '16] [Compère-Fiorucci-Ruzziconi '18]

$$
\begin{aligned}
\delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})} \varphi &= \mathcal{Y}\partial\varphi + \partial\mathcal{Y} \,, \\
\delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})} N^{vac}_{zz} &= (\mathcal{Y}\partial + 2\partial\mathcal{Y}) N^{vac}_{zz} - \partial^3\mathcal{Y} \,. \qquad (2,0)
\end{aligned}
$$

Hard and soft

- Useful derivative operator: [Campiglia '20] [Barnich-Ruzziconi '21] [Donnay-Ruzziconi '21]

$$\mathscr{D}\phi_{h,\bar{h}} = [\partial - h\partial\varphi]\phi_{h,\bar{h}}\,, \quad \bar{\mathscr{D}}\phi_{h,\bar{h}} = [\bar{\partial} - \bar{h}\bar{\partial}\bar{\varphi}]\phi_{h,\bar{h}}\,.$$

- Radiative data can be split into hard and soft. We define $C^{(0)}_{zz}$, $\tilde{C}_{zz}$ and $\tilde{N}_{zz}$ through [Campiglia-Laddha '21] [Donnay-Nguyen-Ruzziconi '22]

$$C_{zz} = u\,N^{vac}_{zz} + C^{(0)}_{zz} + \tilde{C}_{zz}\,, \qquad N_{zz} = N^{vac}_{zz} + \tilde{N}_{zz}\,,$$
$$C^{(0)}_{zz} = -2\mathscr{D}^2 C^{(0)}\,, \qquad C^{(0)} = \tfrac{1}{2}(C_+ + C_-)\,, \qquad N^{(0)} = \tfrac{1}{2}(C_+ - C_-)\,.$$

- $\tilde{N}_{zz}$ is called the "physical news", the non-radiative spacetime condition $\tilde{N}_{zz} = 0$ is BMS invariant. [Compère-Fiorucci-Ruzziconi '18]
- Leading soft graviton $\mathcal{N}^{(0)}_{zz}$ and subleading soft graviton $\mathcal{N}^{(1)}_{zz}$ are defined by [Strominger '13] [He-Lysov-Mitra-Strominger '14] [Kapec-Lysov-Pasterski-Strominger '14]

$$\mathcal{N}^{(0)}_{zz} \equiv \int_{-\infty}^{+\infty} du\, \tilde{N}_{zz} = -4\mathscr{D}^2 N^{(0)}\,, \qquad \mathcal{N}^{(1)}_{zz} \equiv \int_{-\infty}^{+\infty} du\, u\, \tilde{N}_{zz}\,.$$

- Remark: the integral defining $\mathcal{N}^{(1)}_{zz}$ is divergent $\Longrightarrow$ Natural to introduce an IR cut-off. [Compère-Gralla-Wei '23]

Symplectic structure

- Ashtekar-Streubel symplectic structure [Ashtekar-Streubel '81] can be split into hard and soft pieces: [Campiglia-Laddha '21] [Donnay-Nguyen-Ruzziconi '22]

$$\Omega = \frac{1}{32\pi G} \int_{\mathscr{I}^+} du\, d^2 z \, [\delta N_{zz} \wedge \delta C_{\bar{z}\bar{z}} + c.c.] = \Omega^{hard} + \Omega^{soft}$$

where

$$\Omega^{hard} = \frac{1}{32\pi G} \int_{\mathscr{I}^+} du\, d^2 z \left[\delta \tilde{N}_{zz} \wedge \delta \tilde{C}_{\bar{z}\bar{z}} + c.c.\right] ,$$
$$\Omega^{soft} = \frac{1}{32\pi G} \int_{\mathcal{S}} d^2 z \left[\delta \mathcal{N}^{(0)}_{zz} \wedge \delta C^{(0)}_{\bar{z}\bar{z}} + 2\delta \mathcal{N}^{(1)}_{zz} \wedge \delta N^{vac}_{\bar{z}\bar{z}} + c.c.\right] .$$

Where is the log pair in the radiative phase space?

Celestial stress tensor and its $(2,0)$ symplectic partner

- Hard subsector contains finite-energy gravitons: [Pasterski-Puhm-Trevisani '21] [Donnay-Fiorucci-Herfray-Ruzziconi '22]

$$\mathcal{O}_{(\Delta,+2)}(z,\bar{z}) = \kappa^+_\Delta \int_{-\infty}^{+\infty} \frac{du}{(u+i\varepsilon)^{\Delta-1}} \widetilde{N}_{zz}(u,z,\bar{z}), \quad \mathcal{O}^\dagger_{(\Delta,-2)}(z,\bar{z}) = \kappa^-_\Delta \int_{-\infty}^{+\infty} \frac{du}{(u-i\varepsilon)^{\Delta-1}} \widetilde{N}_{zz}(u,z,\bar{z}).$$

- Soft subsector: Memory $\times$ Goldstone

$\phi_{h,\bar{h}}$	$\mathcal{N}^{(0)}_{zz}$	$\mathcal{N}^{(1)}_{zz}$	$C^{(0)}_{zz}$	N^{vac}_{zz}	$\mathscr{D}$	dz
h	$\frac{3}{2}$	1	$\frac{3}{2}$	2	1	-1
$\bar{h}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0

- There are two natural and independent $(2,0)$ operators in the theory:
  1. The Liouville stress tensor: $N^{vac}_{zz}(z) = \frac{1}{2}(\partial\varphi)^2 - \partial^2\varphi$;
  2. The celestial stress tensor: $T(z) = -\frac{6i}{8\pi G}\int \frac{d^2w}{(z-w)^4}\mathcal{N}^{(1)}_{\bar{w}\bar{w}}(w,\bar{w})$. [Kapec-Mitra-Raclariu-Strominger '16]

$$\left\langle T(z)\prod_{i=1}^n \mathcal{O}_{(\Delta_i,J_i)}(z_i,\bar{z}_i)\right\rangle = \sum_{j=1}^n \left[\frac{\partial_j}{z-z_j} + \frac{h_j}{(z-z_j)^2}\right]\left\langle \prod_{i=1}^n \mathcal{O}_{(\Delta_i,J_i)}(z_i,\bar{z}_i)\right\rangle \qquad \text{[subleading soft graviton theorem]}$$

- Comment: shadow transform $\phi_{h,\bar{h}}(z,\bar{z}) \longrightarrow \phi_{1-h,1-\bar{h}}(z,\bar{z}) = \int \frac{d^2w}{(z-w)^{2-2h}(\bar{z}-\bar{w})^{2-2\bar{h}}}\phi_{h,\bar{h}}(w,\bar{w})$.

Logarithmic partner

- At this stage, the two $(2,0)$ operators are standard (quasi-)primaries:

$$\delta_{(\mathcal{Y},\bar{\mathcal{Y}})} T = (\mathcal{Y}\partial + 2\partial\mathcal{Y}) T \,,$$
$$\delta_{(\mathcal{Y},\bar{\mathcal{Y}})} N_{zz}^{vac} = (\mathcal{Y}\partial + 2\partial\mathcal{Y}) N_{zz}^{vac} - \partial^3\mathcal{Y} \,.$$

- However, one could consider the alternative combination: $T(z)$ and $t(z) = :\varphi(z) T(z):$ [Fiorucci-Grumiller-Ruzziconi '23]

$$\delta_{(\mathcal{Y},\bar{\mathcal{Y}})} t = (\mathcal{Y}\partial + 2\partial\mathcal{Y}) t + (\partial\mathcal{Y}) T \,.$$

$\implies$ There is a log partner to the celestial stress tensor identified in the radiative phase space!

- Key ingredient: the anomalous transfomation of the Liouville field [Compère-Long '16] [Compère-Fiorucci-Ruzziconi '18]

$$\delta_{(\mathcal{Y},\bar{\mathcal{Y}})} \varphi = \mathcal{Y}\partial\varphi + \partial\mathcal{Y} \,.$$

Log CCFT as a limit

- In asymptotically flat spacetime, natural to introduce an IR cut-off to regularize the integrals $\Lambda_{IR} \sim \sqrt{G} e^{\frac{1}{\epsilon}}$.
- At finite IR cut-off, the central charge might receive $\mathcal{O}(\epsilon)$ corrections. Simplest case:

$$\langle T(z) T(0) \rangle = -\frac{b\epsilon}{z^4} .$$

- Correlation function of the supertranslation Goldstone mode: [Himwich-Narayanan-Pate-Paul-Strominger '20]

$$\langle C(z,\bar{z}) C(0,0) \rangle = \frac{1}{\epsilon} \frac{2G}{\pi} |z|^2 \ln |z|^2 ,$$

  (cusp anomalous dimension introduced to regularize IR divergences coming from loop diagrams in scattering amplitudes)

- By analogy, since the Liouville field is a Goldstone mode for conformal transformations:

$$\langle \varphi(z) \varphi(0) \rangle = -\frac{2}{\epsilon} \ln z .$$

Uplifted AdS_3/CFT_2 and IR divergence

Justification for the $\langle \varphi(z)\varphi(0)\rangle$ correlation function:

- $\langle \varphi(z)\varphi(0)\rangle \propto \ln z$, usual behaviour for a Liouville scalar field.

- Assuming $\langle \varphi(z)\varphi(0)\rangle = -\frac{2}{\epsilon}\ln z$, and using Wick's contractions:

$$\langle N^{\text{vac}}_{zz}(z) N^{\text{vac}}_{zz}(0)\rangle = \frac{2}{\epsilon^2}\frac{1-6\epsilon}{z^4} \,.$$

$\Longrightarrow$ Divergence ϵ^{-2} consistent with the uplifted AdS_3/CFT_2 dictionary.

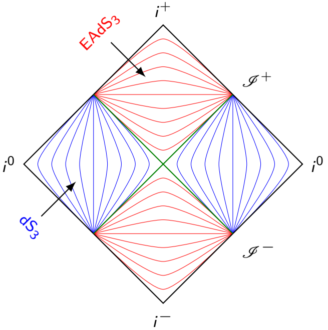
[Cheung-de la Fuente-Sundrum '16] [Ball-Himwich-Narayanan-Pasterski-Strominger '19]

[Pasterski-Verlinde '22] [Nguyen '22]

- The vertex operator $\mathcal{V}_\epsilon(z) \equiv\, :e^{\epsilon\varphi(z)}:$ has conformal weights $(\epsilon, 0)$:

$$\langle \mathcal{V}_\epsilon(z)\mathcal{V}_\epsilon(0)\rangle = -\frac{1}{z^{2\epsilon}} \,.$$

$\Longrightarrow$ The precise factor $-\frac{2}{\epsilon}$ ensures the right exponent in the 2-point function.



- Define $M_\epsilon(z)$ the conformal primary of weight $(2+\epsilon, 0)$

$$M_\epsilon(z) \equiv \, : T(z)\mathcal{V}_\epsilon(z) :$$

  which collides with the stress tensor in the limit: $\lim_{\epsilon\to 0} M_\epsilon(z) = T(z)$.

- Define

$$t_\epsilon(z) = \frac{M_\epsilon(z) - T(z)}{\epsilon} = \, : T(z)\varphi(z) : + \mathcal{O}(\epsilon)\,.$$

  the logarithmic partner in the limit $\epsilon \to 0$.

- Owing to $\langle M_\epsilon(z) T(0)\rangle = 0$, we have

$$\langle T(z) t_\epsilon(0)\rangle = \frac{b}{z^4} \quad \Rightarrow \quad \lim_{\epsilon\to 0}\langle T(z) t_\epsilon(0)\rangle = \frac{b}{z^4}\,,$$

- Moreover, using $\langle M_\epsilon(z) M_\epsilon(0)\rangle = \frac{b\epsilon - \epsilon^2}{z^{4+2\epsilon}} + \mathcal{O}(\epsilon^3)$,

$$\langle t_\epsilon(z) t_\epsilon(0)\rangle = \frac{\langle M_\epsilon(z) M_\epsilon(0)\rangle}{\epsilon^2} + \frac{\langle T(z) T(0)\rangle}{\epsilon^2} = \frac{1}{\epsilon^2}\frac{b\epsilon - \epsilon^2}{z^{4+2\epsilon}} - \frac{1}{\epsilon^2}\frac{b\epsilon}{z^4} + \mathcal{O}(\epsilon)\,.$$

  so that in the limit $\epsilon \to 0$,

$$\lim_{\epsilon\to 0}\langle t_\epsilon(z) t_\epsilon(0)\rangle = -\frac{2b}{z^4}\ln(\mu z)\,, \qquad \text{with } \mu = e^{\frac{1}{2b}}\,.$$

Discussion

- Patterns of Log CFT identified in the CCFT.
  $\Longrightarrow$ Naturally emerges in the IR limit.
  $\Longrightarrow$ Solves the $c = 0$ catastrophe.
  $\Longrightarrow$ Sheds some light on the intrinsic properties of the CCFT.

- How does the Log CFT structure survive with loop-corrected amplitudes?

- Maybe other Jordan blocks to be found. [Bhardwaj-Lippstreu-Ren-Spradlin-Yelleshpur-Volovich '22]

- What is the role of φ in the amplitudes? Dressing field for superrotations at finite cut-off?

- Is there an interplay between Carrollian CFT and Log CCFT?

Thank you!