Ehlers, Carroll and gravitational charges

David Rivera-Betancour

CPHT
École Polytechnique

Based on arXiv:2212.14062,
in collaboration with N. Mittal, P.M. Petropoulos, and M. Vilatte.

Carroll Workshop at Thessaloniki 2023

Motivation

Carrollian manifolds

- $c \to 0$ limit of relativistic manifolds
- The geometry of null hypersurfaces [L. Ciambelli *et al.* (19), Y. Hefray (21)]: null infinity and black hole horizons [R. Penna ('18)] [L. Donnay and C. Marteau ('19)] [J. Redondo-Yuste and L. Lehner ('22)] [L. Freidel and P. Jai-akson ('22)]

Flat holography? [J. Hartong (16), L. Ciambelli *et al.* (18), L. Donnay *et al.* (22)]

- $\mathsf{BMS}_n \equiv \mathsf{CCarr}(n-1)$ [C. Duval *et al.* (14)]
- The dual theory should be $\mathsf{CCarr}(n-1)$-invariant and defined on a Carrollian spacetime

Motivation

The reconstruction of asymptotically flat spacetimes are given in terms of the Carrollian data which are defined on the Carrollian boundary (null infinity) and defines Carrollian conformal dynamics [L. Ciambelli *et al.* (18), A. Campoleoni *et al.* (23)]

In this direction

- What could we learn for their charges using Carrollian-boundary techniques?
- If the Ehlers group is relevant (bulk isometries), how does it act on the Carrollian boundary data?

Carrollian dynamics

Geometry: $\mathrm{d}s^2 = 0 \cdot \upmu^2 + a_{ij}(t, \mathbf{x})\mathrm{d}x^i\mathrm{d}x^j, \quad \upnu = \frac{1}{\Omega}\partial_t, \quad \upmu = -\Omega\mathrm{d}t + b_i\mathrm{d}x^i$

Symmetries of the action: set of momenta

$$\Pi^{ij} = \frac{2}{\sqrt{a}\Omega}\frac{\delta S}{\delta a_{ij}}, \quad \Pi^i = \frac{1}{\sqrt{a}\Omega}\frac{\delta S}{\delta b_i}, \quad \Pi = -\frac{1}{\sqrt{a}}\left(\frac{\delta S}{\delta \Omega} + \frac{b_i}{\Omega}\frac{\delta S}{\delta b_i}\right)$$

- Diffeo invariance $\left(\xi = \xi^t(t,\mathbf{x})\partial_t + \xi^i(\mathbf{x})\partial_i\right) \longrightarrow$ Conservation for Π, Π^i, Π^{ij}
- Weyl invariance $\longrightarrow \Pi^i{}_i = \Pi$
- Carroll boost invariant if $\Pi^i = 0$

$$\frac{1}{\Omega}\hat{\mathscr{D}}_t\Pi + \hat{\mathscr{D}}_i\Pi^i + \Pi^{ij}\xi_{ij} = 0\,, \quad \hat{\mathscr{D}}_i\Pi^i{}_j + 2\Pi^i\varpi_{ij} + \left(\frac{1}{\Omega}\hat{\mathscr{D}}_t\delta^i_j + \xi^i{}_j\right)P_i = 0\,,$$

In the limiting procedure this goes as $T_{00} = \Pi + \mathcal{O}(c^2)$, $cT_0{}^i = \Pi^i + c^2P^i + \mathcal{O}(c^4)$, $T^{ij} = \Pi^{ij} + \mathcal{O}(c^2)$, and then take the expansion of $\nabla_\mu T^{\mu\nu} = 0$

Isometries and (non-)conservations laws

(conformal) Killing fields ξ

- Divergence-free current $I^\mu = T^{\mu\nu}\xi_\nu \longrightarrow \nabla_\mu I^\mu = 0$
- $Q_\xi = \int_\Sigma *\mathsf{I}$ is conserved on-shell
- In 1+2d: $C_{\mu\nu} \Leftrightarrow$ non-conformally flat geometries
- For $I^\mu_{\text{Cott}} = C^{\mu\nu}\xi_\nu$, $Q^{\text{Cott}}_\xi = \int_\Sigma *I_{\text{Cott}}$ is conserved off-shell

Carrollian (conformal) Killing fields $\{\xi^t(t, \mathbf{x}), \xi^i(\mathbf{x})\}$.

- $\mathcal{L}_\xi a_{ij} = -2\lambda a_{ij}$ and $\mathcal{L}_\xi \nu = -\lambda \nu$
- Current: $\kappa = \xi^i P_i - \xi^{\hat{t}}\Pi$, $K^i = \xi^j \Pi_j{}^i - \xi^{\hat{t}}\Pi^i$
- Conserved if: $\Pi^i = 0$ or $\mathcal{L}_\xi \mu = 0$ (strong Killing)
- Electric Charge: $Q_E = \int_\Sigma \mathrm{d}^2x\sqrt{a}\left(\kappa + b_i K^i\right)$
- Magnetic charge: $Q_M = \int_\Sigma \mathrm{d}^2x\sqrt{a}\left(\kappa_{\text{Cott}} + b_i K^i_{\text{Cott}}\right)$

Bulk reconstruction from the boundary (see Simon's talk)

Aymptotically (locally) flat bulk $\Longrightarrow$ null conformal boundary

1. Boundary Carrollian geometry: Ω, b_i, and a_{ij}
2. Cotton Carrollian descendants, example: Taub-NUT-like geometries
3. Bondi shear $\mathscr{C}_{ij}(t, \mathbf{x})$
4. Carrollian and Chthonian data: $\{\Pi, \Pi^i, P^i, \Pi^{ij}\}$ and $\{\Pi_{(s)}, \Pi^i_{(s)}, P^i_{(s)}, \Pi^{ij}_{(s)}\}$
5. Einstein's equations $\Longrightarrow$ Carrollian dynamics on the boundary

Resummation: Algebraically special Petrov type

1. $\mathscr{C}_{ij}(t, \mathbf{x}) = 0$
2. All Chthonian degrees of freedom are discarded
3. Dualization of $\{\Pi^i, P^i, \Pi^{ij}\}$, with the Cotton descendants

$$\mathrm{d}s^2_{\text{res.}} = \upmu \left[2\mathrm{d}r + 2\left(r\varphi_j - *\hat{\mathscr{D}}_j *\varpi\right)\mathrm{d}x^j - \left(r\theta + \hat{\mathscr{K}}\right)\upmu\right] + \rho^2 \mathrm{d}\ell^2 + \frac{\upmu^2}{\rho^2}\left[8\pi G\varepsilon r + *\varpi c\right]$$

with $\rho^2 = r^2 + *\varpi^2$. Solution space: $\{\varepsilon, \Omega, b_i, a_{ij}\}$

Boundary construction of the charges (Resummable)

Observations in the resummable case

- At each $1/r^{2s+1}$, Einstein's Eqs reveal Carrollian dynamics with momenta $\Pi_{(s)}$
- At each $1/r^{2s+2}$, we have Carrollian dynamics with momenta $\Pi_{\mathsf{Cott}(s)}$
- **Isometries in the bulk$\Longleftrightarrow$Carrollian isometries generated by strong Killing vectors**

Multipole moments

- Construction of towers of charges with momenta $\Pi_{(s)} = *\varpi^{2s}\Pi$
- Example: For the Kerr-Taub-NUT family, the "tower" Carrollian charges associated to ∂_t leads to the mass multipolar expansion

$$Q_{E(s)} = M\frac{(n+a)^{2s+1} - (n-a)^{2s+1}}{a(2s+1)}, \quad Q_{M(s)} = n\frac{(n-a)^{2s+1} - (n-a)^{2s+1}}{a(2s+1)}$$

Ehlers hidden symmetry on the boundary [Ehlers ('62) and Geroch ('71)]

Einstein's equations have a hidden symmetry that is revealed upon bulk reduction along isometric orbits

- From 4 to 3 dimensions: Ehlers Möbius group $SL(2,\mathbb{R})$
- This group generates new Ricci-flat spacetimes (going back from 3 to 4 dimensions)

How are these transformations manifested on the boundary?

- We focused on timelike isometries: $\xi = \partial_t$
- These transformations are reflected on the boundary as local transformations for the geometry and the Carrollian "fluid" variables.
- New Carrollian configurations can generate new bulk algebraic Ricci-flat spacetimes
- Example: Type D solutions with $\xi = \partial_t$ (Kerr-Taub-NUT family)

$$SO(2) \longrightarrow -c' + \mathrm{i}8\pi G\varepsilon' = e^{2\mathrm{i}\chi}(-c + \mathrm{i}8\pi G\varepsilon)$$

Some conclusions and future directions

- The action of the hidden Ehlers symmetry of Einstein's equations, as well as the construction of gravitational charges were presented in a Carrollian fashion
- All these by taking advantage of the Carrollian covariant reconstruction of the Ricci-flat spacetime (in its resummed version)

Next steps:

- Complete the picture of the towers of charges and dual charges in a Carrollian set up. Comparison with bulk approaches [H.Godazgar, M.Godazgar and C.N. Pope (18-21)]
- Where are the Newman-Penrose charges in this formalism?
- Boundary Ehlers transformations associated to general Killings (timelike, spacelike and lightlike) and outside of the algebraic special type
- Carrollian reconstruction of Ricci-flat spacetimes in higher dimensions?