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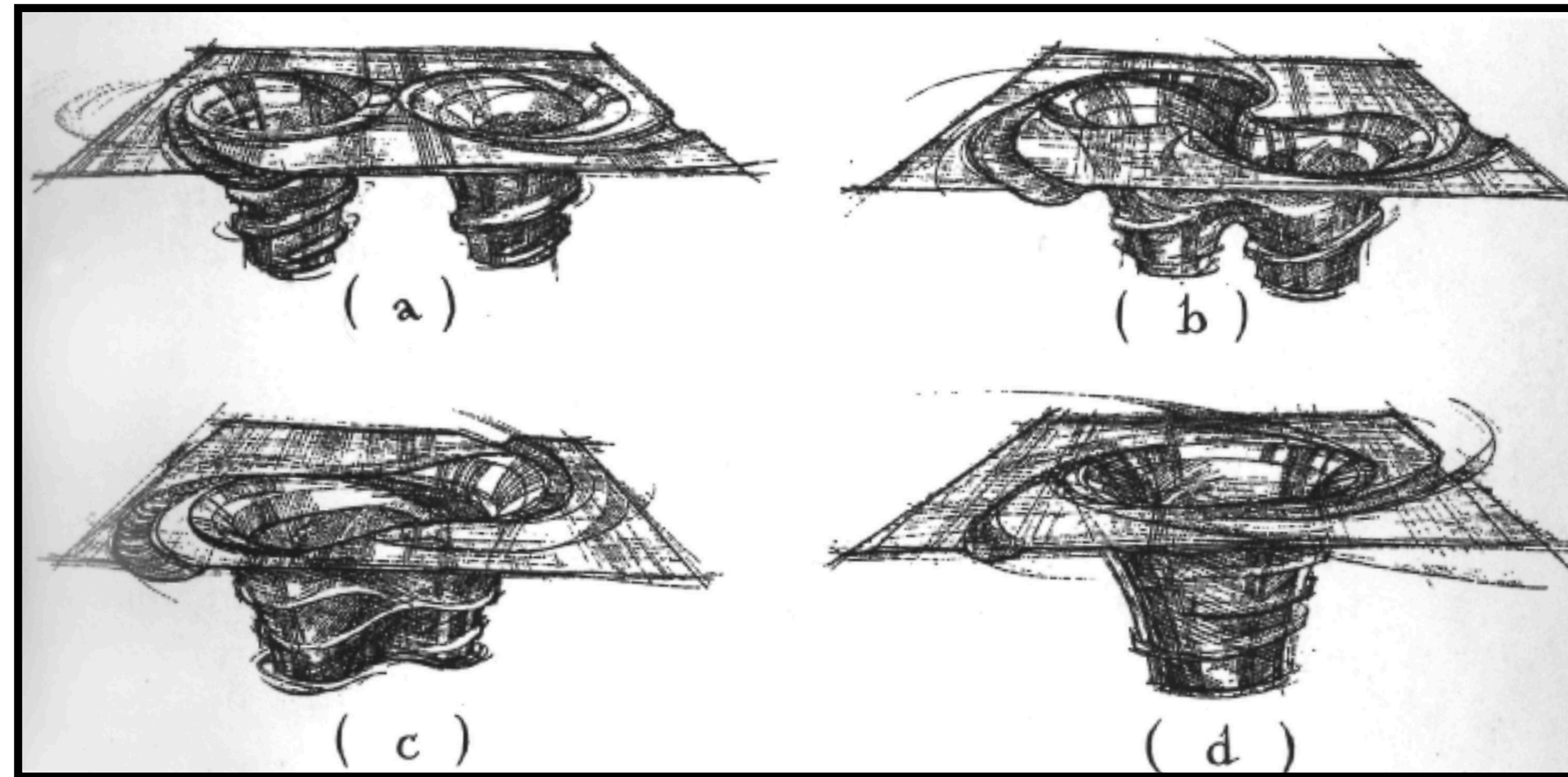
Carrollian insights for GW physics

3rd Carroll Workshop
Θεσσαλονίκη

JAIME REDONDO-YUSTE

(Somewhat) based on
2212.06175 (+L.Lehner)

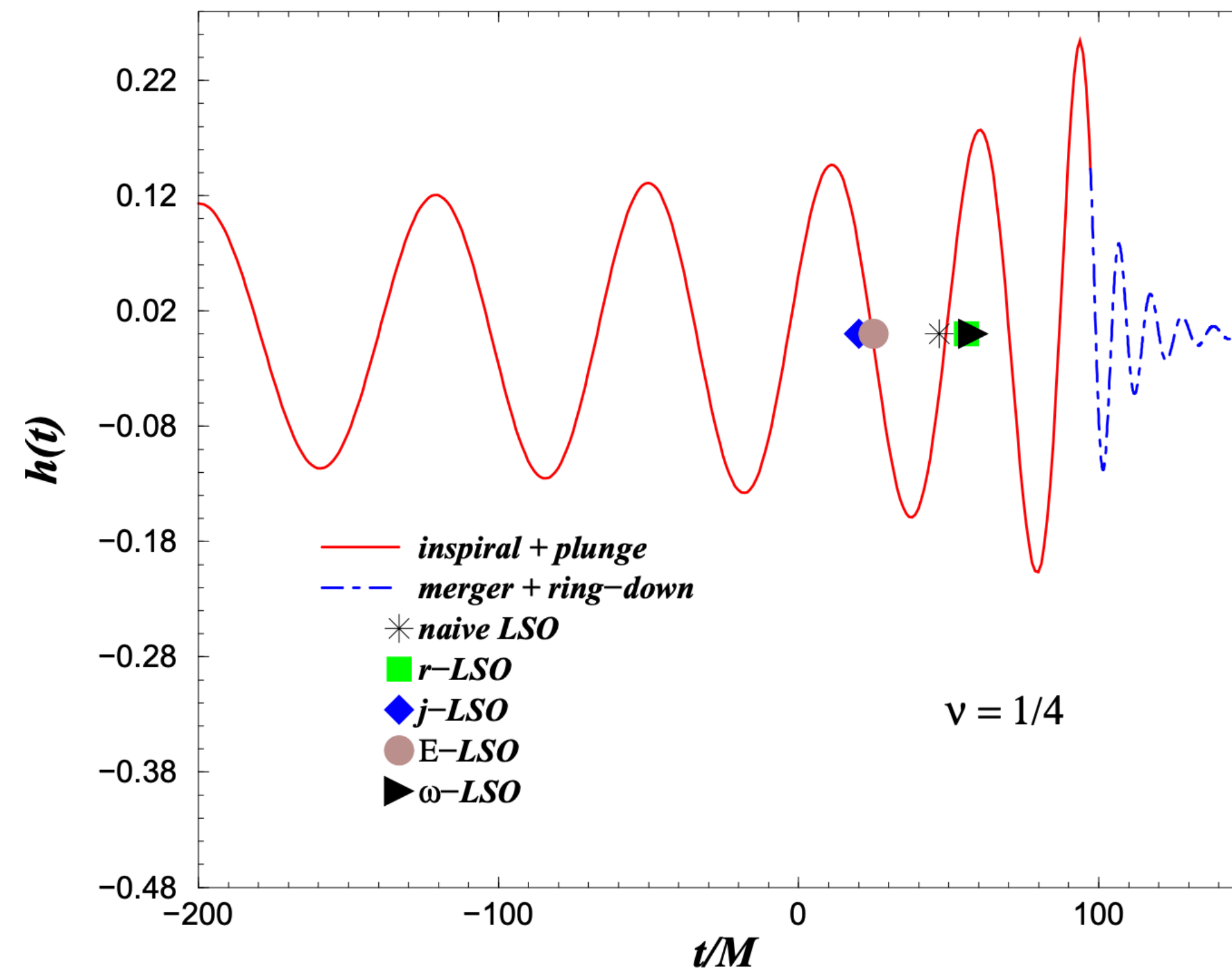
Merger of two BHs: Where are the non-linearities?



When water waves become so high that they “interact with themselves dynamically and nonlinearly,” the result can be the breaking, crashing froth that topples and engulfs surfers — or it can be an enormous tidal wave that travels across oceans at high speed, hits shores, and wreaks havoc. The analogous nonlinear, dynamical behavior of spacetime warpage is largely a mystery today. By combined gravity-wave observations and supercomputer simulations we hope to discover it.

K. Thorne (2002)

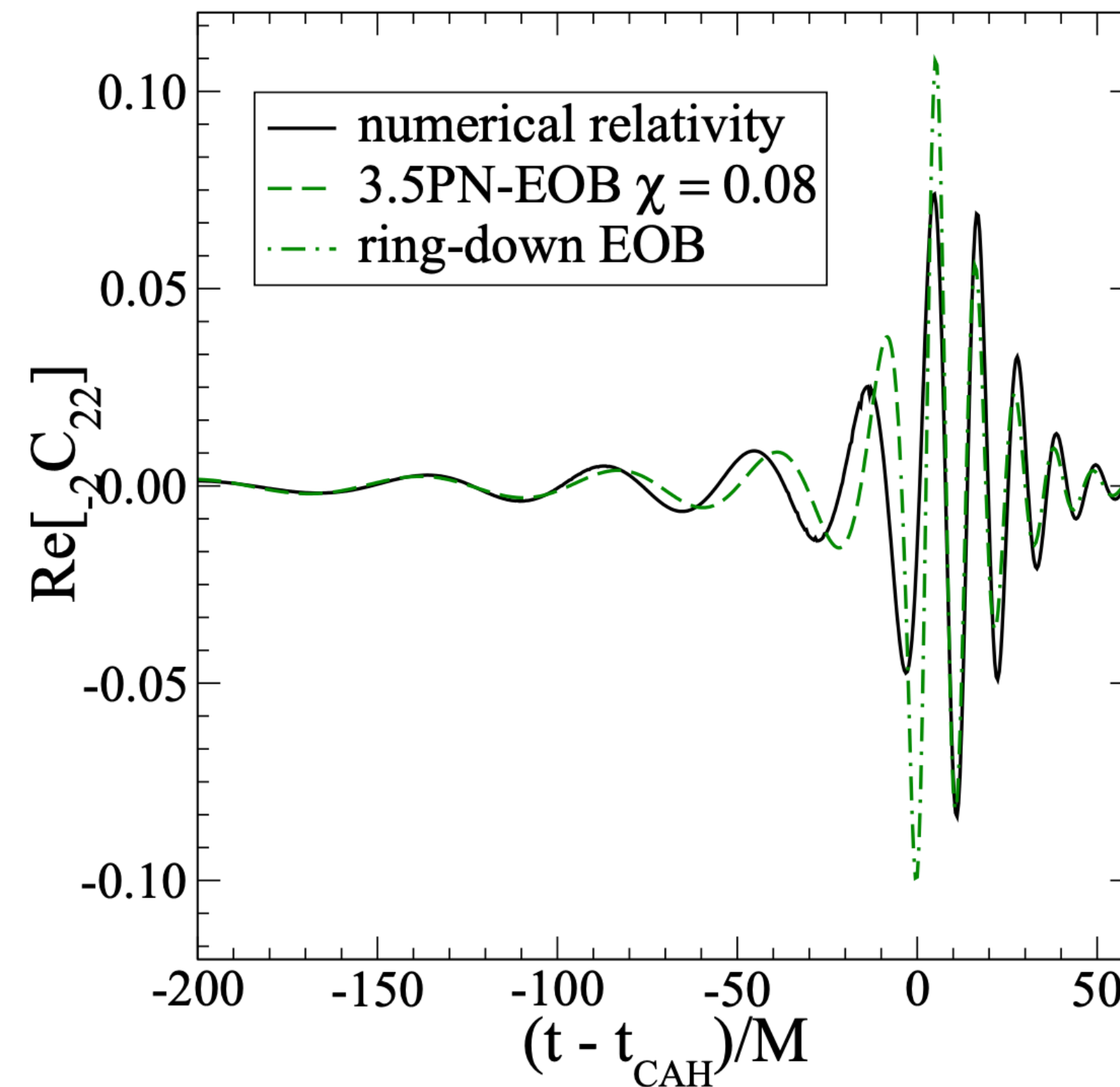
Merger of two BHs: Where are the non-linearities?



Even if our estimate of the waveform is admittedly rough, we think that it can play an important role for defining better filters for the search of signals in LIGO and VIRGO. In particular, two features of this waveform are striking: (i) the ‘plunge’ part of the waveform looks like a continuation of the inspiral part (this is because the orbital motion remains in fact quasi-circular), and (ii) the adiabatic waveform gets significantly out of phase with the exact waveform before crossing the LSO.

T. Damour & A. Buonanno (1998)

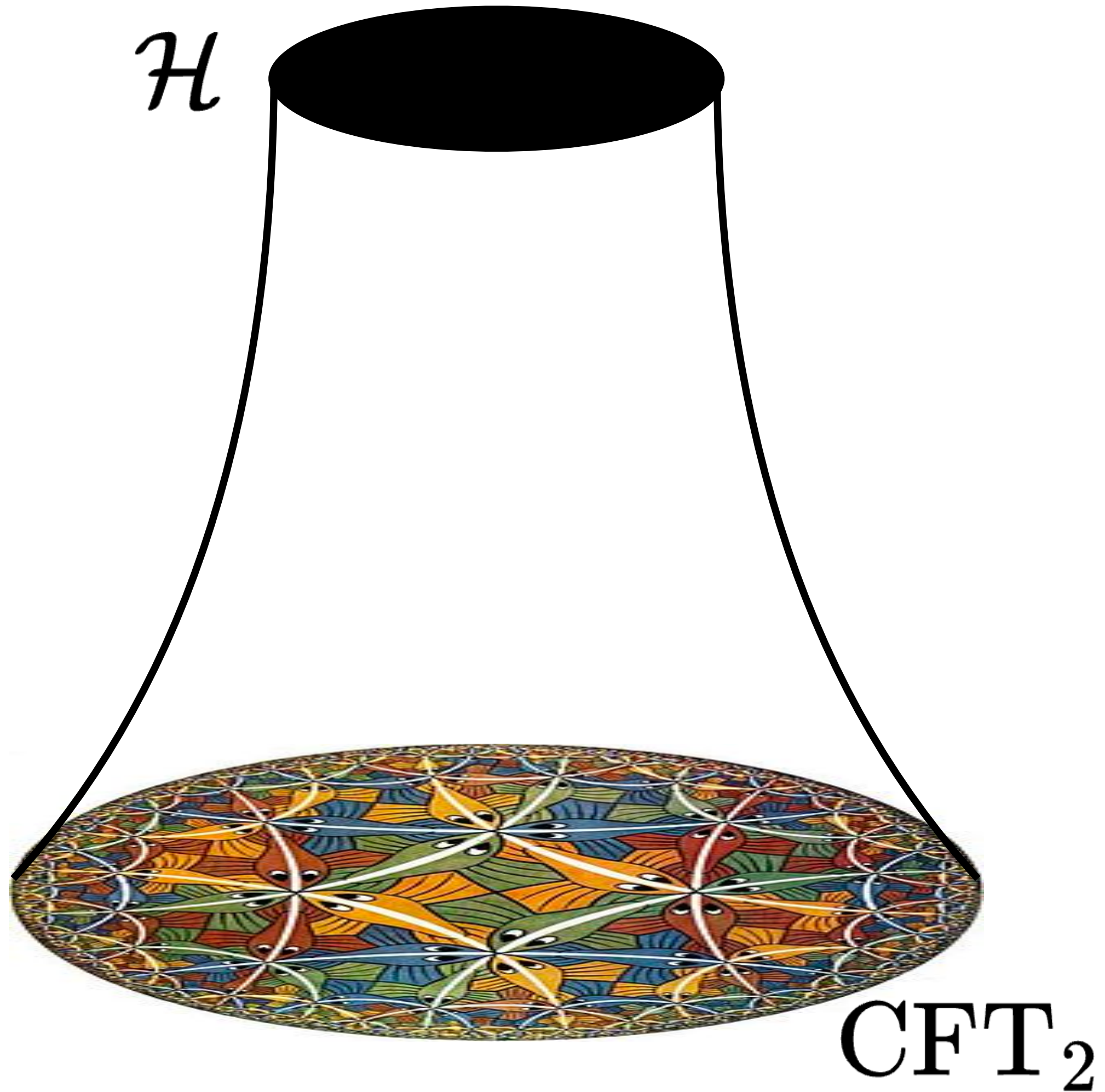
Merger of two BHs: Where are the non-linearities?



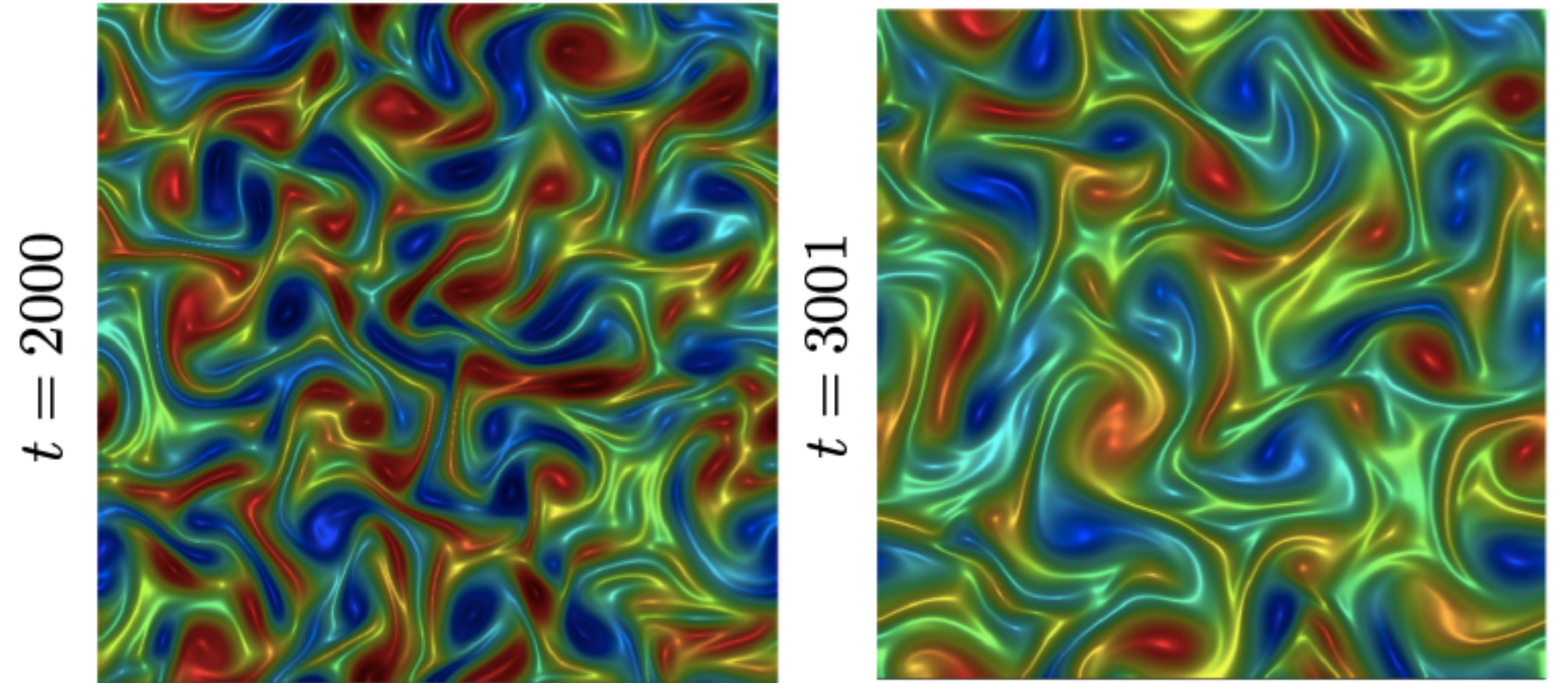
The analysis of the inspiral–merger–ring-down suggests that it should be possible to come up with good hybrid numerical/analytical waveforms, or even complete analytical waveforms where the full numerics guides how we need to patch the inspiral and ring-down waveforms together [...] Of course, all of this will be moot if the relative simplicity of this merger scenario breaks down for more interesting initial conditions

A. Buonanno, G. Cook & F. Pretorius (2006)

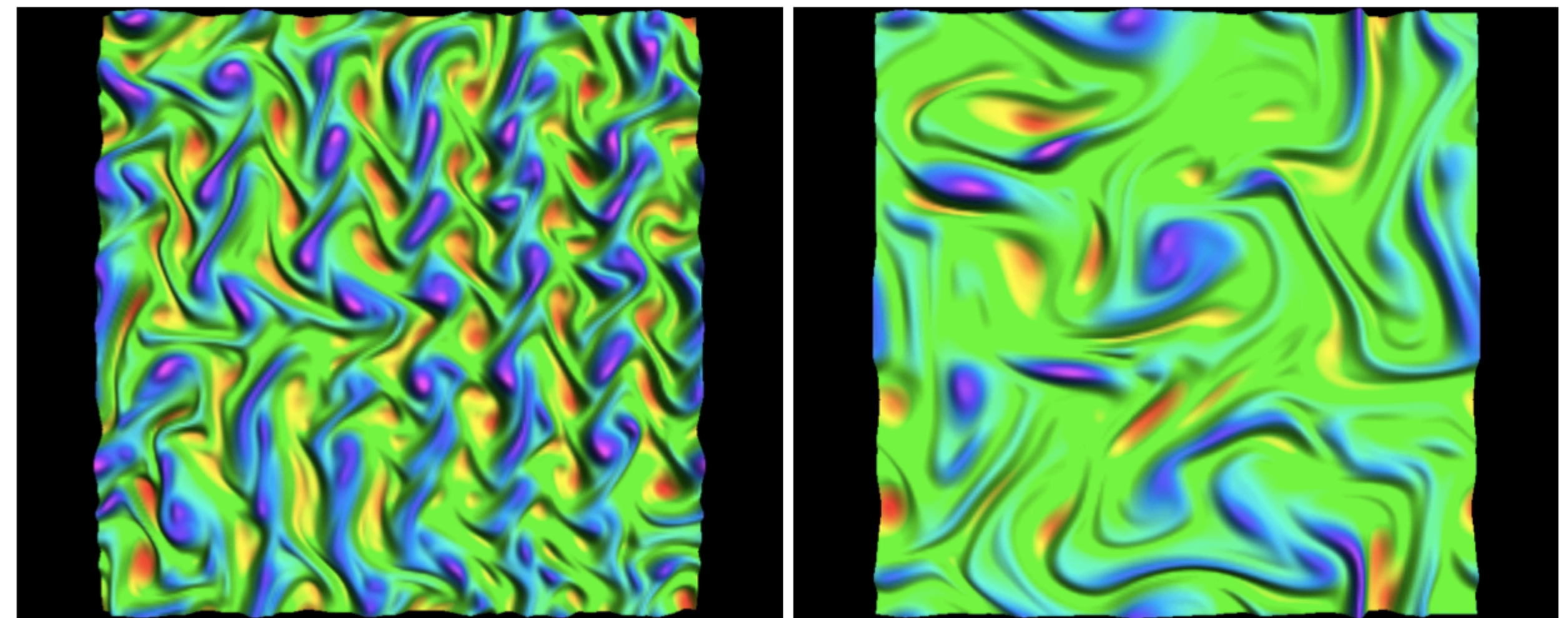
Is this due to Turbulence?



*Bhattacharyya+ (2007),
Van Raamsdonk (2008), Green+ (2014)*

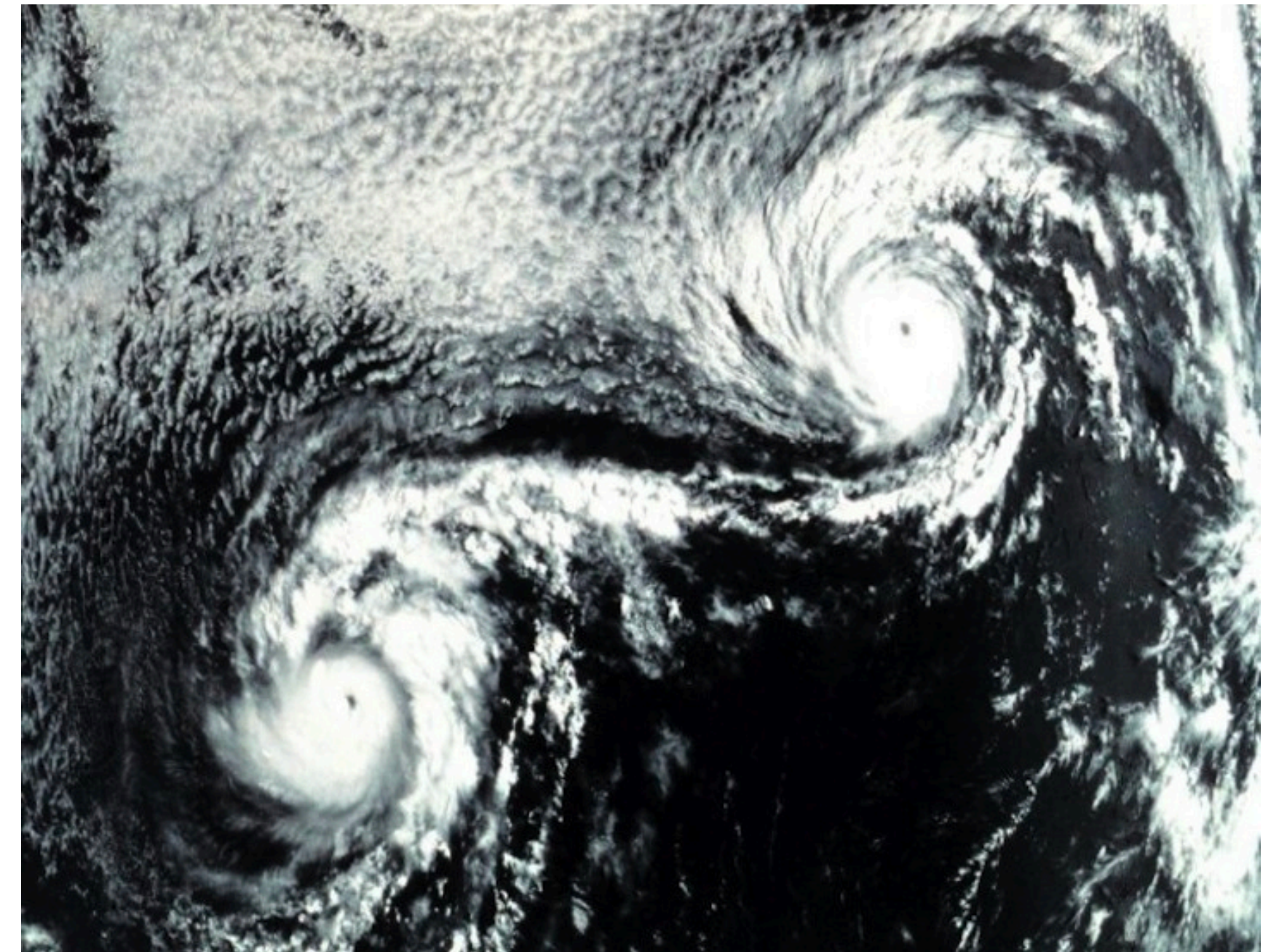
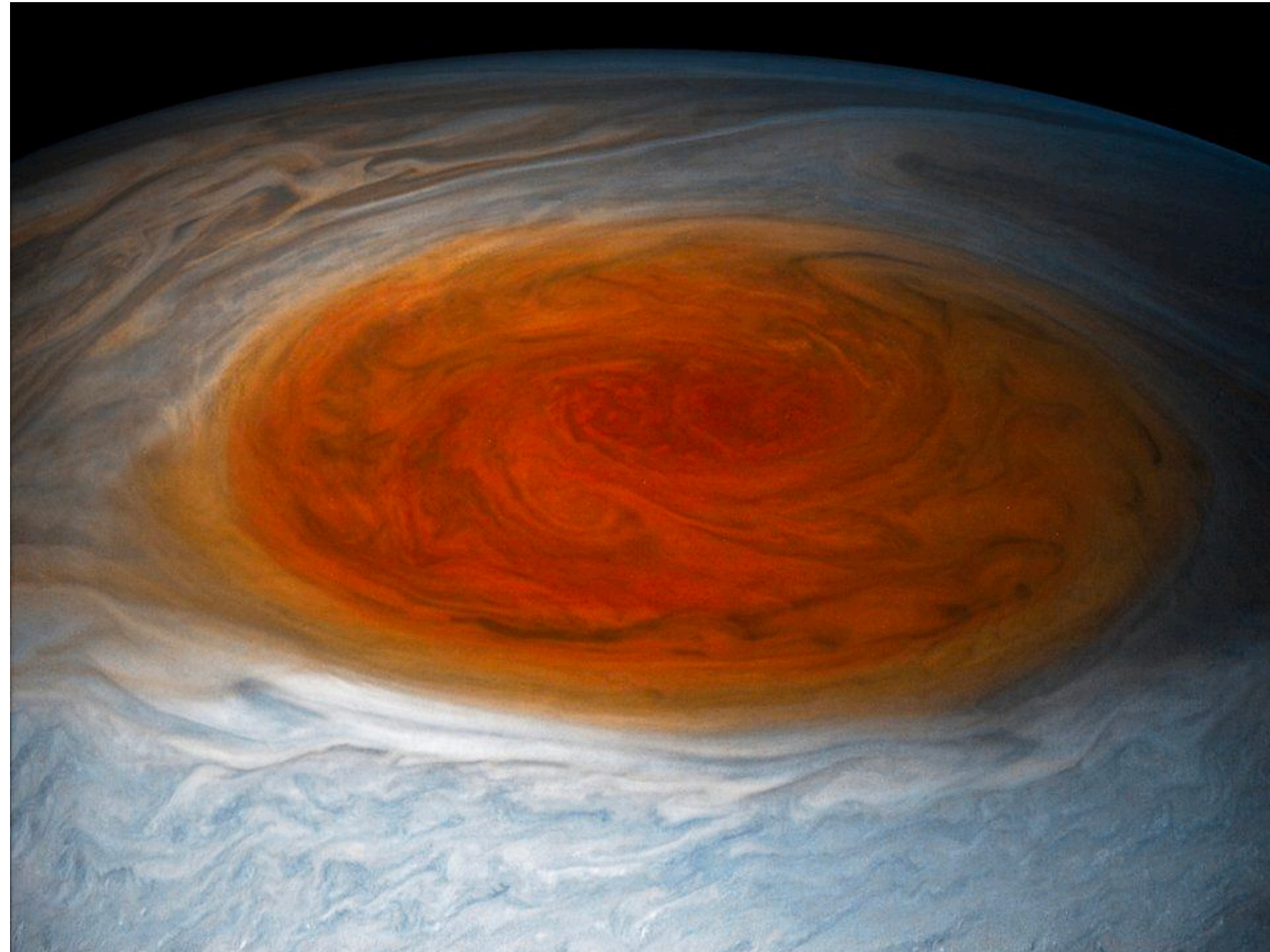


Adams+ (2013)



Carrasco+ (2013)

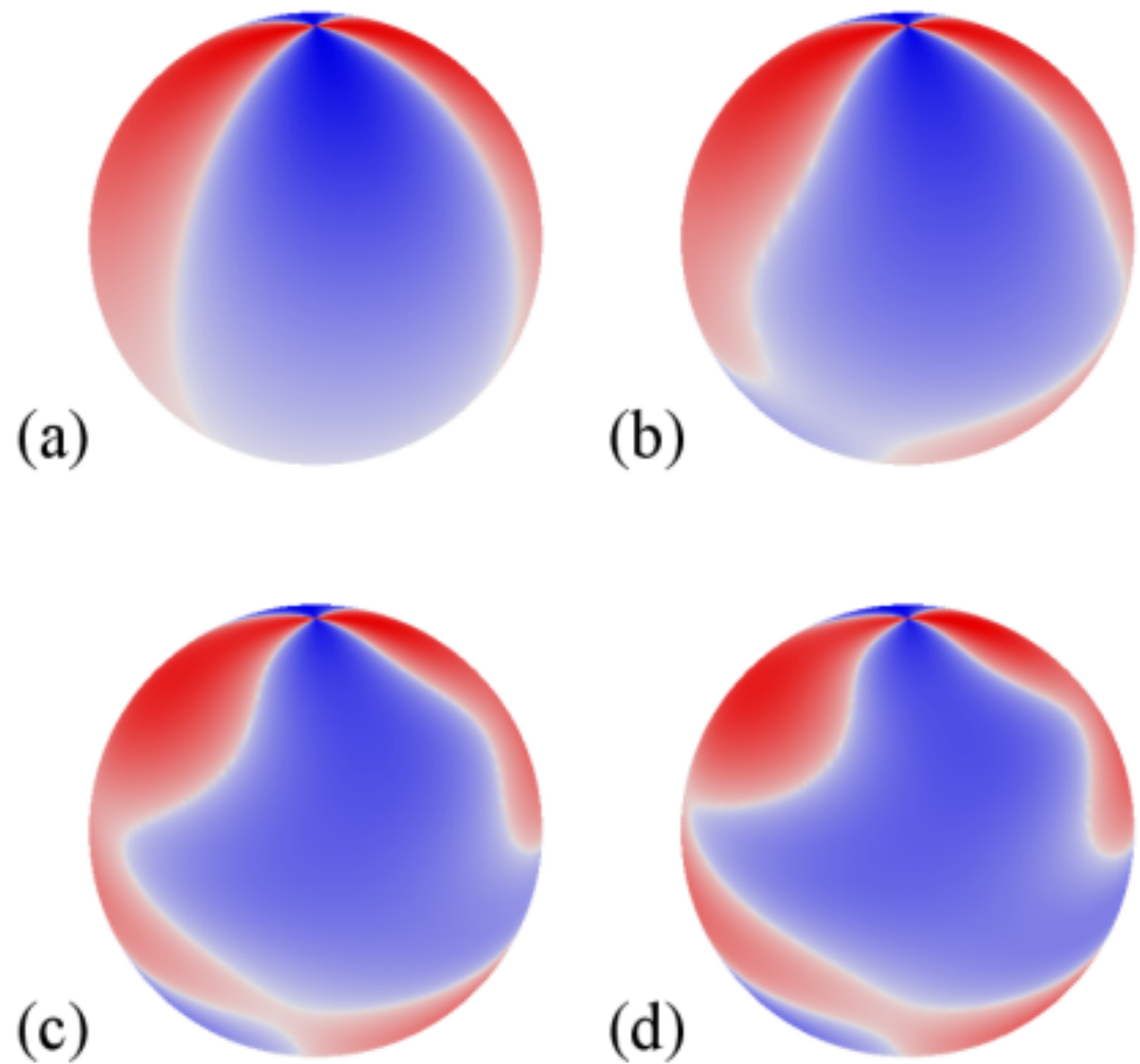
Is this due to Turbulence?



A step towards understanding the distinction between gravity in four and higher dimensions can be obtained by recalling that enstrophy conservation is what drives the hydrodynamics in 2+1 dimensions to exhibit the inverse cascade. One can then exploit the duality to translate the enstrophy into geometrical variables and to understand the implications of its conservation on the gravitational side. That is, the conservation of enstrophy in the fluid description implies a quasi-conserved quantity exists in the bulk gravity theory.

F. Carrasco+ (2013)

We do NOT live in AdS



Yang+ (2014)

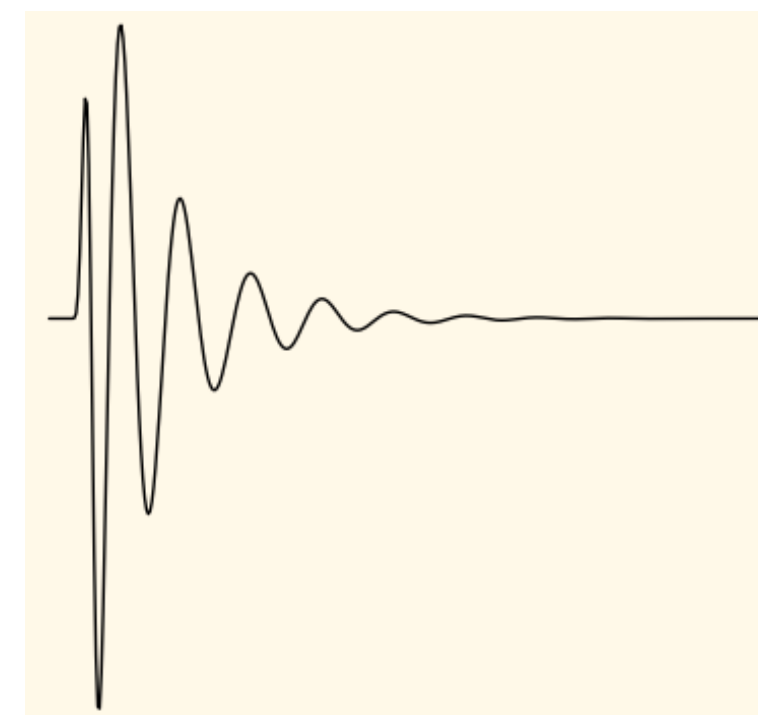
$$g_{ab} = \bar{g}_{ab}^{\text{Kerr}} + \varepsilon h_{ab} + \frac{1}{2} \varepsilon^2 h_{ab}^{(2)}$$

$$\mathcal{T}\Psi^{(1)} = 0$$

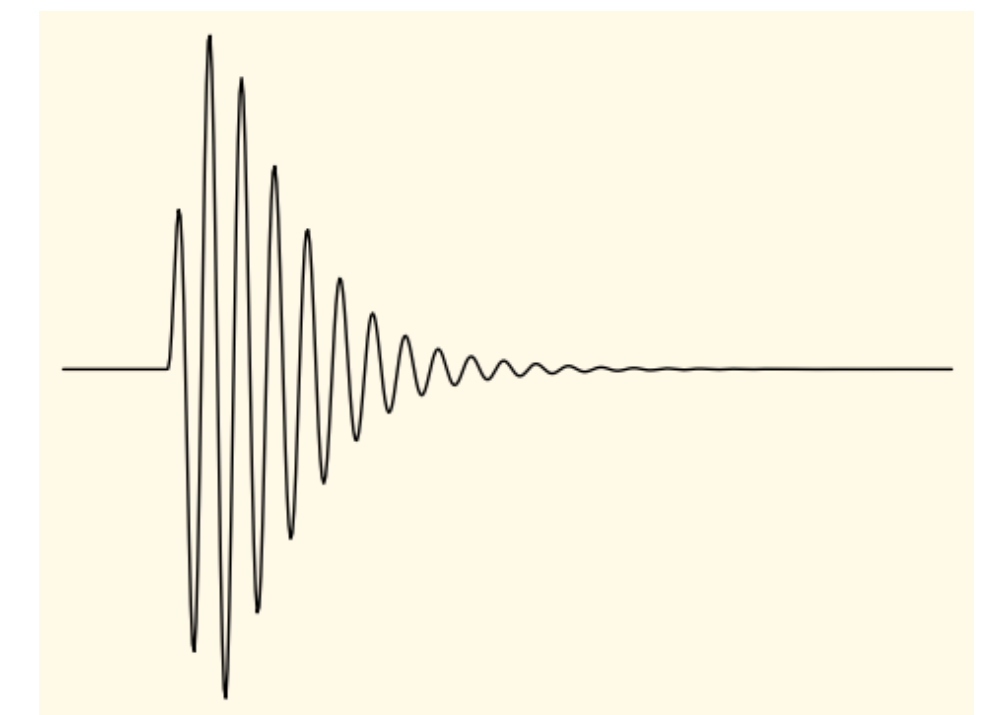
$$\mathcal{T}\Psi^{(2)} = \mathcal{S}[h_{ab}^{(1)}, h_{ab}^{(1)}]$$

$$\Psi^{(1)} \sim e^{i\omega t} \implies \Psi^{(2)} \sim e^{2i\omega t}$$

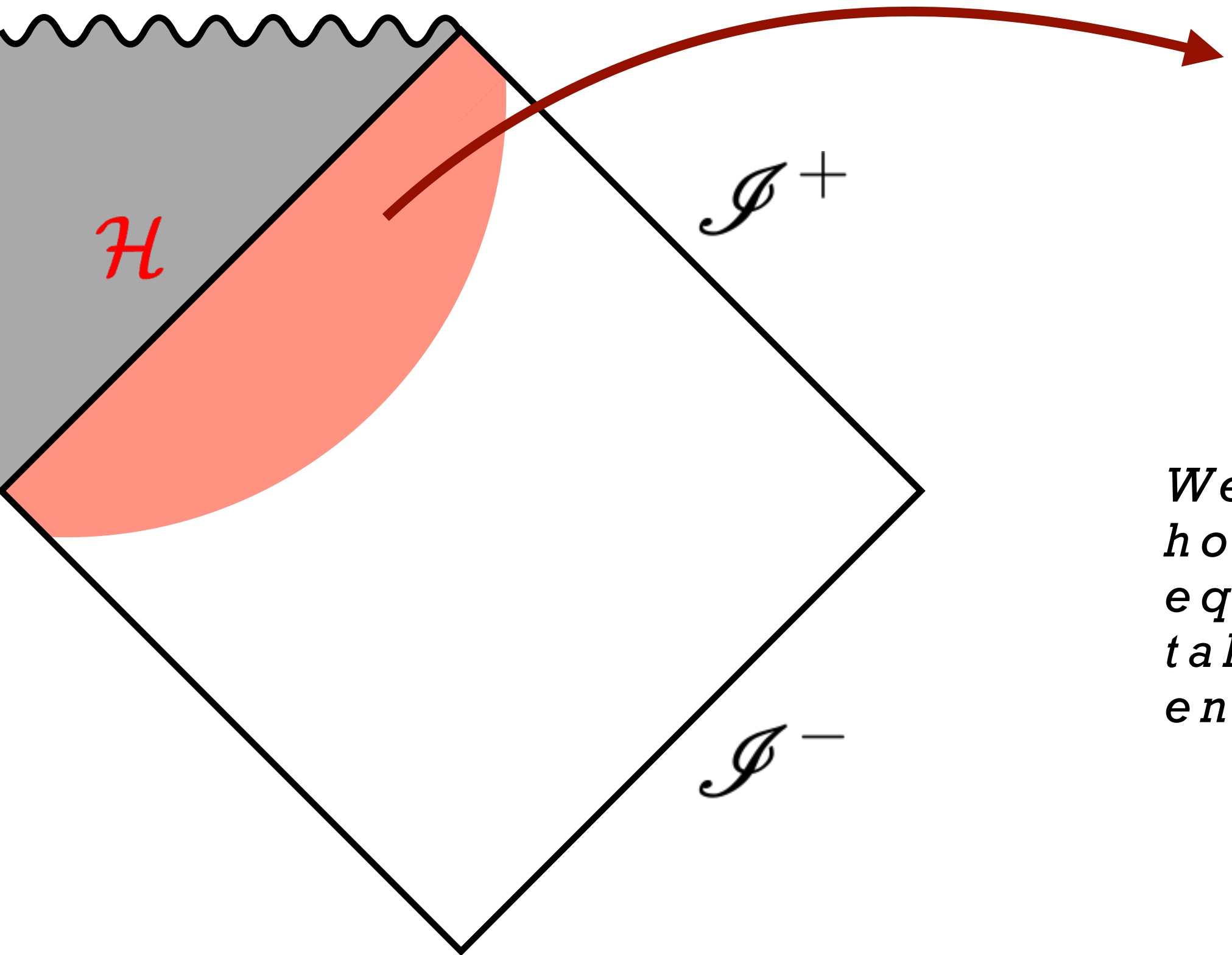
QNM



QQNM



BH-Carrollian Fluid Correspondance



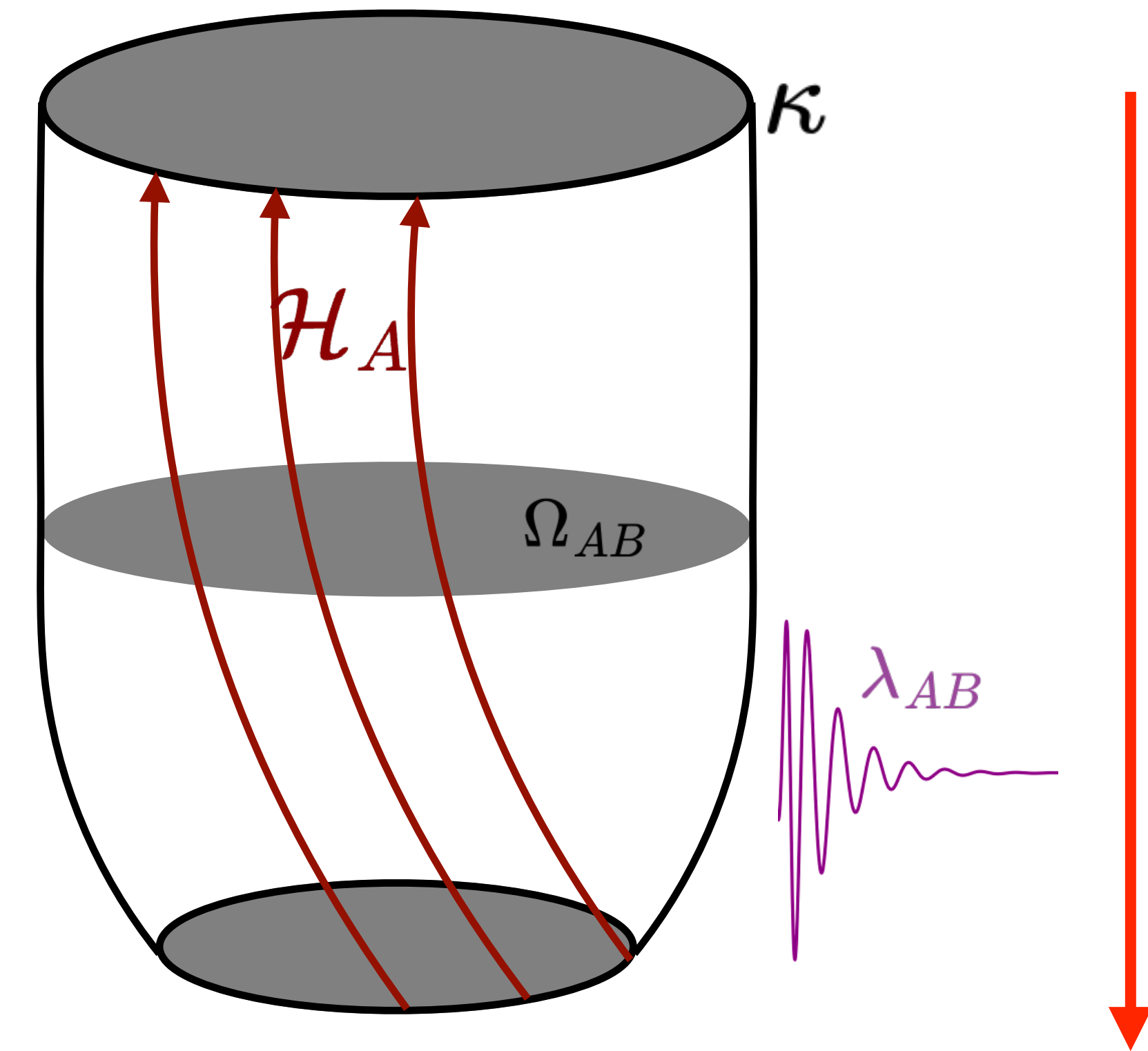
$$ds^2 = -2\rho\kappa dv^2 + 2dv d\rho + 2\rho U_A dv dx^A + (\Omega_{AB} - 2\rho\lambda_{AB}) dx^A dx^B + \mathcal{O}(\rho^2)$$

We prove that the laws governing the dynamics of a black hole horizon, the null Raychaudhuri and Damour equations, are Carrollian conservation laws obtained by taking the ultra-relativistic limit of the conservation of an energy-momentum tensor *L. Donnay & C. Marteau (2020)*

$$t_B^A = -(W_B^A - W\delta_B^A)$$

See also Ciambelli++, Jai-akson+, Chandrasekharan++

Perturbation theory at the horizon



$$\Omega_{AB} = q_{AB} + \epsilon c_{AB} + \epsilon^2 C_{AB} + \dots$$

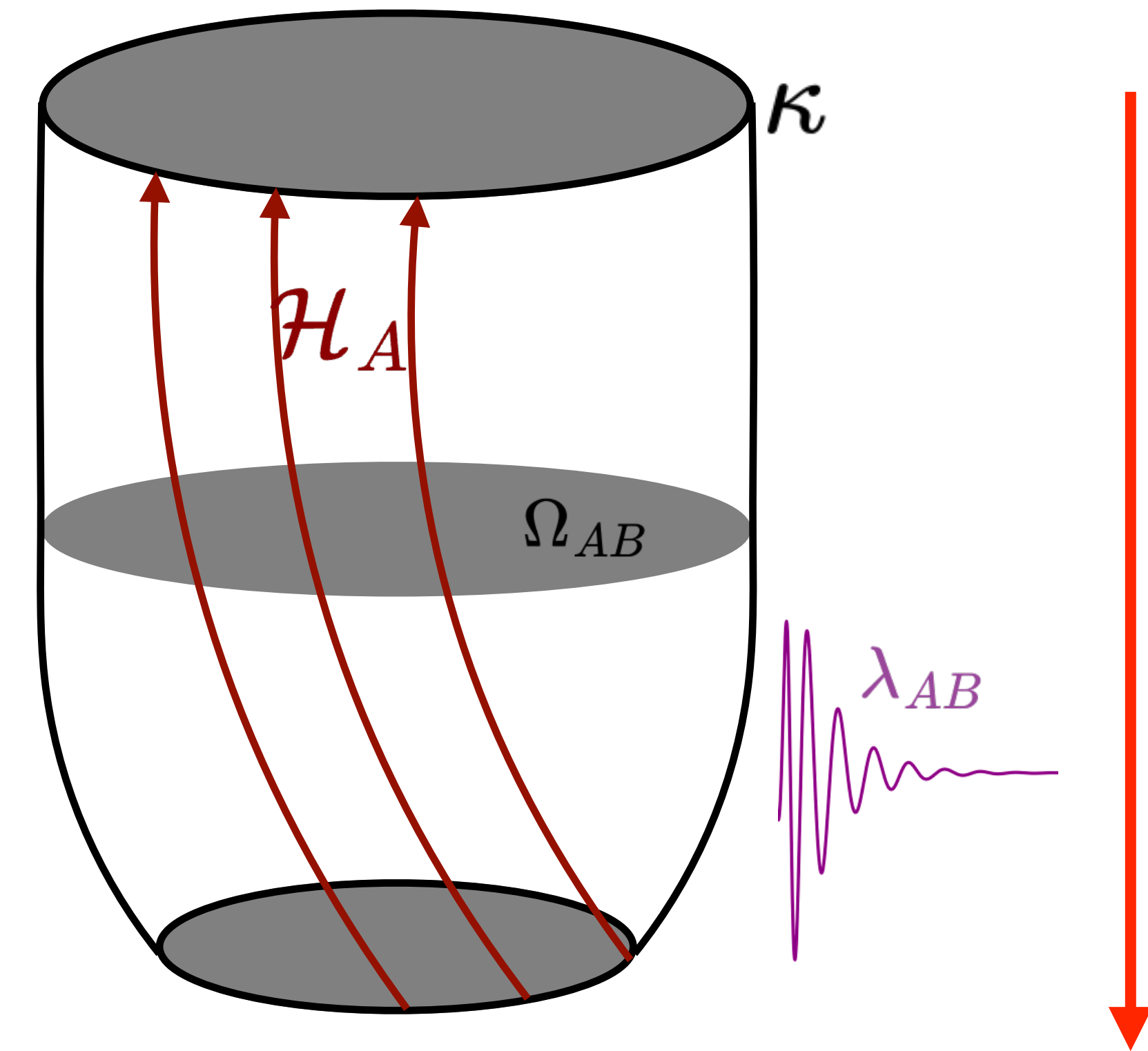
$$\dot{u} = \mathcal{L}[u] + \mathcal{S}^{(1)}$$

$$\dot{U} = \mathcal{L}[U] + \mathcal{Q}[u, u] + \mathcal{S}^{(2)}$$

$$u(v, x^A) = \{h_A, c_{AB}\},$$

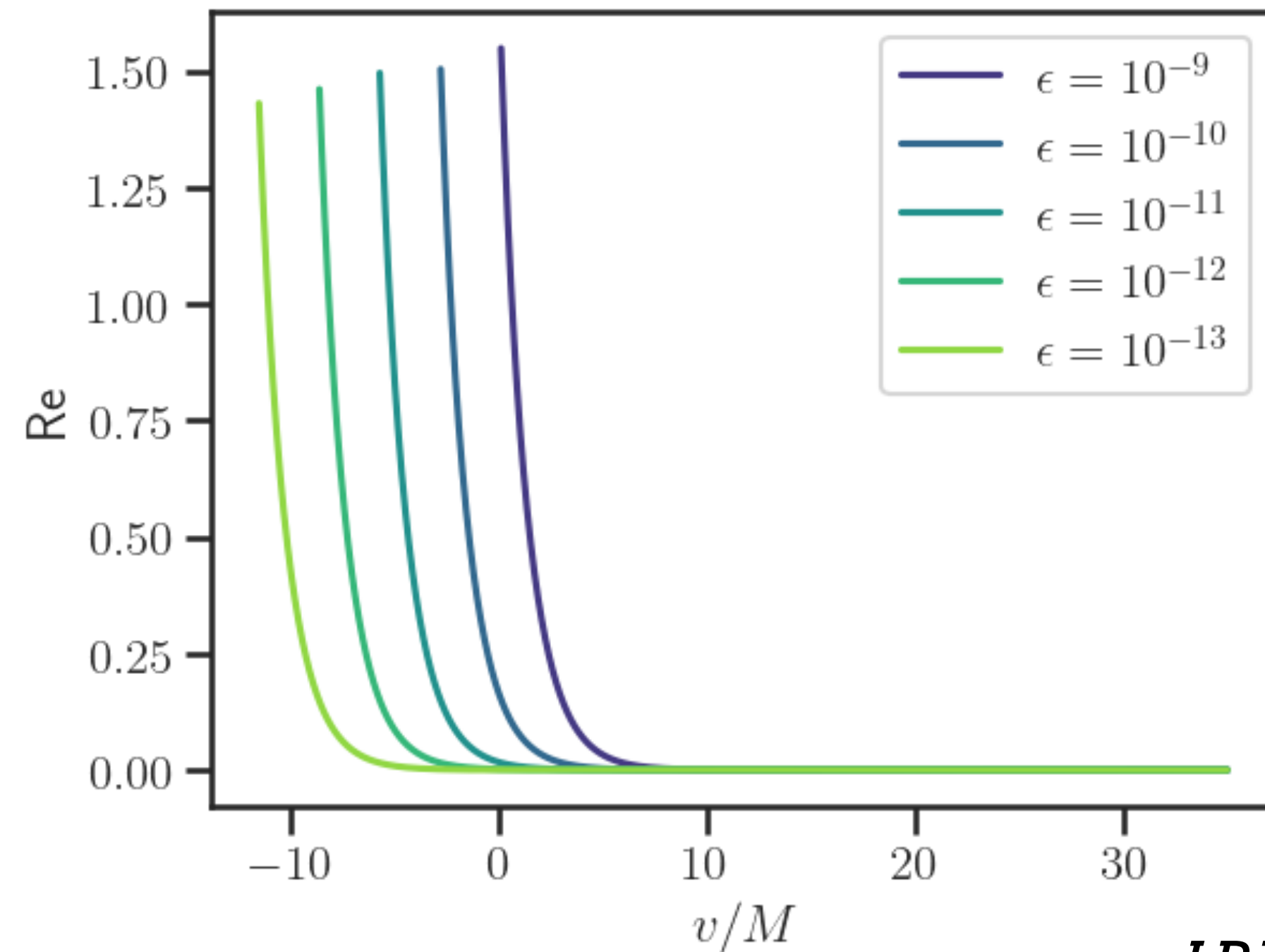
$$U(v, x^A) = \{\vartheta, H_A, C_{AB}\}$$

Perturbation theory at the horizon



$$\dot{H} \sim H''(1 + c) + H'(H + c')$$

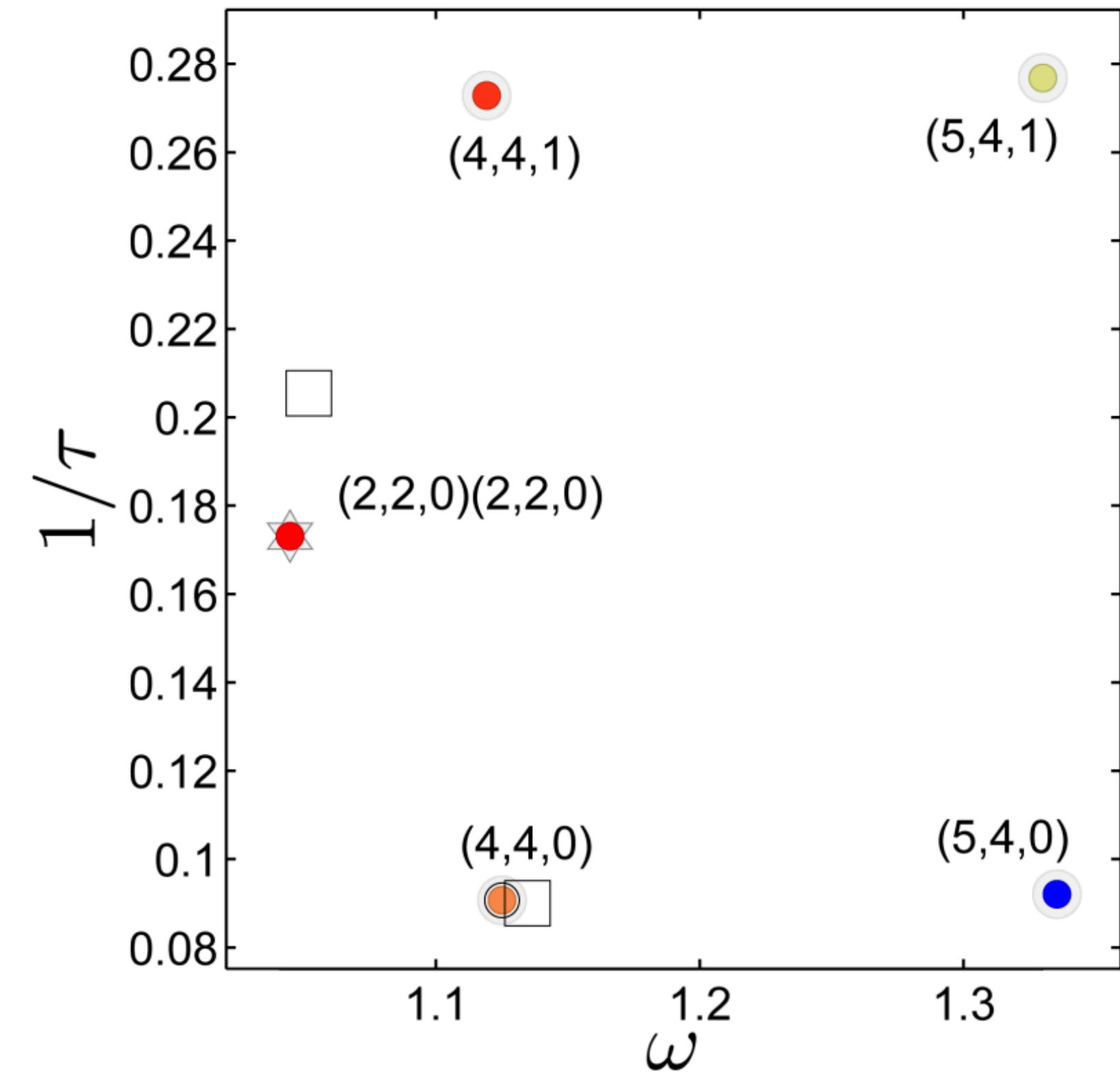
$$\text{Re} = \frac{HL/m + c}{1 + c}$$



JRY & Lehner (2022)

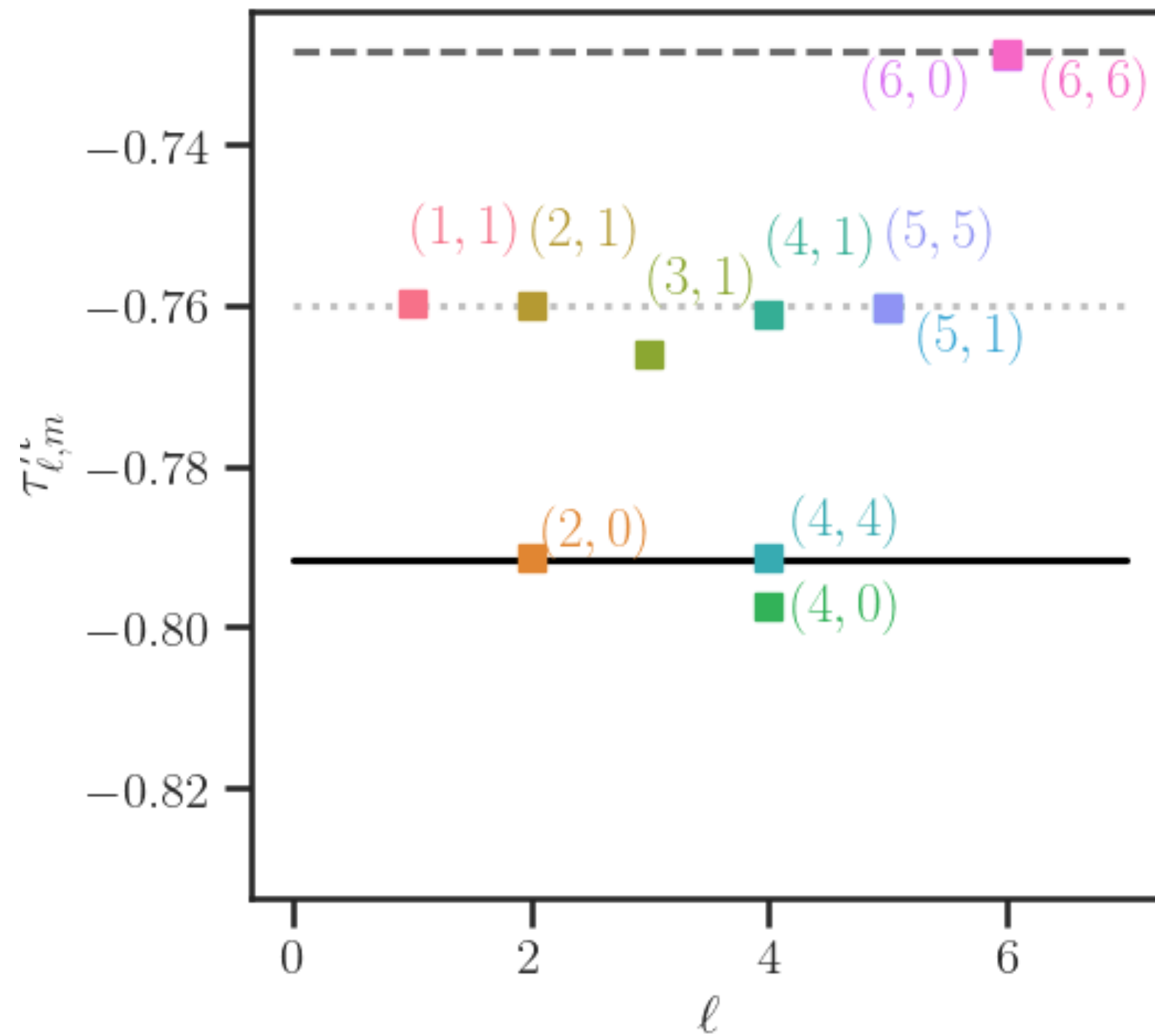
Quadratic Modes — Everywhere

Numerical Relativity (Scri)

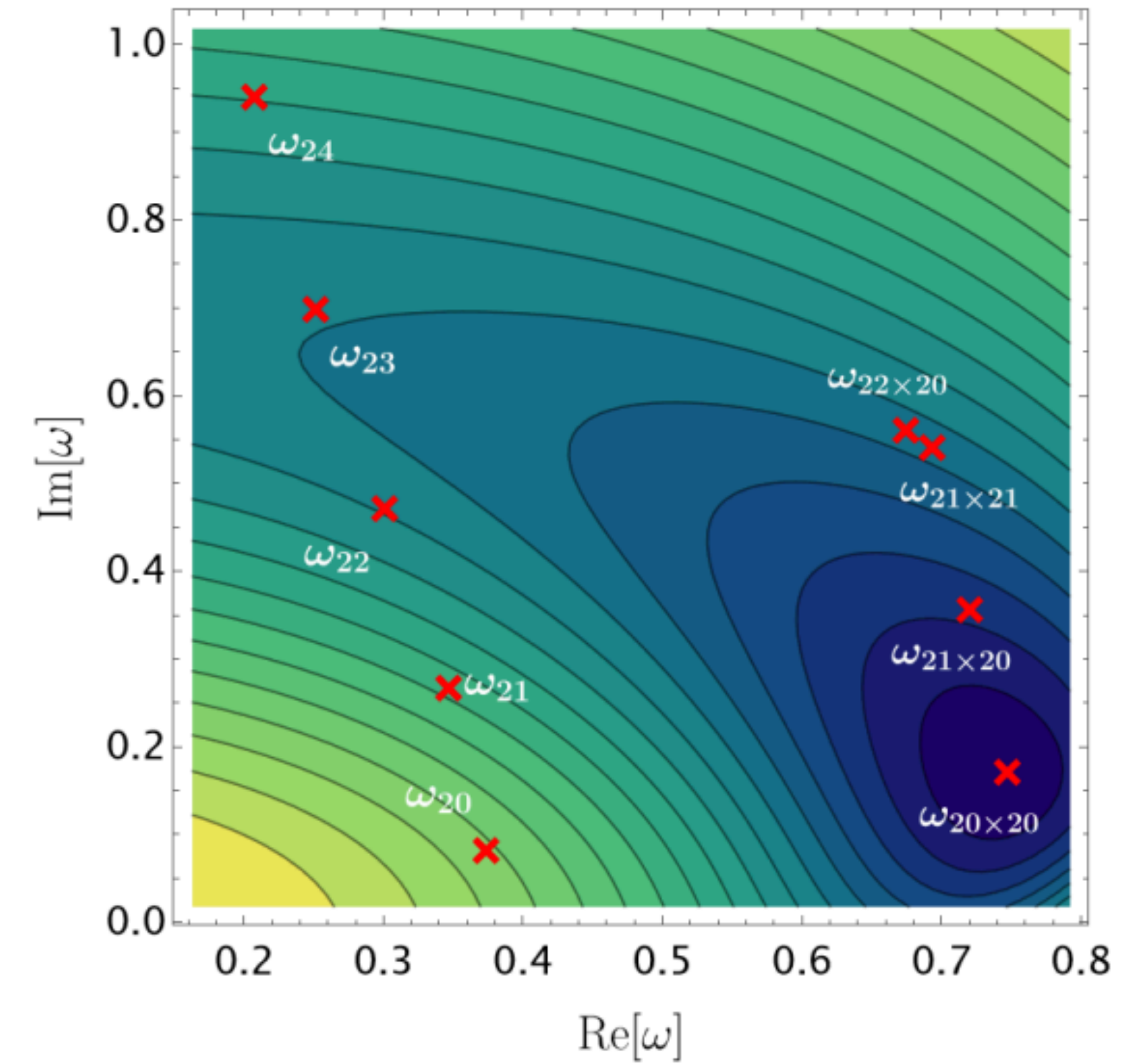


London+ (2014)

Horizon Perturbations



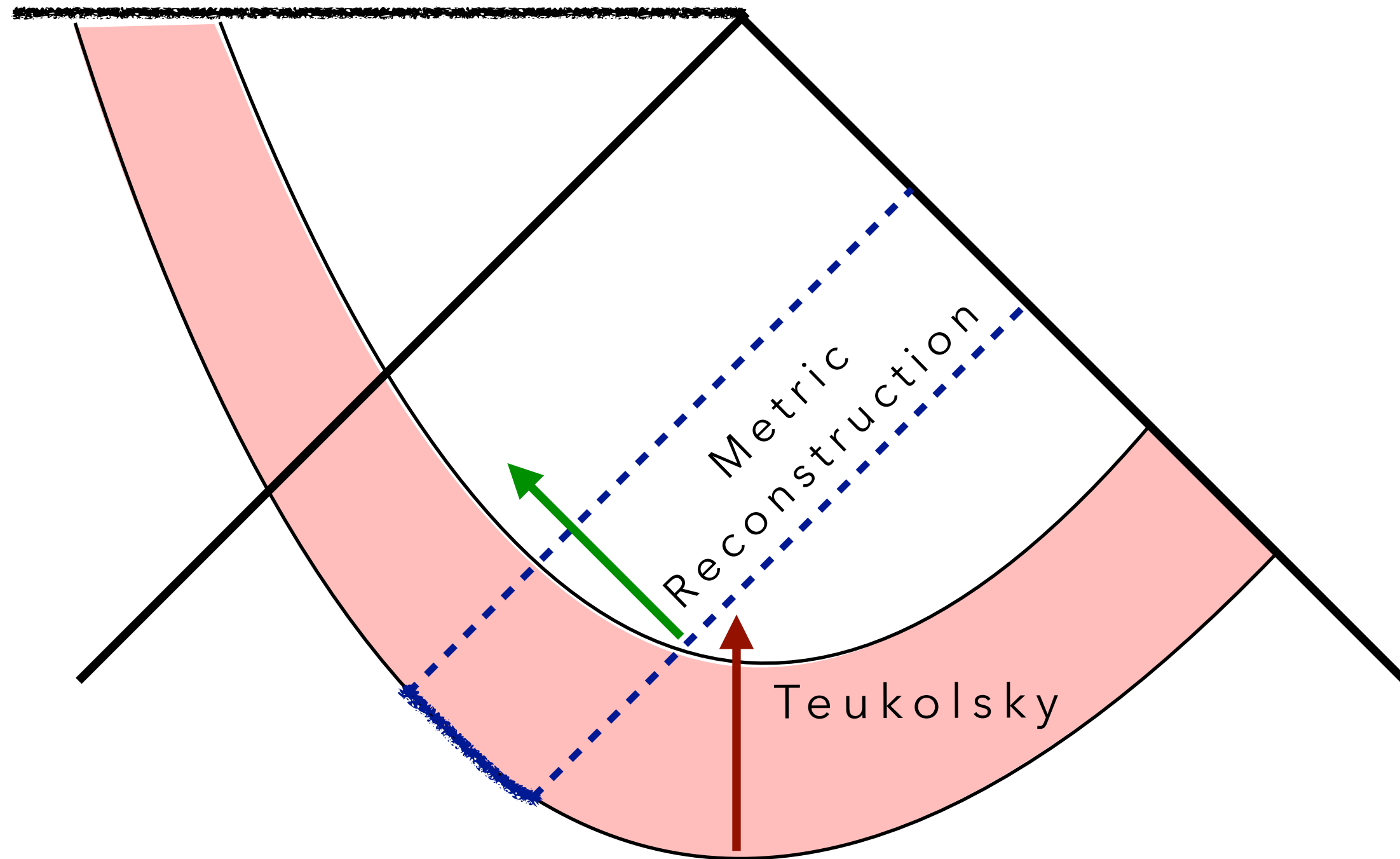
Numerical Relativity (AH)



Khera+ (2023)

See also Ioka++, Pazos++, Ripley+, Cheung+, Mitman+...

Spin dependence of non-linearities

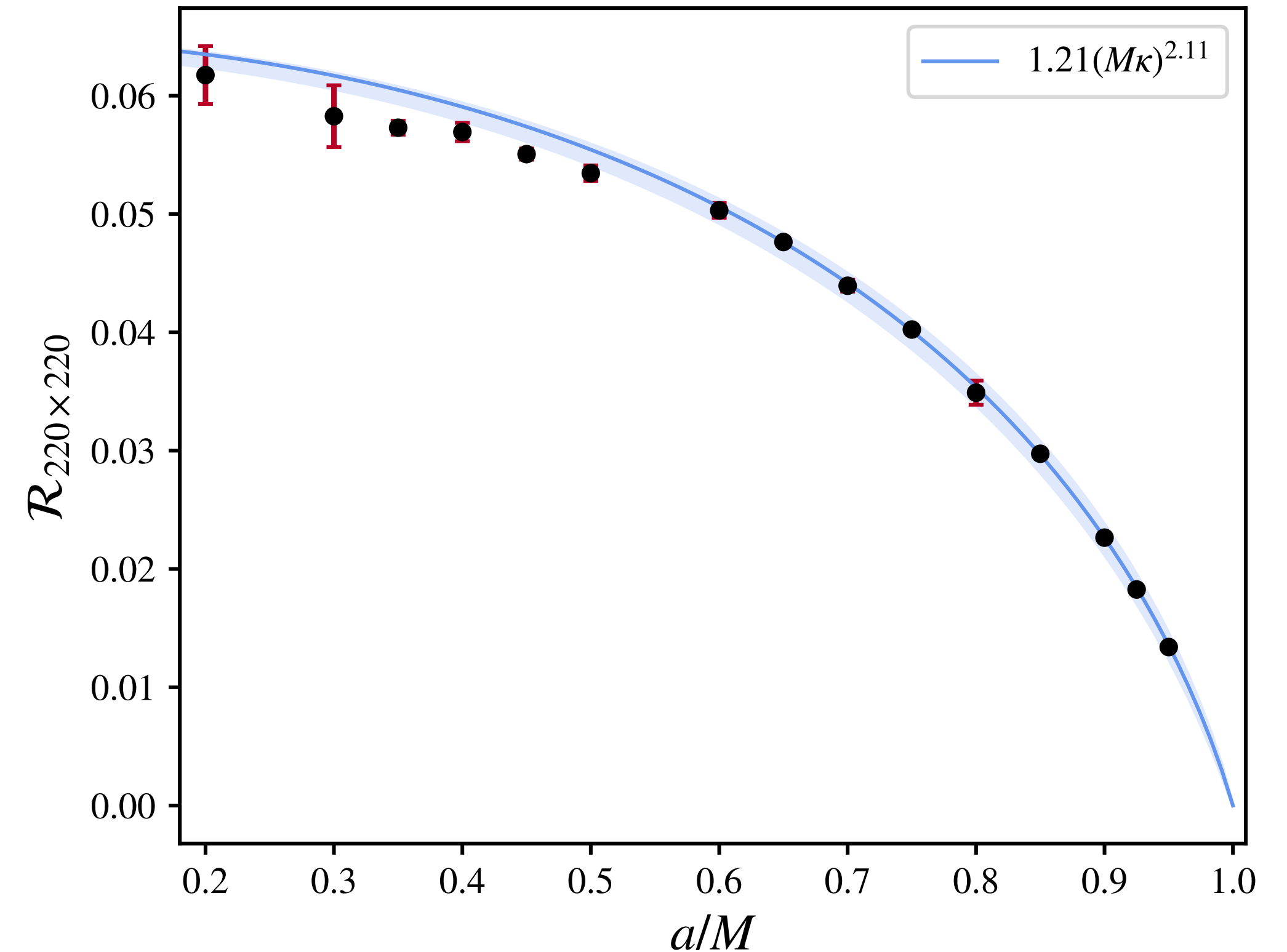


Why do non-linearities become apparently less important at high spins?

What happens for different angular channels?



$$\mathcal{R} = \frac{A_{220 \times 220}}{A_{220}^2}$$



Carrollian insights

What are the conserved quantities and balance laws of 2+1 Carrollian hydro?

Carrollian enstrophy? See Daniele's talk.

Dynamics of (small perturbations) of Carrollian (non-ideal) fluids

Do they admit mode solutions?

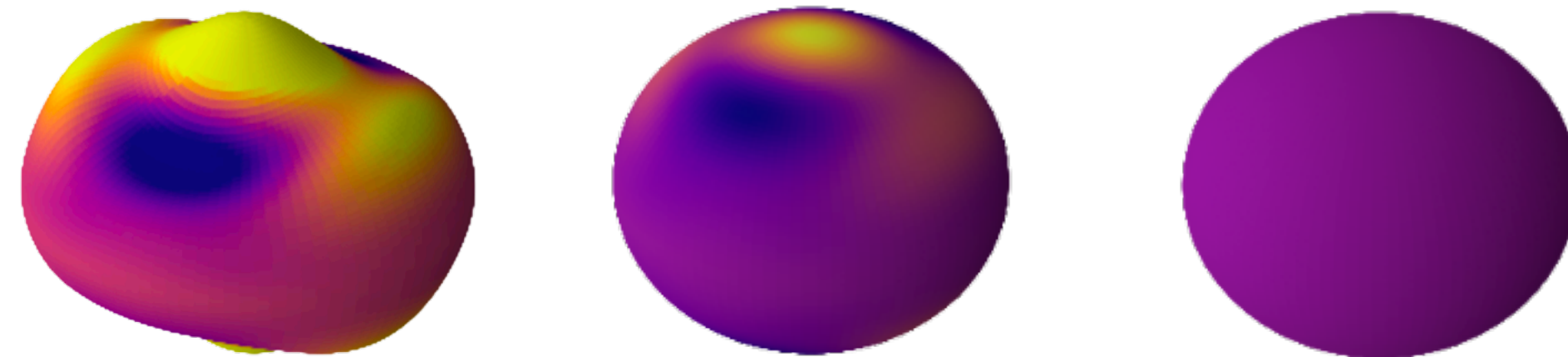
Beyond linear order: non-linear mode coupling?

Turbulence and Kolmogorov scalings?

Schwarzschild is not enough, need to understand near-extremal limit

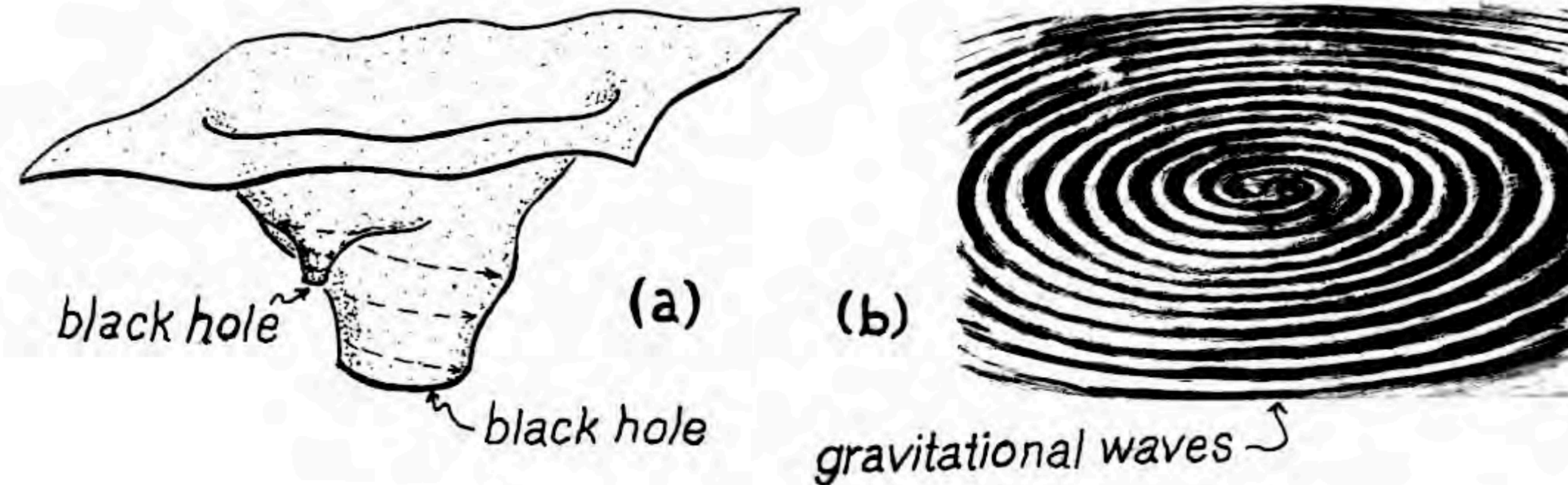
Connection to Carroll from AdS

Can Carrollian hydro predict QQNM spin-dependence?



*De Boer ++ ,
Ciambelli ++ ,
Armas & Have,
+++*

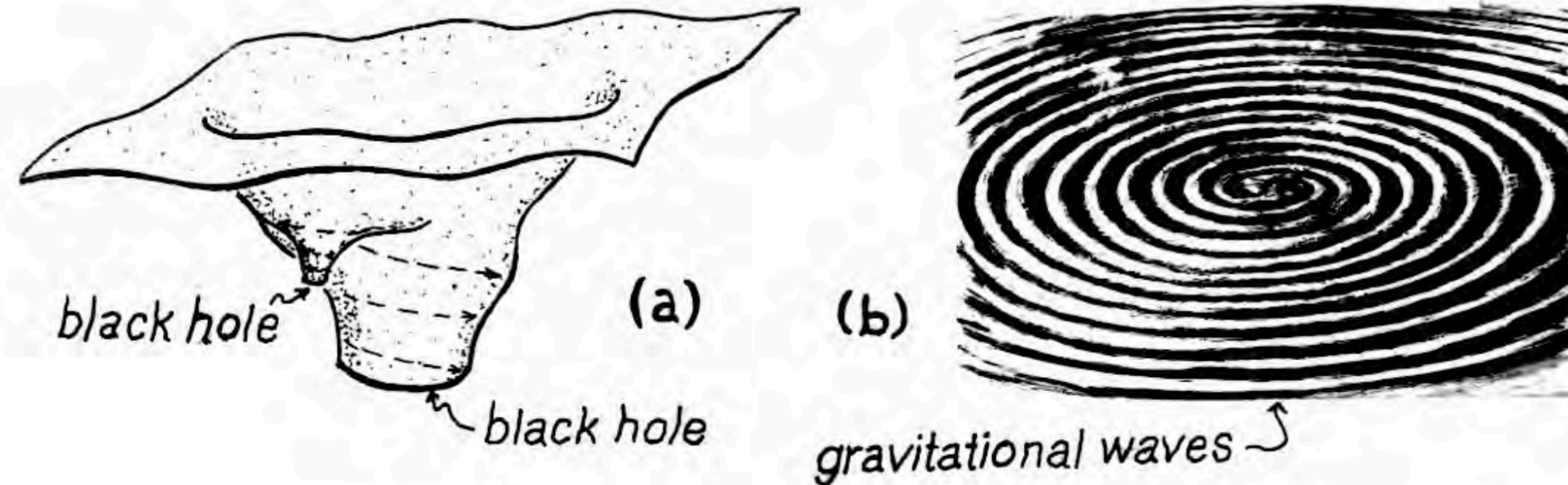
So, what did we learn?



Each hole is like a tornado. Spacetime whirls around its horizon like air whirling around a tornado's core. And as the holes orbit each other, their huge orbital angular momentum also drags spacetime into a whirling motion, so we have two tornados embedded in a third larger tornado and crashing together, and we want to know what happens when the tornados are made not from whirling air, but from a whirling spacetime warpage. To learn the answer will require a three-pronged attack: supercomputer simulations, gravitational-wave observations, and detailed comparison of the simulations and observations.

K. Thorne (2002)

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K. Thorne (2002)