A GEOMETRIC PERSPECTIVE ON FLAT SUPERGRAVITY WITH BOUNDARY AND ITS ASYMPTOTIC SYMMETRIES

Based on Concha, R., Rodríguez, JHEP 01 (2019) 192 and Andrianopoli, R., Universe 7 (2021) 12, 463

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3rd Carroll Workshop, Thessaloniki, 2-6 October 2023

Boundary problem in (super)gravity

"Bdy problem": $\partial\mathcal{M} \to$ Well-def. variational principle, background-independent conserved charges, finiteness?

Gravity and supergravity Lagrangians in the presence of a bdy studied from the early seventies on...

- York, Gibbons, Hawking (1972, 1977): Need of adding a bdy term to the gravity action such as to implement Dirichlet b.c. for the metric field $\to$ Well-def. variational principle
- Horava, Witten (1996): Bdy terms to cancel gauge and grav. anomalies in the Horava-Witten model in $11D$
- AdS/CFT (1997): Bulk fields (metric) diverge at the bdy $\to$ Cured by counterterms at the bdy (Holo. ren.)

General lesson: For $\partial\mathcal{M} \neq 0$, the bulk theory needs to be supplemented by bdy terms

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Aros, Contreras, Olea, Troncoso, Zanelli, PRL 84 (2000) 1647-1650

Gravity: Consider $D = 4$ EH action + negative Λ (AdS gravity) + EGB (topological term)

$$\mathcal{L}_{\text{EGB}} = \mathcal{R}^{ab} \wedge \mathcal{R}^{cd} \epsilon_{abcd} = \mathrm{d}\left(\omega^{ab} \wedge \mathcal{R}^{cd} + \omega^{a}{}_{\ell} \wedge \omega^{\ell b} \wedge \omega^{cd}\right) \epsilon_{abcd} \,, \quad \mathcal{R}^{ab} \equiv \mathrm{d}\omega^{ab} + \omega^{a}{}_{c} \wedge \omega^{cb}$$

$\Rightarrow$ Background-independent def. of Noether charges, without the need of explicitly imposing Dirichlet b.c.

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$\rightsquigarrow \mathcal{L}_{\text{EGB}}$ regularizes the action and the related conserved charges; Reproduces regularization given by holographic regularization (counterterms) $\to$ "Topological regularization"; Full action: MacDowell-Mansouri form

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- Theory given in terms of superfields 1-forms $\mu^{\mathcal{A}}$ def. in superspace $\mathcal{M}_{4|4\mathcal{N}}$ (we will take $\mathcal{N} = 1$, $D = 4$)
- Superspace is spanned by the supervielbein $\{V^a, \psi\}$ (dual to $\{P_a, Q\}$)

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- Action:

$$\mathcal{S} = \int_{\mathcal{M}_4 \subset \mathcal{M}_{4|4\mathcal{N}}} \mathcal{L}[\mu^{\mathcal{A}}]$$

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$$\mathcal{S} = \int_{\mathcal{M}_4 \subset \mathcal{M}_{4|4\mathcal{N}}} \mathcal{L}[\mu^{\mathcal{A}}]$$

- Susy transformations on spacetime are diffeomorphisms in the fermionic directions of superspace:

$$\text{Susy:} \quad \mathcal{M}_{4|4\mathcal{N}}(x,\theta) \rightarrow \mathcal{M}_{4|4\mathcal{N}}(x,\theta+\delta\theta)$$

$\Rightarrow$ Can be described in terms of Lie derivative ℓ_ϵ with fermionic parameter $\epsilon(x,\theta)$ (susy parameter):

$$\ell_\epsilon = \imath_\epsilon \mathrm{d} + \mathrm{d}\imath_\epsilon \,, \qquad \imath_\epsilon(\psi) = \epsilon \,,\; \imath_\epsilon(\mu^{\mathcal{A}}) = 0 \text{ for } \mu^{\mathcal{A}} \neq \psi$$

Sugra theory $\rightarrow$ Invariance of the action under susy transformations: $\delta_\epsilon \mathcal{S} \equiv \int_{\mathcal{M}_4} \delta_\epsilon \mathcal{L} = 0$

- Susy inv. of the superspace Lagrangian:

$$\delta_\epsilon \mathcal{L} = \ell_\epsilon \mathcal{L} = \imath_\epsilon(\mathrm{d}\mathcal{L}) + \mathrm{d}(\imath_\epsilon \mathcal{L}) = 0$$

$\Rightarrow$ Necessary condition for a susy-invariant sugra Lagrangian in superspace:

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- Susy inv. of the action (requires weaker condition on $\mathcal{L}_{\text{bulk}}$):

$$\delta_\epsilon \mathcal{S} = \int_{\mathcal{M}_4} \mathrm{d}(\imath_\epsilon \mathcal{L}_{\text{bulk}}) = \int_{\partial\mathcal{M}_4} \imath_\epsilon \mathcal{L}_{\text{bulk}} = 0 \quad \Rightarrow \quad \imath_\epsilon \mathcal{L}_{\text{bulk}}|_{\partial\mathcal{M}_4} = \mathrm{d}\phi$$

In general. not satisfied by $\mathcal{L}_{\text{bulk}}$ in the presence of non-trivial b.c. on $\partial\mathcal{M}_4 \neq 0$
$\Rightarrow$ Susy inv. requires to add bdy terms $\rightarrow$ Consider the full Lagrangian

$$\mathcal{L}_{\text{full}} = \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\text{bdy}}\,, \quad \mathcal{L}_{\text{bdy}} = \mathrm{d}\mathcal{B}_{(3)} \quad \Rightarrow \quad \imath_\epsilon(\mathrm{d}\mathcal{L}_{\text{full}}) = 0 \quad \text{and} \quad \imath_\epsilon \mathcal{L}_{\text{full}}|_{\partial\mathcal{M}_4} = 0$$

CASE OF PURE SUGRA WITH NEGATIVE COSMOLOGICAL CONSTANT

Andrianopoli, D'Auria, JHEP 08 (2014), 012

Consider pure $\mathcal{N}=1$, $D=4$ sugra with negative cosmological constant $\Lambda=-3/\ell^2$:

$$\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}} = \frac{1}{4}\mathcal{R}^{ab}V^cV^d\epsilon_{abcd} - \bar{\psi}\gamma_5\gamma_a\rho V^a - \frac{\mathrm{i}}{2\ell}\bar{\psi}\gamma_5\gamma_{ab}\psi V^aV^b - \frac{1}{8\ell^2}V^aV^bV^cV^d\epsilon_{abcd}$$

Bulk: $\imath_\epsilon(\mathrm{d}\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}})=0$; But when background spacetime has a non-trivial bdy: $\imath_\epsilon\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}}|_{\partial\mathcal{M}_4}\neq\mathrm{d}\phi \;\Rightarrow\; \delta_\epsilon\mathcal{S}_{\text{bulk}}\neq 0$

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$\Rightarrow$ Add $\mathcal{L}^{\mathcal{N}=1}_{\text{bdy}} = \mathrm{d}\mathcal{B}_{(3)}$ to restore susy inv. (do not alter $\mathrm{d}\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}}$) $\rightarrow$ Possible bdy terms (parity, Lorentz inv.):

$$\mathrm{d}\left(\omega^{ab}\wedge\mathcal{R}^{cd} + \omega^a{}_\ell\wedge\omega^{\ell b}\wedge\omega^{cd}\right)\epsilon_{abcd} = \mathcal{R}^{ab}\wedge\mathcal{R}^{cd}\epsilon_{abcd}\,, \quad \mathrm{d}\left(\bar{\psi}\wedge\gamma_5\rho\right) = \bar{\rho}\wedge\gamma_5\rho - \frac{1}{4}\mathcal{R}^{ab}\wedge\bar{\psi}\gamma_5\gamma_{ab}\wedge\psi$$

$\Rightarrow$ Modify the Lagrangian: $\mathcal{L}^{\mathcal{N}=1}_{\text{bulk}} \rightarrow \mathcal{L}^{\mathcal{N}=1}_{\text{full}} \equiv \mathcal{L}^{\mathcal{N}=1}_{\text{bulk}} + \mathcal{L}^{\mathcal{N}=1}_{\text{bdy}}$, with

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Bdy contributions in the variational principle for $\mathcal{L}_{\text{full}}^{\mathcal{N}=1}$ $\Rightarrow$ Constraint on the supercurvatures at the bdy

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Bdy contributions in the variational principle for $\mathcal{L}^{\mathcal{N}=1}_{\text{full}}$ $\Rightarrow$ Constraint on the supercurvatures at the bdy

α, β such as susy inv. (& $\mathcal{L}^{\mathcal{N}=1}_{\text{bdy}}$ susy EGB) $\Rightarrow$ $\mathcal{L}^{\mathcal{N}=1}_{\text{full}}$ MacDowell-Mansouri in terms of $\mathrm{OSp}(1|4)$-covariant supercurvatures and vanishing of $\mathrm{OSp}(1|4)$ supercurv. at $\partial\mathcal{M}_4$ $\rightarrow$ Bdy enjoys global inv. under $\mathrm{OSp}(1|4)$

Case of "flat" sugra with boundary

MacDowell-Mansouri Lagrangian:

$$\mathcal{L}_{\text{full}}^{\mathcal{N}=1} = -\frac{\ell^2}{8}\mathbf{R}^{ab} \wedge \mathbf{R}^{cd}\epsilon_{abcd} - \mathrm{i}\ell\bar{\boldsymbol{\rho}}\gamma_5 \wedge \boldsymbol{\rho}$$

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- Case where the bdy is placed asymptotically at infinity: BMS group emerges as asymptotic symmetry
- $\exists$ a geometric $\mathcal{L}_{\text{bdy}}$ exhibiting super-BMS symmetry?
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Start from

$$\mathcal{L}_{\text{bulk}}^{\text{flat}} = \frac{1}{4}\mathcal{R}^{ab}V^c V^d\epsilon_{abcd} - \bar{\psi}\gamma_5\gamma_a\rho V^a \quad \leftarrow \quad \text{EH + RS, scale as } \mathrm{L}^2$$

But: Bdy terms that can be constructed using ω^{ab}, V^a, ψ scale as L^0 and L

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Alternative approach:

Add new gauge fields with higher scale-weight: $A^{ab} = -A^{ba}$ (s.w. L^2) and χ (s.w. $\mathrm{L}^{3/2}$)

- Appear only in the bdy Lagrangian ("topological role")
- Act as auxiliary fields under the bulk perspective (off-shell matching of B and F d.o.f.), implementing the Bianchi identities of Lorentz and supersymmetry respectively, associated with ω^{ab} and ψ

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Bdy contributions (not involving a scale parameter):

$$\mathrm{d}\left(A^{ab}\wedge\mathcal{R}^{cd}+\omega^{a}{}_{f}\wedge\omega^{fb}\wedge A^{cd}+2\omega^{a}{}_{f}\wedge A^{fb}\wedge\omega^{cd}+\omega^{ab}\wedge\mathcal{F}^{cd}\right)\epsilon_{abcd}=2\mathcal{R}^{ab}\wedge\mathcal{F}^{cd}\epsilon_{abcd}$$

$$\mathrm{d}\left(\bar{\psi}\gamma_5\wedge\sigma+\bar{\chi}\gamma_5\wedge\rho\right)=2\bar{\sigma}\gamma_5\wedge\rho-\frac{1}{2}\mathcal{R}^{ab}\wedge\bar{\chi}\gamma_5\gamma_{ab}\wedge\psi$$

where $\sigma \equiv \mathcal{D}\chi$ and $\mathcal{F}^{ab} \equiv \mathcal{D}A^{ab}$

Bdy Lagrangian:

$$\mathcal{L}_{\text{bdy}}^{\text{flat}}=\alpha'\left(2\mathcal{R}^{ab}\mathcal{F}^{cd}\epsilon_{abcd}\right)-\mathrm{i}\beta'\left(2\bar{\sigma}\gamma_5\rho-\frac{1}{2}\mathcal{R}^{ab}\bar{\chi}\gamma_5\gamma_{ab}\psi\right)\quad\rightarrow\quad\text{Scale as }\mathrm{L}^2\quad\checkmark$$

α', β': constant dimensionless parameters amounting to the normalization of the auxiliary fields

Full Lagrangian: $\imath_\epsilon(\mathcal{L}^{\text{flat}}_{\text{full}})|_{\partial\mathcal{M}_4} = 0 \leftrightarrow \alpha' \neq 0$, $\beta' \neq 0$; for $\alpha' = -1/8$, $\beta' = 1$ emerging structure transparent:

$$\mathcal{L}^{\text{flat}}_{\text{full}} = \mathcal{L}^{\text{flat}}_{\text{bulk}} + \mathcal{L}^{\text{flat}}_{\text{bdy}} = -\frac{1}{4}\mathcal{R}^{ab} \wedge \hat{\mathcal{F}}^{cd}\epsilon_{abcd} - 2\mathrm{i}\bar{\Xi}\gamma_5 \wedge \rho$$

$\Rightarrow$ "MacDowell-Mansouri-like" Lagrangian, with $\hat{\mathcal{F}}^{ab} \equiv \mathcal{F}^{ab} - V^a V^b - \bar{\chi}\gamma^{ab}\psi\,,\ \Xi \equiv \sigma - \frac{\mathrm{i}}{2}\gamma_a\psi V^a \rightarrow \mathcal{L}^{\text{flat}}_{\text{full}}$ in terms of the so-called (minimal) Maxwell-covariant supercurvatures

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Bdy contributions to the variational principle result in bdy constraints:

$$\mathcal{R}^{ab}|_{\partial\mathcal{M}_4} = 0\,, \quad \hat{\mathcal{F}}^{ab}|_{\partial\mathcal{M}_4} = 0\,, \quad \rho|_{\partial\mathcal{M}_4} = 0\,, \quad \Xi|_{\partial\mathcal{M}_4} = 0$$

$\leftrightarrow$ Super-Maxwell algebra emerges as global symmetry at the bdy
(Consistency of the bulk theory: $R^a = 0 \Rightarrow$ Continuity: $R^a|_{\partial\mathcal{M}_4} = 0$)

Full Lagrangian: $\imath_\epsilon(\mathcal{L}^{\text{flat}}_{\text{full}})|_{\partial\mathcal{M}_4} = 0 \leftrightarrow \alpha' \neq 0$, $\beta' \neq 0$; for $\alpha' = -1/8$, $\beta' = 1$ emerging structure transparent:

$$\mathcal{L}^{\text{flat}}_{\text{full}} = \mathcal{L}^{\text{flat}}_{\text{bulk}} + \mathcal{L}^{\text{flat}}_{\text{bdy}} = -\frac{1}{4}\mathcal{R}^{ab} \wedge \hat{\mathcal{F}}^{cd}\epsilon_{abcd} - 2\mathrm{i}\bar{\Xi}\gamma_5 \wedge \rho$$

$\Rightarrow$ "MacDowell-Mansouri-like" Lagrangian, with $\hat{\mathcal{F}}^{ab} \equiv \mathcal{F}^{ab} - V^a V^b - \bar{\chi}\gamma^{ab}\psi\,,\ \Xi \equiv \sigma - \frac{\mathrm{i}}{2}\gamma_a\psi V^a \to \mathcal{L}^{\text{flat}}_{\text{full}}$ in terms of the so-called (minimal) Maxwell-covariant supercurvatures

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$\mathcal{L}^{\text{flat}}_{\text{full}}$ is the $\ell \to \infty$ limit of a theory orig. from AdS$_4$ sugra (AdS-Lor. cov.), adding A^{ab}, χ (also in bulk) and redef.

$$\omega^{ab} \to \omega^{ab} + \frac{1}{\ell^2}A^{ab} \quad \text{(torsionful)}\,, \quad \psi \to \psi + \frac{1}{\ell}\chi$$

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Ciambelli, Marteau, Petkou, Petropoulos, Siampos, JHEP 07 (2018) 165

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- Starting point: $D = 4$ bulk Einstein st. with $\Lambda = -3\kappa^2 = -3/\ell^2$, dual to a bdy relativistic fluid
- Ricci-flat limit achieved in the limit $\kappa \to 0 \to 2D$ spatial conformal structure emerges at null infinity: Randers-Papapetrou parametrization of the $D = 3$ bdy spacetime metric:

$$\mathrm{d}s^2 = -\kappa^2 \left(\Omega \mathrm{d}t - \mathbf{b}_{\mathfrak{a}} \mathrm{d}x^{\mathfrak{a}}\right)^2 + \mathrm{d}l^2 , \quad \mathrm{d}l^2 = \mathbf{a}_{\mathfrak{a}\mathfrak{b}} \mathrm{d}x^{\mathfrak{a}} \mathrm{d}x^{\mathfrak{b}}$$

For $k \to 0$, time decouples in the bdy geom.; k role of speed of light $\Rightarrow$ Flat limit $k \to 0$ is Carrollian limit
$\Rightarrow$ Carrollian bdy geom.: Spatial surface with positive-def. metric $\mathrm{d}l^2 = \mathbf{a}_{\mathfrak{a}\mathfrak{b}} \mathrm{d}x^{\mathfrak{a}} \mathrm{d}x^{\mathfrak{b}}$ and Carrollian time $t \in \mathbb{R}$
$\rightsquigarrow$ Carrollian surface hosts a conformal Carrollian fluid (holo. dual to Ricci-flat spacetime)

$\rightsquigarrow$ At susy level?

Regarding our construction in this context:

- Role of the "topological auxiliary fields" A^{ab} and χ?
- Relation between super-Maxwell and super-$\mathfrak{bms}_4$ (or super-Carroll)?
  [Known: $\mathfrak{bms}_4 \rightarrow$ super-$\mathfrak{bms}_4$ (finite or infinite number of fermionic generators)]

First step:

Intrinsic description of the bdy Lagrangian for the case of a null bdy geometry

Decomposition of tensorial structures w.r.t. those covariant under the symmetries of the chosen bdy

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Thank you!

Supplementary material

Super-Maxwell algebra

Super-Maxwell algebra ($\{J_{ab}, P_a, Q, Z_{ab}, \Sigma\}$ dual to $\{\omega^{ab}, V^a, \psi, A^{ab}, \chi\}$):

$$[J_{ab}, J_{cd}] \propto \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc} \,, \quad [J_{ab}, P_c] \propto \eta_{bc} P_a - \eta_{ac} P_b \,, \quad [P_a, P_b] \propto Z_{ab}$$

$$[J_{ab}, Z_{cd}] \propto \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}$$

$$[J_{ab}, Q] \propto \gamma_{ab} Q \,, \quad [J_{ab}, \Sigma] \propto \gamma_{ab} \Sigma \,, \quad [P_a, Q] \propto \gamma_a \Sigma \,, \quad \{Q, Q\} \propto C\gamma^a P_a \,, \quad \{Q, \Sigma\} \propto C\gamma^{ab} Z_{ab}$$

The other field eqs. in the flat MacDowell-Mansouri-like sugra case

- E.o.m. of ω^{ab} and ψ:

$$\text{e.o.m. } \omega^{ab} \quad \leftrightarrow \quad \mathcal{D}\hat{\mathcal{F}}^{ab} - 2\mathcal{R}^{[a}{}_{c}A^{c|b]} + \bar{\Xi}\gamma^{ab}\psi - \bar{\chi}\gamma^{ab}\rho = 0$$

$$\text{e.o.m. } \psi \quad \leftrightarrow \quad \mathcal{D}\Xi - \frac{1}{4}\mathcal{R}^{ab}\gamma_{ab}\chi + \frac{\mathrm{i}}{2}\gamma_a\rho V^a = 0$$

- E.o.m. of V^a:

$$\frac{1}{2}V^b\mathcal{R}^{cd}\epsilon_{abcd} - \bar{\psi}\gamma_a\gamma_5\rho = 0$$

Einstein eqs. in superspace (written in the Einstein-Cartan formalism)

Relation with the AVZ model

$\Lambda \neq 0$ case:

Andrianopoli, Cerchiai, D'Auria, Trigiante, JHEP 04 (2018), 007

AVZ $D = 3$ model ("unconventional SUSY") from $\mathcal{N} = 2$, $D = 4$ pure sugra with a $3D$ boundary:

- AVZ (Alvarez, Valenzuela, Zanelli) model: Based on a $3D$ CS Lagrangian with $\mathrm{OSp}(2|2)$ supergroup, but features a Dirac spinor $\chi^{(\mathrm{AVZ})}$ as the only propagating d.o.f.; Important applications in the description of graphene-like systems near the Dirac points
- $\chi^{(\mathrm{AVZ})}$ emerges by imposing the following cond. on the spacetime comp. of the odd CS connection 1-form Ψ:

$$\chi^{(\mathrm{AVZ})}_{\alpha} = \mathrm{i}(\gamma^{i})_{\alpha\beta}\Psi^{\beta}_{\mu}e^{\mu}_{i} \qquad (\alpha, \beta = 1, 2\,,\; i = 0, 1, 2\,,\; \mu = 0, 1, 2)$$

- Correspondence with the CS model of AVZ found for specific choice of the $D = 3$ bdy: Local AdS_3 geometry at spatial infinity of the $D = 4$ theory (asymptotically AdS_4 solutions featuring this boundary geometry comprise the "ultraspinning limit" of AdS_4-Kerr black hole)

Symmetry structure of asymptotically flat spacetimes: BMS and Carroll groups

Maximal set of symmetries admitted by a $D = 4$ theory including gravity with asymptotically locally flat b.c.?

With Dirichlet-type b.c. (non-deg. spatial part of bdy metric: round 2-sphere) $\rightarrow$ Asympt. symm. algebra: $\mathfrak{bms}_4$

Asympt. flat spacetimes at null (i.e., light-like) infinity: Asympt. symm. group is the infinite-dim. BMS group (instead of the Poincaré group)

AdS/CFT duality: A necessary condition is that the asymp. symm. group of the bulk dictates the global symm. of the dual field theory living on the bdy of spacetime

$\rightarrow$ Holographic formulation of quantum gravity in asympt. flat spacetimes: Putative dual field theory expected to be a BMS invariant theory on the null bdy of spacetime ($\rightsquigarrow$ Celestial holography)

Asympt. bdy as a spatial surface at null infinity $\Rightarrow$ Null surfaces have in general a deg. metric
$\rightarrow$ Any given holographic model has to deal with a consistent definition of the $2D$ induced spatial metric

$\Rightarrow$ Consider a systematic singular limit where an infinite boost is implemented on a space-like surface of a relativistic field theory: $c \rightarrow 0$ in the field theory $\rightarrow$ Contraction of the Poincaré group to the Carrollian group

$\rightsquigarrow$ As fields on a null hypersurface of spacetime necessarily propagate at the speed of light and they must therefore be massless, consider a conformal extension of the Carroll group

$$\text{BMS}\,(D) \cong \text{conformal Carroll}(D-1)$$