Entanglement, soft modes and celestial CFT

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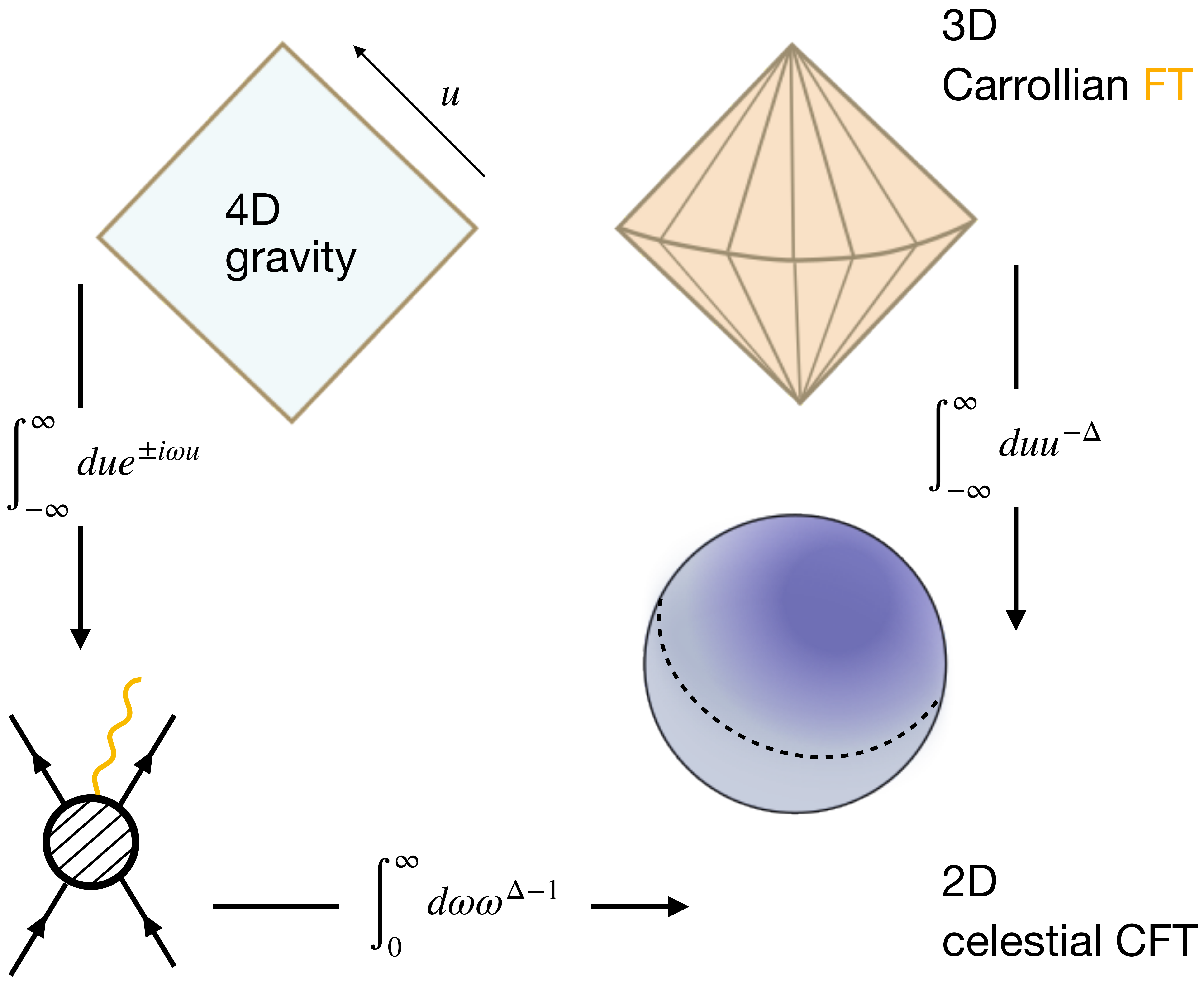
Based on 2308.12341 with Vincent Chen and Robert Myers

Motivation

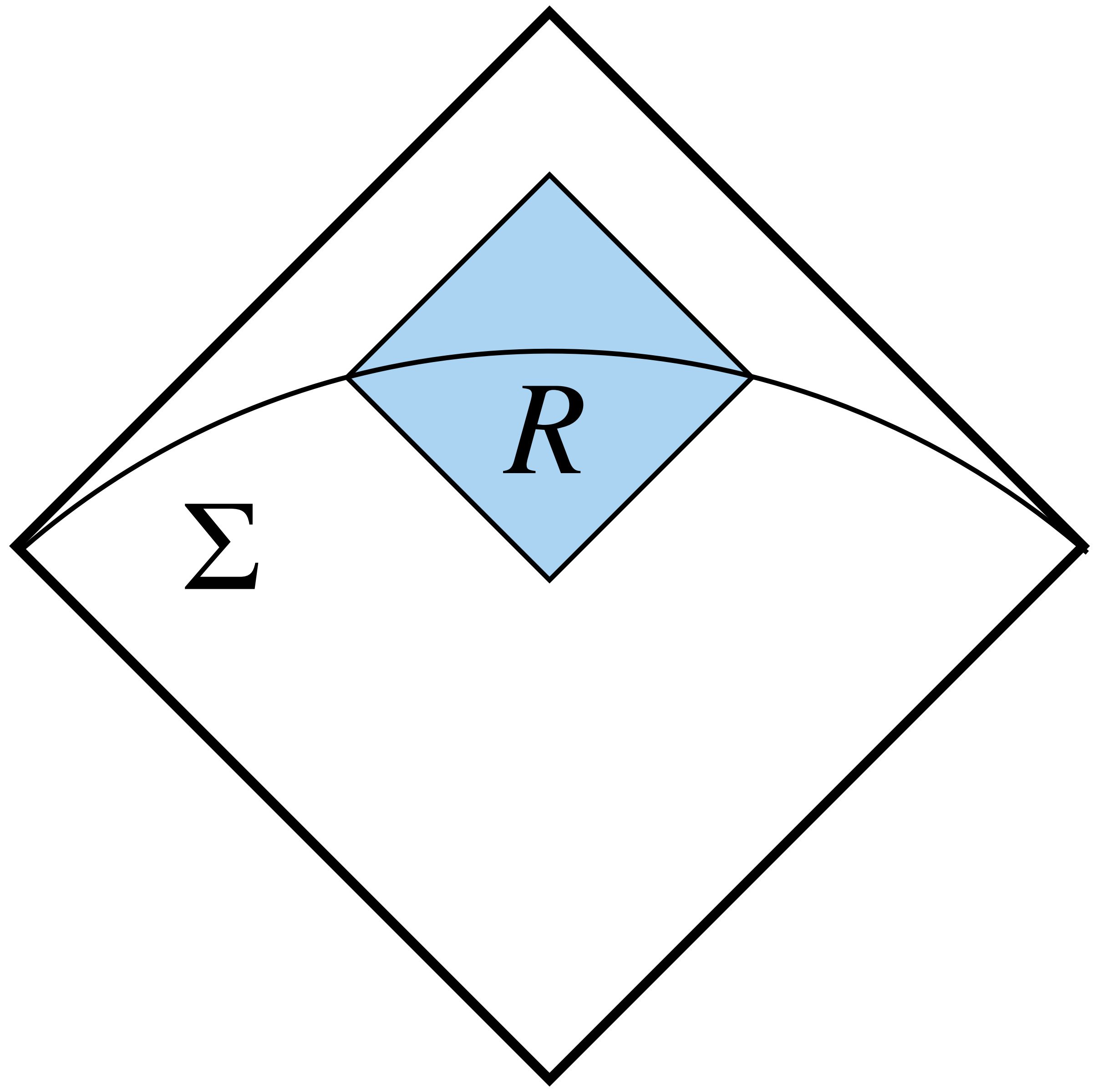


Flat space holography?

Massless particles:

- 4D asymptotic symmetries - 2D currents
- Collinear limits - 2D OPEs?
- Most entries in the holographic dictionary so far are kinematic…

Motivation



Flat space holography?

Basic missing entries:

- Geometry — entanglement?
- Subregion duality?

Let ρ be a pure state on a Cauchy slice Σ and $R \subset \Sigma$

Entanglement entropy: $S_{\text{vN}}({}^{R}\rho) \equiv -\,\text{Tr}\left({}^{R}\rho \log {}^{R}\rho\right), \quad {}^{R}\rho \equiv \text{Tr}_{\bar{R}}\rho$

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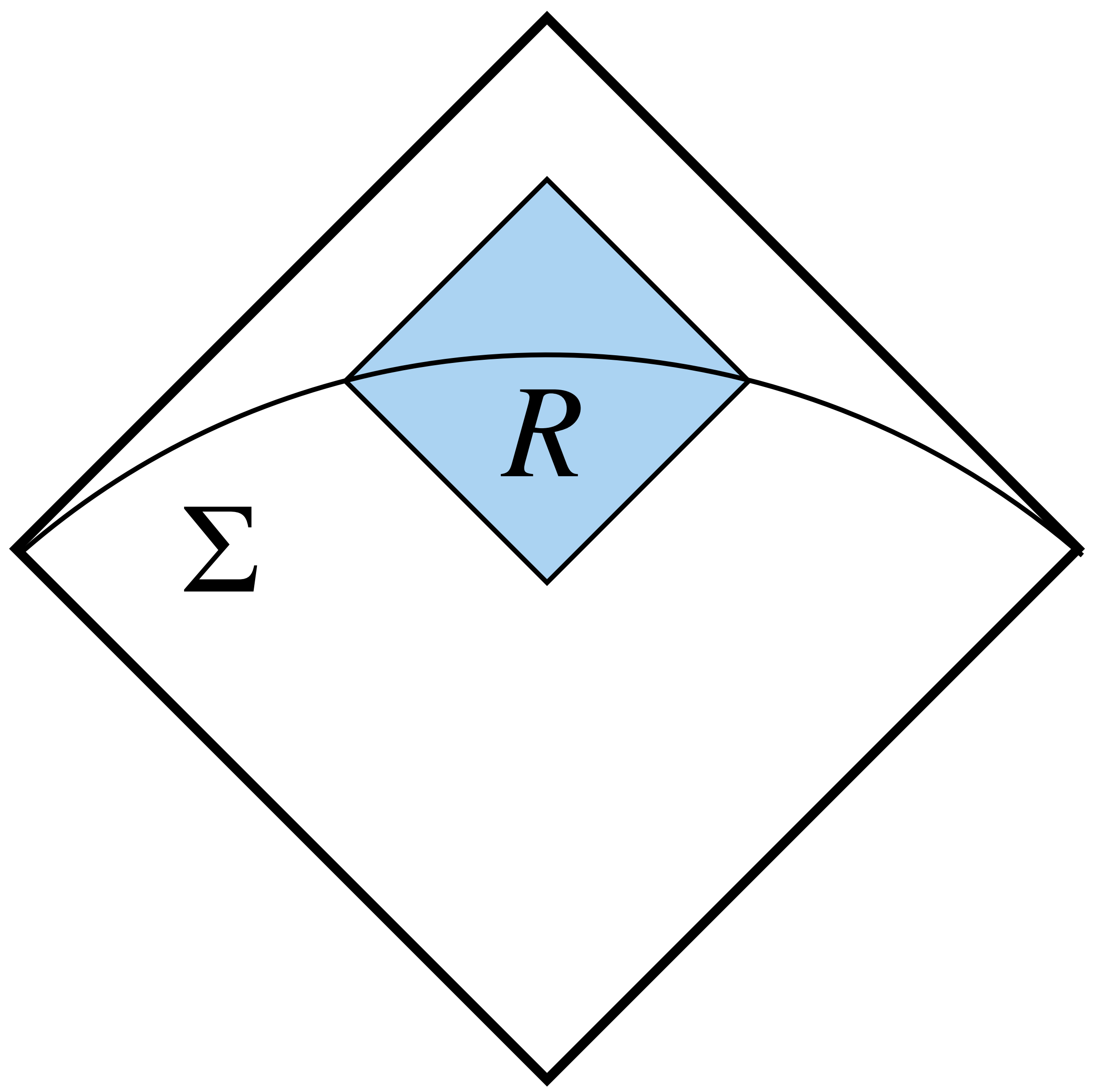

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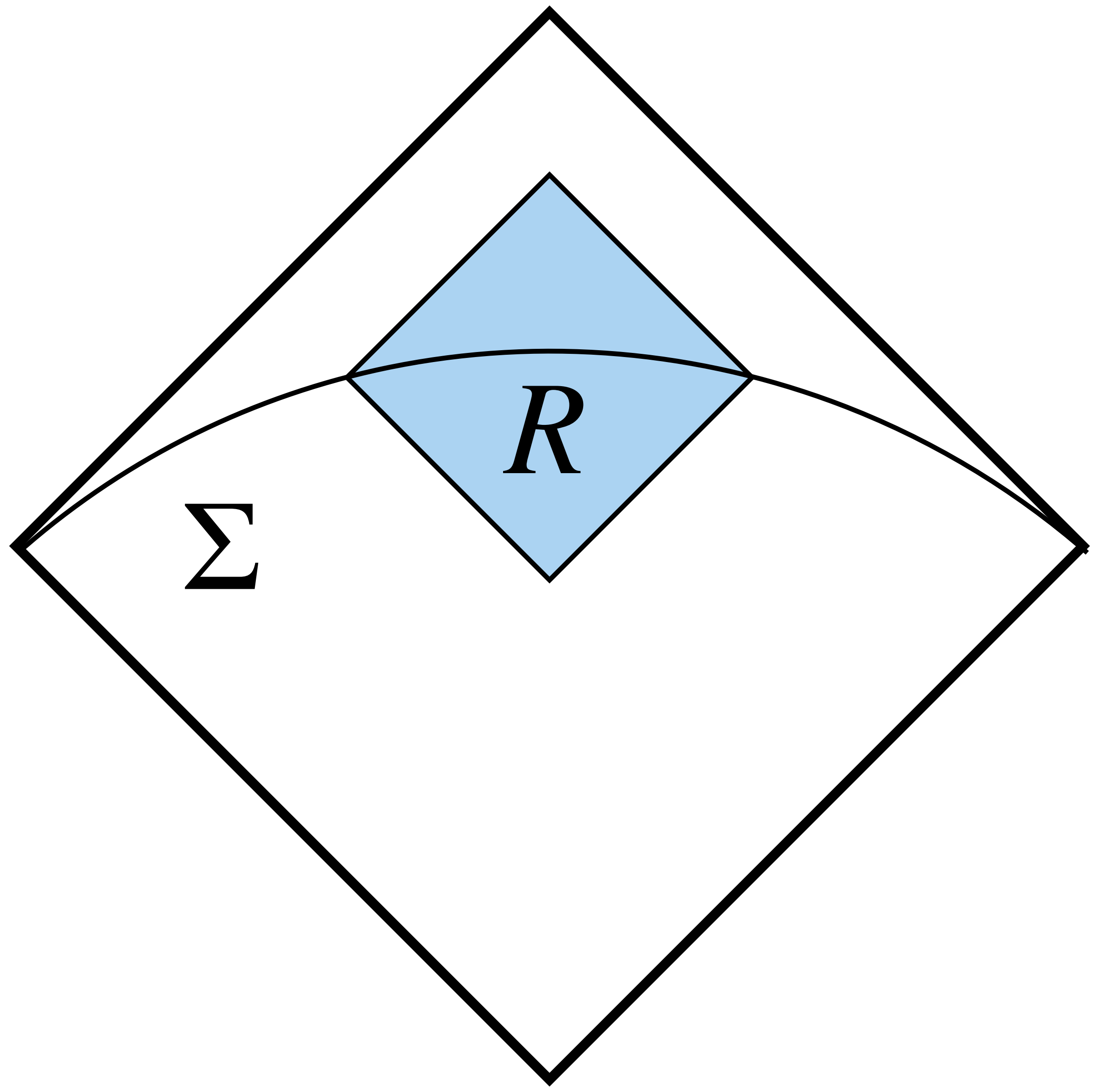
- If theory is a CFT, conformal transformation maps spacetime to R and vacuum to thermal state

entanglement entropy = thermal entropy [Casini, Huerta, Myers '11]

Broad goals/questions



Does R admit a holographic description?

For gauge and gravity theories constraint on $\Sigma \Longrightarrow$
``edge mode” contribution to entanglement entropy

[Donnelly, Wall ’14; Donnelly, Freidel ’16]

- Are ``soft modes” related to the DW ``edge modes”?
- Entanglement entropy across a cut of $\mathscr{I}^+$?

[Kapec, A.R., Strominger ’16]

Outline

1. Setup: conformal primary wavefunctions, Einstein static universe
2. Subregions, inversions and the thermofield double
3. Soft modes and constraints
4. Entanglement
5. Outlook

Setup

Free Maxwell theory in 4D Minkowski spacetime:

1. Rich asymptotic symmetry structure
2. Weyl invariance

Setup: soft sector

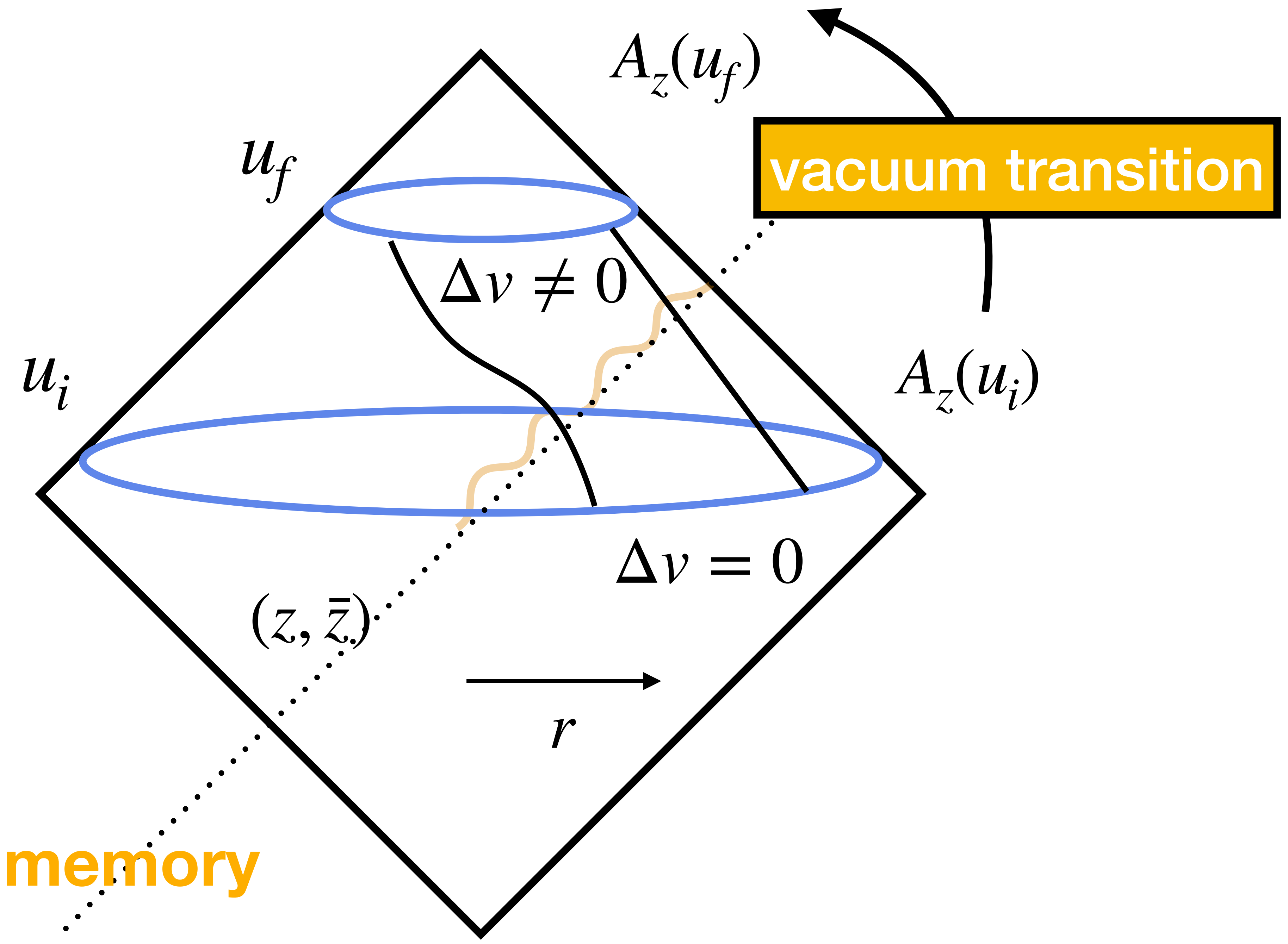
Free Maxwell theory in 4D Minkowski spacetime:

1. Rich asymptotic symmetry structure
2. Weyl invariance

- Infinity of large gauge charges: $Q_{LG} = \int_{S^2} \epsilon(z,\bar{z}) \star F$
- Constraint at $\mathscr{I}^+$: $\partial_u F^{(2)}_{ru} + D^z F^{(0)}_{uz} + D^{\bar{z}} F^{(0)}_{u\bar{z}} = 0 \implies$

$$Q_{LG} = \int_{S^2} d^2 z \gamma_{z\bar{z}} D^B \epsilon \Delta A_B \equiv Q_{\text{soft}}$$

$\Delta A_B \equiv \int du F^{(0)}_{uB} \neq 0 \implies$ radiative zero-mode ~ soft photon ~ memory



Setup: soft sector

Free Maxwell theory in 4D Minkowski spacetime:

1. Rich asymptotic symmetry structure
2. Weyl invariance

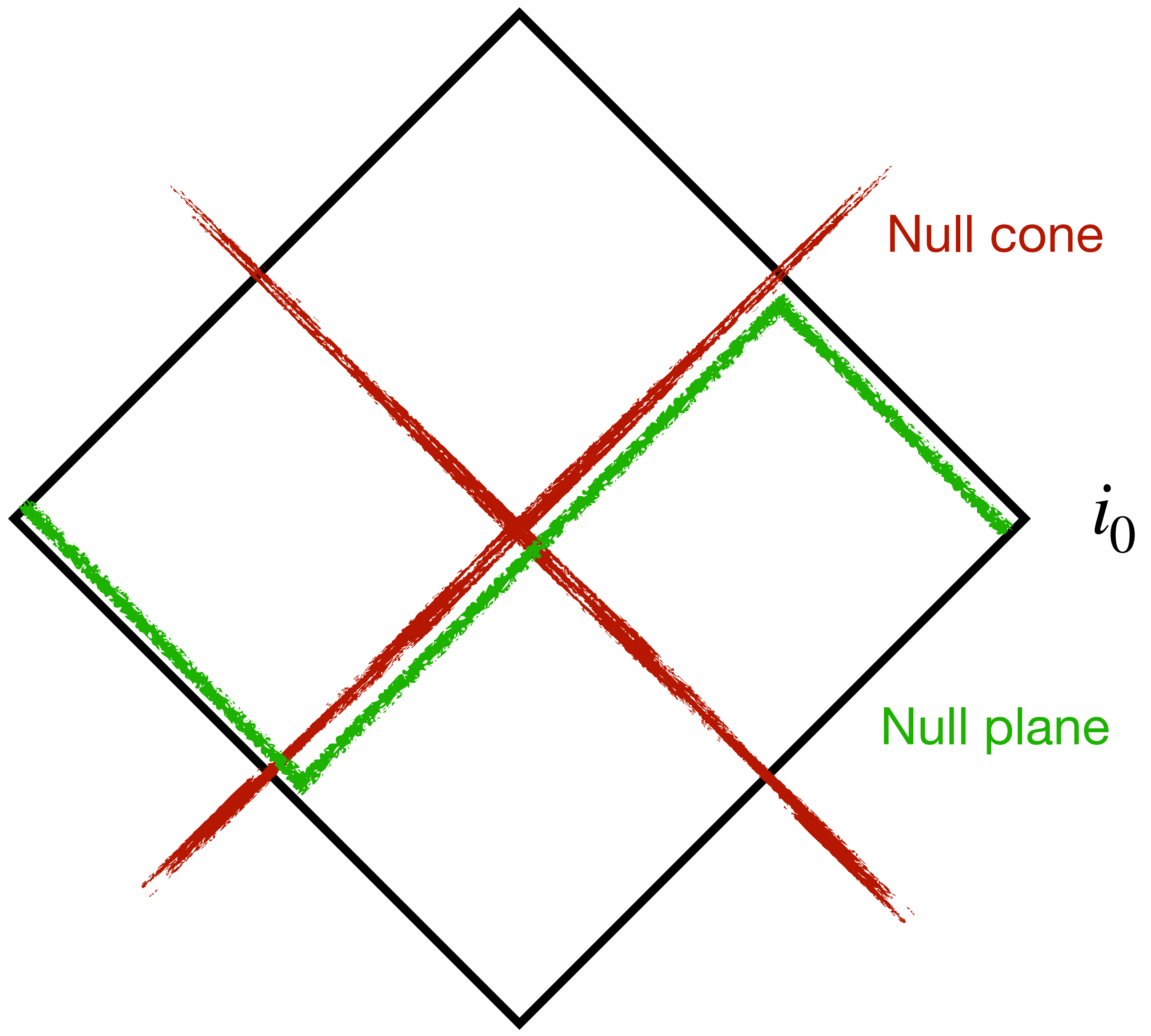
- ``Memory" solutions to the vacuum [no charges] Maxwell equations

$$\partial_u F^{(2)}_{ru} + D^z F^{(0)}_{uz} + D^{\bar{z}} F^{(0)}_{u\bar{z}} = 0$$

have both Coulomb and radiative modes:

$$F^{(0)}_{uz} \sim \delta(u),$$

$$F^{(2)}_{ru} \sim \theta(-u)$$ matched across i_0



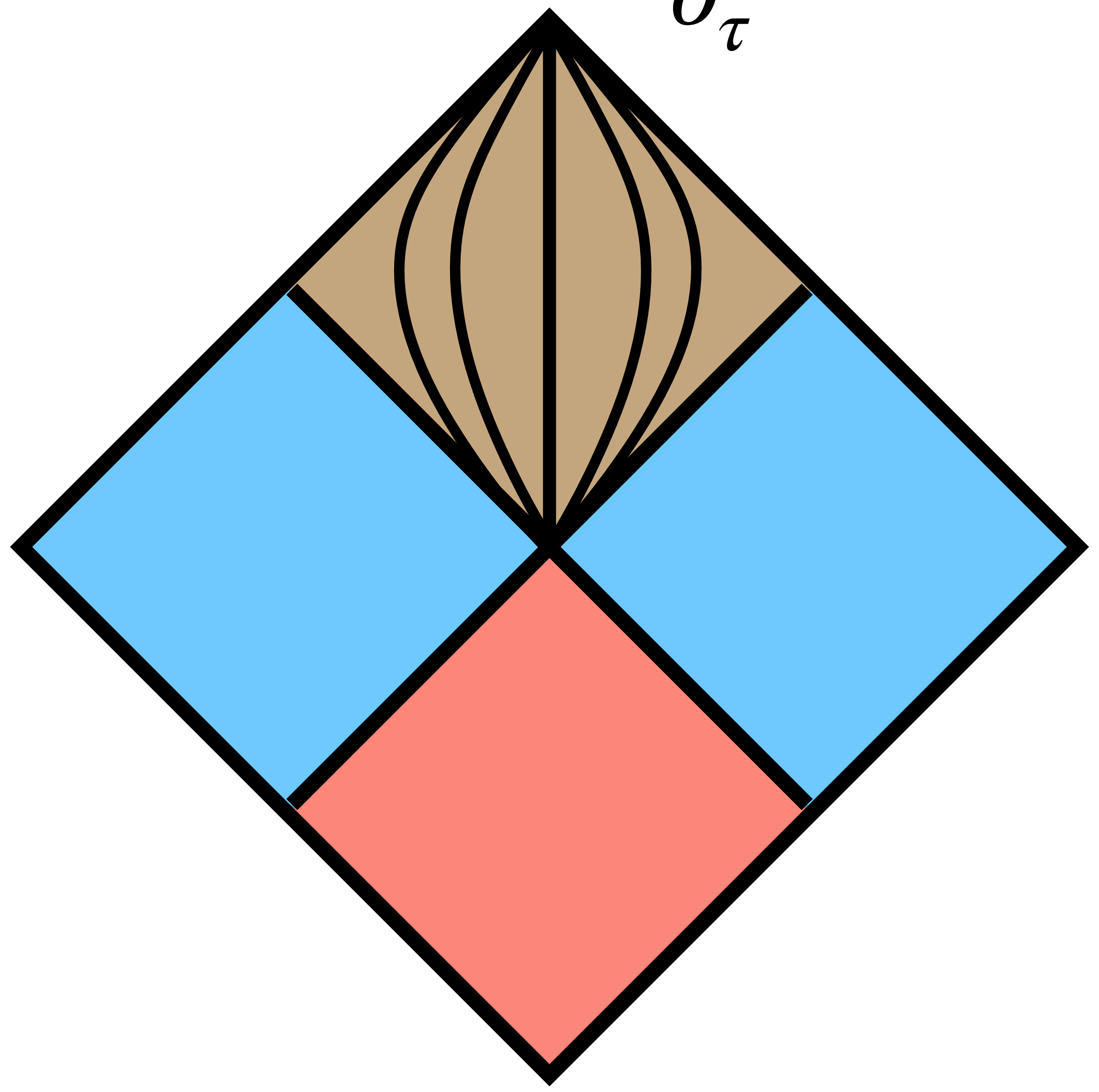
- Field strength in a conformal primary basis

  [diagonalizing boosts towards $(z, \bar{z})$] associated with

  $\Delta = 1$ wavefunctions

[Donnay, Puhm, Strominger '18]

Conformal primary wavefunctions



Solutions to the free Maxwell equations

$$A^{\Delta,\pm}_{a;\mu}(\mathbf{w};X) = \frac{m^{\pm}_{a;\mu}(\mathbf{w};X)}{(-\hat{q}\cdot X_\pm)^\Delta}, \qquad m^{\pm}_{a;\mu}(\mathbf{w};X) = \varepsilon_{a;\mu} + \frac{\varepsilon_a \cdot X_\pm}{-\hat{q}\cdot X_\pm}\hat{q}_\mu \quad X^\mu_\pm = X^\mu \mp i\epsilon n^\mu \,.$$

- Eigenstates of Milne time translation: $\partial_\tau A^{1+i\lambda} = -\, i\lambda A^{1+i\lambda}, \qquad \tau = \frac{1}{2}\log(-X^2)$
- Soft wavefunctions have $\lambda = 0$:

$$\begin{cases} A^{\rm G}_a(\mathbf{w},X) \equiv \dfrac{A^{1,+}_a(\mathbf{w};X) + A^{1,-}_a(\mathbf{w};X)}{2} = d_X \alpha^{\rm G}_a(\mathbf{w},X) \\ A^{\rm CS}_a(\mathbf{w},X) \equiv \dfrac{A^{\log,+}_a(\mathbf{w},X) - A^{\log,-}_a(\mathbf{w},X)}{2\pi i} \end{cases}$$

[Goldstone, pure gauge]

[Conformally soft, memory]

$$A^{\log,\pm}_a \equiv \lim_{\Delta\to 1} \partial_\Delta \left[A^{\Delta,\pm}_a + \tilde{A}^{2-\Delta,\pm}_a\right] = -\log(-X^2_\pm)A^{1,\pm}_a$$

[Donnay, Puhm, Strominger ’18]

Setup: soft sector

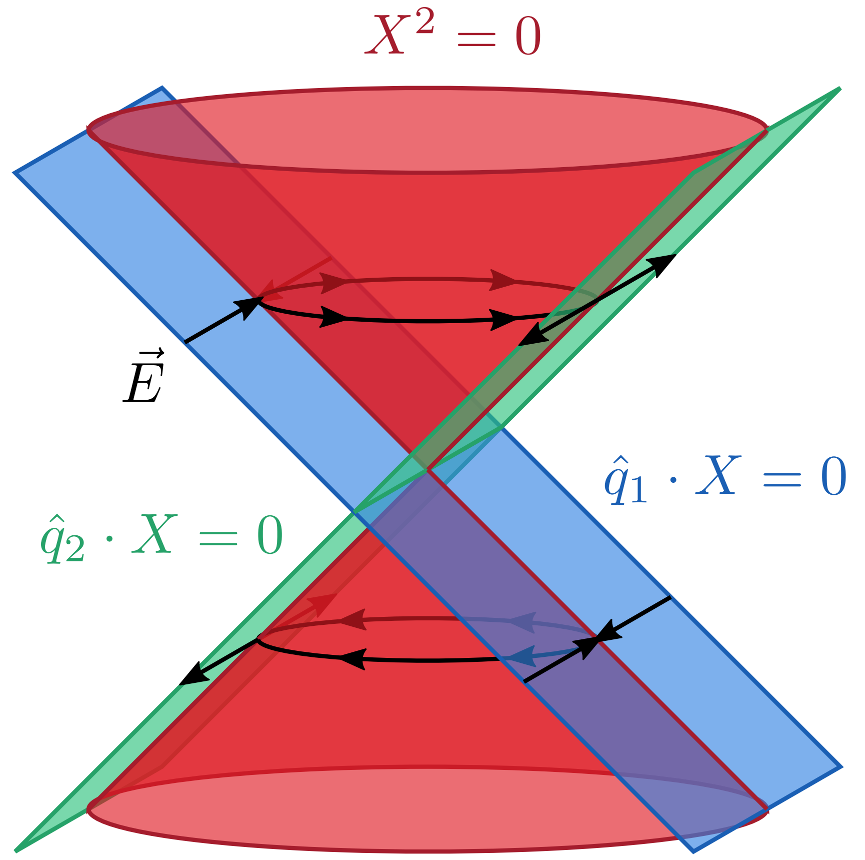
Free Maxwell theory in 4D Minkowski spacetime:

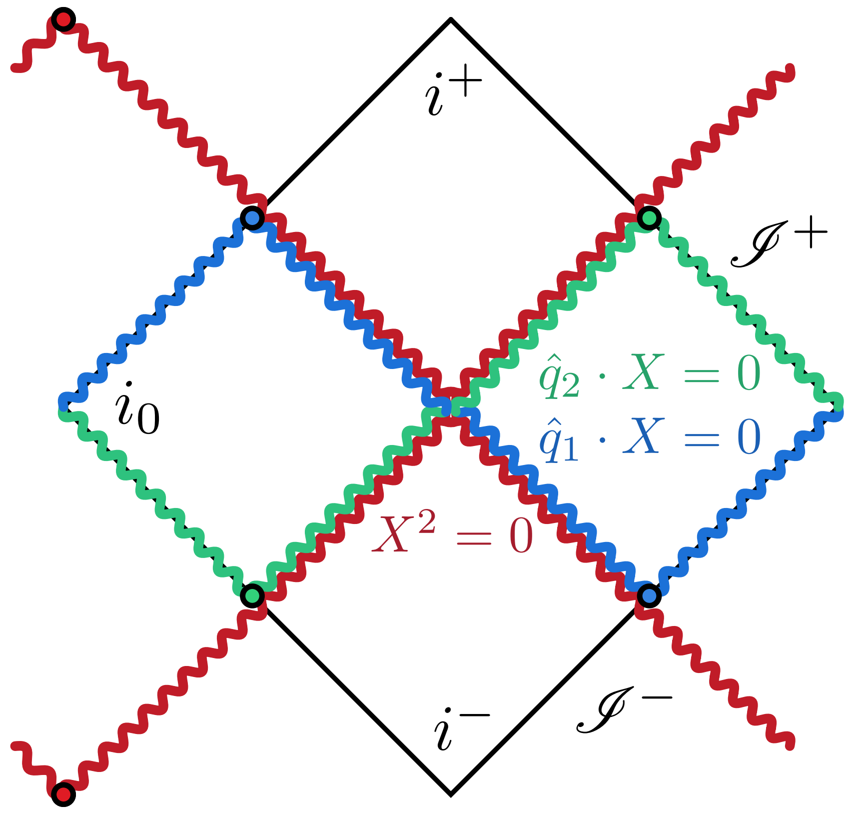
1. Rich asymptotic symmetry structure
2. Weyl invariance

- Conformally soft/memory mode A^{CS} is exact in celestial space $\Longrightarrow$ integrated mode is path independent

$$A^{\mathrm{CSI}}(\mathbf{w_1}, \mathbf{w_2}; X) \equiv \int_{\mathbf{w}_1}^{\mathbf{w}_2} A^{\mathrm{CS}}(\mathbf{w}; X)$$

- Field strength:

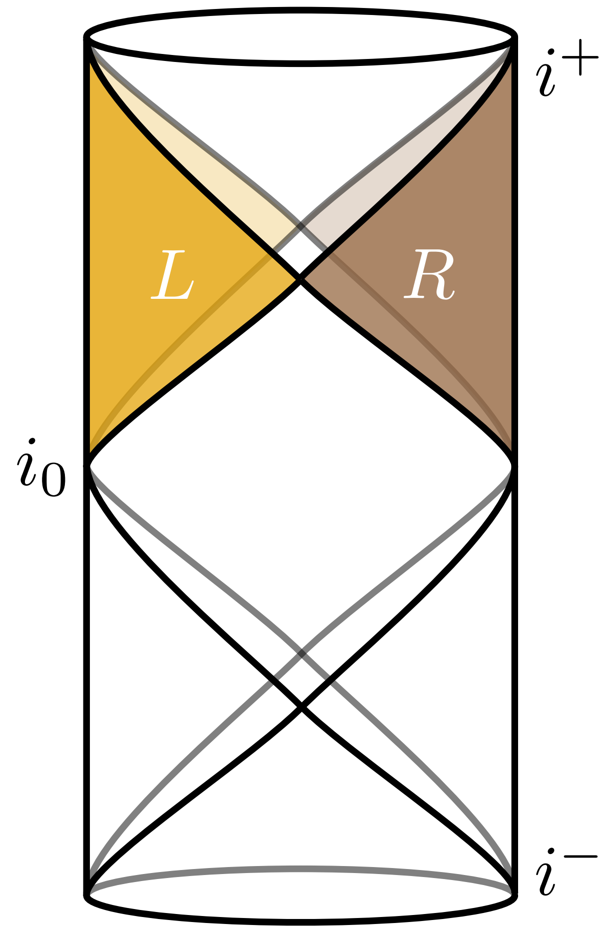


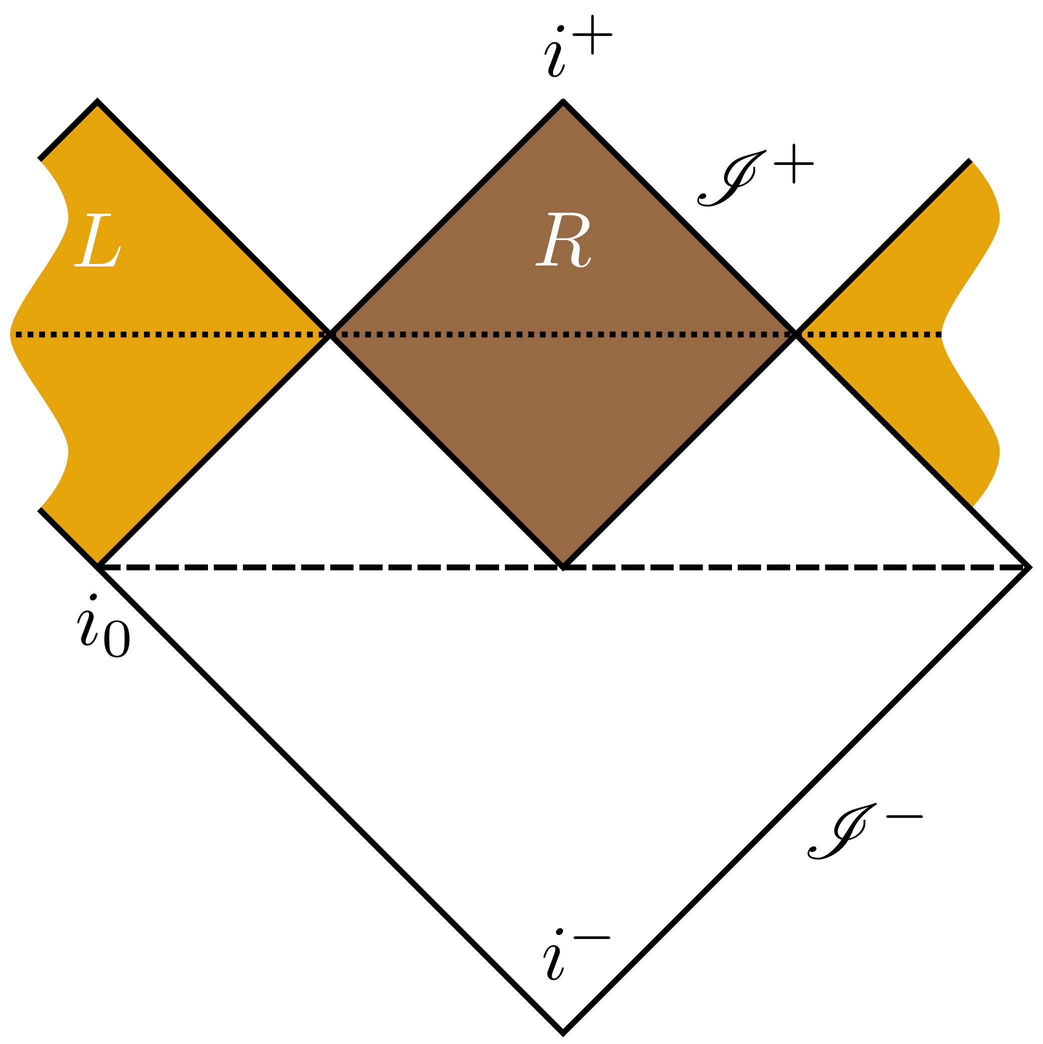


$$Q_{LG} \neq 0$$

Setup: embedding inside the Einstein static universe

Free Maxwell theory in 4D Minkowski spacetime:





1. Rich asymptotic symmetry structure
2. Weyl invariance

$$\nabla^\mu F_{\mu\nu} = 0 \xrightarrow[ds^2 \to \Omega^{-2} ds^2]{} \underline{\nabla}^\mu \underline{F}_{\mu\nu} = 0$$

- $\mathscr{I}^+$ Cauchy slice inside the Einstein static universe

- Entanglement across $\mathscr{I}^+$ ~ entanglement across S^3 in Einstein static universe
- Domains of dependence of partitions are the future Milne patches (R) of the original Minkowski geometry and another one (L) related by **conformal inversion**

Inversions vs. shadow transforms

A is weight 0 under Weyl rescalings:

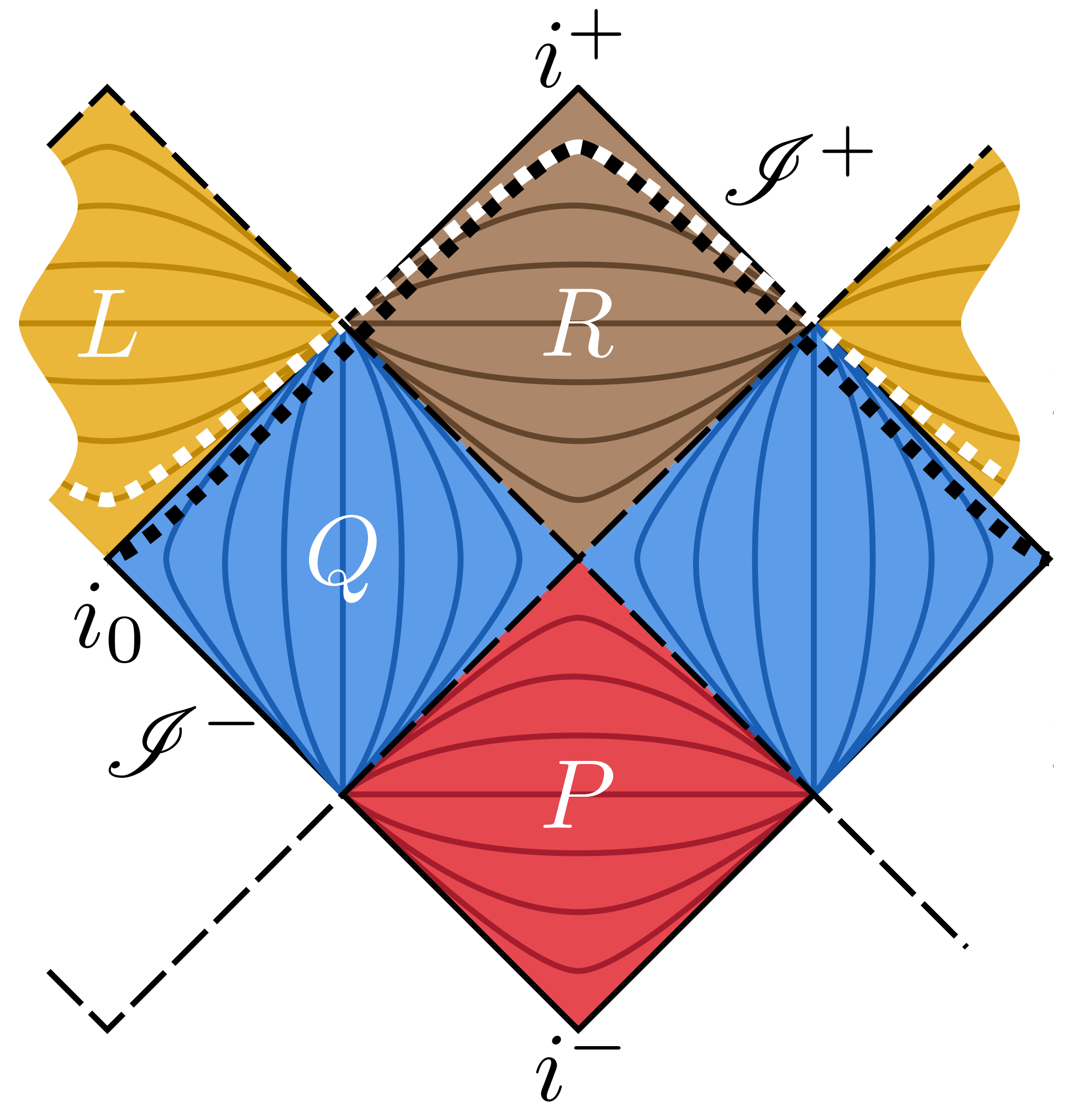
$$\underline{A}_\mu(\underline{X}) = \frac{\partial X^\nu}{\partial \underline{X}^\mu} A_\nu(X), \quad X^2 > 0$$ [in Q, then analytically continue outside]

- Applying to CPW and using that $I_\mu^\nu(X)\, m^{\pm}_{a;\nu}(\mathbf{w}; X) = m^{\pm}_{a;\nu}(\mathbf{w}; \underline{X})$

$$\underline{A}^{\Delta,\pm}_{a,\mu}(\mathbf{w}; \underline{X}) = (\underline{X}^2_\pm)^{\Delta-1} A^{\Delta,\pm}_{a,\mu}(\mathbf{w}; \underline{X}) = e^{\pm i\pi(\Delta-1)} \tilde{A}^{\Delta,\pm}_{a,\mu}(\mathbf{w}; \underline{X})$$

- $\tilde{A}^\Delta$ is the **shadow wavefunction** of dimension Δ

$$\tilde{A}^{\Delta,\pm}_a = (-X^2_\pm)^{\Delta-1} A^{\Delta,\pm}_a$$



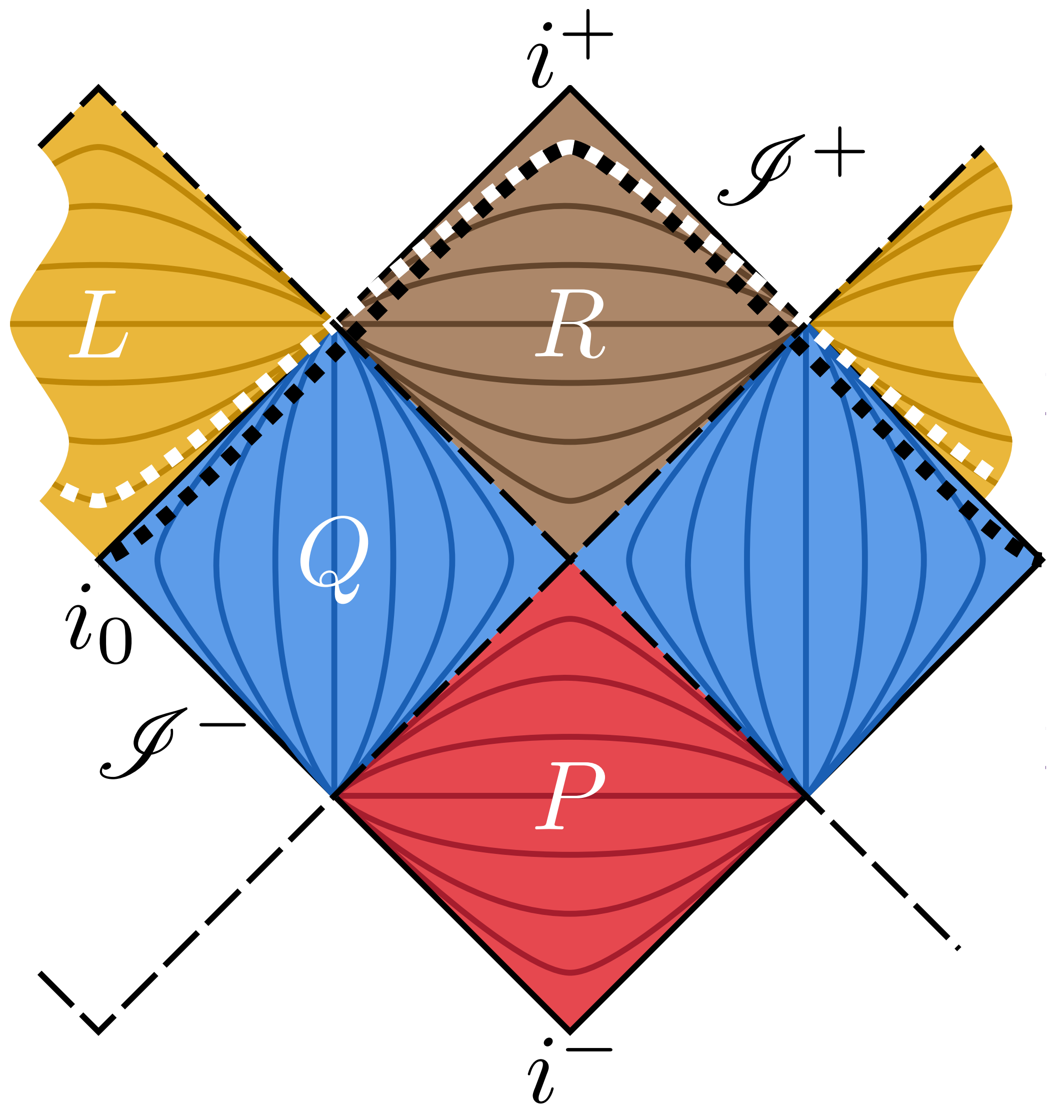
Inversion: $\underline{X}^\mu = \dfrac{X^\mu}{X^2}$

$$\frac{\partial X^\nu}{\partial \underline{X}^\mu} = X^2 I_\mu^\nu(X) = \underline{X}^{-2} I_\mu^\nu(\underline{X})$$

$$I_\mu^\nu(X) = \delta_\mu^\nu - \frac{2X_\mu X^\nu}{X^2} = I_\mu^\nu(\underline{X})$$

Inversions vs. shadow transforms

$\underline{A}^{\Delta,\pm}_{a,\mu}(\mathbf{w};\underline{X}) = e^{\pm i\pi(\Delta-1)}\tilde{A}^{\Delta,\pm}_{a,\mu}(\mathbf{w};\underline{X})$ in inverted Minkowski patch $\Longrightarrow$



- Decompose conformal primary wavefunctions (cpw) supported on S^3 slice of cylinder [dotted white] in terms of cpw supported on the L and R Milne patches:

$$A^{\Delta,\pm} = e^{\pm i\pi(\Delta-1)}{}^L\tilde{A}^{\Delta} + {}^R A^{\Delta}, \quad \Delta \notin \mathbb{Z} \quad ✽$$

$\left\{ \begin{array}{l} \tilde{A}^{\Delta} \text{ has opposite Milne frequency to } A^{\Delta}\ (\Delta = 1 + i\lambda) \\ \textbf{Span}\{\text{positive frequency plane waves}\} = \textbf{Span}\{\text{positive Milne energy cpw}\} \end{array} \right\} \Longrightarrow$

✽ Global definite energy modes decompose into positive and negative energy Milne modes

Compare to Rindler decomposition [decompositions related by time translation on the cylinder]

Minkowski vacuum as TFD

$$A^{\Delta,\pm} = e^{\pm i\pi(\Delta-1)}\,{}^{L}\tilde{A}^{\Delta} + {}^{R}A^{\Delta}, \quad \Delta \notin \mathbb{Z}$$

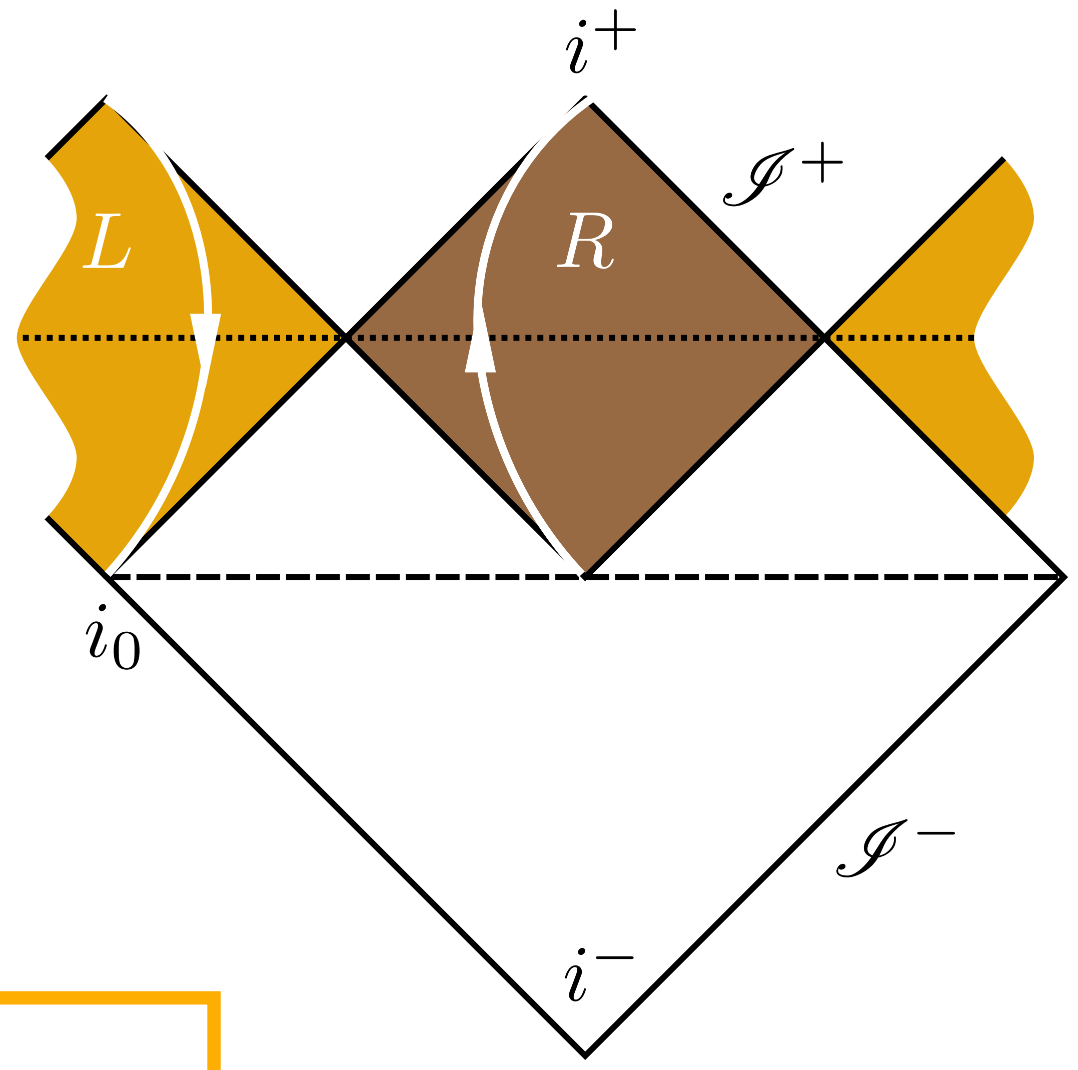
``Hard” mode decomposition

$$A(X) = \int \epsilon^{(2)}(\mathbf{w}) \int_{\mathbb{R}-i0^{+}} d\lambda N(\lambda) \left[{}^{L}\tilde{A}^{1+i\lambda} \cdot {}^{L}\tilde{O}^{\dagger}_{1+i\lambda} + {}^{R}A^{1+i\lambda} \cdot {}^{R}O^{\dagger}_{1+i\lambda} \right] + [\text{soft}]$$

$$\begin{cases} {}^{R}O^{\dagger}_{1+i\lambda} = \#\,{}^{R}a^{\dagger}_{1+i\lambda}\Theta(-\lambda) + \#\,{}^{R}a_{1+i\lambda}\Theta(\lambda) \\ {}^{L}\tilde{O}^{\dagger}_{1+i\lambda} = \#\,{}^{L}\tilde{a}^{\dagger}_{1+i\lambda}\Theta(\lambda) + \#\,{}^{L}\tilde{a}_{1+i\lambda}\Theta(-\lambda) \end{cases}$$

${}^{R}a, {}^{R}a^{\dagger}$ and ${}^{L}\tilde{a}, {}^{L}\tilde{a}^{\dagger}$

are canonically conjugate in R and L



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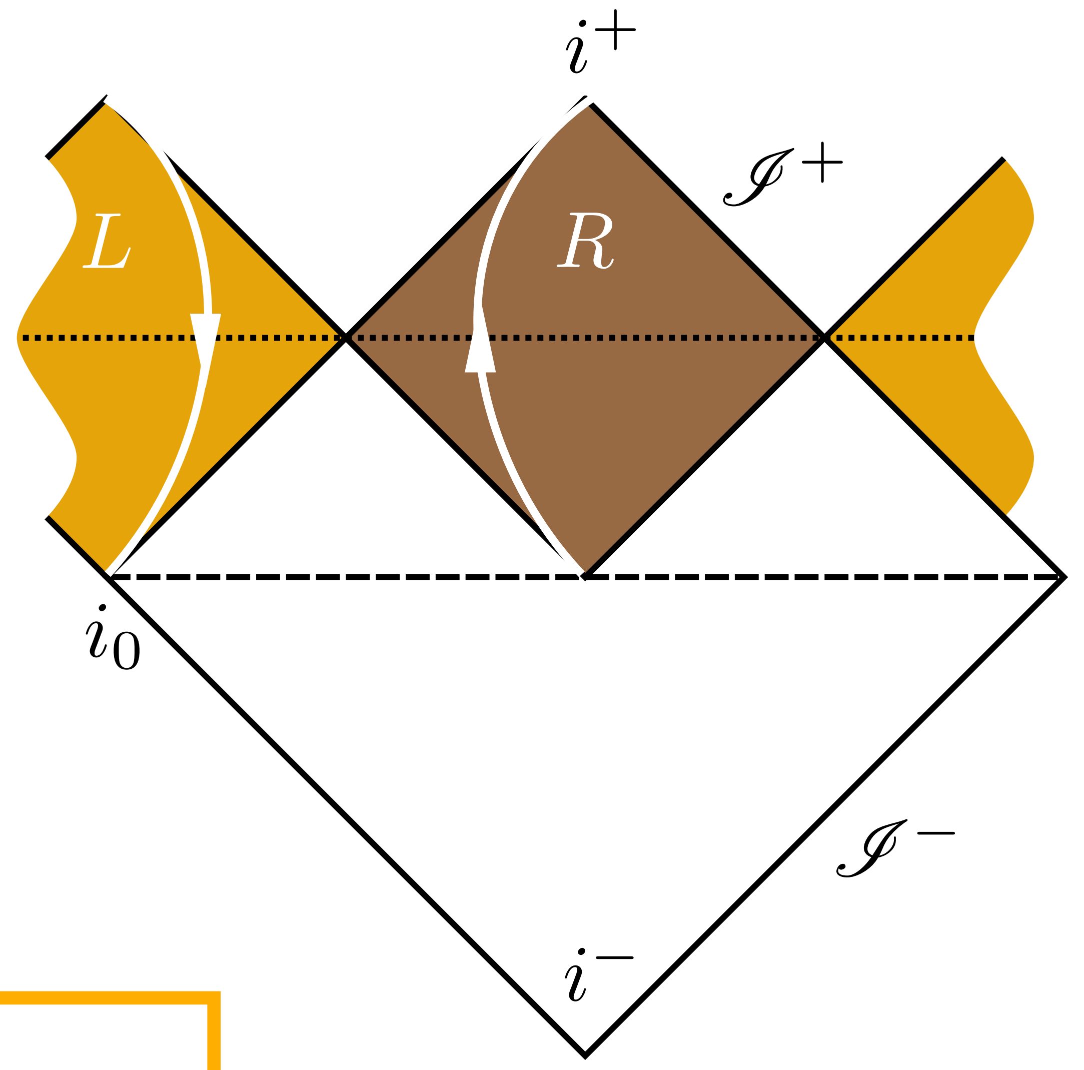

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${}^Ra, {}^Ra^{\dagger}$ and ${}^L\tilde{a}, {}^L\tilde{a}^{\dagger}$

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L (R) decompose into +/- Minkowski modes: ${}^L\tilde{A}^{\Delta} = i\dfrac{A^{\Delta,+} - A^{\Delta,-}}{2\sin\pi\Delta}$, ${}^RA^{\Delta} = i\dfrac{e^{-i\pi\Delta}A^{\Delta,+} - e^{i\pi\Delta}A^{\Delta,-}}{2\sin\pi\Delta}$

$\Longrightarrow$ Vacuum is TFD with respect to L and R Milne patches $|0\rangle = \exp\left\{\int \epsilon^{(2)} \int_{\lambda>0} d\lambda \left[\log(1-e^{-2\pi\lambda}) + e^{-\pi\lambda}\gamma^{a\bar{b}}\tilde{a}^{L\dagger}_{a,\lambda}a^{R\dagger}_{b,\lambda}\right]\right\}|0\rangle_L|0\rangle_R$

Soft sector of subregions

$$A^{\Delta,\pm} = e^{\pm i\pi(\Delta-1)} {}^{L}\tilde{A}^{\Delta} + {}^{R}A^{\Delta}, \quad \Delta \notin \mathbb{Z}$$

- L/R decompositions degenerate for $\Delta = 1 \Longrightarrow$ need independent construction of L/R soft sectors

$$A^{\mathrm{G}} = \lim_{\Delta \to 1} \frac{A^{\Delta,+} + A^{\Delta,-}}{2} = {}^{L}\tilde{A}^{1} + {}^{R}A^{1} \implies A^{\mathrm{G}} = {}^{L}A^{\mathrm{G}} + {}^{R}A^{\mathrm{G}}$$

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- Appropriate definitions of conformally soft modes found by considering inversion of A^{CS}:

$$\underline{A}^{\mathrm{CS}}(\underline{X}) = A^{1+} + A^{1-} - A^{\mathrm{CS}}(\underline{X}) = 2A^{\mathrm{G}}(\underline{X}) - A^{\mathrm{CS}}(\underline{X})$$

$\implies$ R, L components of conformally soft wf from restriction of A^{CS} and its inverse to the R, L Milne patches:

$$A^{\mathrm{CS}} = 2{}^{L}A^{\mathrm{G}} + \underbrace{{}^{L}A^{\mathrm{E}} + {}^{R}A^{\mathrm{E}}}$$

``edge modes”

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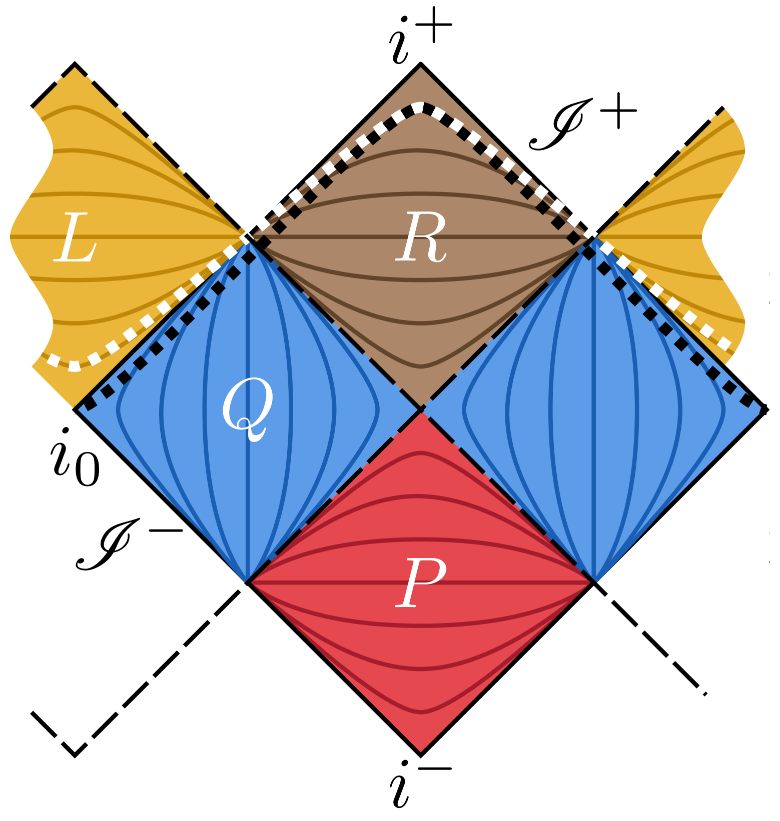
$$A^{\rm CS} = 2\,{}^{L}A^{\rm G} + \underbrace{{}^{L}A^{\rm E} + {}^{R}A^{\rm E}}$$

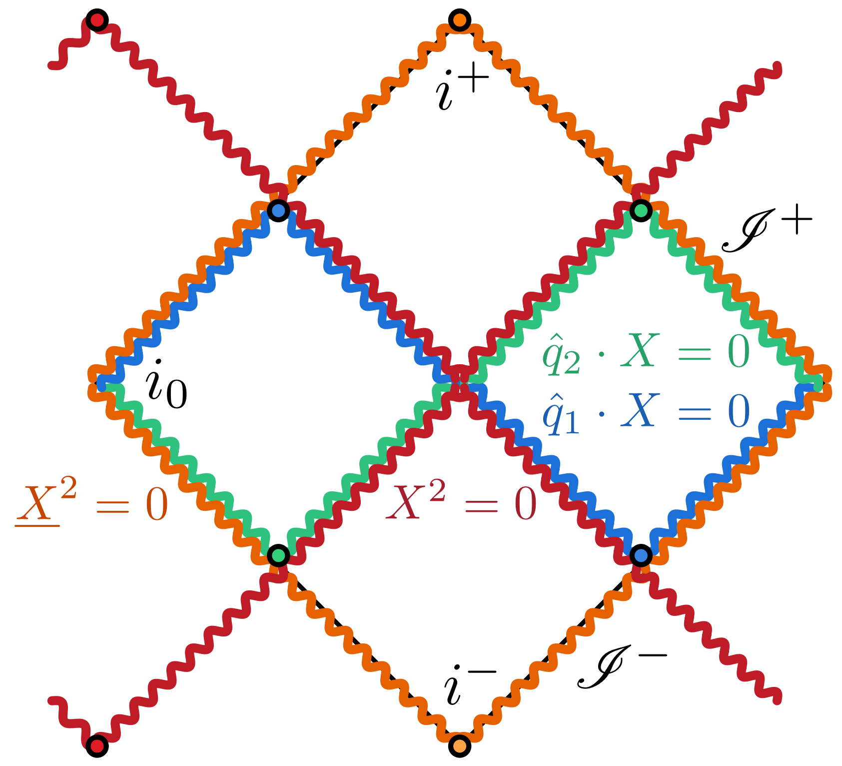
``edge modes”

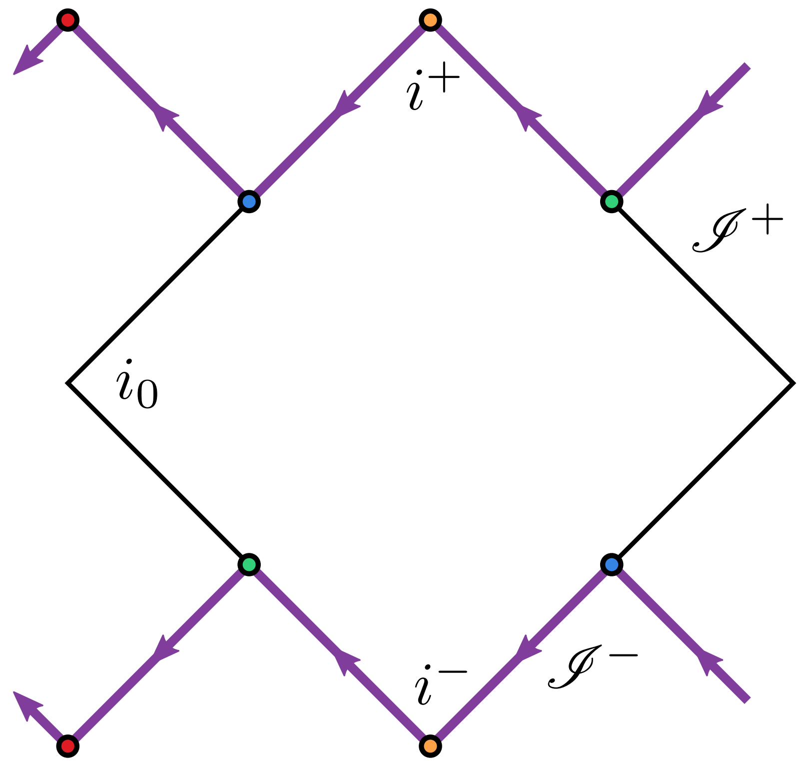
- ${}^{L}A^{G}, {}^{L}A^{\rm E}$ and R counterparts are canonically conjugate in the respective patches (for $\epsilon \to 0$)

$$\langle \bullet, \bullet \rangle = {}^{L}\langle \bullet, \bullet \rangle + {}^{R}\langle \bullet, \bullet \rangle$$



Sources in inverted patch





Extension of the conformally soft field configuration obtained by inversion coincides with the Lienard Wiechert fields of sources outside the two Minkowski patches

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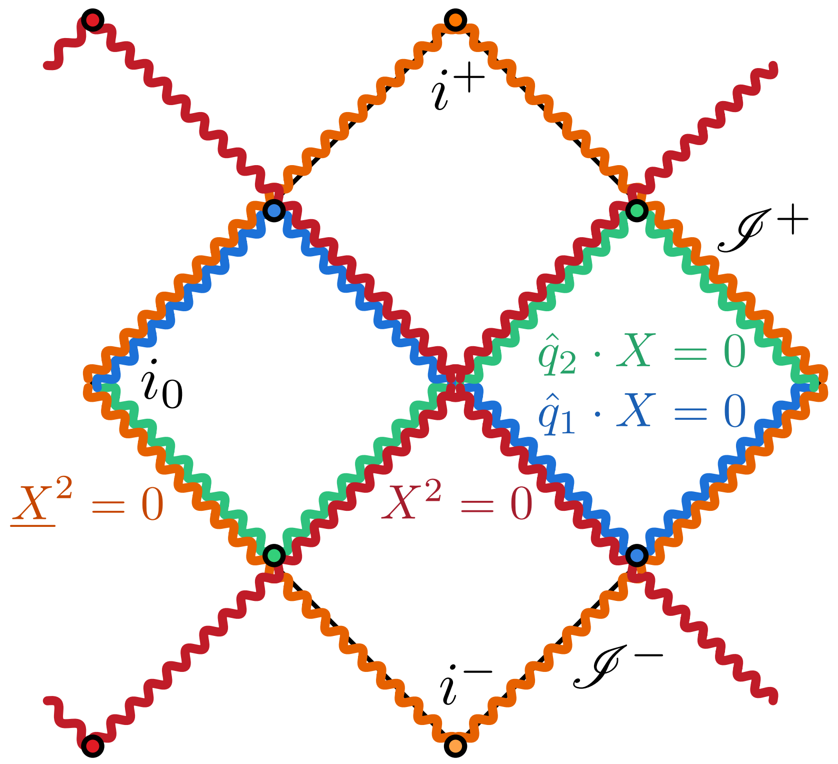

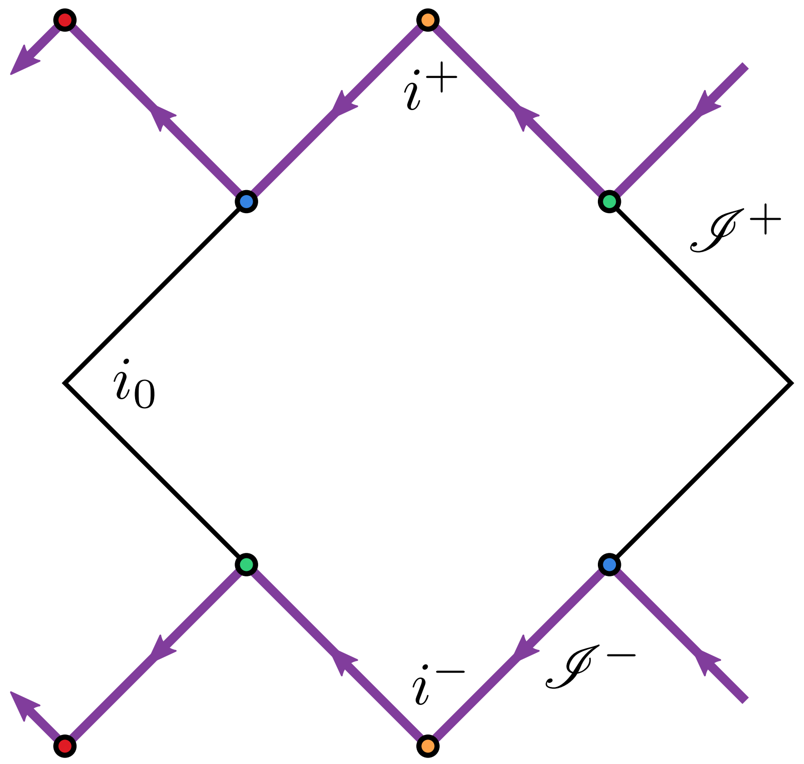

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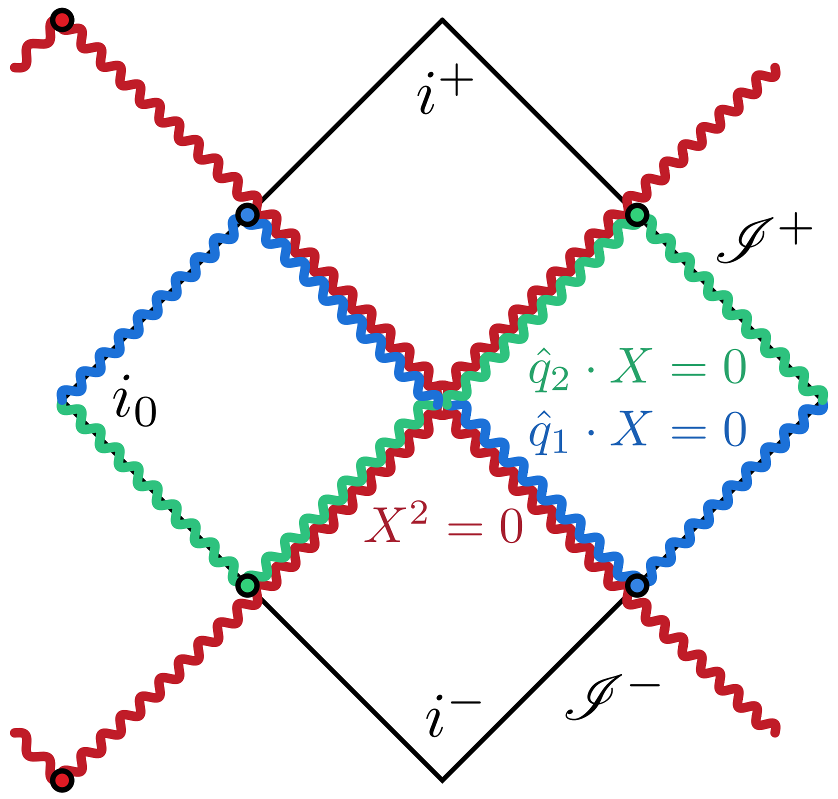
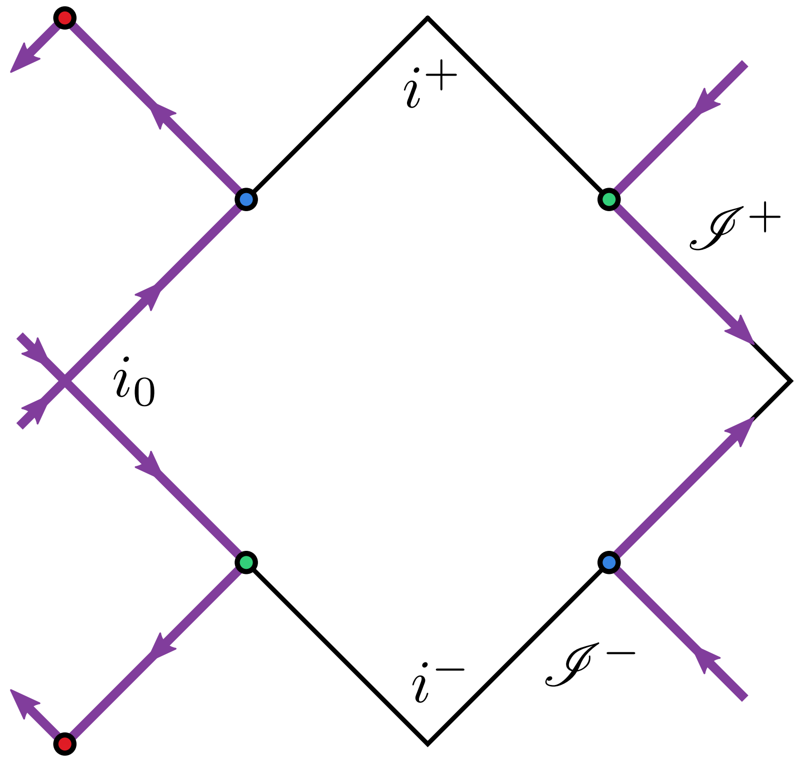




Extension of conformally soft field configuration with sources inside the inverted patch is indistinguishable from Minkowski perspective

- Can also construct fully sourceless extension to the cylinder

Constraints

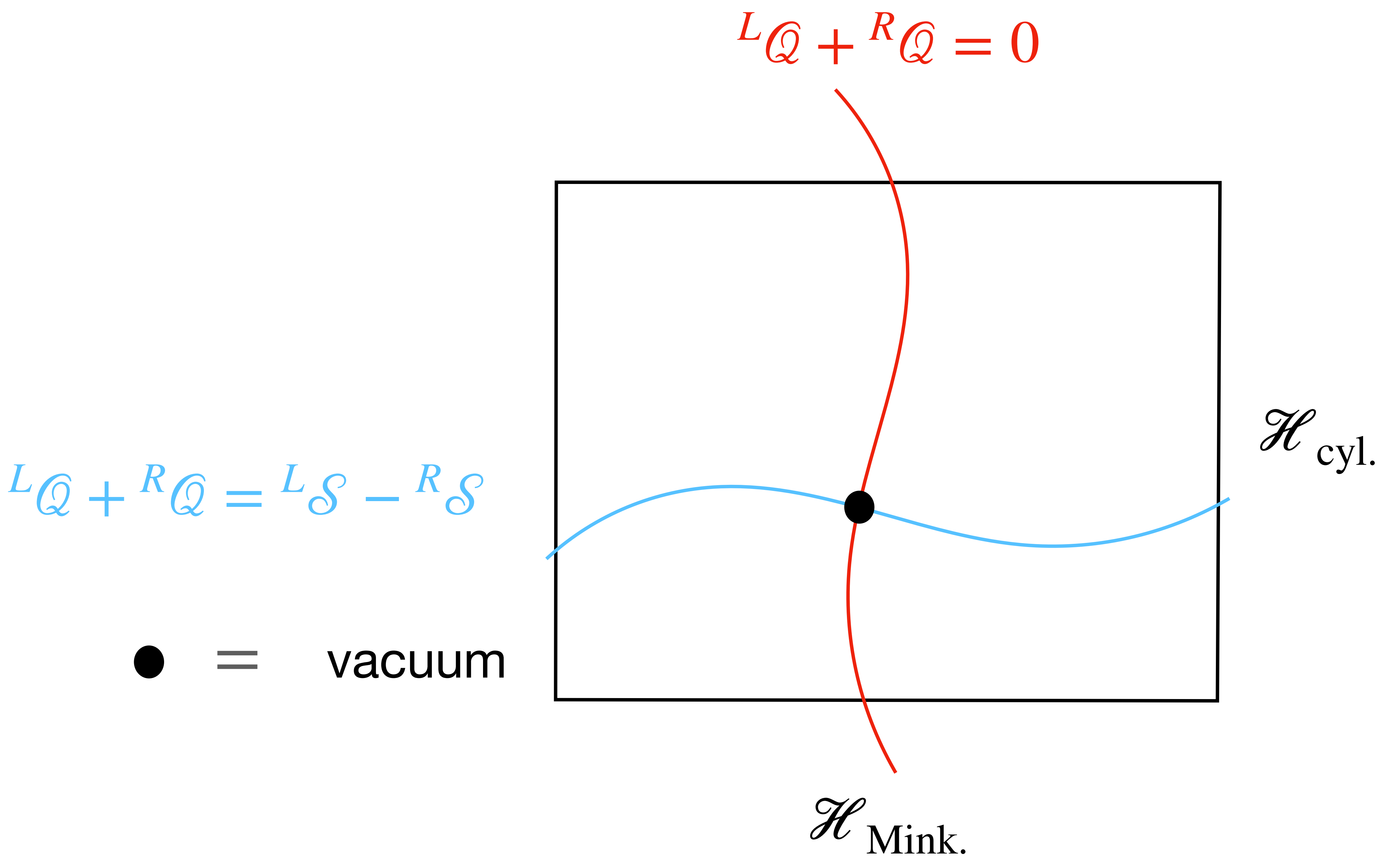
Gauss law imposes constraints on physical state space:

$$\mathscr{H} = \ker \mathscr{Q}^{\text{ent}} \implies \mathscr{H} \subset \mathscr{H}_L \otimes \mathscr{H}_R$$

- Cauchy slices in Einstein static universe are compact $\Longrightarrow$ large gauge charges vanish

$$^{\text{Ein.}}\mathscr{Q}^{\text{ent}}(\mathbf{w}) = {}^L\mathscr{Q}(\mathbf{w}) + {}^R\mathscr{Q}(\mathbf{w})$$

[Freidel, Donnelly '16]



$${}^R\mathscr{Q} \equiv -i\langle {}^RA^{\text{G}}, A\rangle$$

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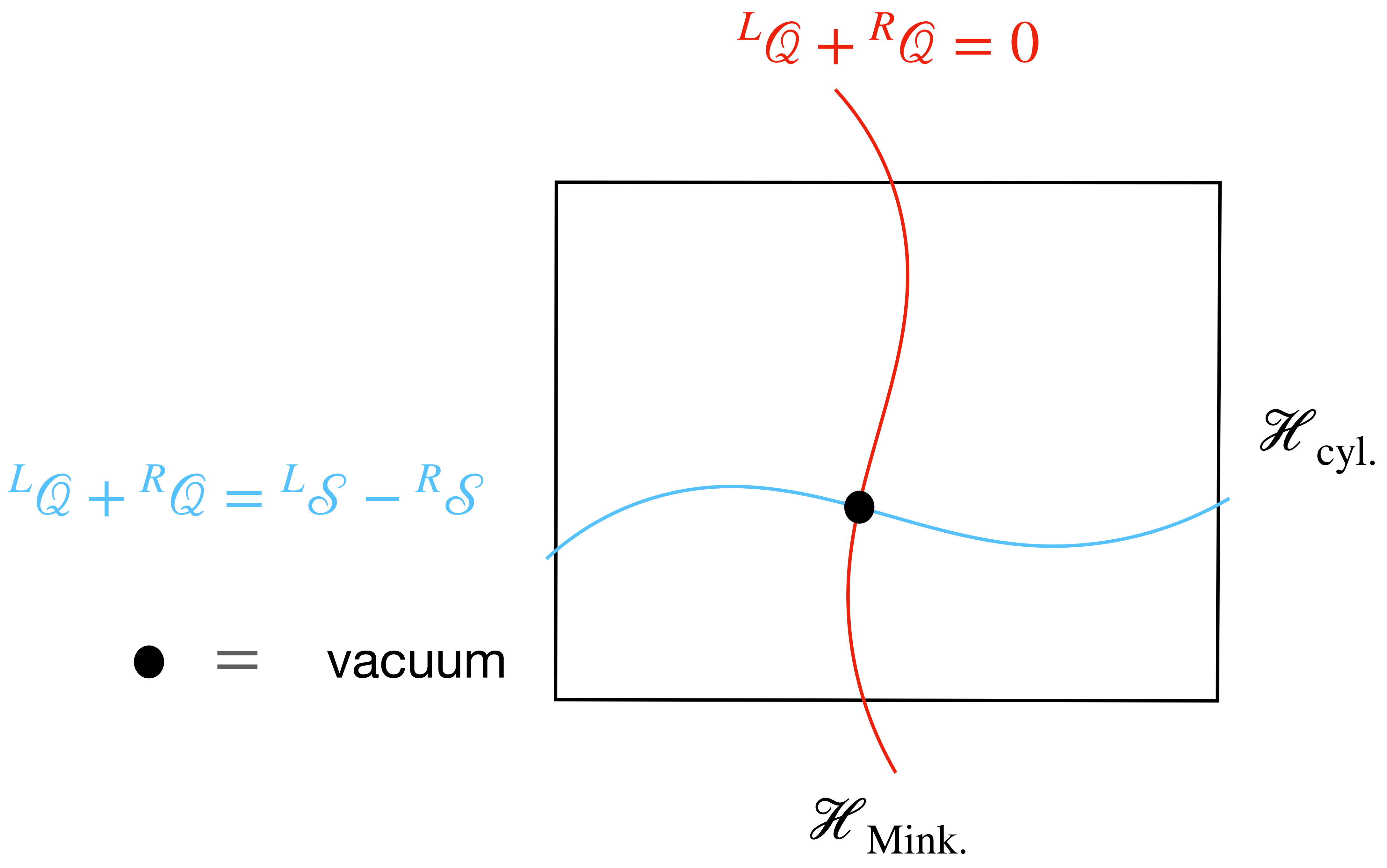
[Freidel, Donnelly '16]

- On the other hand, large gauge charges in Minkowski space may be non-zero:

$${}^{\text{Mink.}}\mathscr{Q}^{\text{ent}}(\mathbf{w}) = {}^{L}\mathscr{Q}(\mathbf{w}) + {}^{R}\mathscr{Q}(\mathbf{w}) + \boxed{{}^{L}\mathscr{S}(\mathbf{w}) - {}^{R}\mathscr{S}(\mathbf{w})}$$

— picked out by the condition that it commutes with the Minkowski CS and G modes



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$$\boxed{{}^{R}\mathscr{S} \equiv -i\langle {}^{R}A^{\text{E}}, A\rangle}$$

$$A^{\text{CS}} = 2\,{}^{L}A^{\text{G}} + {}^{L}A^{\text{E}} + {}^{R}A^{\text{E}}$$

$$A^{\text{G}} = {}^{L}A^{\text{G}} + {}^{R}A^{\text{G}}$$

Physical state space

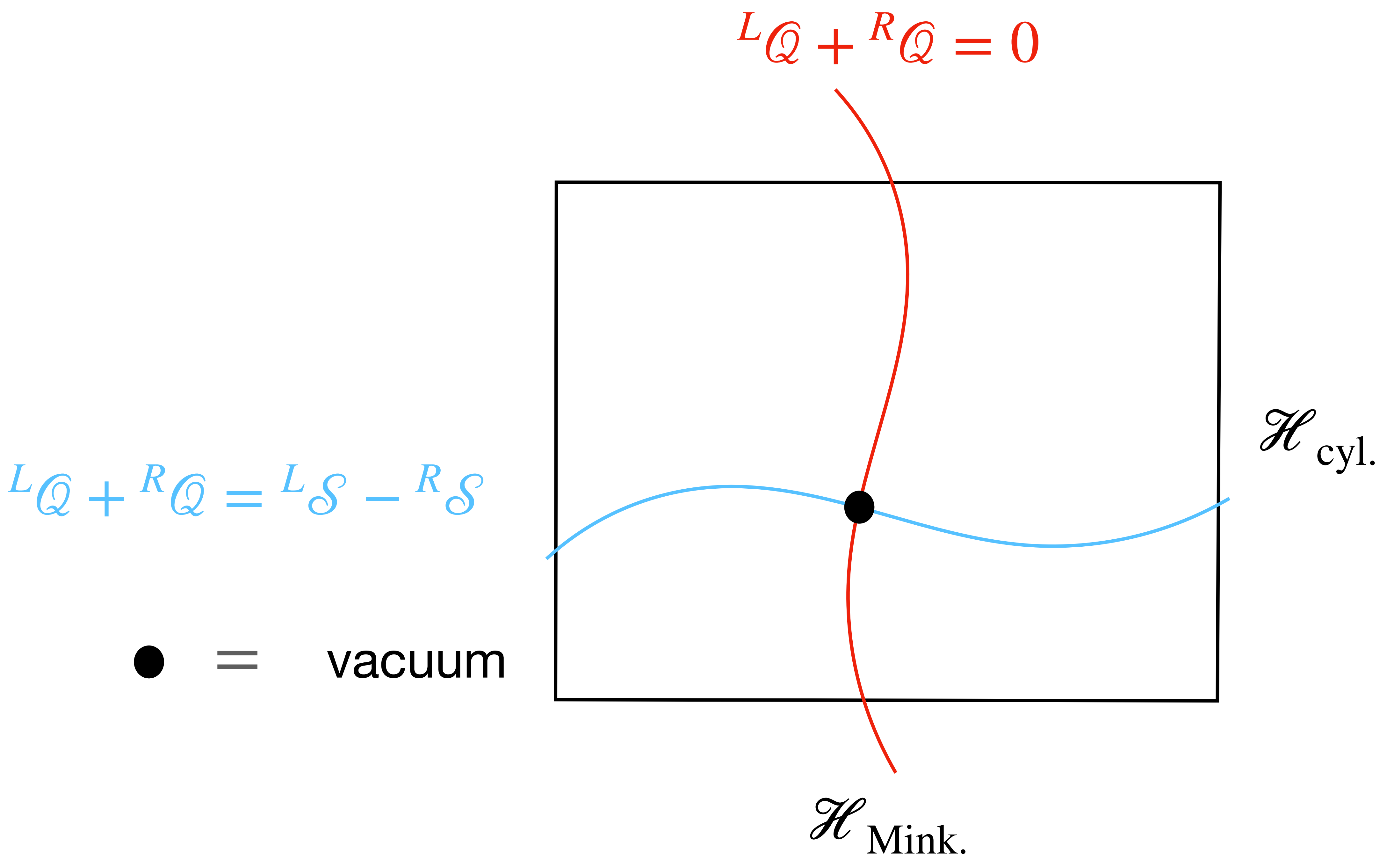
Minkowski theory

- Define the vacuum state $\mathcal{Q}_{LG}|0\rangle = 0$

- Use $\mathcal{S}$ to generate states of arbitrary large gauge charge:

$$|q\rangle = e^{i\mathcal{S}[q]}|0\rangle$$

$|q\rangle$ is physical: ${}^{\text{Mink.}}\mathcal{Q}^{\text{ent}}(\mathbf{w})|q\rangle = 0$



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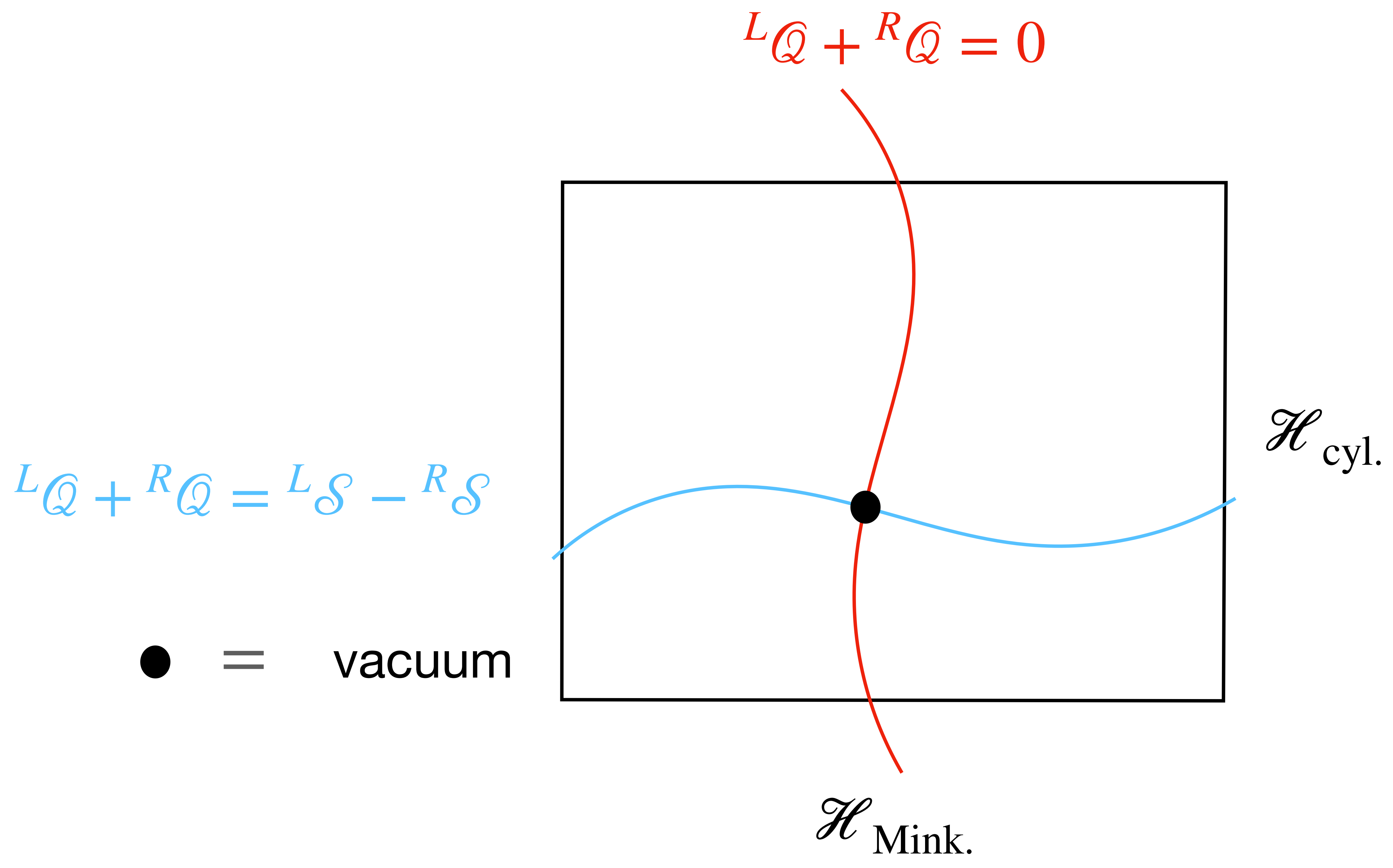

2D Green's function

$$\langle A^{\text{G}}(\mathbf{w}), A^{\text{CS}}(\mathbf{w}')\rangle = -(4\pi)^2 i \mathrm{d}_{\mathbf{w}} \mathrm{d}_{\mathbf{w}'} \mathrm{G}(\mathbf{w}, \mathbf{w}')$$

$$\mathcal{S}[q] = -i\langle A^{\text{CSI}}[q], A\rangle,$$

$$A^{\text{CSI}}[q] = \frac{1}{4\pi}\int \epsilon^{(2)}(\mathbf{w}) q(\mathbf{w}) A^{\text{CSI}}(\mathbf{w}, \infty)$$

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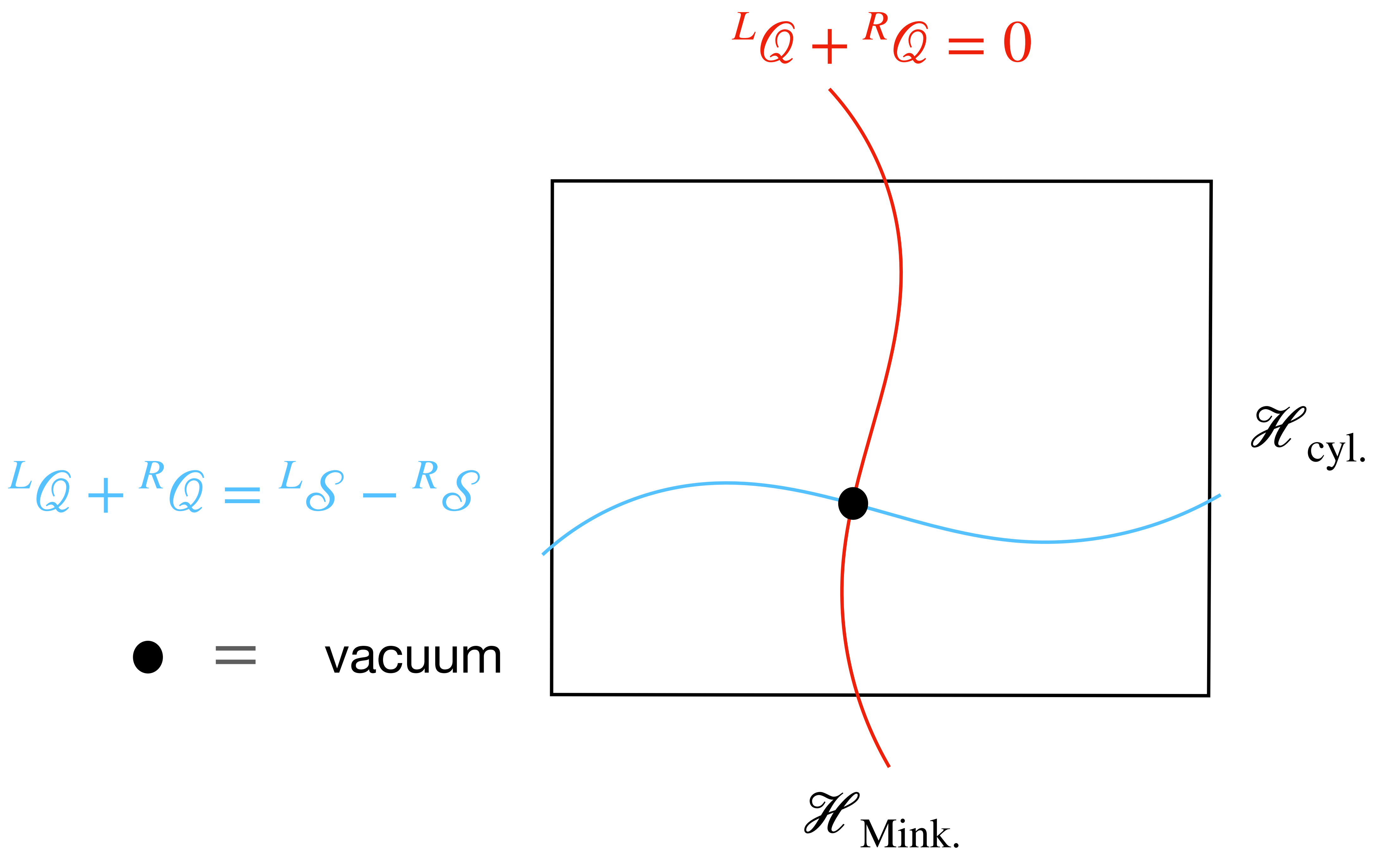
- Define ${}^{R}\rho[q] = \text{Tr}_{{}^{L}\mathcal{H}}|q\rangle\langle q|$

- Goldstone dressing admits decomposition in terms of L and R

Goldstone and edge modes $\Longrightarrow$ ${}^{R}\rho[q]$, ${}^{R}\rho[0]$ have the same von Neumann entropy



$$\mathcal{S}[q] = \frac{1}{2}{}^{R}\mathcal{S} + {}^{L}(\cdots)$$

Entanglement in the soft sector

$\rho_{\text{vac.}} = |0\rangle\langle 0|$ satisfies $[\mathcal{Q}, \rho_{\text{vac}}] = 0 \implies [{}^L\mathcal{Q}, \rho_{\text{vac}}] = -[{}^R\mathcal{Q}, \rho_{\text{vac}}]$

- tracing over the left sector we find $[{}^R\mathcal{Q}, {}^R\rho[0]] = 0$ so ${}^R\rho[0]$ decomposes into blocks of definite R charge

$$ {}^R\rho[0] = \int \mathscr{E}[q] p[q] {}^R\rho[0,q] $$

$$ \rho[0] = \begin{pmatrix} \rho[\mathbf{0}, \mathbf{q}_1] & \mathbf{0} & \cdots & \\ \mathbf{0} & \rho[\mathbf{0}, \mathbf{q}_2] & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \rho[\mathbf{0}, \mathbf{q}_3] & \cdots \\ \cdots & & & \end{pmatrix} $$

- associated von Neumann entropy receives two contributions

$$ S_{\text{vN}}\left({}^R\rho[0]\right) = S_{\text{Sh}}(p) + \int \mathscr{E}[q] p[q] S_{\text{vN}}\left({}^R\rho[0,q]\right) $$

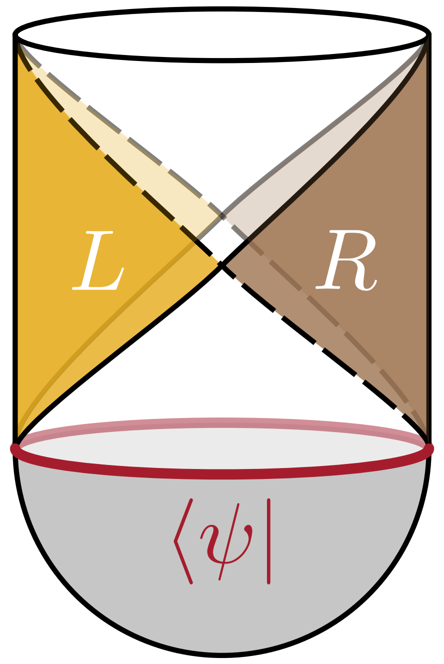
associated with classical probability distribution

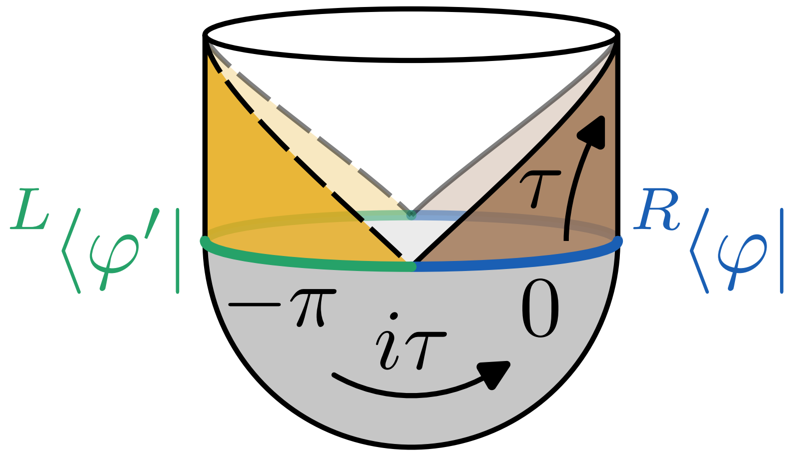
$\implies$ It remains to determine $p[q]$

Entanglement in the soft sector

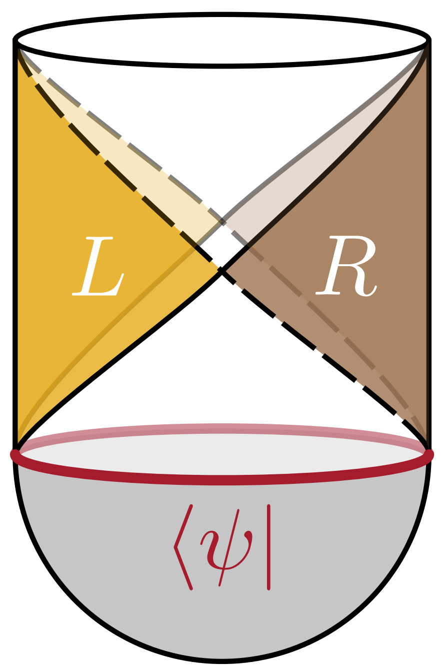
Consider correlator of R operators in the global vacuum:

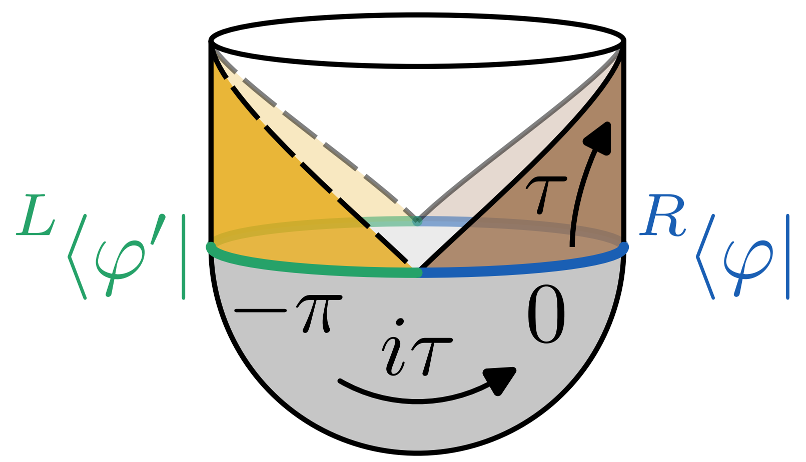
$$\langle 0 | {}^R\mathcal{O} \dots {}^R\mathcal{O} | 0 \rangle = \frac{\int_{i\tau=0}^{i\tau=2\pi} \mathscr{E}[A]\, e^{-I_{2\pi}[A]}\ {}^R\mathcal{O} \dots {}^R\mathcal{O}}{{}^R Z_{2\pi}} = \int \mathscr{E}[q]\, p[q]\, \mathrm{tr}\left({}^R\mathcal{O} \dots {}^R\mathcal{O}\, {}^R\rho[0,q]\right)$$





Entanglement in the soft sector





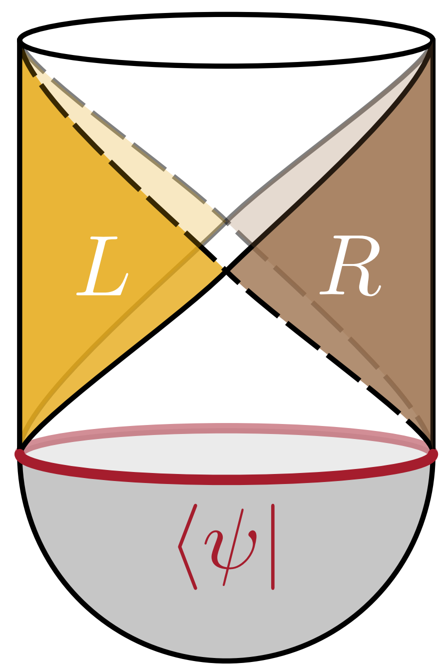
Consider correlator of R operators in the global vacuum:

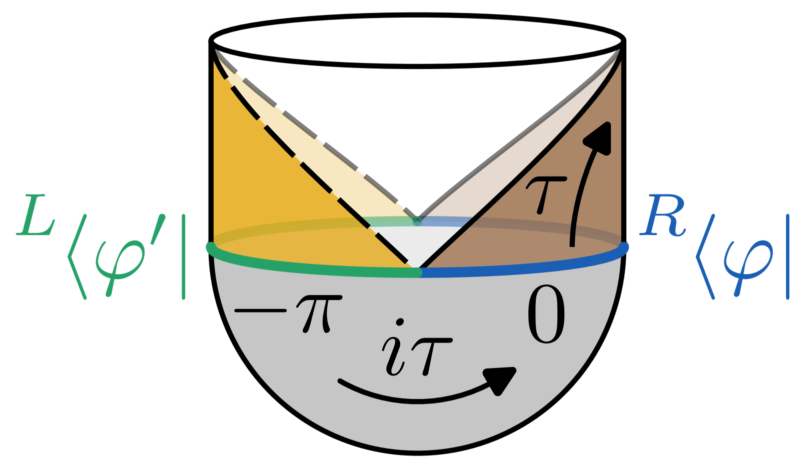
$$\langle 0|\,{}^R\mathcal{O}\ldots{}^R\mathcal{O}\,|0\rangle \underset{(1.)}{=} \frac{\int_{i\tau=0}^{i\tau=2\pi}\mathscr{E}[A]\,e^{-I_{2\pi}[A]}\;{}^R\mathcal{O}\ldots{}^R\mathcal{O}}{{}^RZ_{2\pi}} = \int \mathscr{E}[q]\,p[q]\,\mathrm{tr}\left({}^R\mathcal{O}\ldots{}^R\mathcal{O}\,{}^R\rho[0,q]\right)$$

- insert the identity in (1.) as an integral over sectors of definite ${}^R\mathcal{Q}$

$$1 = \int \mathscr{E}[q]\delta(q - {}_{\mathbf{z}}\star\,\pi^*_{\partial^R\Sigma}\star F)$$

Entanglement in the soft sector





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- insert the identity in (1.) as an integral over sectors of definite ${}^R\mathcal{Q}$

$$1 = \int \mathscr{E}[q] \delta(q - {}_{\mathbf{z}} \star \pi^*_{\partial^R\Sigma} \star F)$$

- shift the integration variable $A = A' + {}^R A^{\mathrm{E}}$

$$\langle 0 | {}^R\mathcal{O} \dots {}^R\mathcal{O} | 0 \rangle = \int \mathscr{E}[q] e^{-I_{2\pi}[{}^R A^{\mathrm{E}}]}\, {}^R\mathscr{C}\Big|_{\pi^*_{\partial R_\Sigma} * F = 0}, \quad {}^R\mathscr{C}\Big|_{\pi^*_{\partial R_\Sigma} * F = 0} = \frac{\int_{i\tau=0}^{i\tau=2\pi} \mathscr{E}[A]\, e^{-I_{2\pi}[A]} \delta(\pi^*_{\partial^R\Sigma} * F)\, {}^R\mathcal{O}' \dots}{{}^R Z_{2\pi}}.$$

Entanglement in the soft sector

$$\langle 0| {}^R\mathcal{O}\cdots{}^R\mathcal{O}|0\rangle = \int \mathcal{E}[q] e^{-I_{2\pi}[{}^R A^{\mathrm{E}}]}\, {}^R\mathscr{C}\Big|_{\pi^*_{\partial^R\Sigma} {}^*F=0} = \int \mathcal{E}[q]\, p[q]\, \mathrm{tr}\left({}^R\mathcal{O}\cdots{}^R\mathcal{O}\, {}^R\rho[0,q]\right)$$

(2.)

Setting: $${}^R\mathcal{O} = I \implies {}^R Z_\beta = {}^R Z^{\mathrm{E}}_\beta\, {}^R Z_\beta[0], \quad {}^R Z^{\mathrm{E}}_\beta = \int \mathcal{E}[q] e^{-I_\beta[{}^R A^{\mathrm{E}}]}$$

$${}^R\mathcal{O} = {}^R\mathcal{Q} \implies p[q] = \frac{e^{-I_{2\pi}[{}^R A^{\mathrm{E}}[q]]}}{{}^R Z^{\mathrm{E}}_\beta}$$

Path integral in zero charge sector:

$${}^R\mathscr{C}\Big|_{\pi^*_{\partial^R\Sigma} {}^*F=0} = \frac{\int_{i\tau=0}^{i\tau=2\pi} \mathcal{E}[A]\, e^{-I_{2\pi}[A]} \delta(\pi^*_{\partial^R\Sigma} * F)^R\mathcal{O}'\cdots}{{}^R Z_{2\pi}}$$

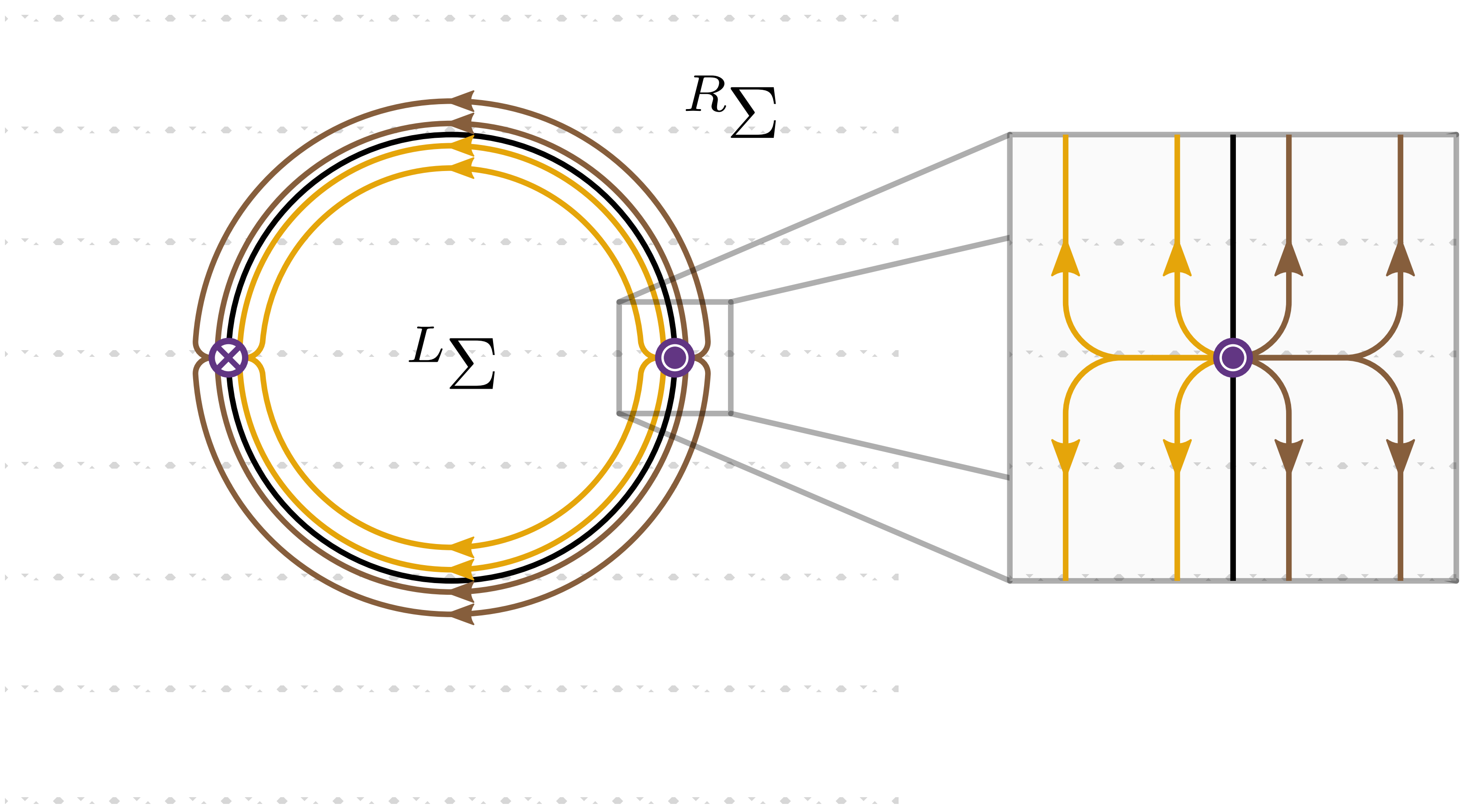
Entanglement in the soft sector

$$\langle 0 | {}^R\mathcal{O} \cdots {}^R\mathcal{O} | 0 \rangle = \int \mathcal{E}[q] e^{-I_{2\pi}[{}^R A^{\mathrm{E}}]} \, {}^R\mathscr{C} \Big|_{\pi^*_{\partial R_\Sigma} * F = 0} = \int \mathcal{E}[q] \, p[q] \, \mathrm{tr} \left({}^R\mathcal{O} \cdots {}^R\mathcal{O} \, {}^R\rho[0,q] \right)$$

(2.)

Setting: ${}^R\mathcal{O} = I \implies {}^R Z_\beta = {}^R Z^{\mathrm{E}}_\beta \, {}^R Z_\beta[0], \quad {}^R Z^{\mathrm{E}}_\beta = \int \mathcal{E}[q] e^{-I_\beta[{}^R A^{\mathrm{E}}]}$

$${}^R\mathcal{O} = {}^R\mathcal{Q} \implies p[q] = \frac{e^{-I_{2\pi}[{}^R A^{\mathrm{E}}[q]]}}{{}^R Z^{\mathrm{E}}_\beta}$$



- Classical probability distribution of ``edge modes” given by on-shell action of conformally soft modes!

- Donnelly-Wall ``static” edge modes in R patch related to the log-mode constituents of CS modes by gauge transformation

Path integral in zero charge sector:

$${}^R\mathscr{C} \Big|_{\pi^*_{\partial R_\Sigma} * F = 0} = \frac{\int_{i\tau=0}^{i\tau=2\pi} \mathcal{E}[A] \, e^{-I_{2\pi}[A]} \delta(\pi^*_{\partial^R \Sigma} * F) {}^R\mathcal{O}' \cdots}{{}^R Z_{2\pi}}$$

Bulk entanglement from CCFT

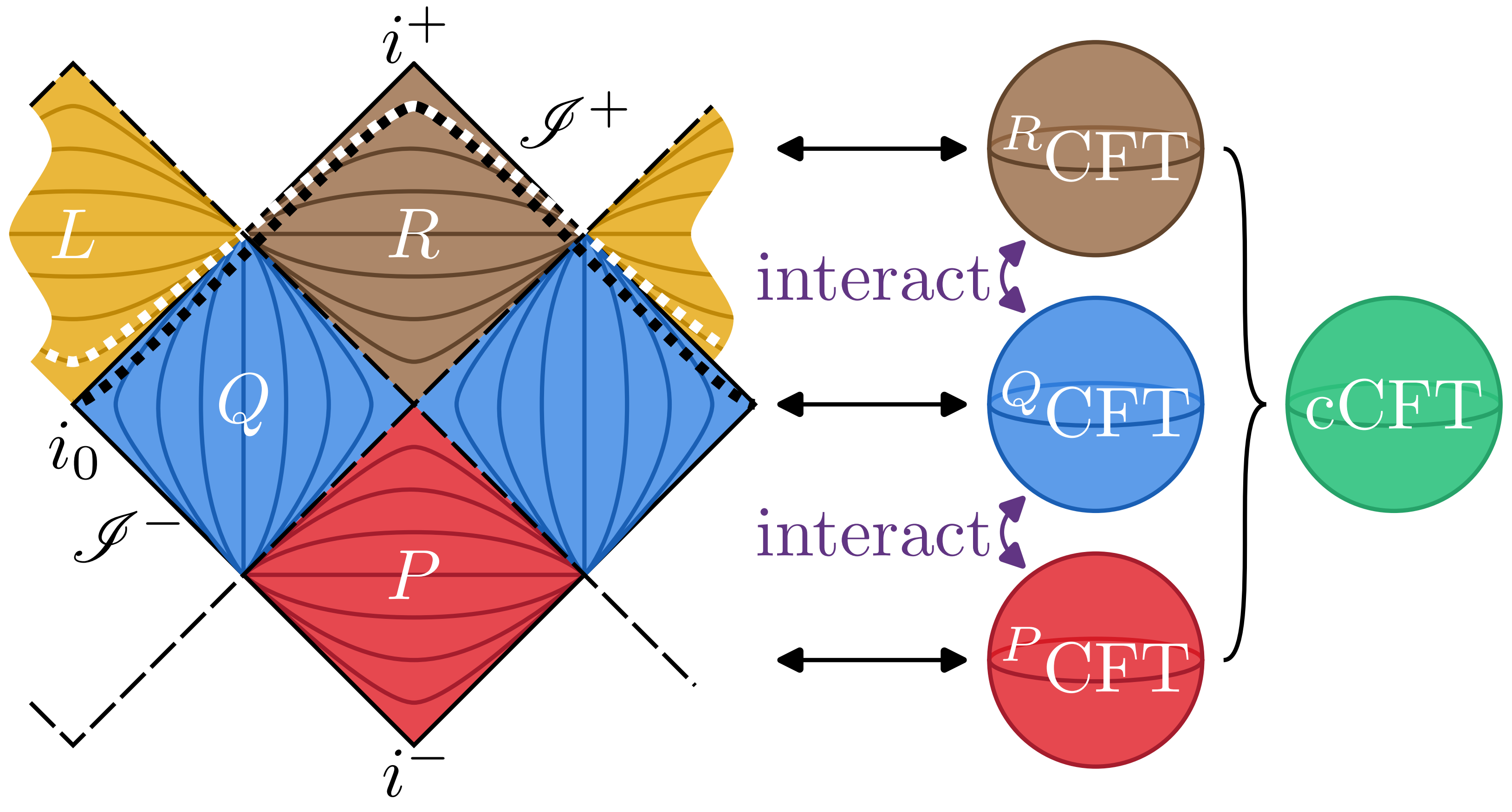
Thermofield double vacuum $\Longrightarrow$ celestial amplitudes decompose in terms of L and R CFT correlators:

$$\langle 0|\bullet|0\rangle = \langle e^{-\mathscr{K}^+ - (\mathscr{K}^+)^\dagger}\bullet\rangle_{{}^L\mathrm{CFT},{}^R\mathrm{CFT}}$$

- entangling operator: $\mathscr{K}^+ = -\int_0^\infty \frac{d\lambda\, e^{-\pi\lambda}}{(2\pi)^3}\frac{1+\lambda^2}{2\lambda}\int \epsilon^{(2)}\ {}^L\tilde{\mathcal{O}}^{1-i\lambda}\cdot{}^R\mathcal{O}^{1+i\lambda}$

- Bulk subregion eg . $\bullet$ are in R; tracing L out $\Longrightarrow$ $\langle e^{-{}^R\mathscr{K}}\ \bullet\rangle_{{}^R\mathrm{CFT}}$

$${}^R\mathscr{K} = -\int_0^\infty \frac{d\lambda\, e^{-2\pi\lambda}}{(2\pi)^3}\frac{1+\lambda^2}{2\lambda}\int \epsilon^{(2)}\ {}^R\mathcal{O}^{1-i\lambda}\cdot{}^R\mathcal{O}^{1+i\lambda}$$



- Build up flat spacetime/celestial amplitudes from interacting 2D CFTs?

Summary

- Conformal primary modes in subregions partitioning $\mathscr{I}^+$ into two halves

- In 4D Maxwell theory = CFT. In CFT inversions $\sim$ shadows: - $i\epsilon$ matters!

  - asymptotic expansions and matching matter!

- Minkowski vacuum = TFD with respect to subregions in the non-soft sector

- Soft modes $\Longrightarrow$ constraint relating ``asymptotic” charges ${}^L\mathcal{Q}, {}^R\mathcal{Q}$ of the subregions

- In vacuum - fluctuations in ${}^R\mathcal{Q}$ lead to Donnelly-Wall edge mode entropy; log CS modes $\sim$ edge modes

Outlook

- Generalize to (3+1)-d gravity — not conformal, but similar ``conformally soft” modes present
- Edge modes and entanglement in Carroll FT?
- Infrared divergences and soft effective actions?
- Spacetime fluctuations?
- Implications for black hole information paradox…?

Thank you!