Higher spin ‘superrotations’ from the BBB action

Sucheta Majumdar

Laboratoire de Physique, ENS de Lyon

3rd Carroll Workshop, Thessaloniki

October 2-6, 2023

Motivation

- What is the Bengtsson-Bengtsson-Brink (BBB) action?

  A higher spin action in the $\underbrace{\text{light-cone formalism}}_{\text{light-cone coordinates+ light-cone gauge}}$ with cubic interaction vertices

[Bengtsson-Bengtsson-Brink, 1983]

Motivation

- What is the Bengtsson-Bengtsson-Brink (BBB) action?

  A higher spin action in the $\underbrace{\text{light-cone formalism}}_{\text{light-cone coordinates+ light-cone gauge}}$ with cubic interaction vertices

  [Bengtsson-Bengtsson-Brink, 1983]

- Why consider higher spins?

  Some interesting results for higher spin supertranslations and superrotations at null infinity

  [Campoleoni-Francia-Heissenberg]

  More recently, higher spin Carrollian algebras and field theories

  [Bekaert, Campoleoni, Nyugen, Oblak, Pekar West, ...]

- Why the BBB action?

  An interacting model for studying HS generalizations of supertranslations/ superrotations

Motivation

- What is the Bengtsson-Bengtsson-Brink (BBB) action?

  A higher spin action in the $\underbrace{\text{light-cone formalism}}$ with cubic interaction vertices

  light-cone coordinates+ light-cone gauge

  [Bengtsson-Bengtsson-Brink, 1983]

- Why consider higher spins?

  Some interesting results for higher spin supertranslations and superrotations at null infinity

  [Campoleoni-Francia-Heissenberg]

  More recently, higher spin Carrollian algebras and field theories

  [Bekaert, Campoleoni, Nyugen, Oblak, Pekar West, ...]

- Why the BBB action?

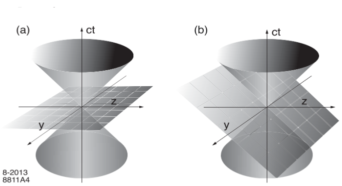
  An interacting model for studying HS generalizations of supertranslations/ superrotations

  Another motivation: Andrea and Dario had asked me once

Focus of the talk:

A particular example of higher spin theories in a reduced phase space of on-shell modes

Light-cone coordinates: The front form



DOI:10.1016/j.physrep.2015.05.001

"Forms of relativistic dynamics" [Dirac '49]

(a) Instant form: time x^0

Initial data on a spatial hyperplane ($x^0 = 0$)

(b) Front form: time $x^+ = \frac{x^0+x^3}{\sqrt{2}}$

Initial data on a null hyperplane ($x^+ = 0$)

- Poincaré generators in the instant form: $(P_\mu, M_{\mu\nu})$

$$[P, P] \sim 0\,, \quad [P, M] \sim P\,, \quad [M, M] \sim M$$

$(P_0, M_{0i}) \;\rightarrow\;$ four "Hamiltonians"

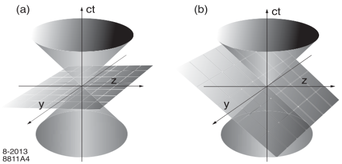
- Poincaré generators in light-cone coordinates, $x^\mu = (x^+, x^-, x^i)\,, \quad i = 1, 2$

Kinematical $K = \{P_i, P_-, M_{ij}, M_{i-}, M_{+-}\}\,,$

Dynamical $D = \{P_+, M_{i+}\} \rightarrow$ three "Hamiltonians"

Non-linear corrections to $\mathbb{D}$ give us the dynamics of the interacting theory

Light-cone coordinates: The front form



DOI:10.1016/j.physrep.2015.05.001

"Forms of relativistic dynamics" [Dirac '49]

(a) Instant form: time x^0

Initial data on a spatial hyperplane ($x^0 = 0$)

(b) Front form: time $x^+ = \frac{x^0+x^3}{\sqrt{2}}$

Initial data on a null hyperplane ($x^+ = 0$)

- Poincaré generators in the instant form: $(P_\mu, M_{\mu\nu})$

$$[P,P] \sim 0\,, \quad [P,M] \sim P\,, \quad [M,M] \sim M$$

$(P_0, M_{0i}) \;\rightarrow\;$ four "Hamiltonians"

- Poincaré generators in light-cone coordinates, $x^\mu = (x^+, x^-, x^i)\,, \quad i = 1,2$

Kinematical $K = \{P_i, P_-, M_{ij}, M_{i-}, M_{+-}\}\,,$

Dynamical $D = \{P_+, M_{i+}\} \rightarrow$ three "Hamiltonians"

Non-linear corrections to $\mathbb{D}$ give us the dynamics of the interacting theory

$\rightarrow$ a key advantage when the interactions are not known

Light-cone Poincaré algebra in $d = 4$

- Non-vanishing commutators of the Poincaré algebra

$$J^+ \;=\; \frac{J^{+1} + iJ^{+2}}{\sqrt{2}}\,, \quad \bar{J}^+ \;=\; \frac{J^{+1} - iJ^{+2}}{\sqrt{2}}\,, \quad J \;=\; J^{12}\,, \quad H \;=\; P_+ = -P^- \,.$$

$$\begin{aligned} [H, J^{+-}] &= -iH\,, & [H, J^+] &= -iP\,, & [H, \bar{J}^+] &= -i\bar{P} \\ [P^+, J^{+-}] &= iP^+\,, & [P^+, J^-] &= -iP\,, & [P^+, \bar{J}^-] &= -i\bar{P} \\ [P, \bar{J}^-] &= -iH\,, & [P, \bar{J}^+] &= -iP^+\,, & [P, J] &= P \end{aligned}$$

... and many more

[Bengtsson-Bengtsson-Brink '83]

- Underlying Carrollian structure

Rotation $\mathbb{J} = \{J^{12}, J^{+-}, J^+, \bar{J}^+\}$, Boosts $\mathbb{K} = \{J^-, \bar{J}^-\}$

Translations $\mathbb{P} = \{P, \bar{P}, P_-\}$, Hamiltonian $\mathbb{H} = P_+$

$$\begin{aligned} [\mathbb{J}, \mathbb{J}] &= \mathbb{J}\,, & [\mathbb{J}, \mathbb{P}] &= \mathbb{P}\,, & [\mathbb{J}, \mathbb{K}] &= \mathbb{K} \\ [\mathbb{J}, \mathbb{H}] &= 0\,, & [\mathbb{H}, \mathbb{P}] &= 0\,, & [\mathbb{H}, \mathbb{K}] &= 0 \\ [\mathbb{P}, \mathbb{P}] &= 0\,, & [\mathbb{K}, \mathbb{K}] &= 0\,, & [\mathbb{P}, \mathbb{K}] &= \mathbb{H} \end{aligned}$$

Light-cone Poincaré algebra in $d = 4$

- Non-vanishing commutators of the Poincaré algebra

$$J^+ = \frac{J^{+1} + iJ^{+2}}{\sqrt{2}}\,, \quad \bar{J}^+ = \frac{J^{+1} - iJ^{+2}}{\sqrt{2}}\,, \quad J = J^{12}\,, \quad H = P_+ = -P^-\,.$$

$$\begin{array}{lll} [H, J^{+-}] = -iH\,, & [H, J^+] = -iP\,, & [H, \bar{J}^+] = -i\bar{P} \\ [P^+, J^{+-}] = iP^+\,, & [P^+, J^-] = -iP\,, & [P^+, \bar{J}^-] = -i\bar{P} \\ [P, \bar{J}^-] = -iH\,, & [P, \bar{J}^+] = -iP^+\,, & [P, J] = P \end{array}$$

... and many more

[Bengtsson-Bengtsson-Brink '83]

- Underlying Carrollian structure

Rotation $\mathbb{J} = \{J^{12}, J^{+-}, J^+, \bar{J}^+\}$, Boosts $\mathbb{K} = \{J^-, \bar{J}^-\}$

Translations $\mathbb{P} = \{P, \bar{P}, P_-\}$, Hamiltonian $\mathbb{H} = P_+$

$$\begin{array}{lll} [\mathbb{J}, \mathbb{J}] = \mathbb{J}\,, & [\mathbb{J}, \mathbb{P}] = \mathbb{P}\,, & [\mathbb{J}, \mathbb{K}] = \mathbb{K} \\ [\mathbb{J}, \mathbb{H}] = 0\,, & [\mathbb{H}, \mathbb{P}] = 0\,, & [\mathbb{H}, \mathbb{K}] = 0 \\ [\mathbb{P}, \mathbb{P}] = 0\,, & [\mathbb{K}, \mathbb{K}] = 0\,, & [\mathbb{P}, \mathbb{K}] = \mathbb{H} \end{array}$$

- In terms of the Kinematical-Dynamical split

$$\mathbb{K} = \{P_i, P_-, M_{ij}, M_{+-}\}\,, \quad \mathbb{D} = \{P_+, M_{i+}\}$$

$$[\mathbb{K}, \mathbb{K}] = \mathbb{K}\,, \quad [\mathbb{K}, \mathbb{D}] = \mathbb{D}\,, \quad [\mathbb{D}, \mathbb{D}] = 0$$

Spin 1 as a prelude

- Light-cone gauge: Set the lower minus components to zero

$$A_- = -A^+ = -\frac{A^0 + A^3}{\sqrt{2}} = 0$$

- Maxwell equations: $\partial_\mu F^{\mu\nu}$ = 0 a) Constraint

$$(\nu = +): \quad \partial_-^2 A^- + \partial_i \partial_- A^i = 0 \quad \Rightarrow \quad A^- = -\frac{\partial_i A^i}{\partial_-} + \alpha(x^+, x^i)\, x^- + \beta(x^+, x^i)$$

b) Trivial equation

$$(\nu = -): \quad \text{relates } \alpha \text{ and } \beta \quad \Rightarrow \quad \text{only one arbitrary constant}$$

A further choice: set the constants to zero $\rightarrow$ more on this later

c) Dynamical equation $(\nu = i)$

$$(2\partial_-\partial_+ - \partial_i\partial^i)A^j = \Box_{lc} A^j = 0 \Rightarrow \text{two propagating modes of the photon}$$

The "inverse derivative" operator [Mandelstam '83, Leibbrandt '83]

$$\partial_- f(x^-) = g(x^-) \quad \Rightarrow \quad f(x^-) = \frac{1}{\partial_-} g(x^-) = -\int \epsilon(x^- - y^-)\, g(y^-)\, dy^- + \text{“}constant\text{”}$$

Electromagnetism in light-cone formalism

- Complexify the x^i

$$x = \frac{x^1 + ix^2}{\sqrt{2}}\,, \quad \bar{x} = \frac{x^1 - ix^2}{\sqrt{2}} \quad \partial_i \to (\partial, \bar{\partial})$$

$$A^i \to (A, \bar{A}) : \quad \pm 1 \text{ helicity states of the photon}$$

- Light-cone action for electromagnetism

$$\mathcal{S} = \frac{1}{2} \int d^4x \; \bar{A} \, \Box_{lc} \, A = \int d^4x \; \bar{A} \, (\partial_+ \partial_- - \partial \bar{\partial}) \, A$$

$\rightarrow$ lc_2 formalism of electromagnetism

Electromagnetism in light-cone formalism

- Complexify the x^i

$$x = \frac{x^1 + ix^2}{\sqrt{2}}\,, \quad \bar{x} = \frac{x^1 - ix^2}{\sqrt{2}} \quad \partial_i \to (\partial, \bar{\partial})$$

$$A^i \to (A, \bar{A}) : \quad \pm 1 \text{ helicity states of the photon}$$

- Light-cone action for electromagnetism

$$\mathcal{S} = \frac{1}{2} \int d^4x\, \bar{A}\, \Box_{lc}\, A = \int d^4x\ \bar{A}\, (\partial_+ \partial_- - \partial \bar{\partial})\, A$$

$\rightarrow$ lc_2 formalism of electromagnetism

- Hamiltonian and Poisson brackets (recall: x^+ is time)

$$\pi = \frac{\delta \mathcal{L}}{\delta(\partial_+ A)} = -\,\partial_- \bar{A}\,, \quad \bar{\pi} = \frac{\delta \mathcal{L}}{\delta(\partial_+ \bar{A})} = -\,\partial_- A$$

$(\pi, \bar{\pi})$ not independent variables $\Rightarrow$ Half the d.o.f than in the 3+1 formalism

- Poisson brackets

$$[A(x), \bar{A}(y)] = \epsilon(x^- - y^-)\, \delta^{(2)}(x - y)\,, \quad [\, A(x), A(y)\,] = [\, \bar{A}(x), \bar{A}(y)\,] = 0\,.$$

Two paths to light-cone action for interacting theories

1) Gauge-fixing a covariant action

- Spin 1: Maxwell or Yang-Mills action

$$A_- = 0$$

- Spin 2: Einstein-Hilbert or Fierz-Pauli action

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \quad h_{-\mu} = 0$$

- Higher spins: Fronsdal action

  Spin 3: symmetric rank-3 tensor $\phi_{\mu\nu\rho}$

$$\phi_{-\mu\nu} - \frac{1}{4}\eta_{\mu\nu}\phi^{\sigma}{}_{\sigma-} = 0$$

  Spin λ: symmetric rank-λ tensor

$$\phi_{-\mu_1\mu_2\ldots\mu_{\lambda-1}} - \frac{1}{4}\eta_{(\mu_1\mu_2}\phi^{\sigma}_{\mu_3\ldots\mu_{\lambda-1})\sigma-} = 0$$

2) Light-cone deformation procedure

$\rightarrow$ Deform the free LC action and derive interaction vertices from closure of Poincaré algebra

$\rightarrow$ Gauge constraints may be solved to eliminate off-shell modes from the theory

Many successes: Higher spins, Quintic action for LC gravity, Super Yang-Mills, Supergravity, etc.

[Ananth, Akshay, Brink, Hesse, Kim, Kovacs, Majumdar, Mali, Ramond, Shah, ...]

The Bengtsson-Bengtsson-Brink (BBB) action

A higher spin action in the light-cone gauge with cubic interaction vertices

- for even spins,

$$S[\phi,\bar{\phi}] = \int d^4x \left(\frac{1}{2}\bar{\phi}\Box\phi + \alpha \sum_{n=0}^{\lambda} (-1)^n \binom{\lambda}{n} \bar{\phi}(\partial_-)^{(\lambda-1)} \left[\frac{\bar{\partial}^{\lambda-n}}{\partial_-^{\lambda-n}}\phi \frac{\bar{\partial}^{\lambda}}{\partial_-^{\lambda}}\phi \right] + c.c. \right)$$

The Bengtsson-Bengtsson-Brink (BBB) action

A higher spin action in the light-cone gauge with cubic interaction vertices

- for even spins,

$$S[\phi,\bar{\phi}] = \int d^4x \left(\frac{1}{2}\bar{\phi}\Box\phi + \alpha \sum_{n=0}^{\lambda} (-1)^n \binom{\lambda}{n} \bar{\phi}(\partial_-)^{(\lambda-1)} \left[\frac{\bar{\partial}^{\lambda-n}}{\partial_-^{\lambda-n}}\phi \frac{\bar{\partial}^{\lambda}}{\partial_-^{\lambda}}\phi \right] + c.c. \right)$$

- for odd spins, closure of Poincaré algbera for odd spins demands a structure constant

$$S[\phi^a,\bar{\phi}^a] = \int d^4x \left(\frac{1}{2}\bar{\phi}^a\Box\phi^a + \alpha\, f^{abc} \sum_{n=0}^{\lambda} (-1)^n \binom{\lambda}{n} \bar{\phi}^a(\partial_-)^{(\lambda-1)} \left[\frac{\bar{\partial}^{\lambda-n}}{\partial_-^{\lambda-n}}\phi^b \frac{\bar{\partial}^{\lambda}}{\partial_-^{\lambda}}\phi^c \right] + c.c. \right)$$

The Bengtsson-Bengtsson-Brink (BBB) action

A higher spin action in the light-cone gauge with cubic interaction vertices

- for even spins,

$$S[\phi,\bar\phi] = \int d^4x \left(\frac{1}{2}\bar\phi\Box\phi + \alpha \sum_{n=0}^{\lambda} (-1)^n \binom{\lambda}{n} \bar\phi(\partial_-)^{(\lambda-1)} \left[\frac{\bar\partial^{\lambda-n}}{\partial_-^{\lambda-n}}\phi \frac{\bar\partial^{\lambda}}{\partial_-^{\lambda}}\phi \right] + c.c. \right)$$

- for odd spins, closure of Poincaré algbera for odd spins demands a structure constant

$$S[\phi^a,\bar\phi^a] = \int d^4x \left(\frac{1}{2}\bar\phi^a\Box\phi^a + \alpha\, f^{abc} \sum_{n=0}^{\lambda} (-1)^n \binom{\lambda}{n} \bar\phi^a(\partial_-)^{(\lambda-1)} \left[\frac{\bar\partial^{\lambda-n}}{\partial_-^{\lambda-n}}\phi^b \frac{\bar\partial^{\lambda}}{\partial_-^{\lambda}}\phi^c \right] + c.c. \right)$$

Some nice features:

- Involves physical degrees of freedom only: No ghosts, auxiliary fields
- Perturbative approach: symmetries are non-linearly realized on physical fields
- Action written in a helicity basis:

$$h[\phi] = \lambda \quad h[\bar\phi] = -\lambda$$

Closely related to on-shell physics, scattering amplitudes, (anti) self-dual sectors, etc.

[Ananth, Akshay, Brink, Kovacs, Pant, Pandey, Parikh , Theisen ...]

Canonical realization of light-cone Poincaré algebra

Poisson brackets

$$[\phi(x),\bar\phi(y)] \;=\; \epsilon(x^- - y^-)\,\delta^{(2)}(x-y)\,, \quad [\,\phi(x),\phi(y)\,] \;=\; [\,\bar\phi(x),\bar\phi(y)\,] \;=\; 0\,.$$

Poincaré generators in terms of the fields ϕ and $\bar\phi$

$$H = P_+ = \int d^3x\,\partial_-\bar\phi\,\frac{\partial\bar\partial}{\partial_-}\phi \;+\; \text{cubic terms}\,,$$

$$J^- \;=\; \int d^3x\partial_-\bar\phi\left(x\frac{\partial\bar\partial}{\partial_-}\phi + x^-\partial\phi - \lambda\frac{\partial}{\partial_-}\phi\right) \;+\; \text{cubic terms}\,, \quad \cdots$$

$$P = \int d^3x\partial_-\bar\phi\,\partial\phi\,, \quad P_- = d^3x\partial_-\bar\phi\partial_-\phi\,, \quad \cdots$$

$$J = i\int d^3x\partial_-\bar\phi\,(x\bar\partial - \bar x\partial - \lambda)\phi\,,, \quad \cdots$$

which satisfy

$$[H, J^{+-}] \;=\; -iH\,, \qquad [H, J^+] = -iP\,, \qquad [H, \bar J^+] \;=\; -i\bar P$$
$$[P^+, J^{+-}] \;=\; iP^+\,, \qquad [P^+, J^-] \;=\; -iP\,, \qquad [P^+, \bar J^-] \;=\; -i\bar P$$
$$[P, \bar J^-] \;=\; -iH\,, \qquad [P, \bar J^+] \;=\; -iP^+\,, \qquad [P, J] \;=\; P$$

and many more

Canonical realization of light-cone Poincaré algebra

Poisson brackets

$$[\phi(x),\bar\phi(y)] \,=\, \epsilon(x^- - y^-)\,\delta^{(2)}(x-y)\,, \quad [\,\phi(x),\phi(y)\,] \,=\, [\,\bar\phi(x),\bar\phi(y)\,] \,=\, 0\,.$$

Poincaré generators in terms of the fields ϕ and $\bar\phi$

$$H = P_+ = \int d^3x\,\partial_-\bar\phi\,\frac{\partial\bar\partial}{\partial_-}\phi \,+\, \text{cubic terms}\,,$$

$$J^- \,=\, \int d^3x\partial_-\bar\phi\left(x\frac{\partial\bar\partial}{\partial_-}\phi + x^-\partial\phi - \lambda\frac{\partial}{\partial_-}\phi\right) \,+\, \text{cubic terms}\,, \quad \cdots$$

$$P = \int d^3x\partial_-\bar\phi\,\partial\phi\,, \quad P_- \,= d^3x\partial_-\bar\phi\partial_-\phi\,, \quad \cdots$$

$$J = i\int d^3x\partial_-\bar\phi\,(x\bar\partial - \bar x\partial - \lambda)\phi\,,\,, \quad \cdots$$

which satisfy

$$[H, J^{+-}] \,=\, -iH\,, \qquad [H, J^+] = -iP\,, \qquad [H,\bar J^+] \,=\, -i\bar P$$

$$[P^+, J^{+-}] \,=\, iP^+\,, \qquad [P^+, J^-] \,=\, -iP\,, \qquad [P^+,\bar J^-] \,=\, -i\bar P$$

$$[P,\bar J^-] \,=\, -iH\,, \qquad [P,\bar J^+] \,=\, -iP^+\,, \qquad [P,J] \,=\, P$$

and many more

What are the residual gauge symmetries of the BBB action?

The spin 3 case

- BBB action for spin 3:

$$S[\phi^a, \bar{\phi}^a] = S^{free} + S^{(\alpha)}\,, \quad S^{free} = \int d^4x \frac{1}{2} \bar{\phi}^a \Box \phi^a$$

$$S^{(\alpha)} = \alpha f^{abc} \int d^4x\, \bar{\phi}^a \partial_-{}^2 \left[\frac{\bar{\partial}^3}{\partial_-^3} \phi^b \phi^c - 3 \frac{\bar{\partial}^2}{\partial_-^2} \phi^b \frac{\bar{\partial}}{\partial_-} \phi^c + 3 \frac{\bar{\partial}}{\partial_-} \phi^b \frac{\bar{\partial}^2}{\partial_-^2} \phi^c - \phi^b \frac{\bar{\partial}^3}{\partial_-^3} \phi^c \right] + c.c.$$

- Gauge symmetry $\phi_{\mu\nu\rho} = \partial_{(\mu} \varepsilon_{\nu\rho)}$

$$\text{Ansatz:} \quad \phi^a = \partial \varepsilon^a + \dots\,, \quad \partial_- \varepsilon^a = 0, \quad \varepsilon^a \to \text{spin-2 parameter}$$

The spin 3 case

- BBB action for spin 3:

$$S[\phi^a, \bar{\phi}^a] = S^{free} + S^{(\alpha)}\,, \quad S^{free} = \int d^4x \frac{1}{2}\bar{\phi}^a \Box \phi^a$$

$$S^{(\alpha)} = \alpha f^{abc} \int d^4x\, \bar{\phi}^a \partial_-{}^2 \left[\frac{\bar{\partial}^3}{\partial_-^3}\phi^b\phi^c - 3\frac{\bar{\partial}^2}{\partial_-^2}\phi^b \frac{\bar{\partial}}{\partial_-}\phi^c + 3\frac{\bar{\partial}}{\partial_-}\phi^b \frac{\bar{\partial}^2}{\partial_-^2}\phi^c - \phi^b \frac{\bar{\partial}^3}{\partial_-^3}\phi^c \right] + c.c.$$

- Gauge symmetry $\phi_{\mu\nu\rho} = \partial_{(\mu}\varepsilon_{\nu\rho)}$

$$\text{Ansatz:} \quad \phi^a = \partial\varepsilon^a + \ldots\,, \quad \partial_-\varepsilon^a = 0, \quad \varepsilon^a \to \text{spin-2 parameter}$$

Are these symmetries of the free action, S^{free}?

Yes, if

- $\partial_+\epsilon^a = 0 \quad \Rightarrow \quad \epsilon^a(x^+, x, \bar{x}) = K^a(x, \bar{x})$
- At large x^-, field ϕ^a decays as $1/x^-$ or faster
- $\partial\bar{\partial}\epsilon^a = 0 \Rightarrow \quad K^a(x, \bar{x}) = K^a(x) + \bar{K}^a(\bar{x}) \quad \to$ Is this suprising?

Revisitng elcetromagnetism in lc_2 formalism

Light-cone action for electromagnetism

$$\mathcal{S} = \frac{1}{2}\int d^4x\, \bar{A}\,\Box_{lc}\, A = \int d^4x\; \bar{A}\,(\partial_+\partial_- - \partial\bar{\partial})\, A$$

Residual gauge symmetries:

$$A \to A + \partial\epsilon, \quad \bar{A} \to \bar{A} + \bar{\partial}\epsilon$$

and

$$\partial_-\epsilon = 0, \quad \partial_+\epsilon = 0, \quad \partial\bar{\partial}\epsilon = 0$$

To recover all the residual gauge transformations, a boundary mode (zero mode in x^-) is needed

Modified light-cone action

$$\mathcal{S}[A,\bar{A},\Phi] = \int dx^+ \left\{ \int_\Sigma d^3x\, \bar{A}\,(\partial_+\partial_- - \partial\bar{\partial})\, A - \int_{\partial\Sigma} dx\, d\bar{x}\; \dot{\Phi}\triangle\Phi \right\}$$

Phase space extended to include the boundary d.o.f. $\Phi \to$ a.k.a. lc_4 formalism

Amounts to relaxing the boundary conditons at large x^-

[SM, arXiv:2212.10637]

Complete set of residual gauge symmetries

- Proper: $\delta_\epsilon A = \partial\epsilon,\ \delta_\epsilon\Phi = 0$ with $\Delta\epsilon = 0$
- Improper: $\delta_\epsilon A = 0\ \delta_\epsilon\Phi = \epsilon$ with $\Delta\epsilon \neq 0$

Revisitng elcetromagnetism in lc_2 formalism

Light-cone action for electromagnetism

$$\mathcal{S} = \frac{1}{2}\int d^4x\, \bar{A}\,\Box_{lc}\, A = \int d^4x\ \bar{A}\,(\partial_+\partial_- - \partial\bar{\partial})\,A$$

Residual gauge symmetries:

$$A \to A + \partial\epsilon, \quad \bar{A} \to \bar{A} + \bar{\partial}\epsilon$$

and

$$\partial_-\epsilon = 0, \quad \partial_+\epsilon = 0, \quad \partial\bar{\partial}\epsilon = 0$$

To recover all the residual gauge transformations, a boundary mode (zero mode in x^-) is needed

Modified light-cone action

$$\mathcal{S}[A,\bar{A},\Phi] = \int dx^+ \left\{ \int_\Sigma d^3x\, \bar{A}\,(\partial_+\partial_- - \partial\bar{\partial})\,A - \int_{\partial\Sigma} dx\, d\bar{x}\ \dot{\Phi}\triangle\Phi \right\}$$

Phase space extended to include the boundary d.o.f. $\Phi \to$ a.k.a. lc_4 formalism

Amounts to relaxing the boundary conditons at large x^-

[SM, arXiv:2212.10637]

Complete set of residual gauge symmetries

- Proper: $\delta_\epsilon A = \partial\epsilon,\ \delta_\epsilon\Phi = 0$ with $\Delta\epsilon = 0$
- Improper: $\delta_\epsilon A = 0\ \delta_\epsilon\Phi = \epsilon$ with $\Delta\epsilon \neq 0$

Maybe we need similar boundary modes for higher spin case too?

Residual gauge symmetries for arbitrary spins

- Spin 3:

$$\delta_K \phi^a = \partial K^a(x)\,, \quad \delta_{\bar{K}} \bar{\phi}^a = \bar{\partial} \bar{K}^a(\bar{x})$$

- For even spins:

$$\delta_K \phi^{(\lambda)} = \partial K^{(\lambda-1)}\,, \quad \delta_{\bar{K}} \bar{\phi}^{(-\lambda)} = \bar{\partial} \bar{K}^{(-\lambda+1)}$$

  with

$$K^{(\lambda)} = K^{(\lambda)}(x)\,, \quad \bar{K}^{(-\lambda)} = \bar{K}^{(-\lambda)}(\bar{x})\,, \quad \text{for all } \lambda$$

- For odd spins:

$$\delta_K \phi^{a(\lambda)} = \partial K^{a(\lambda-1)}\,, \quad \delta_{\bar{K}} \bar{\phi}^{a(-\lambda)} = \bar{\partial} \bar{K}^{a(-\lambda+1)}\,, \quad \text{same for } K^a, \bar{K}^a$$

$\rightarrow$ Two infinite towers of residual gauge symmetries for higher spin fields

Residual gauge symmetries for arbitrary spins

- Spin 3:

$$\delta_K \phi^a = \partial K^a(x)\,, \quad \delta_{\bar{K}} \bar{\phi}^a = \bar{\partial} \bar{K}^a(\bar{x})$$

- For even spins:

$$\delta_K \phi^{(\lambda)} = \partial K^{(\lambda-1)}\,, \quad \delta_{\bar{K}} \bar{\phi}^{(-\lambda)} = \bar{\partial} \bar{K}^{(-\lambda+1)}$$

  with

$$K^{(\lambda)} = K^{(\lambda)}(x)\,, \quad \bar{K}^{(-\lambda)} = \bar{K}^{(-\lambda)}(\bar{x})\,, \quad \text{for all } \lambda$$

- For odd spins:

$$\delta_K \phi^{a(\lambda)} = \partial K^{a(\lambda-1)}\,, \quad \delta_{\bar{K}} \bar{\phi}^{a(-\lambda)} = \bar{\partial} \bar{K}^{a(-\lambda+1)}\,, \quad \text{same for } K^a, \bar{K}^a$$

$\rightarrow$ Two infinite towers of residual gauge symmetries for higher spin fields

- Going to the cubic order

$$S^{(\alpha)} = \alpha\, f^{abc} \int d^4x\, \bar{\phi}^a \partial_-{}^2 \left[\frac{\bar{\partial}^3}{\partial_-^3} \phi^b \phi^c - 3 \frac{\bar{\partial}^2}{\partial_-^2} \phi^b \frac{\bar{\partial}}{\partial_-} \phi^c + 3 \frac{\bar{\partial}}{\partial_-} \phi^b \frac{\bar{\partial}^2}{\partial_-^2} \phi^c - \phi^b \frac{\bar{\partial}^3}{\partial_-^3} \phi^c \right] + c.c.$$

  Spin 3 residual gauge invariance

$$\delta_{K,\bar{K}} \phi^a = \partial K^a(x) - 2\alpha\, f^{abc} \frac{\bar{\partial}^2}{\partial_-} \phi^b K^c(x) + 2\alpha\, f^{abc} \frac{1}{\partial_-^3} \left(\partial_-^2 \partial^2 \phi^b \bar{K}^c(\bar{x}) \right)$$

  and $\delta_{K,\bar{K}} \bar{\phi}^a = (\delta_{K,\bar{K}} \phi^a)^*$

But the algebra doesn't seem to close!

Why call them superrotations?

HS supertranslations and superrotations at null infinity $(r, u, z, \bar{z})$

[Campoleoni-Francia-Heissenberg]

Bondi-like gauge:

$$\phi_{r\mu\nu} = 0, \quad \gamma^{ij}\phi_{ij\mu} = 0\,, \quad \phi_{ijk} \sim \mathcal{O}(r^2)\,, \quad \dots$$

Large gauge transformations:

$$\delta\phi_{ijk} = \partial_{(i}\varepsilon_{jk)}\,, \quad \varepsilon_{ij} \sim r^2\partial_i\partial_j F + r\partial_i\rho_j + K_{ij}$$

HS superrotation parameter satisfies similar conditions as the LC K parameter

$$K^{zz} = K(z)\,, \quad K^{\bar{z}\bar{z}} = \bar{K}(\bar{z}), \quad K^{z\bar{z}} = 0$$

Why call them superrotations?

HS supertranslations and superrotations at null infinity $(r, u, z, \bar{z})$

[Campoleoni-Francia-Heissenberg]

Bondi-like gauge:

$$\phi_{r\mu\nu} = 0, \quad \gamma^{ij}\phi_{ij\mu} = 0\,, \quad \phi_{ijk} \sim \mathcal{O}(r^2)\,, \quad \dots$$

Large gauge transformations:

$$\delta\phi_{ijk} = \partial_{(i}\varepsilon_{jk)}\,, \quad \varepsilon_{ij} \sim r^2\partial_i\partial_j F + r\partial_i\rho_j + K_{ij}$$

HS superrotation parameter satisfies similar conditions as the LC K parameter

$$K^{zz} = K(z)\,, \quad K^{\bar{z}\bar{z}} = \bar{K}(\bar{z}), \quad K^{z\bar{z}} = 0$$

Some concluding remarks...

- Are these the same HS superrotations? Large $r \neq$ Large x^-
- Can we find all the HS supertranslations and superrotations by adding more boundary d.o.f.?

Why call them superrotations?

HS supertranslations and superrotations at null infinity $(r, u, z, \bar{z})$

[Campoleoni-Francia-Heissenberg]

Bondi-like gauge:

$$\phi_{r\mu\nu} = 0, \quad \gamma^{ij}\phi_{ij\mu} = 0\,, \quad \phi_{ijk} \sim \mathcal{O}(r^2)\,, \quad \ldots$$

Large gauge transformations:

$$\delta\phi_{ijk} = \partial_{(i}\varepsilon_{jk)}\,, \quad \varepsilon_{ij} \sim r^2\partial_i\partial_j F + r\partial_i\rho_j + K_{ij}$$

HS superrotation parameter satisfies similar conditions as the LC K parameter

$$K^{zz} = K(z)\,, \quad K^{\bar{z}\bar{z}} = \bar{K}(\bar{z}), \quad K^{z\bar{z}} = 0$$

Some concluding remarks...

- Are these the same HS superrotations? Large $r \neq$ Large x^-
- Can we find all the HS supertranslations and superrotations by adding more boundary d.o.f.?
- Does the cubic action exist in that case?
- Do these symmetries truly extend to the interacting higher spin theory?

THANK YOU!

APPENDIX

Gauge-fixing the Fronsdal action

Spin 3:

$$\phi_{-\mu\nu} - \frac{1}{4}\eta_{\mu\nu}\phi^{\sigma}{}_{\sigma -} = 0$$

Constraint equations yield

$$\phi^{++-} = \phi^{+11} = \phi^{+22} = 0$$

$$\phi^{-ij} = \frac{1}{\partial_-}\partial_k\phi^{kij}\,, \quad \phi^{--i} = \frac{1}{\partial_-^2}\partial_i\partial_k\phi^{ijk}$$

$$\phi^{-ij} = \frac{1}{\partial_-^3}\partial_i\partial_j\partial_k\phi^{kij}\,, \quad \phi^{11i} = -\phi^{22i}$$

Finally

$$\phi = \frac{\phi^{111} + i\phi^{112}}{\sqrt{2}} \quad \bar{\phi} = \frac{\phi^{111} - i\phi^{112}}{\sqrt{2}}$$

[Ananth-Akshay]