Carroll Structures and Null Quantization

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References

- **primary reference:**
  - Null Raychaudhuri: Canonical Structure and the Dressing Time
    - 2309.03932 (with Luca Ciambelli, Laurent Freidel)
- **related:**
  - Isolated Surfaces and Symmetries of Gravity
    - 2104.07643 (with Luca Ciambelli)
  - Embeddings and Integrable Charges for Extended Corner Symmetry
    - 2111.13181 (with Luca Ciambelli, Pin-Chun Pai)
  - Universal Corner Symmetry and the Orbit Method for Gravity
    - 2207.06441 (with Luca Ciambelli)
  - Extended Phase Spaces in General Gauge Theories
    - 2303.06786 (with Marc Klinger, Pin-Chun Pai)
  - Crossed Products, Extended Phase Spaces and the Resolution of Entanglement Singularities
    - 2306.09314, 2310.xx (with Marc Klinger)

References

- **primary reference:**
  - Null Raychaudhuri: Canonical Structure and the Dressing Time
    - 2309.03932 (with Luca Ciambelli, Laurent Freidel)

see Luca's talk for many details

- **other key players:**
  - Isolated Surfaces and Symmetries of Gravity
    - 2104.07643 (with Luca Ciambelli)
  - Embeddings and Integrable Charges for Extended Corner Symmetry
    - 2111.13181 (with Luca Ciambelli, Pin-Chun Pai)
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corner symmetry

extended phase space

Null Hypersurfaces

- consider some theory in a classical spacetime
  - could be a simple field theory, a gauge theory, and/or a diffeomorphism-invariant theory
- hypersurfaces embedded/immersed in spacetime are crucial structures:
  - in classical theory, we build symplectic structure on a hypersurface
    - might be space-like or null
  - in quantizing a classical theory similarly, a choice of hypersurface is part of the process
    - conversely, we might suppose that a classical spacetime is not required in this context, but might emerge as a semi-classical structure
      - to capture the emergent data (intrinsic and extrinsic), would need to formulate in such a way that all of the symplectic data is present
    - here we study the null case
      - will leverage our knowledge of Carroll structures
      - will uncover interesting inter-relationships to recently constructed extended phase space methods
      - the latter is closely related to a crossed-product construction in a corresponding quantum operator algebraic construction, so apparently amenable to quantization

Corners

- in the above discussion, I focused on codimension-1 surfaces (hypersurfaces)
  - from the point of view of symmetries, these are important for another reason
    - by Noether's (first) theorem, symmetry currents can be regarded as codim-1 forms, and symmetry charges are obtained by integrating them over codim-1 surfaces
- but higher codimension surfaces are also important in many contexts
  - in any gauge theory (including diff-invariant theories), Noether's second theorem reports that the current is of the form

$$J_{\underline{\lambda}} = M_{\underline{\lambda}} + dq_{\underline{\lambda}}$$

  - in the case of a gauge symmetry, $\underline{\lambda}$ is an element of the Lie algebra
  - $M_{\underline{\lambda}}$ is the constraint, which vanishes in some sense (classically, on-shell)
  - $q_{\underline{\lambda}}$ is the charge density or aspect, whose integration over a codim-2 surface gives the (generally non-vanishing) gauge charge

Corner Symmetry

- such a codimension-2 surface (possibly a component of the boundary of a hypersurface or a subregion on a hypersurface) is referred to as a corner
    - in the context of any diff-invariant theory, although diff(M) is the full gauge symmetry, only a closed subalgebra $\mathcal{A}_2$ supports non-zero charges [Ciambelli, RGL]
        - this is true irrespective of a choice of a background, depends only on the geometric structure in a neighborhood of a corner
        - in any given context, this algebra is (a subalgebra of) the universal corner symmetry
    - classically, these charges are integrable (i.e., represented properly on the phase space) if one includes the embedding of the corner into the phase space

$$\varphi : S \hookrightarrow M$$

$$H_{\underline{\xi}} = \int_S \varphi^*(q_{\underline{\xi}})$$

$$\left\{ H_{\underline{\xi}}, H_{\underline{\eta}} \right\} = H_{[\underline{\xi},\underline{\eta}]}$$

extended phase space

[Donnelly,Freidel, Geiller,Speranza, Pai,Klinger,RGL, Pranzetti,Wieland, Jai-akson,Moosavian,…]

Corners

- in fact in a general theory with local symmetries (internal or diff), there is a convenient formulation using (Atiyah) Lie algebroids, in which gauge symmetries and diffs appear on an equal footing as morphisms of Lie algebroids [Klinger, RGL, Pai]
    - Noether's theorem then becomes a more general statement

$$\mathfrak{J}_{\underline{\chi}} = \mathfrak{M}_{\underline{\chi}} + \hat{d}\mathfrak{q}_{\underline{\chi}}$$

    - locally, the section $\underline{\chi}$ can be understood as $\underline{\chi} \sim (\underline{\lambda}, \underline{\xi})$
    - locally, $\hat{d} \sim \delta + \sigma$, that is it contains both free variation (exterior derivative on the field space) and BRST
    - embedding of a surface of any codimension-k extends to a Lie algebroid morphism, containing locally an embedding map and gauge fixing
        - **constraint algebra closes off-shell and charge algebra closes on-shell**

Null Hypersurfaces

- **I'm going to focus on an example: null hypersurfaces**
  - no reference to a spacetime in which they are embedded, so quite flexible
  - won't make assumptions about its nature, so applies to any hypersurface
    - can specialize to constrained examples, such as Killing horizons or asymptotic
- **the physics of a null hypersurface can be generally addressed because the content can be understood geometrically**
  - a Carroll structure: a line bundle over a Euclidean space
  - along with a choice of symplectic structure
    - can think of this in two (equivalent) ways:
      - as an induced non-Riemannian geometry from a bulk theory
      - or, as a `Carrollian fluid', possessing a stress-energy tensor

Ruled Carroll Structures

- **recall a Carroll structure $\mathcal{N} \to \mathcal{S}$ is a line bundle endowed with geometric data $(q, \underline{\ell})$** (n+1)-dimensional, mostly n=2
  - $\underline{\ell}$ is a 'vertical' vector field which we will often think of as $\underline{\partial}_v$ with v a null 'time' coordinate
  - q is a non-degenerate time-dependent metric on a section $\mathcal{S}$, with $\iota_{\underline{\ell}} q = 0$
  - there is no metric on $\mathcal{N}$, but one can introduce an Ehresmann connection k satisfying $\iota_{\underline{\ell}} k = 1$ and construct a rigging, essentially a projector from $T\mathcal{N}$ to $T\mathcal{S}$

$$\Pi = 1 - \underline{\ell} \otimes k \qquad\qquad \Pi(\underline{\ell}) = 0$$

  - $(q, \underline{\ell}, k)$ defines a ruled Carroll structure
    - **any vector field can be decomposed as** $\underline{\xi} = f\underline{\ell} + \underline{Y}$ **with** $\iota_{\underline{Y}} k = 0$
  - q determines a volume $\varepsilon_{\mathcal{S}}$, and k determines a volume $\varepsilon_{\mathcal{N}}$, with $\iota_{\underline{\ell}} \varepsilon_{\mathcal{N}} = \varepsilon_{\mathcal{S}}$

$$\varepsilon_{\mathcal{N}} = k \wedge \varepsilon_{\mathcal{S}}$$

  - one defines the (horizontal) expansion tensor

expansion

$$\theta_{ab} = \tfrac{1}{2}(\mathcal{L}_{\underline{\ell}} q)_{ab} \qquad\qquad \theta^a{}_b = \tfrac{1}{n}\,\theta\,\Pi^a{}_b + \sigma^a{}_b$$

Ruled Carroll Structures

- finally, one introduces a Carroll connection D
  - assume it preserves the ruling, $D\Pi = 0$
    - can be thought of as induced from a bulk L-C connection
    - we'll take the horizontal components of torsion and metricity to vanish
  - the Carroll connection determines a 1-form ω via

$$D_{\underline{\xi}}\varepsilon_{\mathcal{N}} = \left(\iota_{\underline{\xi}}\omega\right)\varepsilon_{\mathcal{N}}$$

  - and one can decompose it as

$$\omega = \kappa k + \pi, \qquad \iota_{\underline{\ell}}\pi = 0$$

inaffinity (κ) — Hájiček connection (π)

  - it's convenient to extract the 'area' Ω

$$\Omega := \sqrt{\det q}, \qquad q = \Omega^{2/n}\bar{q}, \qquad \det\bar{q} = 1, \qquad \varepsilon_{\mathcal{N}} = \Omega\,\varepsilon^{(0)}_{\mathcal{N}}$$

Carroll Fluids

- given a choice of local coordinates, construct a stress-energy tensor

$$T_a{}^b := \frac{1}{8\pi G_N}\left(D_a\ell^b - \Pi_a{}^b D_c\ell^c\right)$$

- this can be decomposed into

[Petropoulos,Petkou,Freidel,Donnay,Marteau,RGL, Speranza,Flanagan,Obers,Oling, Sybesma,Vandoren,deBoer,…]

$$T_a{}^b = \tau_a\ell^b + \tau_a{}^b$$

- where

$$\tau_a = \frac{1}{8\pi G_N}\left(\pi_a - \theta\, k_a\right),$$
$$\tau_a{}^b = \frac{1}{8\pi G_N}\left(\sigma_a{}^b - \mu\, q_a{}^b\right)$$

$$\mu := \kappa + \frac{n-1}{n}\,\theta$$

$$(\Omega, \mu), \quad (\underline{\ell}, \pi), \quad (\bar{q}, \sigma)$$

spin-0 spin-1 spin-2

Symplectica

- **the presymplectic potential is**

$$\Theta^{\text{can}} = \int_{\mathcal{N}} \theta^{\text{can}} = \int_{\mathcal{N}} \varepsilon_{\mathcal{N}} \Big(\frac{1}{2} \tau^{ab} \delta q_{ab} - \tau_a \delta \ell^a \Big)$$

- this gives rise to diffeomorphism Noether currents

$$\hat{I}_{\underline{\hat{\xi}}} \Theta^{can} = - \int_{\mathcal{N}} \varepsilon_{\mathcal{N}} \xi^a D_b T_a{}^b + \int_{\mathcal{C}} \varepsilon_b \xi^a T_a{}^b$$

- e.g.,

$$J_{f\underline{\ell}} = \frac{1}{8\pi G_N} \int_{\mathcal{N}} \varepsilon_{\mathcal{N}} \left(f \sigma_a{}^b \sigma_b{}^a - \theta(\underline{\ell} + \mu)[f] \right) \hat{=} - \frac{1}{8\pi G_N} \int_{\mathcal{S}} \varepsilon_{\mathcal{S}}\, f \theta.$$

- importantly, there is also a boost symmetry which rescales $\underline{\ell}$

$$J_\lambda = \frac{1}{8\pi G_N} \int_{\mathcal{S}} \varepsilon_{\mathcal{S}}\, \lambda.$$

- κ (and μ) transforms as a connection under this boost

see Luca's talk for details

- the presymplectic form can be explicitly inverted to kinematical Poisson brackets by rewriting the metric in terms of Beltrami differentials
- the constraint algebra properly closes only when the contributions of all spins are taken into account

Extended Phase Space

- the closure of the constraint algebra is an instance of a general property of extended phase space [Klinger,RGL,Pai '22]
    - indeed, in the extended phase space formalism, the current algebra closes off-shell generically for any gauge theory
- importantly, it is not really an 'extension' — the required fields (spin-0 and more generally spin-1) are already present in the off-shell theory
    - this is also an automatic property within the Lie algebroid formalism
- this is in fact closely related to a feature of a quantum theory, where gauge symmetries appear as inner automorphisms via a crossed-product construction [Klinger,RGL '23]

$$\begin{array}{ccc} (X,\Omega) & \xrightarrow{GQ} & \mathcal{M} \\ \downarrow{\scriptstyle EPS} & & \downarrow{\scriptstyle CP} \\ (X_{ext},\Omega_{ext}) & \xrightarrow{GQ} & \mathcal{R}(\mathcal{M},G) \end{array}$$

Carrollian Diffs

- the Carrollian diffeomorphisms are those that preserve the Carroll structure (in particular $\underline{\ell}$)

- these are a combination of diffs and boosts

$$\mathcal{L}'_{\hat{f}} := \mathcal{L}_f + \mathcal{L}_{\lambda_f}, \qquad \lambda_f = \partial_v f$$

- the corresponding constraint/charge is

$$M'_f = \frac{1}{8\pi G_N} \int_{\mathcal{N}} \varepsilon^{(0)}_{\mathcal{N}} f \, C_{Ray}, \qquad H'_f = \frac{1}{8\pi G_N} \int_{\mathcal{S}} \varepsilon^{(0)}_{\mathcal{S}} (\Omega \partial_v f - f \partial_v \Omega)$$

- where the Raychaudhuri constraint appears

$$C_{Ray} = \partial_v^2 \Omega - \mu \partial_v \Omega + \Omega(\sigma_a{}^b \sigma_b{}^a + 8\pi G_N \, T^{\text{mat}}_{vv})$$

More on Constraints

- the Carroll structure can be thought of as a null congruence, a bundle of curves through each point on $\mathcal{S}$
  - as we saw, the Carrollian constraint is closely related to Raychaudhuri
  - in our notation, with $\underline{\ell}$ reduced to $\underline{\partial}_v$ for simplicity, the Raychaudhuri constraint is

$$C_{Ray} = \partial_v^2 \Omega - \mu \partial_v \Omega + \Omega({\sigma_a}^b {\sigma_b}^a + 8\pi G_N T_{vv}^{\text{mat}})$$

spin-0   shear   matter

  - this equation is exact (no weak field expansion invoked)
    - **it contains contributions from arbitrary gravitational stress-energy and arbitrary matter**
  - it is valid whenever $\Omega > 0$:
    - **that is, away from caustics (focal points) or singularities**

Raychaudhuri and Clocks

- we are employing here the fibre coordinate v as a null time variable
- we would like to, in some sense, set $C_{Ray} \to 0$
  - can the constraint be solved?
    - traditionally this would be attacked perturbatively in G_N
  - can it be solved non-perturbatively?
    - *yes*, and the solution is interesting for several reasons
- the basic mechanism is to replace the coordinate time by a dynamical variable
  - it is often supposed that one should use affine time, which corresponds to taking the null congruence to be geodesic
    - this is the correct thing under certain limited circumstances
    - but the imposition of affine time is *not background-independent*, and so should not be expected to be universally relevant/appropriate

Dressing Time

- to introduce a dynamical clock, we perform a local diffeomorphism, together with a compensating local boost so that $\underline{\ell}$ is invariant

$$V \to V(v, \sigma)$$

- each field can be rewritten as a function of V

$$\Omega(v) = \tilde{\Omega}(V(v))$$

- or formally

$$\Omega = \tilde{\Omega} \circ V, \quad q_{ab} = \tilde{q}_{ab} \circ V, \quad \sigma_{ab} = \partial_v V(\tilde{\sigma}_{ab} \circ V), \quad \mu = \partial_v V(\tilde{\mu} \circ V) + \frac{\partial_v^2 V}{\partial_v V}$$

- the non-linear transformation of μ will be crucial
  - recall it is a boost connection
  - a tensor of boost weight s transforms as

$$O = (\partial_v V)^s(\tilde{O} \circ V), \qquad (\partial_v - s\mu)O = (\partial_v V)^{s+1}[(\partial_V - s\tilde{\mu})\tilde{O}] \circ V$$

Raychaudhuri and Clocks

- **there is a choice of dynamical clock variable that is canonically conjugate to the Raychaudhuri constraint**
    - in particular, its choice renders the symplectic form of the spin-0 sector as vanishing on the constraint surface, where (classically!) the constraint is rendered as a non-dynamical variable
    - this is a description of the theory that is analogous to finding a canonical transformation to 'action-angle variables'
- **we call this the *dressing clock***
    - writing the theory in terms of this clock has the additional effect of rendering all other dynamical variables invariant
        - **that is, they are dressed to diffeomorphisms (here, the null time reparameterizations)**
    - one can regard the dressing clock as an 'observer'
        - **here it is a built-in feature of gravity, is a background-independent notion, and not introduced as an *ad hoc* device**

Dressing Time

- **the non-linear transformation allows us to take $\tilde{\mu} = 0$ which is the defining property of the dressing time**

$$\mu = \partial_v V(\tilde{\mu} \circ V) + \frac{\partial_v^2 V}{\partial_v V} \longrightarrow \mu = \frac{\partial_v^2 V}{\partial_v V}$$

- note that this coincides with affine time iff the expansion vanishes

$\mu = \kappa + \frac{1}{2}\theta$

  - e.g., Killing horizon

- the clock is a Wilson line, relating values on corners at different times

$$\partial_v V(v, \sigma) = \partial_v V(b, \sigma) \exp\left(\int_b^{v'} dv' \mu(v', \sigma)\right)$$

residual affine diffs preserving clock = BMSW

- the Raychaudhuri constraint is a tensor of boost weight-2

$$C_{Ray} = (\partial_v V)^2 \tilde{C}_{Ray} \circ V$$

$$\tilde{C}_{Ray} = \partial_V^2 \tilde{\Omega} - \tilde{\mu}\partial_V \tilde{\Omega} + \tilde{\Omega}(\tilde{\sigma}_a{}^b \tilde{\sigma}_b{}^a + 8\pi G_N \tilde{T}_{VV}^{\text{mat}})$$

Solving Ray

- **the Raychaudhuri constraint can be solved non-perturbatively if we compare dressing time to some other notion of time**
  - for example, with non-zero expansion, consider the *areal time*

$$\Omega : v \to \Omega(v), \qquad V = \bar{V} \circ \Omega$$

  - this is well-defined as long as $\partial_v \Omega \neq 0$ (i.e., the areal clock does not stop)

$$\mu = \frac{\partial_v^2 V}{\partial_v V} = \frac{\partial_v^2 \Omega}{\partial_v \Omega} + \partial_v \Omega \left(\frac{\partial_\Omega^2 \bar{V}}{\partial_\Omega \bar{V}} \right) \circ \Omega$$

  - this can be rewritten in terms of the Raychaudhuri constraint

$$0 = \frac{C_{Ray}}{\partial_v \Omega} - \frac{\Omega(\sigma^2 + 8\pi G_N T^{\mathrm{mat}}_{vv})}{\partial_v \Omega} + \partial_v \Omega \left(\frac{\partial_\Omega^2 \bar{V}}{\partial_\Omega \bar{V}} \right) \circ \Omega$$

  - and integrated, when the constraint vanishes, to

$$\partial_\Omega \bar{V}(\Omega) = \partial_\Omega \bar{V}(\Omega(b)) \ \exp \int_b^\Omega d\Omega' \, \Omega' (\bar{\sigma}^2 + 8\pi G_N T^{\mathrm{mat}}_{\Omega\Omega})(\Omega')$$

Solving Ray

- **note that this is precisely the boost generator in areal time**

$$\partial_\Omega \bar{V}(\Omega) = \partial_\Omega \bar{V}(\Omega(b))\ \exp \int_b^\Omega d\Omega'\, \Omega'(\bar{\sigma}^2 + 8\pi G_N T^{\text{mat}}_{\Omega\Omega})(\Omega')$$

  - this is *non-perturbative* in G_N

$$\partial_v V = (\partial_v \Omega)\partial_\Omega \bar{V} \circ \Omega$$

  - in the absence of shear, the RHS involves the matter boost Hamiltonian
    - **tantalizing relationship with a *modular Hamiltonian* and thus ANEC**
  - although this discussion is classical so far, it is closely related to focussing theorems, covariant entropy bounds, etc.
    - **may be considered as a refinement of the usual discussions to arbitrary hypersurfaces**
    - **e.g., for a linear boost about v=0, returning to dressing time**

$$M'_v \hat{=} \frac{1}{8\pi G_N} \int_{\mathcal{S}} \varepsilon^{(0)}_{\mathcal{S}} \tilde{\Omega} \circ V \geq 0 \qquad \partial^2_V M'_v = -\frac{1}{8\pi G_N} \int_{\mathcal{S}} \varepsilon^{(0)}_{\mathcal{S}} \tilde{\Omega}(\tilde{\sigma}^2 + 8\pi G \tilde{T}^{\text{mat}}_{VV}) \circ V \leq 0$$

    - **in vacuum, charge is conserved — a proper notion of energy, even if expanding**
    - **in any other clock, these conclusions are generally false**
  - would be interesting to explore its relevance for generalized second law, quantum focussing, quantum entropy bounds, etc.

[Bousso,Wall,…]

Symplectica

- the symplectic form can be rewritten

(dressed) spin-2 model matter

$$\Omega^{\rm tot}_{\mathcal{N}} = \int_{\mathcal{N}} \varepsilon^{(0)}_{\mathcal{N}} \partial_v V \left(\frac{1}{16\pi G_N} \delta(\tilde{\Omega}\tilde{\sigma}^{ab}) \wedge \delta \tilde{q}_{ab} + \delta \left(\tilde{\Omega} \partial_V \tilde{\varphi} \right) \wedge \delta \tilde{\varphi} \right) \circ V$$
$$+ \frac{1}{8\pi G_N} \int_{\mathcal{N}} \varepsilon^{(0)}_{\mathcal{N}} \delta \left(\partial_v V [\tilde{C} \circ V] \right) \wedge \delta V$$

(C,V) conjugate

- the spin-0 terms have completely disappeared, replaced by the *(C,V)* symplectic pair
    - any other choice of a clock would not generally have this feature
        - there would be residual left-over spin-0 terms in the bulk symplectic form

Some Quantum Aspects

- **this eventuality is crucial in understanding how to implement the gauge constraints**
  - classically, we can proceed directly to the constraint surface, by freezing the constraint to zero
  - notice that the dressing of physical degrees of freedom happens automatically in the course of passing to the dressing time
  - the formalism of extended phase space in fact leads to several equivalent ways to think about a corresponding quantum theory [Klinger,RGL; to appear]
    - **e.g., there is a geometric version of the Fadeev-Popov procedure in the context of path integral quantization for which this canonical transformation is a relevant change of integration variables that leads to integration over physical variables**

- **expect that this structure is true more generally**
    - in the null context, extends to Damour [Ciambelli, Freidel, RGL; to appear]
- **if we can think of the dressing time as defining an observer's clock, then we should expect that unraveling Damour will lead to further 'measurement apparati'**
    - this notion of observer and measurement is built into the theory
        - **comes from properly organizing "pure gauge" spin-0 (and spin-1 in the case of Damour) degrees of freedom**
    - QG should then be capable of 'measuring itself', with no 'meta-observers'
        - **measurements are relational**
    - compare to other recent suggestions for observers (in the context of semiclassical gravity), which are ad hoc additions designed to find non-trivial operator algebras
        - **relation between extended phase space and crossed product algebras guarantees non-trivial algebras in our more general context**

Further Quantum Musings

- **it seems natural to ask if this perspective might lead us to a useful place in terms of quantum gravity/geometry.**
  - the detailed relationship between the classical notion of extended phase space and crossed product von Neumann algebras seems promising in general
  - in fact, these concepts unify all of the usual notions of quantization in gauge theories [Klinger,RGL; to appear]
- **our study of gravity on null hypersurfaces suggests a new interpretation of a quantum theory:**
  - there is a *chiral Carroll CFT* which captures the principle features
    - one reinterprets the vanishing of the Raychaudhuri constraint as gauging reparameterization and boost invariance of the Carroll structure
      - that is, reinterpret as the vanishing of a stress tensor
        - (very natural given the form of C_{Ray})

- such a CFT can be thought of as a certain chiral 3+1 CFT
  - given the fibration structure, it can be thought of as a chiral 2d CFT at each point of the corner S
    - there is a natural WZW coset model whose target includes the space of Beltrami differentials, i.e., the space of 2d corner metrics
  - this model has more or less trivially an infinite central charge, obtained as a finite value per point on S
    - from this perspective, it seems natural to regard the smooth geometry of S as a classical approximation, with some underlying 'atomic geometric structure' (perhaps related to an operator algebra of Type < III)
- more generally should presumably be interested in intersecting null hypersurfaces —> non-chiral CFT
  - this is where dynamics really enters

Outlook

- Carroll symmetry governs and organizes the symplectic structure on null hypersurfaces
- kinematical Poisson brackets for gravitational modes can be solved for non-perturbatively
  - inclusion of off-shell modes closes the constraint and charge algebras
    - an instance of the power of extended phase space
- a canonical transformation that chooses a clock recasts the theory in such a way that the constraints can be easily managed
  - we expect that a similar story pertains when all of the constraints (Raychaudhuri and Damour) are taken into account
- interesting hints at quantum version