Carroll black holes

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Carroll@Thessaloniki, October 2023

2308.10947 with Ecker, Hartong,
Pérez, Prohazka, and Troncoso

Outline

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Poem by ChatGPT on “Carrollian physics”

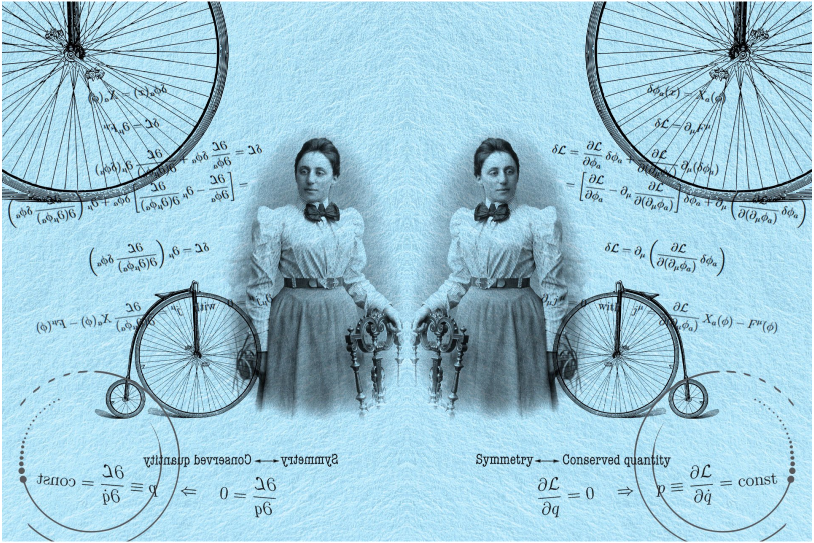
Carrollian black holes, dark and deep,
With symmetries that cannot be beat,
As they mirror the Poincaré set,
In the limit, where light is not yet.

This world of madness, is not just fun,
But has applications, one by one,
From condensed matter, to cosmology,
It’s a source of knowledge, a revelatory key.

So let us explore, this world unknown,
And be swept away, by its whims and tone,
For in the land of Carrollian physics,
We may find a truth, that’s truly cryptic.

Symmetries ubiquitious in constraining physics

- Kinematics & Dynamics



Carrollian symmetries

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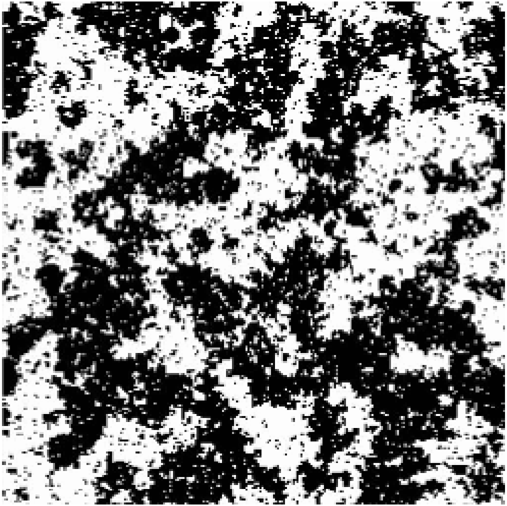
- Correlations functions



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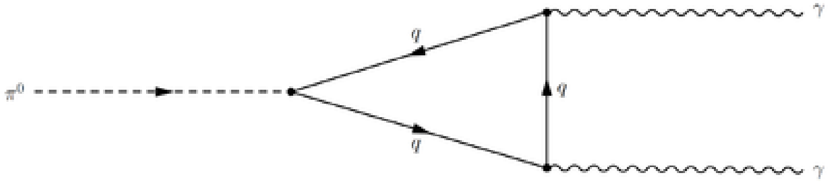
- Decay channels



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- Density of states

$$S_{\mathrm{BH}} = S_{\mathrm{Cardy}}$$

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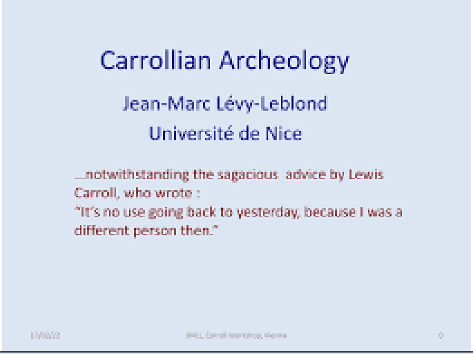
Carrollian symmetries arise in various contexts

- Formally $c \to 0$ limit of Poincaré



The Red Queen offers advice to Alice, who finds herself running intensely, but not actually moving forward: "Now, here, you see," says the Red Queen, "it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

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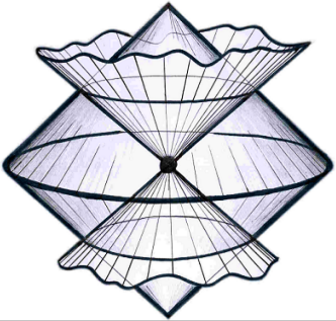

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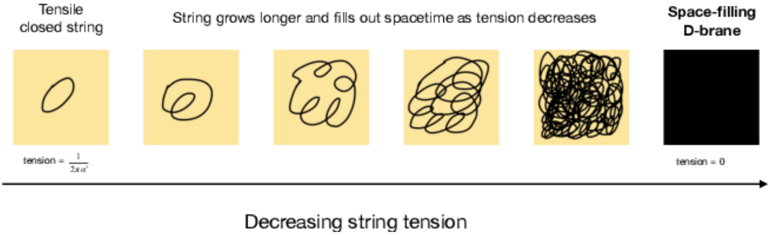

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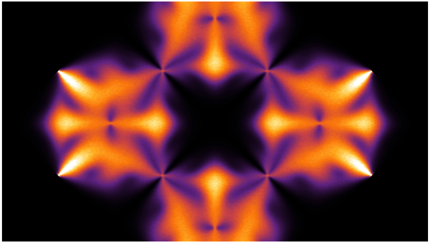

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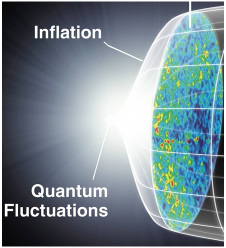

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Following history from SR to GR: natural to gauge Carroll algebra

- Gravity actions (but with Carroll boost invariance)

$$\tilde{\mathcal{L}} = E\left[\frac{1}{4}\Pi^{\mu\nu}\Pi^{\rho\sigma}T_{\mu\rho}T_{\nu\sigma} + \sigma\Pi^{\mu\nu}\overset{(C)}{R}_{\mu\nu} - \sigma^2 T^\mu T^\nu \overset{(C)}{R}_{\mu\nu}\right]$$

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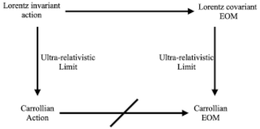

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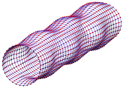

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google “carroll black hole” images; 17th result is song ‘Black Hole’ by Mackin Carroll

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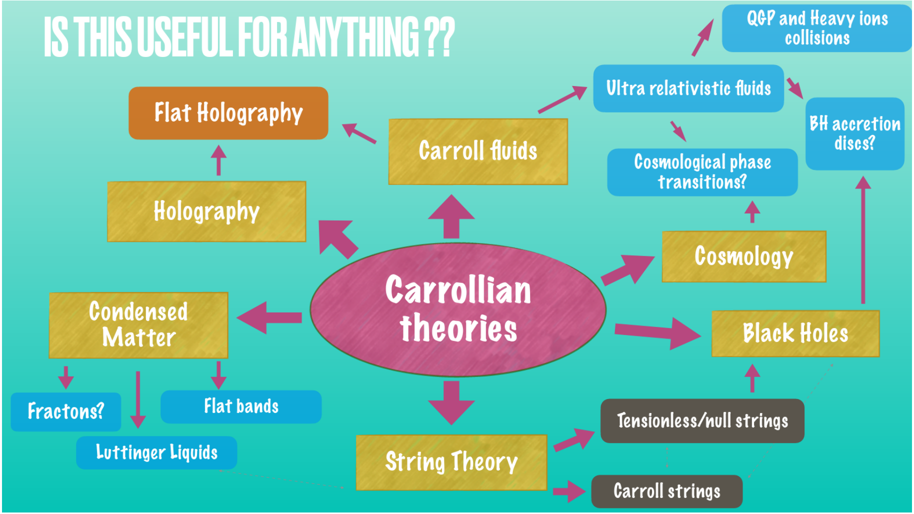
Carrollian symmetries key in numerous recent developments

Landscape of applications of Carrollian physics



slide provided by Arjun Bagchi in Edinburgh 2023

Outline

Formally: take $c \to 0$ limit of Poincaré symmetries

Analogous to Galilean limit but with reversed roles of space and time

- Unchanged: translations $H = \partial_t$, $P_i = \partial_i$, rotations $J_{ij} = x_i\partial_j - x_j\partial_i$

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- Changed: boosts

$$B_i = c^2 t\,\partial_i - x_i\,\partial_t \quad \overset{c\to 0}{\to} \quad B_i = -x_i\,\partial_t$$

Carrollian boosts shift time but do not affect space:

$$\text{Carroll boost: } t' = t - \vec{b}\cdot\vec{x} \qquad \vec{x}' = \vec{x}$$

This behavior is opposite to well-known Galilean boosts (limit $c \to \infty$):

$$\text{Galilei boost: } t' = t \qquad \vec{x}' = \vec{x} - \vec{v}\,t$$

Therefore, the Carrollian limit is often dubbed "ultra-relativistic"

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Carrollian algebra like Poincaré, except for boosts:

- Hamiltonian commutes with Carrollian boosts Hamiltonian in center of Carroll algebra

$$[B_i,\, H] = 0$$

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$$[B_i,\, P_j] = \delta_{ij}\, H$$

boosts and translations generate subalgebra of Carroll algebra

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$$[B_i,\, P_j] = \delta_{ij}\, H$$

- Angular rotations do not commute with Carrollian boosts vector trafo

$$[B_k,\, J_{ij}] = \delta_{k[i}\, B_{j]}$$

Carrollian limit of Minkowski metric

- Metric degenerates to spatial metric:

$$\mathrm{d}s^2 = \eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -c^2\,\mathrm{d}t^2 + \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j \quad \stackrel{c\to 0}{\to} \quad \mathrm{d}s^2 = \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

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- Inverse metric degenerates to temporal bi-vector:

$$-c^2\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -c^2\delta^{ij} \end{pmatrix} \quad \stackrel{c\to 0}{\to} \quad v^\mu v^\nu \qquad \text{with } v^\mu = \delta^\mu_t$$

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- Carroll spacetimes require specification of Carrol metric $h_{\mu\nu}$ with signature $(0, +, +, \dots, +)$ and time-like Carroll vector v^μ with

$$h_{\mu\nu}\,v^\nu = 0$$

could envisage generalization to metrics with signature $(0, \dots, 0, -, \dots, -, + \dots, +)$

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- Carroll symmetries preserve this Carroll structure

$$\mathcal{L}_\xi h_{\mu\nu} = 0 = \mathcal{L}_\xi v^\mu$$

Carroll symmetries generated by vector ξ^μ through Lie derivative

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Carroll gravity in Cartan-like variables

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  temporal einbein τ, spatial einbein e, Carroll boost connection ω

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translation to metric formulation:

$$v^\mu \tau_\mu = -1 \qquad \mathrm{d}s^2 = h_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = e_\mu e_\nu\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

as required, the metric $h_{\mu\nu}$ has signature $(0,+)$ and obeys $h_{\mu\nu}v^\nu = 0$

additionally, we have the orthonomality relations

$$v^\mu e_\mu = 0 = e^\mu \tau_\mu \qquad e^\mu e_\mu = 1$$

where e^μ is the inverse spatial einbein

trafo under Carroll boosts:

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- curvature: $R = \mathrm{d}\omega$
- intrinsic torsion: $K = \mathrm{d}e$

  the word "intrinsic" means independence from the Carroll boost connection ω

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$$I_{\text{CJT}} \sim \int \big(X\, R + X_{\text{H}}\, T + X_{\text{P}}\, K - \tau \wedge e\, \Lambda X \big)$$

DG, Hartong, Prohazka, Salzer '20; Gomis, Hidalgo, Salgado-Rebolledo '20

X: dilaton field

X_{H}: boost invariant auxiliary scalar

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Λ: model parameter (comparable to cosmological constant)

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$\tau \wedge e$: volume form

- on-shell: (intrinsic) torsion vanishes; constant curvature

Generic Carroll dilaton gravity in two dimensions

DG, Hartong, Prohazka, Salzer '20

- Most general bulk action:

$$I_{\mathrm{CDG}} = \frac{k}{2\pi} \int \big(X\, R + X_{\mathrm{H}}\, T + X_{\mathrm{P}}\, K + \tau \wedge e\, \mathcal{V}(X,\, X_{\mathrm{H}})\big)$$

for connoisseurs:
models above equivalent to Poisson-sigma model (PSM) with Poisson tensor

$$P^{IJ}_{\mathrm{Carroll}} = \begin{pmatrix} 0 & 0 & X_{\mathrm{H}} \\ 0 & 0 & \mathcal{V}(X,\, X_{\mathrm{H}}) \\ -X_{\mathrm{H}} & -\mathcal{V}(X,\, X_{\mathrm{H}}) & 0 \end{pmatrix}$$

to be contrasted with Poisson tensor of Lorentzian dilaton gravity

$$P^{IJ}_{\mathrm{Lorentz}} = \begin{pmatrix} 0 & -X^+ & X^- \\ X^+ & 0 & \mathcal{V}(X,\, X^+X^-) \\ -X^- & -\mathcal{V}(X,\, X^+X^-) & 0 \end{pmatrix}$$

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- Transformation under Carroll boosts:

$$\delta_\lambda X = 0 \qquad \delta_\lambda X_{\text{H}} = 0 \qquad \delta_\lambda X_{\text{P}} = X_{\text{H}}\,\lambda$$
$$\delta_\lambda \omega = \mathrm{d}\lambda \qquad \delta_\lambda \tau = -e\,\lambda \qquad \delta_\lambda e = 0$$

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- Two additional gauge symmetries:

$$\delta_{\lambda_t} X = 0 \qquad \delta_{\lambda_t} X_{\text{H}} = 0 \qquad \delta_{\lambda_t} X_{\text{P}} = \mathcal{V}\,\lambda_t$$
$$\delta_{\lambda_r} X = -\,\lambda_r \qquad \delta_{\lambda_r} X_{\text{H}} = -\mathcal{V}\,\lambda_r \qquad \delta_{\lambda_r} X_{\text{P}} = 0$$
$$\delta_{\lambda_t} \omega = -(\partial_X \mathcal{V})\,e\lambda_t \qquad \delta_{\lambda_t} \tau = \mathrm{d}\lambda_t - (\partial_{\text{H}} \mathcal{V})\,e\lambda_t \qquad \delta_{\lambda_t} e = 0$$
$$\delta_{\lambda_r} \omega = (\partial_X \mathcal{V})\,\tau\lambda_r \qquad \delta_{\lambda_r} \tau = (\partial_{\text{H}} \mathcal{V})\,\tau\lambda_r \qquad \delta_{\lambda_t} e = \mathrm{d}\lambda_r$$

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- Two additional gauge symmetries $(A_I = (\omega,\,\tau,\,e),\, \lambda_I = (\lambda,\,\lambda_t,\,\lambda_r))$:

$$\delta_{\lambda_t} X = 0 \qquad \delta_{\lambda_t} X_{\text{H}} = 0 \qquad \delta_{\lambda_t} X_{\text{P}} = \mathcal{V}\,\lambda_t$$
$$\delta_{\lambda_r} X = -\,\lambda_r \qquad \delta_{\lambda_r} X_{\text{H}} = -\mathcal{V}\,\lambda_r \qquad \delta_{\lambda_r} X_{\text{P}} = 0$$
$$\delta_{\lambda_t} \omega = -(\partial_X \mathcal{V})\,e\lambda_t \qquad \delta_{\lambda_t} \tau = \mathrm{d}\lambda_t - (\partial_{\text{H}} \mathcal{V})\,e\lambda_t \qquad \delta_{\lambda_t} e = 0$$
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- On-shell they generate diffeomorphisms, $\lambda_I = A_{I\,\mu}\,\xi^\mu$ like CS-form of 3d gravity

Equations of motion

Variation of the bulk action

$$I_{\text{CDG}} = \frac{k}{2\pi} \int \big(X\, R + X_{\text{H}}\, T + X_{\text{P}}\, K + \tau \wedge e\, \mathcal{V}(X,\, X_{\text{H}})\big)$$

yields the equations of motion

δX	Carroll curvature:	$R = \mathrm{d}\omega = -\partial_X \mathcal{V}(X,\, X_{\text{H}})\, \tau \wedge e$
δX_{H}	Carroll torsion:	$T = \mathrm{d}\tau + \omega \wedge e = -\partial_{\text{H}} \mathcal{V}(X,\, X_{\text{H}})\, \tau \wedge e$
δX_{P}	No intrinsic torsion:	$K = \mathrm{d}e = 0$
$\delta\omega$	Carroll metric:	$\mathrm{d}X + X_{\text{H}}\, e = 0$
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moreover: constant curvature, vanishing torsion; boring solutions!

Generic solutions ("linear dilaton vacua")

Florian Ecker et al. 2308.10947

Solution algorithm inspired by Lorentzian 2d dilaton gravity (see DG, Kummer, Vassilevich '02)

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and solve it for X_{H} as function of dilaton X $\left(w(X) := \int^X V(y)\,\mathrm{d}y\right)$

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- solve absence of intrinsic torsion by $e = \mathrm{d}r$ ("radial coordinate")
- remaining equations yield dilaton, timelike vector field, and metric

$$\mathrm{d}r = -\frac{\mathrm{d}X}{X_{\mathrm{H}}} \qquad\qquad v = \frac{1}{X_{\mathrm{H}}}\,\partial_t \qquad\qquad \mathrm{d}s^2 = \mathrm{d}r^2$$

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- pick $V(X) = \frac{1}{\ell^2}\,X$ with Carroll-AdS radius $\ell > 0$

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$$X_{\mathrm{P}} = 0 \qquad\qquad e = \mathrm{d}r\,.$$

or in metric variables

$$v = \frac{2\ell\, e^{-r/\ell}}{1 - 2M\ell^2\, e^{-2r/\ell}}\, \partial_t \qquad\qquad \mathrm{d}s^2 = \mathrm{d}r^2$$

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- three qualitatively different classes of solutions:
  1. $M < 0$: reminiscent of global AdS_2 in JT
  2. $M = 0$: reminiscent of Poincaré patch AdS_2 in JT
  3. $M > 0$: reminiscent of black hole sector of JT

Example: Carroll–Jackiw–Teitelboim model

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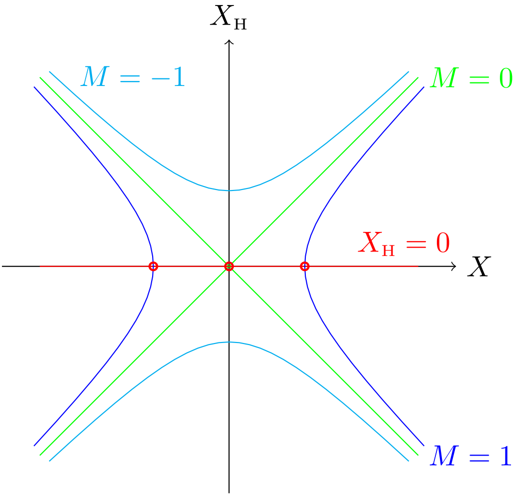
key feature: locus with $X_{\mathrm{H}} = 0$ part of solution?



Outline

Carroll energy

- codimension-2 charge variation for generic PSM:

$$\delta Q[\lambda] = \frac{k}{2\pi}\,\lambda_I\,\delta X^I$$

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- define Carroll energy as charge associated with unit time translations

$$E = \frac{k}{2\pi}\,M$$

Carroll temperature

- demand disk topology (with center at $X_{\text{H}} = 0$)

$$2\pi \overset{!}{=} \int_{\mathcal{M}} \mathrm{d}\omega - \int_{\partial\mathcal{M}} \omega$$

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- Interpreting $\beta = T^{-1}$ as inverse Carroll temperature yields

$$T = \frac{w'(X_{\text{min}})}{2\pi}$$

  formally identical to Hawking temperature in Lorentzian case

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$$S = k\,X_{\text{min}}$$

concurrent with the Lorentzian result for the Wald entropy ("dilaton evaluated at the horizon")

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- e.g. view Carrollian theories as limits of Carrollian expansions where speed of light still present

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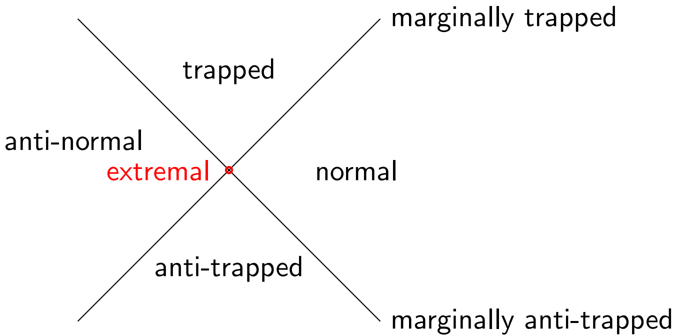
signs	$X^+ > 0$	$X^+ < 0$	$\boldsymbol{X^+ = 0}$
$X^- > 0$	anti-trapped	anti-normal	marginally anti-trapped
$X^- < 0$	normal	trapped	marginally trapped
$\boldsymbol{X^- = 0}$	marginally anti-trapped	marginally trapped	**extremal**



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- action of Lorentzian boosts on $X^{\pm}$:

$$\delta_\lambda X = 0 \qquad\qquad \delta_\lambda X^{\pm} = \mp\lambda X^{\pm}$$

- same result evaluated at extremal surface:

$$\delta_\lambda X|_{\text{ext}} = 0 \qquad\qquad \delta_\lambda X^{\pm}|_{\text{ext}} = 0$$

Extremal surfaces are boost invariant loci!

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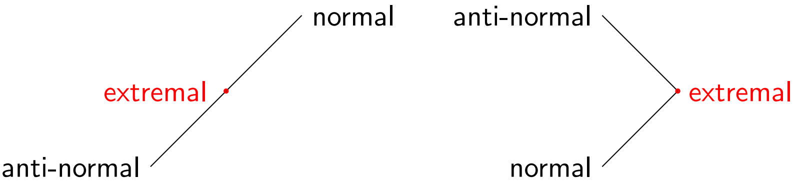

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Proposal for Carroll black hole definition in 2d

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Carroll black holes are defined to have all of these properties:

1. (exact) solution to some Carroll gravity
2. Carroll thermal state (finite temperature and entropy)
3. must have (isolated) CES

Outline

Selected list of 2d Carroll dilaton gravity models $\mathcal{V} = V(X) - \frac{1}{2}\,X_{\mathrm{H}}^2\,U(X)$

Model	$U(X)$	$V(X)$
1. Carroll–Schwarzschild	$-\frac{1}{2X}$	$-\lambda^2$
2. Carroll–Jackiw–Teitelboim	0	ΛX
3. Carroll–Witten BH	$-\frac{1}{X}$	$-2b^2 X$
4. Carroll–CGHS	0	λ
5. Carroll–Schwarzschild–Tangherlini	$-\frac{D-3}{(D-2)X}$	$-\lambda^2 X^{(D-4)/(D-2)}$
6. All above: Carroll ab-family	$-\frac{a}{X}$	$B X^{a+b}$
7. Carroll–Liouville gravity	a	$b e^{\alpha X}$
8. Carroll–Reissner–Nordström	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
9. Carroll–Schwarzschild-(A)dS	$-\frac{1}{2X}$	$-\lambda^2 + \Lambda X$
10. Carroll–Katanaev–Volovich	α	$\beta X^2 - \Lambda$
11. Carroll–Achúcarro–Ortiz	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
12. Carroll 2D type 0A string BH	$-\frac{1}{X}$	$-2b^2 X + \frac{b^2 q^2}{8\pi}$

- CSBH given by 2d Carroll dilaton gravity with potentials

$$U(X) = -\frac{1}{2X} \qquad V(X) = \frac{\lambda^2}{4}$$

yielding the solutions

$$X_{\text{H}} = -\sqrt{4X - 4M\sqrt{X}} \qquad \tau = -\frac{X_{\text{H}}}{2\sqrt{X}}\,\mathrm{d}t \qquad e = \mathrm{d}r$$

Carroll–Schwarzschild black hole
2d Carroll dilaton gravity perspective

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- in second order variables we get ($X = \mathrm{r}^2$, $M = 2m$)

$$v = -\frac{1}{\sqrt{1-\frac{2m}{\mathrm{r}}}}\,\partial_t \qquad h = \frac{\mathrm{dr}^2}{1-\frac{2m}{\mathrm{r}}}$$

Carrollian curvature scalar singular at origin, $R = -4m/\mathrm{r}^3$

- CSBH given by 2d Carroll dilaton gravity with potentials

$$U(X) = -\frac{1}{2X} \qquad V(X) = \frac{\lambda^2}{4}$$

yielding the solutions

$$X_{\text{H}} = -\sqrt{4X - 4M\sqrt{X}} \qquad \tau = -\frac{X_{\text{H}}}{2\sqrt{X}}\,\mathrm{d}t \qquad e = \mathrm{d}r$$

- in second order variables we get ($X = \mathrm{r}^2$, $M = 2m$)

$$v = -\frac{1}{\sqrt{1-\frac{2m}{\mathrm{r}}}}\,\partial_t \qquad h = \frac{\mathrm{dr}^2}{1-\frac{2m}{\mathrm{r}}}$$

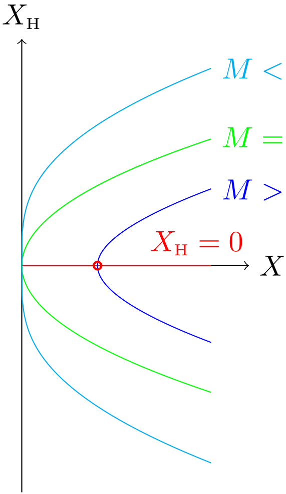
Carrollian curvature scalar singular at origin, $R = -4m/\mathrm{r}^3$

- Carroll thermodynamics yields

$$E = \frac{km}{\pi} \qquad T = \frac{1}{8\pi m} \qquad S = 4km^2$$

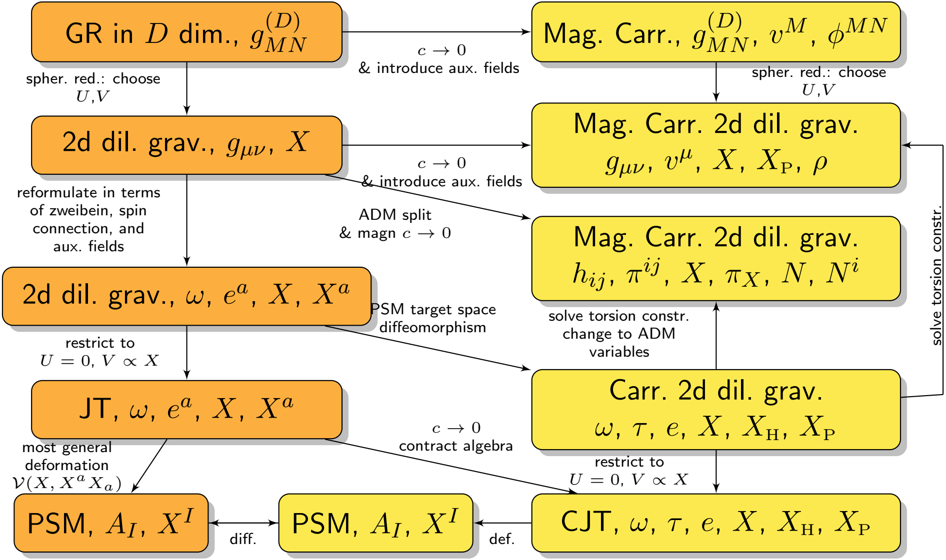
Carroll–Schwarzschild black hole

PSM target space perspective



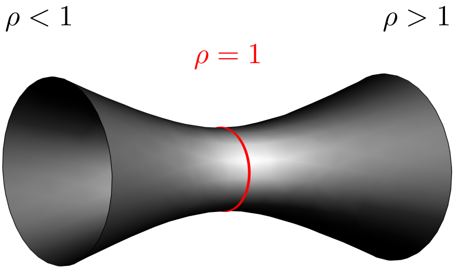
Carroll–Schwarzschild black hole

Map of perspectives (orange = Lorentzian, yellow = Carrollian)



Carroll–Schwarzschild black hole

4d Carroll gravity perspective



wormhole coordinates

$$\mathrm{r} = \frac{m}{2}\left(\rho + \frac{1}{\rho} + 2\right)$$

yield Carrollian structure (note $\rho \to 1$)

$$v = -\frac{\rho+1}{\rho-1}\,\partial_t$$

$$h = 4m^2\left(\frac{(\rho+1)^2}{4\rho^2}\right)^2\left(\mathrm{d}\rho^2 + \rho^2\,\mathrm{d}\Omega^2\right)$$

- magnetic $c \to 0$ limit of Schwarzschild metric

$$\mathrm{d}s^2 = -c^2\left(1 - \frac{2m}{\mathrm{r}}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - \frac{2m}{\mathrm{r}}} + \mathrm{r}^2\,\mathrm{d}^2\Omega$$

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4d Carroll gravity perspective on CES

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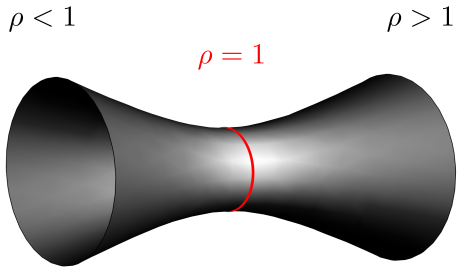
- CES precisely at throat of the wormhole!



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$$S_{\text{BH}} = \frac{4\pi c^3\, m^2}{\hbar G} \qquad T_{\text{BH}} = \frac{\hbar c}{8\pi\, m} \qquad E_{\text{BH}} = \frac{c^4}{G}\, m$$

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- of course, the first law holds: $\delta E = T\,\delta S$

Outline

Charged Carroll black holes

- adding a Maxwell field A to 2d Carroll gravity straightforward

$$\mathcal{L} = Y\,\mathrm{d}A + X\,\mathrm{d}\omega + X_{\text{H}}\big(\mathrm{d}\tau + \omega\wedge e\big) + X_{\text{P}}\,\mathrm{d}e + \mathcal{V}(X,\,X_{\text{H}},\,Y)\,\tau\wedge e$$

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  $$|q_e| \leq m$$

Rotating Carroll black holes

Carroll limit of spherically reduced BTZ black hole (a.k.a. Carroll–Achúcarro–Ortiz)

- Kaluza–Klein reduction of AdS_3 Einstein gravity

$$\mathrm{d}s^2 = g_{\alpha\beta}(x^\gamma)\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta + X^2(x^\gamma)\,\big(\mathrm{d}\varphi + A_\alpha(x^\gamma)\,\mathrm{d}x^\alpha\big)^2$$

leads to Achúcarro–Ortiz model (2d dilaton gravity) with potential

$$V_{\mathrm{AO}}(X,\,Y) = \frac{X}{\ell^2} - \frac{Y^2}{X^3}$$

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$$\mathrm{d}s^2 = \mathrm{d}r^2 = \frac{\mathrm{d}X^2}{X_{\text{H}}^2} \qquad v = \frac{1}{X_{\text{H}}}\,\partial_t \qquad X_{\text{H}} = \pm\sqrt{X^2 - \frac{J^2}{X^2} - 2M}$$

with gauge field $A = \frac{J}{X^2}\,\mathrm{d}t$ and $Y = J = \text{const.}$

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- have again two CES

$$X_\pm^2 = M \pm \sqrt{M^2 - J^2}$$

  and BPS-bound $|J| \leq M$

Other generalizations and applications

- mathematics of Carroll black holes
  - Carrollian structure singularities ($v \to \infty$, $v \to 0$)
  - topologies of Carroll manifolds
  - Carrollian singularity theorems?
  - sharper/alternative definitions of CES and Carroll black holes?
  - second and third law?

Other generalizations and applications

- mathematics of Carroll black holes
- rotating and/or supersymmetric Carroll black holes
  - Carroll supergravity
  - Carroll dilaton supergravity
  - BPS bounds
  - Killing spinors
  - rotating Carroll black holes in 4d and higher?

Other generalizations and applications

- mathematics of Carroll black holes
- rotating and/or supersymmetric Carroll black holes
- adding matter
  - electric vs. magnetic matter couplings
  - backreactions
  - loop effects
  - Carroll black hole formation?
  - Hawking-like effect?

Other generalizations and applications

- mathematics of Carroll black holes
- rotating and/or supersymmetric Carroll black holes
- adding matter
- fracton gravity
  - see 2308.10947
    simple map/reinterpretation of 2d Carroll dilaton gravity fields H, P, Q, D (energy, momentum, charge, dipole moment)

    $$[D,\, P] = Q$$

    fraction BF action = 2d Carroll dilaton gravity (without potential)

    $$\mathcal{L} = X_H\,\mathrm{d}A^H + X_P\,\mathrm{d}A^P + X_Q(\mathrm{d}A^Q + A^D \wedge A^P) + X_D\,\mathrm{d}A^D$$

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I conclude:

> Carroll black holes are fun — feel free to join the adventure