Radiative asymptotic symmetries of 3d Einstein–Maxwell

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One-sentence summary

At null infinity, 3d Einstein–Maxwell $\sim$ 4d vacuum gravity

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- Lots of recent exciting developments
  - Carrollian physics [Bagchi, Ecker, Grumiller, Hartong, Obers, Pérez, Prohazka, . . .]
  - celestial/Carrollian holography [Donnay, Herfray, Petropoulos, Puhm, Raclariu, Strominger, . . .]
  - classical and quantum soft theorems [Campiglia, He, Laddha, Lysov, Mitra, Sen, . . .]
  - covariant phase space [Barnich, Ciambelli, Freidel, Pranzetti, Speranza, Speziale, Wieland, . . .]
  - dual charges [Godazgar, Godazgar, Long, Oliveri, Pope, . . .]
  - extensions to (A)dS [Compère, Fiorucci, Pool, Ruzziconi, Skenderis, Taylor, Zwikel, . . .]
  - extensions to FLRW [Bonga, Enriquez-Rojo, Heckelbacher, Oliveri, Prabhu, Schroeder, . . .]
  - horizon tomography [Ashtekar, Khera, Kolanowski, Lewandowski, . . .]
  - log terms [Chrusciel, Mac Callum, Fuentealba, Henneaux, Singleton, Troessaert, Valiente Kroon, . . .]
  - new memory effects [Flanagan, Grant, Nichols, Oblak, Pasterski, Seraj, . . .]
  - inclusion of matter [Bonga, Grant, Majumdar, Mao, Oblak, Prabhu, . . .]
  - $w_{1+\infty}$ and twistors [Adamo, Costello, Mason, Paquette, Penrose, Sharma, . . .]

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- One can enrich this theory in a number of ways
  - relax the boundary conditions [Ciambelli, Delfante, Ruzziconi, Zwikel] [Goeller, MG, Zwikel]
  - couple to a perfect fluid with $p=w\rho$ to describe FLRW cosmologies: leads to $\mathfrak{bms}_3^w$ [Bonga, Prabhu] [Enriquez-Rojo, Heckelbacher, Oliveri] [MG, Portella]

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- We will follow [Barnich, Lambert, Mao], but relax their boundary conditions

Setup

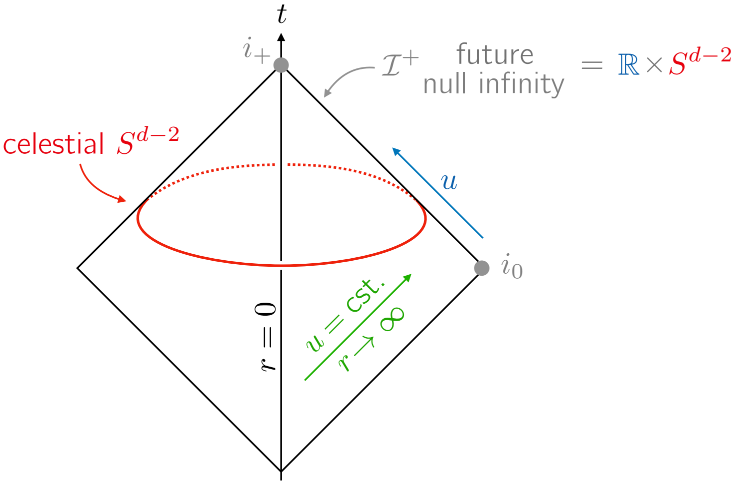
Geometry

Setup

Geometry

- Minkowski

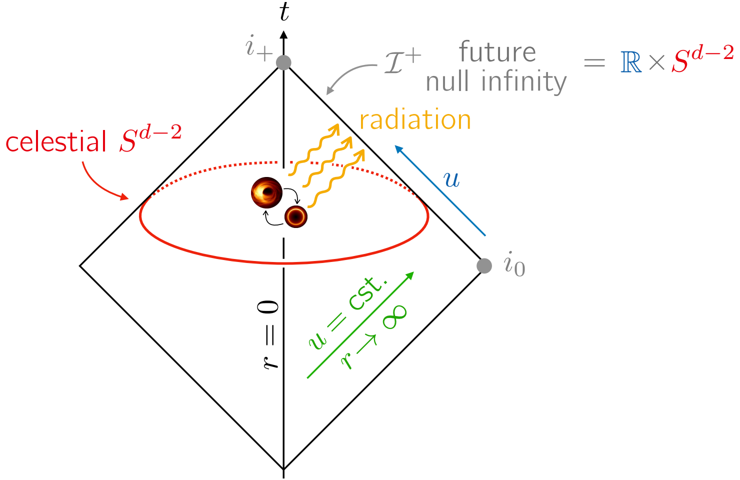
$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\,\mathrm{d}r + g_{ab}\mathrm{d}x^a\mathrm{d}x^b$$



Geometry

- Minkowski + radiation

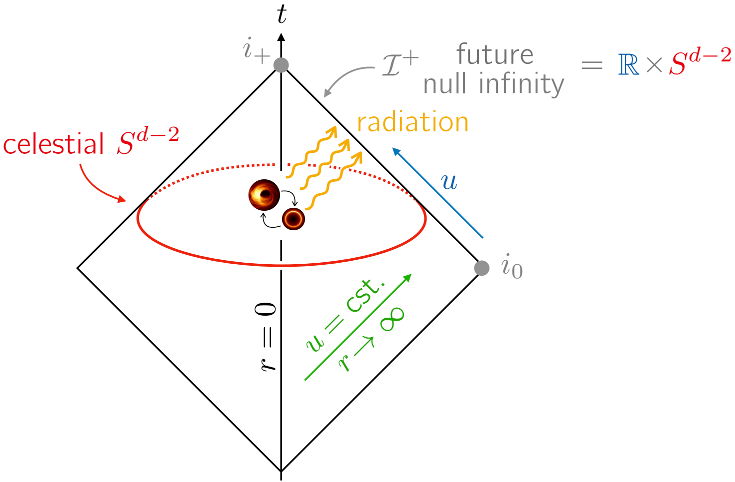
$$\mathrm{d}s^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\,\mathrm{d}r + g_{ab}\mathrm{d}x^a\mathrm{d}x^b + \mathcal{O}(?)$$



Setup

Geometry

- Minkowski + radiation ⇒ Bondi gauge



Setup

Geometry

- 4d Bondi gauge

$$\mathrm{d}s^2 = V B \,\mathrm{d}u^2 - 2B \,\mathrm{d}u \,\mathrm{d}r + g_{ab}(\mathrm{d}x^a - U^a \mathrm{d}u)(\mathrm{d}x^b - U^b \mathrm{d}u)$$

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- As a reminder, in the 4d vacuum case we have

$$g_{ab} = r^2 q_{ab} + r\,C_{ab} + D_{ab} + \frac{1}{r}\,E_{ab} + \mathcal{O}(r^{-2})$$

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3d vacuum gravity

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with mass $M(u,\phi)$ and angular momentum $P(u,\phi)$

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  with mass $M(u, \phi)$ and angular momentum $P(u, \phi)$
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- Adding matter will source $G_{\mu\nu} = T_{\mu\nu}$ and bring physics into the metric $\Rightarrow$ but which matter?

Setup

Maxwell field

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- Motivated by the radiative solutions of 3d Maxwell theory, we consider the gauge $A_r = 0$ and

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- A_ϕ plays the role of g_{ab} in 4d, and contains the simplest form of news available

	4d	3d
Einstein	$\dot{C}_{ab}$	$\emptyset$
Maxwell	$\dot{C}_a$	$\dot{C}$

Solution space

Solving the Einstein–Maxwell field equations

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- u-dependent data also appears in 4d when we relax the boundary metric $(\sqrt{q},\ U_0,\ B_0,\ \dots)$
  [Barnich, Troessaert] [Compere, Fiorucci, Ruzziconi] [MG, Zwikel] [MG, Goeller, Zwikel]

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- This also suggests an analogy with the Ψ_0/multipolar structure of 4d gravity [Freidel, Pranzetti, Raclariu] [Blanchet, Compère, Faye, Oliveri, Seraj]

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$\nabla^\mu F_{\mu\phi}\big\|_{\text{lead}} = 0 \quad \Rightarrow \quad \Lambda C = 0$	$G_{ab}\big\|_{\text{lead}} = 0 \quad \Rightarrow \quad \Lambda C_{ab} = (\partial_u - \partial_u \ln\sqrt{q}) q_{ab}$
$\sqrt{r}^{\,2\mathbb{N}+1}$ terms have to vanish	$\ln(r)$ terms have to vanish
$\nabla^\mu F_{\mu\phi}\big\|_{\text{sub}} = 0 \quad \Rightarrow \quad \dot{A}^n_\phi = \Lambda A^{n+1}_\phi + \dots$	$G_{ab}\big\|_{\text{sub}} = 0 \quad \Rightarrow \quad \dot{g}^n_{ab} = \Lambda g^{n+1}_{ab} + \dots$
$\mathcal{I}^+$ data $(C, G)\big\|_{\Lambda=0}$ $\mathcal{I}^+$ data $(A^\ell_\phi, A^0_\phi, G)\big\|_{\Lambda\neq 0}$	$\mathcal{I}^+$ data $C_{ab}\big\|_{\Lambda=0}$ $\mathcal{I}^+$ data $(q_{ab}, E_{ab})\big\|_{\Lambda\neq 0}$
infinite $\mathcal{I}^+_-$ data $(M, P, Q, A^n_\phi)\big\|_{\Lambda=0}$ finite $\mathcal{I}^+_-$ data $(M, P, Q)\big\|_{\Lambda\neq 0}$	infinite $\mathcal{I}^+_-$ data $(M, P_a, g^n_{ab})\big\|_{\Lambda=0}$ finite $\mathcal{I}^+_-$ data $(M, P_a)\big\|_{\Lambda\neq 0}$

- This also suggests an analogy with the Ψ_0/multipolar structure of 4d gravity [Freidel, Pranzetti, Raclariu] [Blanchet, Compère, Faye, Oliveri, Seraj]
- Let us set $\Lambda = 0$ again

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Asymptotic Killing vectors

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- With ξ and ε we know how any quantity transforms, in particular

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- One option used later will be to set

$$G = 0 \quad \Rightarrow \quad \alpha(u,\phi) = \alpha_0(\phi) + ug'Q$$

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- Recall that the Noether charge arises from the Lagrangian as

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- Under a change of split $\delta H_\xi + \Xi_\xi[\delta] = \delta(H_\xi + S_\xi) + \Xi_\xi[\delta] - \delta S_\xi$ the cocycle is modified :(

Noether split and bracket [Freidel, Oliveri, Pranzetti, Speziale]

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- Unfortunately this k_ξ is not conserved in any "natural" vacuum (e.g. absence of news)

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- H_ξ is conserved in the absence of news. Its evolution gives a purely radiative flux $\dot{H}_\xi = F_\xi\big[\dot{C}\big]$

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- The field-dependency of K seems robust, but what about the split ambiguity?

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- This leads once again to the field-dependent K_{ξ_1,ξ_2} above

Electromagnetic memory

Integrating the Lorentz force law

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- For 2 nearby particles with same mass, charge, and relative 3-velocity $\delta v = v_2 - v_1$, we get

$$\delta \dot{v}^\mu = F^\mu{}_\nu \delta v^\nu \qquad\qquad v = v_u \partial_u + v_r \partial_r + \frac{v_\phi}{r} \partial_\phi$$

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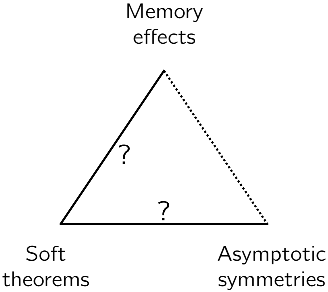
- The IR triangle [Strominger et al.] does not close because of radiative / Coulombic missmatch



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- 3d circle holography with matter fields
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Zero-mode solutions

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- Turning off the modes of A and keeping only Q, the EOM $\nabla^\mu F_{\mu\phi} = 0$ leads to $QP = 0$
- Instead, we can try to start from the charged BTZ [Martinez, Teitelboim, Zanelli]

$$
\begin{aligned}
A_\mu \mathrm{d}x^\mu &= \frac{Q}{\sqrt{2(1-P^2/\ell^2)}}(\ln r)\big(\mathrm{d}t - P\,\mathrm{d}\phi\big) \\
\mathrm{d}s^2 &= \left(N_r \frac{r^2}{\rho^2} + \rho^2 N_\phi^2\right)\mathrm{d}t^2 - N_r^{-1}\mathrm{d}r^2 + 2\rho^2 N_\phi\,\mathrm{d}t\,\mathrm{d}\phi + \rho^2 \mathrm{d}\phi^2 \\
N_r &= -\frac{r^2}{\ell^2} + M + Q^2(\ln r) \\
N_\phi &= -\frac{P}{\rho^2(1-P^2/\ell^2)}\big(M + Q^2(\ln r)\big) \\
\rho^2 &= r^2 + \frac{P^2}{1-P^2/\ell^2}\big(M + Q^2(\ln r)\big)
\end{aligned}
$$

- Taking the flat limit $\ell \to \infty$ yields

$$
\mathrm{d}s^2 = N_r\,\mathrm{d}t^2 - N_r^{-1}\mathrm{d}r^2 - 2PN_r\,\mathrm{d}t\,\mathrm{d}\phi + (r^2 + P^2 N_r)\mathrm{d}\phi^2
$$

- Mapping to Bondi gauge removes the angular momentum! (finite BMS_3 transformation)

$$
\mathrm{d}s^2 = N_r\,\mathrm{d}u^2 - 2\mathrm{d}u\,\mathrm{d}r + r^2\mathrm{d}\phi^2
$$