Memory effects in de Sitter and the Λ-BMS group

Geoffrey Compère

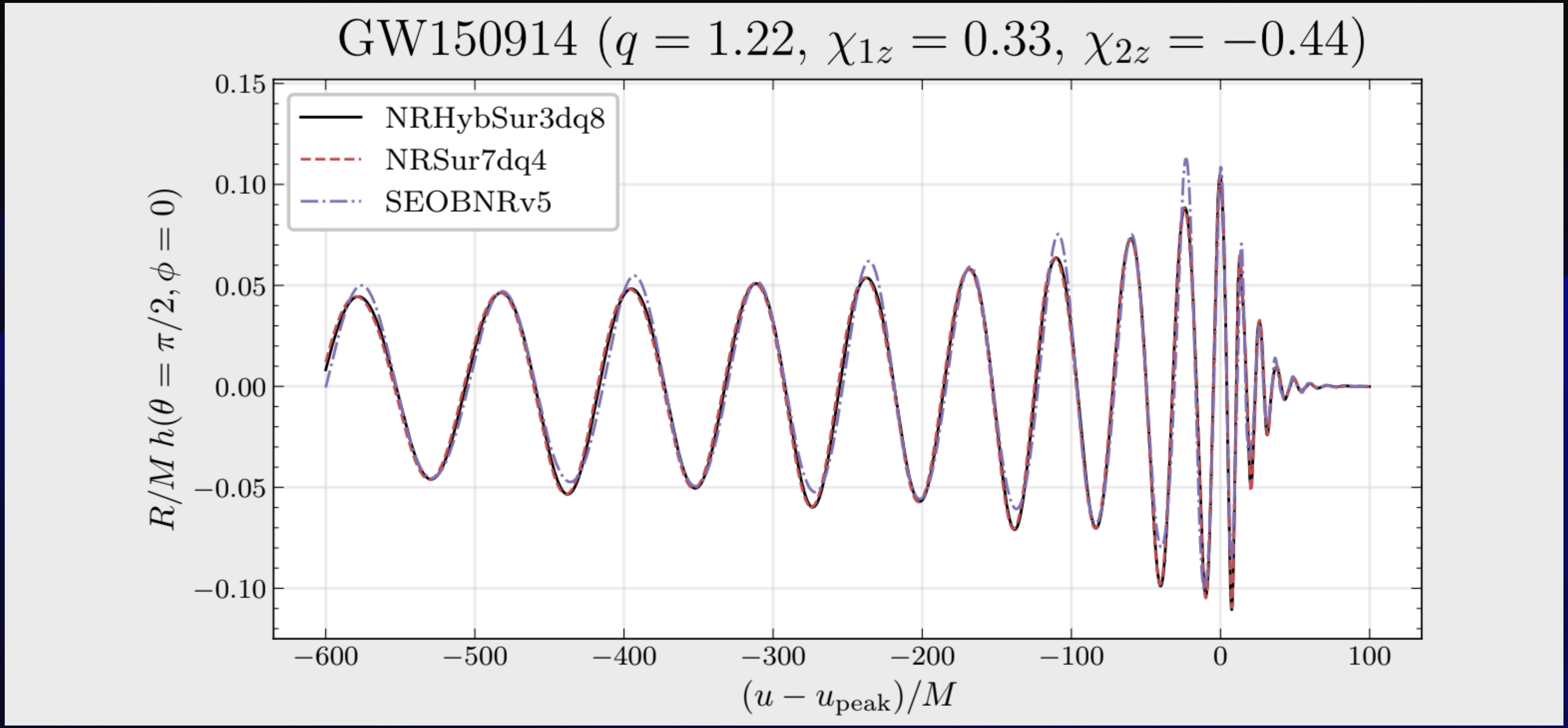
Based on 2309.02081 with Sk Jahanur Hoque and Emine Seyma Kutluk

3rd Carroll Workshop, Thessaloniki, October 4th 2023

Plan

1. Memory/BMS in flat spacetime
2. $\mathscr{I}^+$ in dS, Λ-BMS group
3. Linear fields in dS
4. Memory & Λ-BMS

The templates for GW150914 : state-of-the-art in 2015



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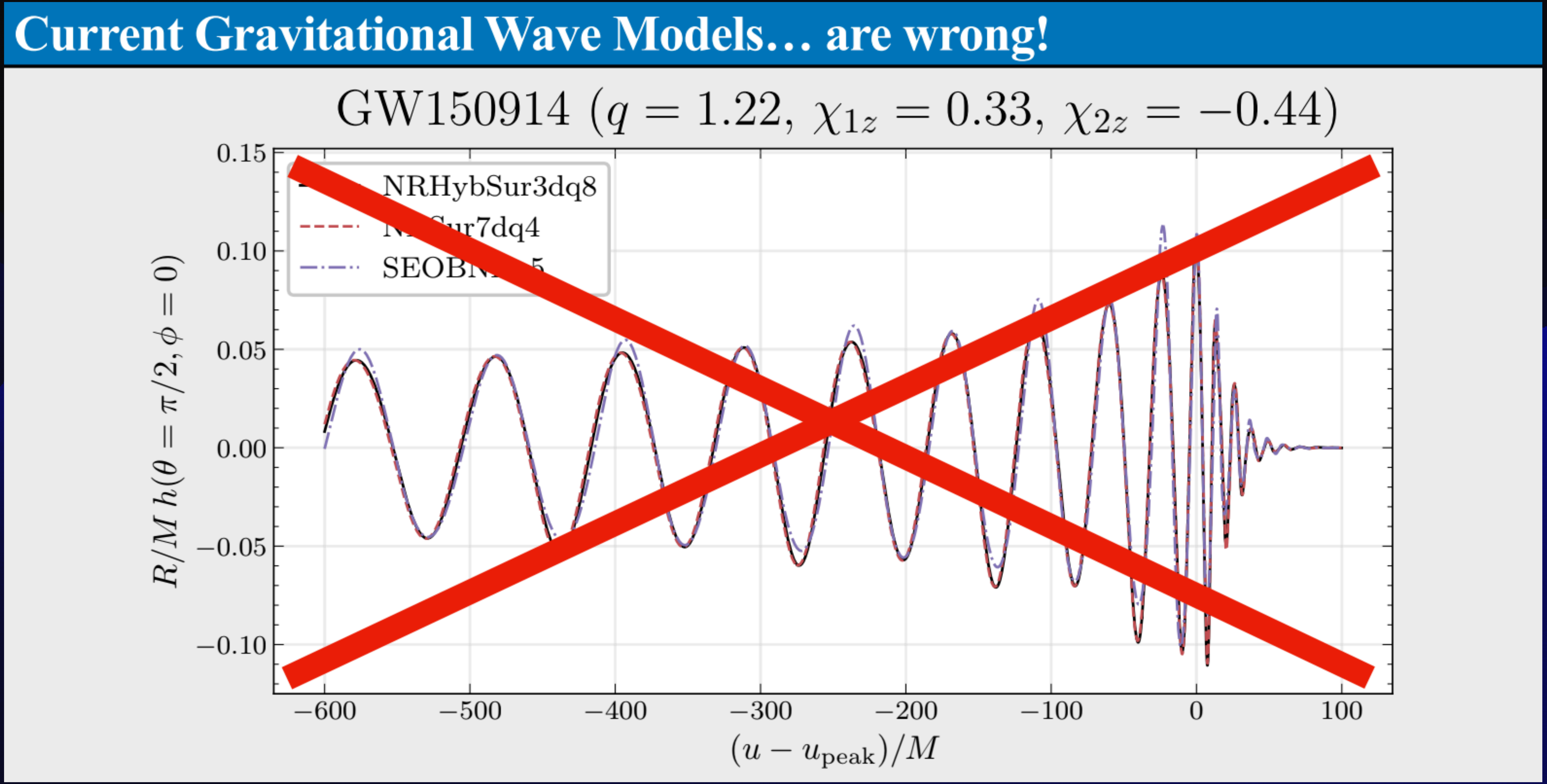
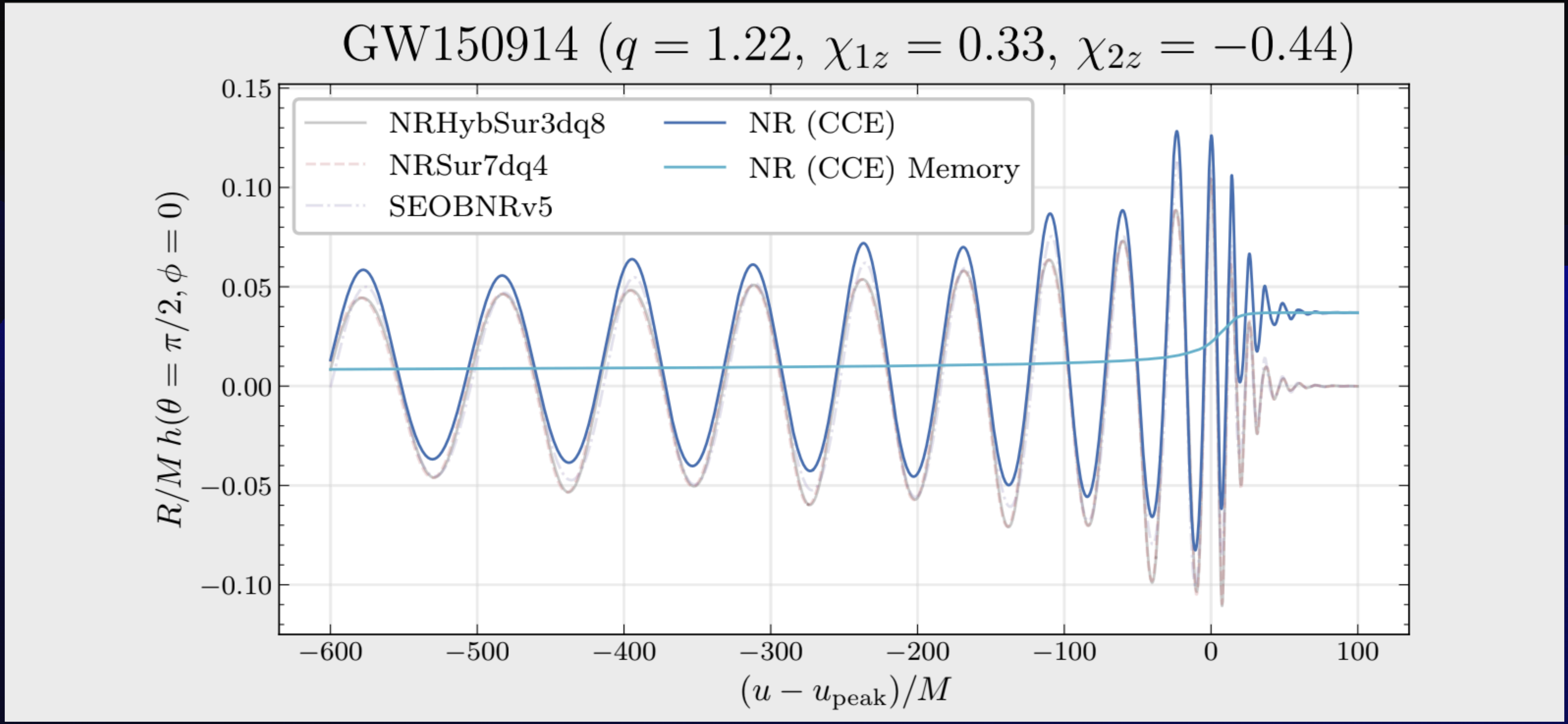


Keefe Mitman (w/ Jooheon Yoo and Leo Stein)
Gravitational Memory Effects: From Theory to Observation
Queen Mary University of London, June 7th, 2023

The templates for GW150914 : state-of-the-art in 2023

Current Gravitational Wave Models… are wrong!



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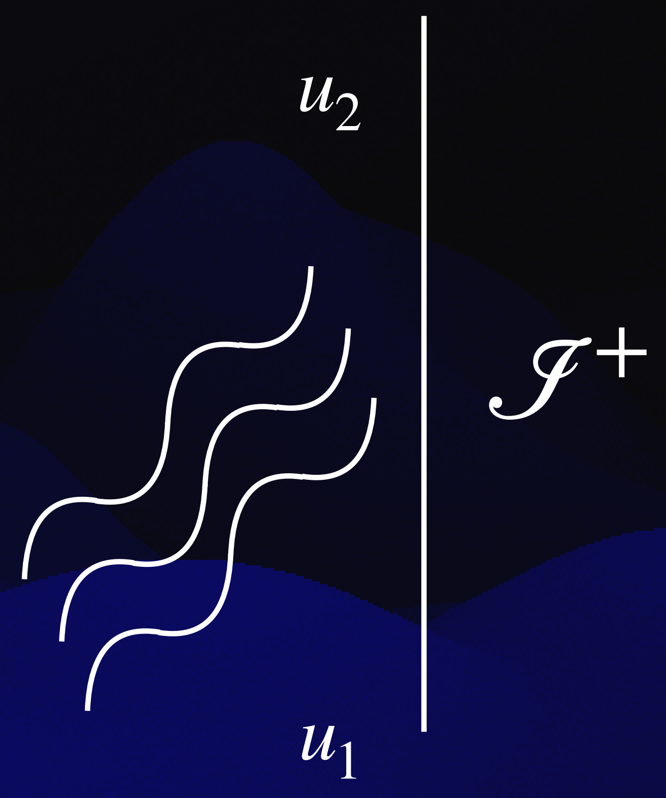
The displacement memory effect in General Relativity for $\Lambda = 0$ in a nutshell

$$\partial_u m = -\frac{1}{8}\partial_u C_{AB}\partial_u C^{AB} - \frac{1}{4}\nabla^2(\nabla^2 + 2)\partial_u C - 4\pi T^{(2)}_{uu}$$

$$ds^2 = \cdots + (r^2\gamma_{AB} + rC_{AB} + \ldots)dx^A dx^B$$

$$C_{AB} = (-2\nabla_A\nabla_B + \gamma_{AB}\nabla^2)C + \varepsilon_{C(A}\nabla_{B)}\nabla^C\Psi$$



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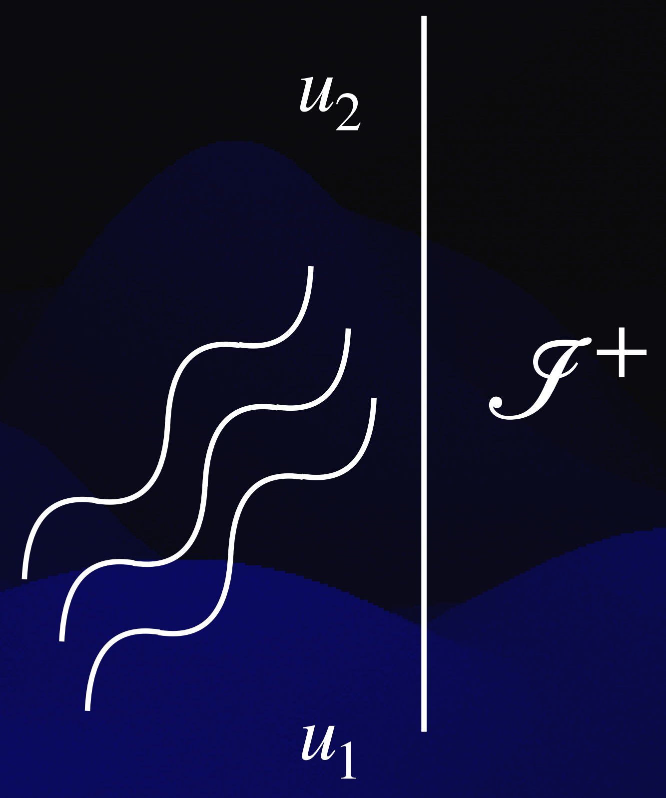
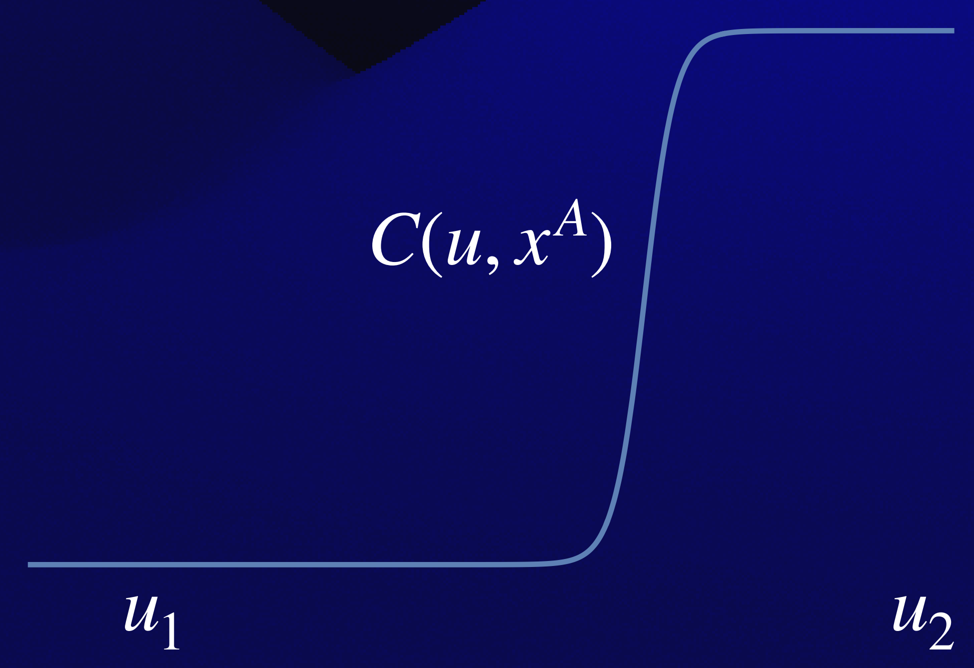


$$\int_{u_1}^{u_2} \quad \Longrightarrow \quad C_{\mathscr{I}^+} - C_{\mathscr{I}^+_-} = m_{\mathscr{I}^+} - m_{\mathscr{I}^+_-} + \mathscr{D}^{-1}\left(-\frac{1}{8}\partial_u C_{AB}\partial_u C^{AB} - 4\pi T^{(2)}_{uu}\right)$$



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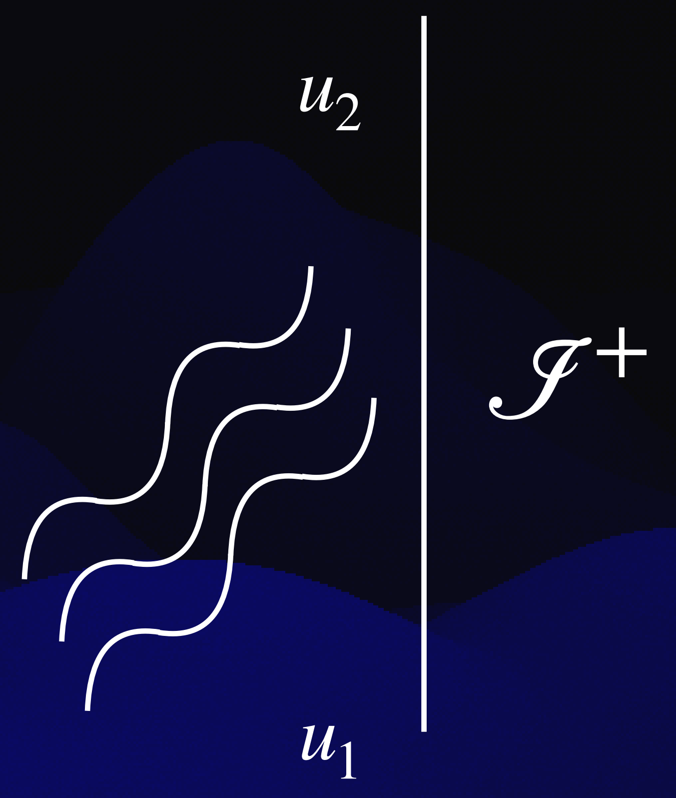

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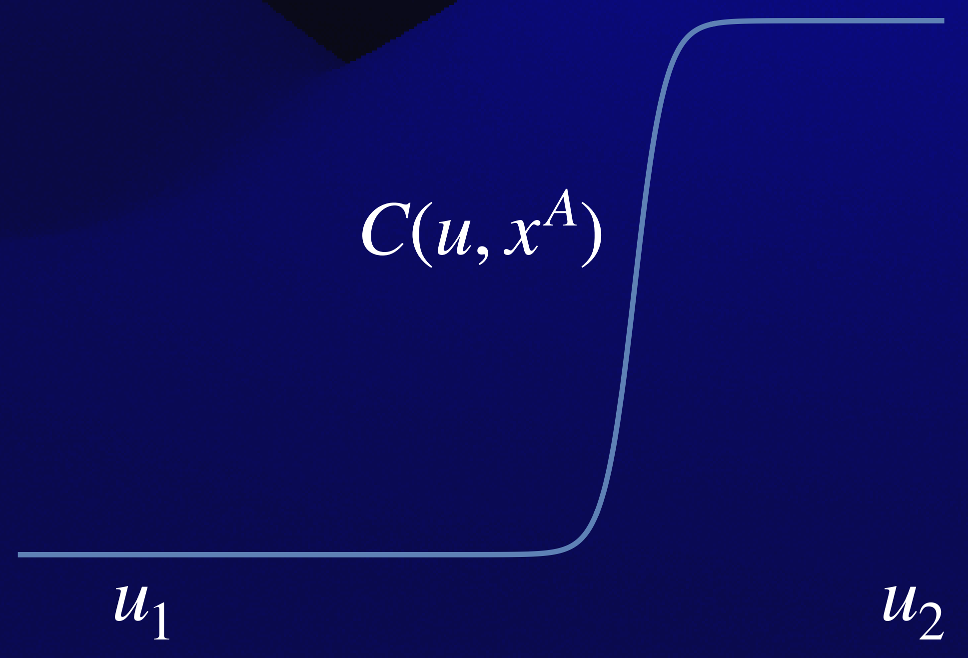
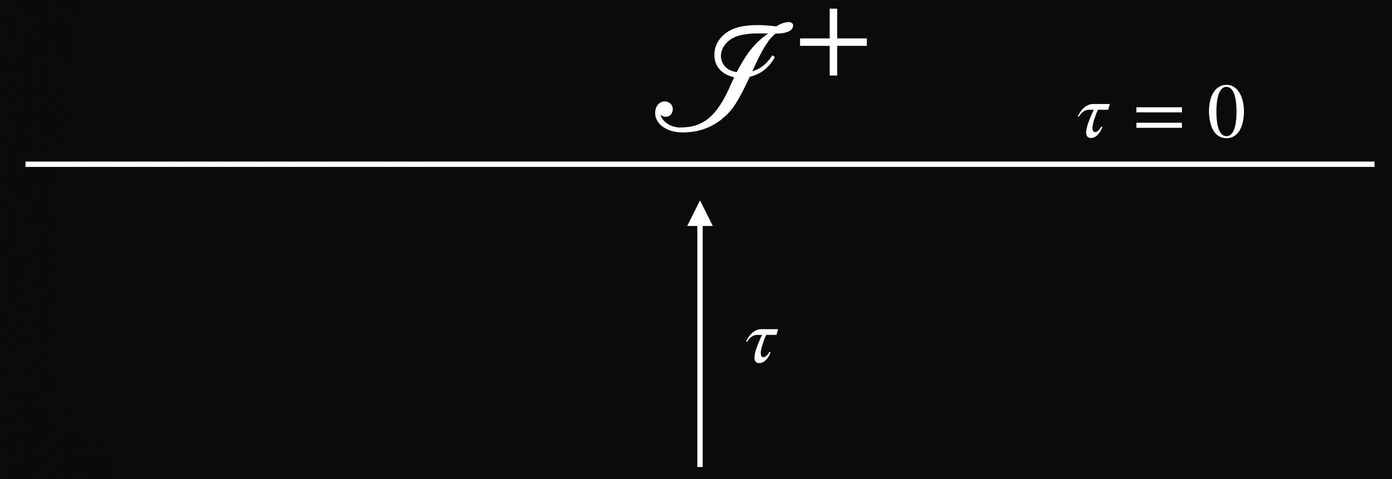


Features:

- An observer can fix a radiation gauge up to a residual BMS transformation that (s)he cannot fix.
- BMS supertranslations: $\delta_T C = T(x^A)$ "vacuum transition"
- In linear theory without matter: trivial u integration
- Generalized BMS group uncorrelated with memory for localized sources

The asymptotic structure of $\mathscr{I}^+$ for $\Lambda > 0$ in a nutshell



$$H = \sqrt{\frac{\Lambda}{3}}$$ ($= k$ in Arnaud's talk)

Starobinsky / Fefferman-Graham gauge :

$$ds^2 = -d\tau^2 + \tau^2(g^{(0)}_{ab}(x^c) + \ldots + \tau^{-3}T_{ab}(x^c) + \ldots)dx^a dx^b \qquad T^a_a = 0, \qquad D^a_{(0)}T_{ab} = 0$$

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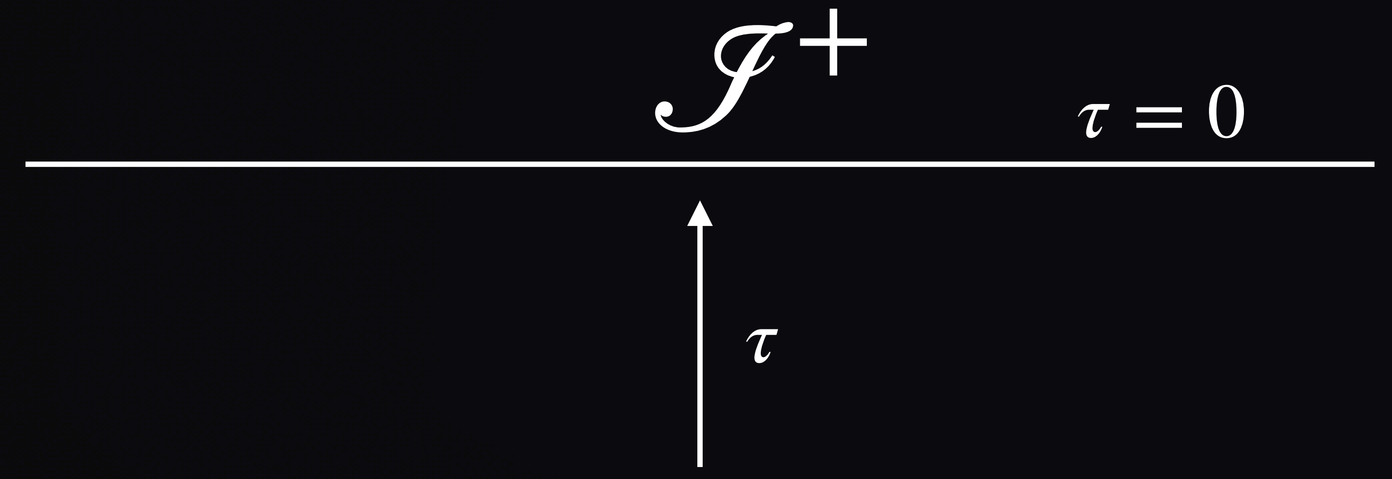

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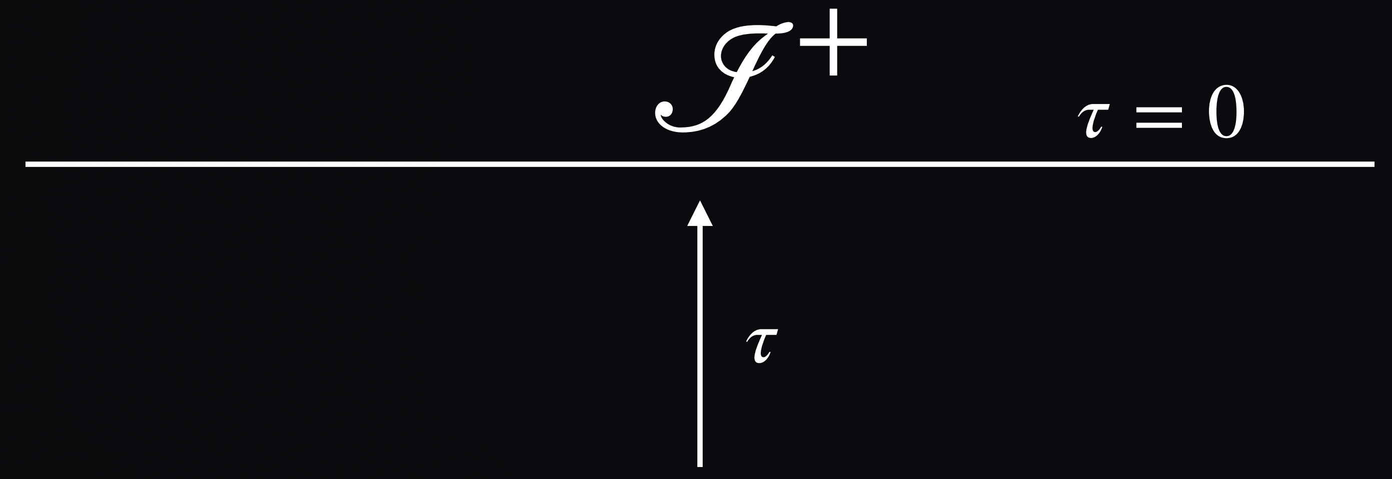

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We can further gauge fix the boundary metric :

$$g^{(0)}_{ab}dx^a dx^b = H^2 du + q_{AB}(u, x^C)dx^A dx^B \qquad \text{det}(q_{AB}) = \text{det}(\mathring{q}_{AB})$$

The residual gauge transformations are spanned by 3 functions of $x^A = (\theta, \phi)$.
They form the Λ-BMS algebroid whose structure constants depend upon the phase space field q_{AB} .

In the presence of radiation, an observer located close to $\mathscr{I}^+$ cannot gauge fix the diffeomorphism group any further. The Λ-BMS symmetries reflect the freedom at setting up a detector at $\mathscr{I}^+$ in asymptotically de Sitter. The same symmetries appear in Bondi gauge fixing as long as the boundary metric is gauged fixed.

The asymptotic structure of $\mathscr{I}^+$ for $\Lambda > 0$ in a nutshell

"3d" presentation of the Λ-BMS generators :

$$\xi^u = U(u, x^A) \qquad \partial_u U = -\frac{1}{2} D_A Y^A$$

$$\xi^A = Y^A(u, x^A) + O(r^{-1}) \qquad \partial_u Y^A = -H^2 q^{AB} \partial_B U$$

Algebroid :

$$[(U, Y^A), (U', Y'^A)] = (U'', Y''^A)$$

$$U'' = Y^A \partial_A U' + \frac{1}{2} U D_A Y'^A - (\leftrightarrow)$$

$$Y''^A = Y^B \partial_B Y'^A - H^2 U q^{AB} \partial_B U' - (\leftrightarrow)$$

[GC, Fiorucci, Ruzziconi, 2019]

In the flat limit, the algebra reduces to the generalized BMS algebra diff(S^2) + vect(S^2)

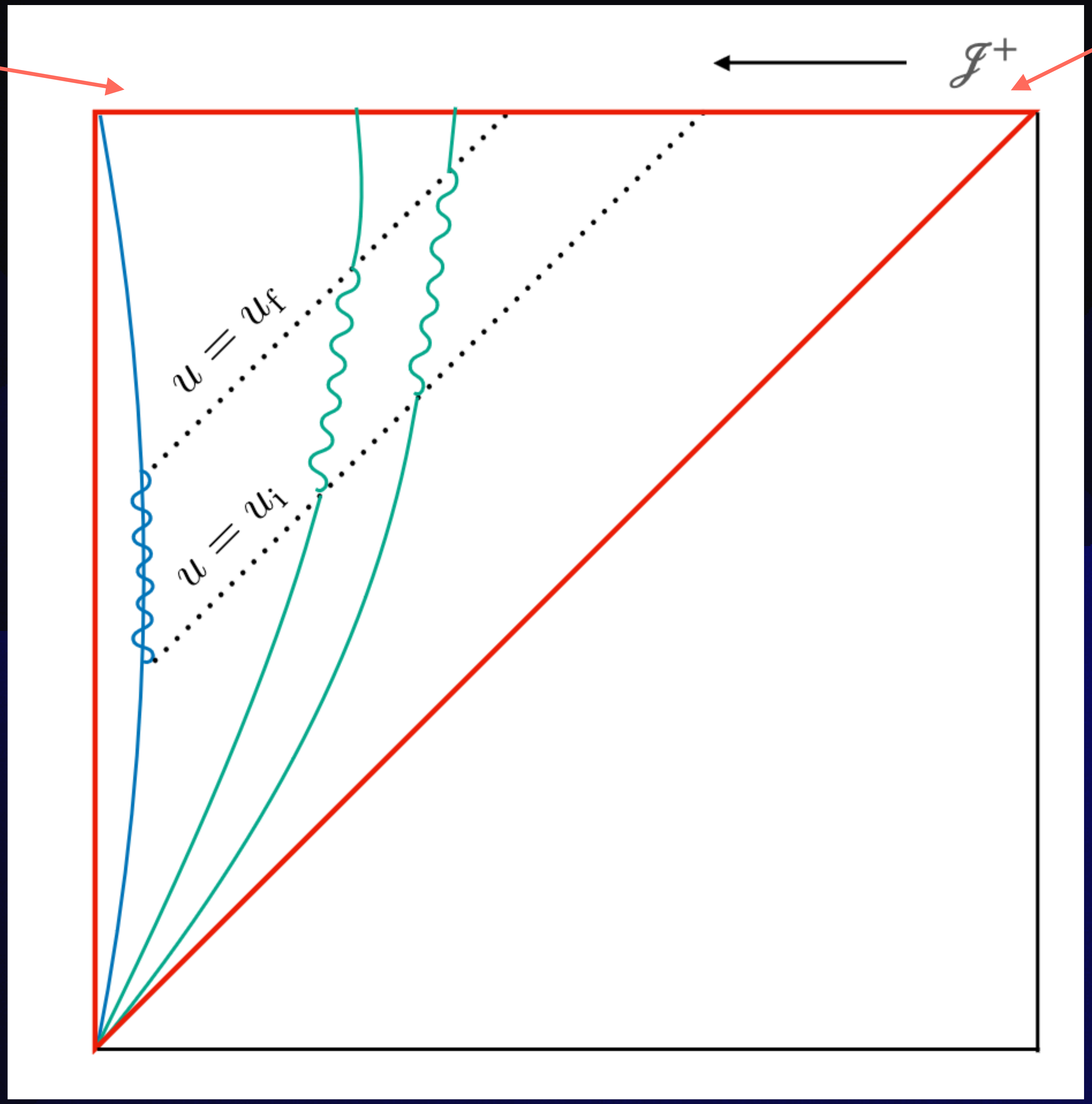
[Barnich, Troessaert, 2010]
[Campiglia, Laddha, 2015]

When $q_{AB}(u, x^A) = \mathring{q}_{AB}(x^A)$, the Λ-BMS algebroid becomes the Λ-BMS algebra that contains the $SO(4,1)$ algebra of exact symmetries of de Sitter.

Two boundary condition at $\mathscr{I}^+$

No radiation at $\mathscr{I}^+_+$: $q_{AB}(u, x^A) = q_{AB}(x^A)\,|_{u_\mathrm{f}}$

No radiation at $\mathscr{I}^+_-$: $q_{AB}(u, x^A) = q_{AB}(x^A)\,|_{u=u_\mathrm{i}}$



Note: The topology at $\mathscr{I}^+$ is S^3 minus 2 points : $\mathbb{R} \times S^2$.

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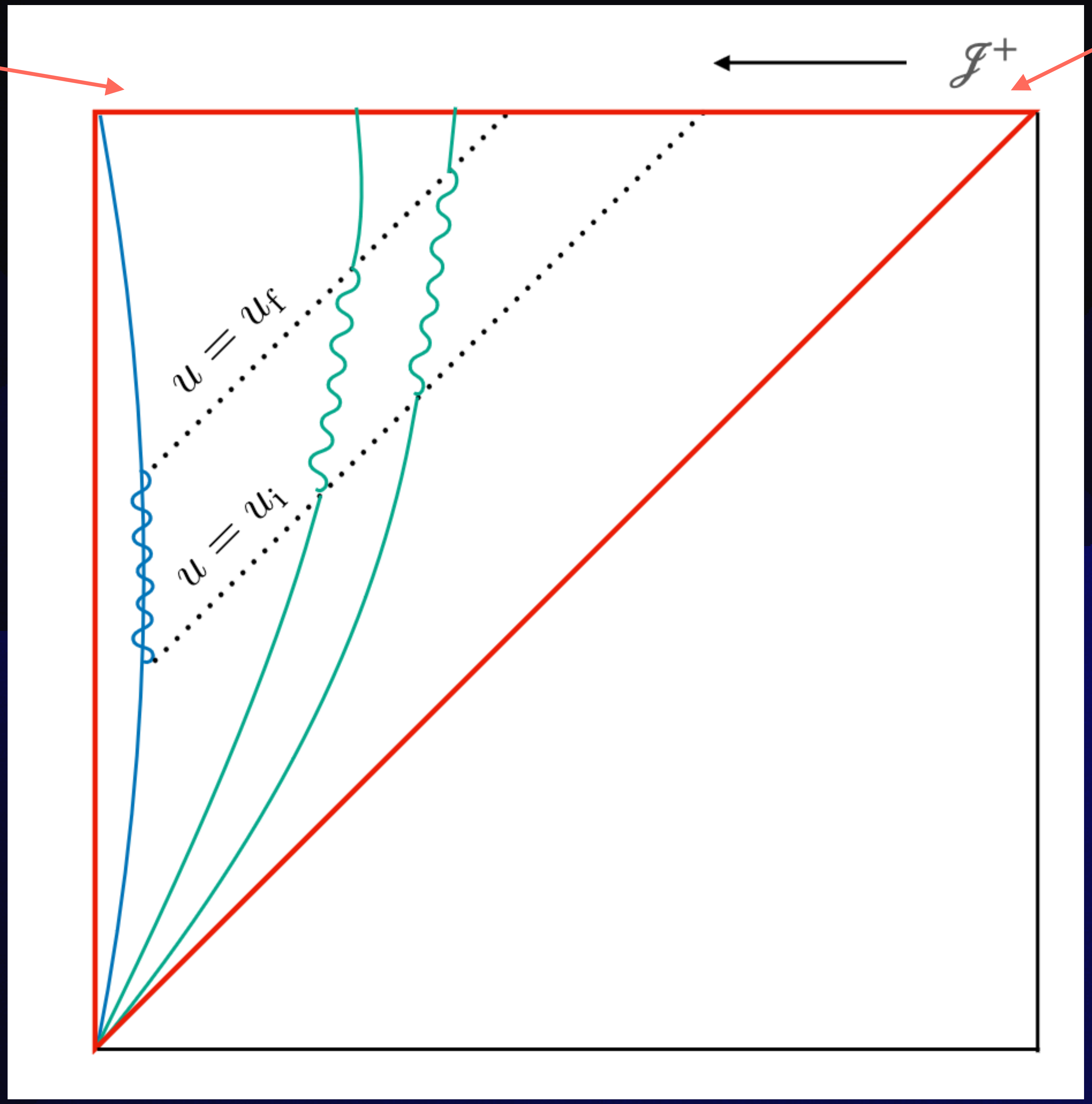

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What is the metric $q_{AB}(u, x^A)$ for a localized event below the Hubble scale?

Is there a Λ-BMS group transition after the passage of the gravitational wave strain?

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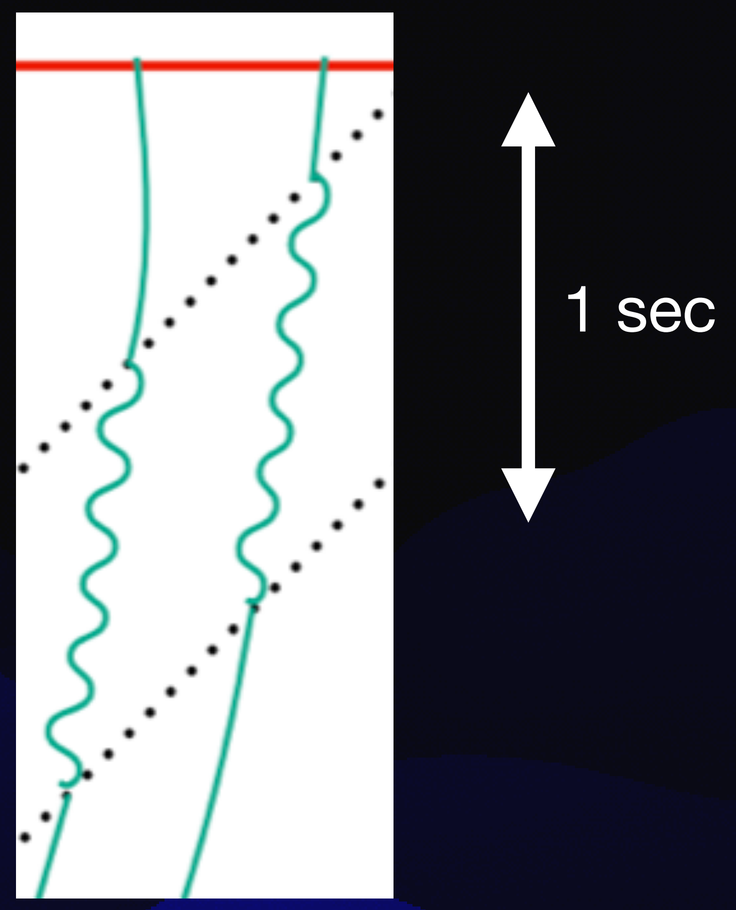

$$\partial_u q_{AB} = H^2 C_{AB}$$

$$q_{AB}|_{u_2} - q_{AB}|_{u_1} = H^2 \int_{u_1}^{u_2} du C_{AB}$$

This flux-balance law is specific to de Sitter.

Expect qualitative differences from the flat case !

What is the metric $q_{AB}(u, x^A)$ for a localized event below the Hubble scale?

Is there a Λ-BMS group transition after the passage of the gravitational wave strain?

A Carrollian thought

For $\Lambda = 0$, the 5 boundaries $(i^0, i^-, i^+, \mathscr{I}^+, \mathscr{I}^-)$ transform under a single BMS group. [GC, Gralla, Wei, 2023]

The BMS group can be described from a Carrollian structure consisting of

γ_{ab} : non-invertible metric of signature (0,+,+) of coordinates $x^a = (u, x^A)$
n^a : vector degenerate direction $n^a \gamma_{ab} = 0$

which is left invariant under the transformations $\mathscr{L}_\xi \gamma_{ab} = 2\alpha(u, x^A)\gamma_{ab}$, $\mathscr{L}_\xi n^a = -\alpha(u, x^A) n^a$. [Duval, Gibbons, Horvathy, 2014]. Carroll structures describe the entire boundary of flat spacetimes [Figueroa-O'Farrill, Have, Prohazka, Salzer, 2021]

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For $\Lambda > 0$, $\mathscr{I}^+$ is not described by a Carrollian structure. The flat Carrollian structure arises as the $H \mapsto 0$ limit :

$$g^{(0)}_{ab} dx^a dx^b = H^2 du + q_{AB}(u, x^C) dx^A dx^B$$

This limit "from a spacelike structure" is distinct from the $c \mapsto 0$ "from a timelike structure" limit.

[GC, Fiorucci, Ruzziconi, 2021]
[Campoleoni, Delfante, Pekar, Petropoulos, Rivera-Betancour, Vilatte, 2023]

Linear spin 2 field on de Sitter

Solved in the cosmology literature using a time-Fourier analysis.

However, this is very unsuitable to describe individual localized sources.

Instead, a multipolar decomposition is appropriate, as for $\Lambda = 0$ in the multipolar PN/PM formalism.

Starting point: de Sitter in the Poincaré patch

$$\bar{g}_{\alpha\beta}dx^\alpha dx^\beta = a^2(-d\eta^2 + d\vec{x}^2), \qquad a(\eta) = -\frac{1}{H\eta} \qquad \eta = -\frac{1}{H}e^{-Ht}$$

Perturbations $h_{\alpha\beta}$ are described using good variables: $\chi_{\mu\nu} = a^{-2}(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\alpha_\alpha), \quad \hat{\chi} = \chi_{00} + \chi_{ii} \ , \quad \chi_{0i}, \ \chi_{ij}$

and a good gauge "Generalized harmonic gauge" : $\partial^\alpha \chi_{\alpha\mu} + \frac{1}{\eta}(2\chi_{0\mu} + \delta^0_\mu \chi^\alpha_\alpha) = 0.$ [de Vega, Ramirez, Sanchez, 98]

Linear spin 2 field on de Sitter

Result:

$$\begin{aligned}
\Box\left(\frac{\hat{\chi}}{\eta}\right) &= -\frac{16\pi G\hat{T}}{\eta}, \\
\Box\left(\frac{\chi_{0i}}{\eta}\right) &= -\frac{16\pi G T_{0i}}{\eta}, \\
\left(\Box + \frac{2}{\eta^2}\right)\left(\frac{\chi_{ij}}{\eta}\right) &= -\frac{16\pi G}{\eta}T_{ij},
\end{aligned}$$

$$\hat{T} := T_{00} + T^{i}{}_{i}.$$

$$\Box = -\partial_\eta^2 + \partial_i^2$$

Scalar and vectors modes are similar to flat space.

Tensor mode depends upon the de Sitter potential. There is propagation inside the lightcone.

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Tensor Green function known [Ford, Parker, 77] [Weylen, 78]

$$G_R(\eta, x; \eta' x') = \frac{\Lambda}{3}\eta\eta' \frac{1}{4\pi}\frac{\delta(\eta-\eta'-|x-x'|)}{|x-x'|} + \frac{\Lambda}{3}\frac{1}{4\pi}\theta(\eta-\eta'-|x-x'|) .$$

We will express the solution in terms of multipole moments of the stress-energy tensor

$$Q_L^{(\rho)}(\eta) := \int d^3\bar{x} T_{\bar{0}\bar{0}} \bar{x}_L = \int d^3x a^{\ell+1}(\eta) T_{00} x_L,$$
$$P_{i|L}(\eta) := \int d^3\bar{x} T_{\bar{0}\bar{i}} \bar{x}_L = \int d^3x a^{\ell+1}(\eta) T_{0i} x_L,$$
$$S_{ij|L}(\eta) := \int d^3\bar{x} T_{\bar{i}\bar{j}} \bar{x}_L = \int d^3x a^{\ell+1}(\eta) T_{ij} x_L.$$

$$Q_L^{(p)}(\eta) = \int d^3\bar{x} \eta^{\bar{i}\bar{j}} T_{\bar{i}\bar{j}} \bar{x}_L = \int d^3x a^{\ell+1}(\eta) \delta_{ij} T_{ij} x_L = S_{ii|L}$$

The stress-energy tensor is conserved. This is equivalent to

$$\partial_t Q_L^{(\rho)} = H(\ell Q_L^{(\rho)} - Q_L^{(p)}) - \ell P_{(i_1|i_2 \cdots i_\ell)} \ ,$$
$$\partial_t P_{i|L} = (\ell-1) H P_{i|L} - \ell S_{i(i_1|i_2 \cdots i_\ell)},$$

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$$S_{i(j|L)} = -\frac{1}{\ell+1}(\partial_t - \ell H) P_{i|jL}.$$

$$S_{(ij|L)} = \frac{1}{(\ell+1)(\ell+2)}(\partial_t - \ell H)\left((\partial_t - (\ell+2)H) Q_{ijL}^{(\rho)} + H Q_{ijL}^{(p)}\right).$$

Consistent quadrupolar truncation

$$\int d^3x a^{\ell+1} T_{\mu\nu} x^L = 0, \qquad \forall \ell > 2.$$

The conservation equations imply

$$P_{(i|jk)} = 0, \qquad P_{i|jkl} = 0, \qquad S_{i(j|kl)} = 0.$$

Solution in terms of SO(3) irreducible tensors (dipoles $P_{i|kk}$, $Q_i^{(p)}$, odd parity quadrupoles J_{ij}, K_{ij}, even parity quadrupole $Q_{ij}^{(p)}$):

$$\begin{aligned}
P_{i|jk} &= \frac{1}{2}\epsilon_{li(j}J_{k)l} - \frac{1}{2}\delta_{i(k}P_{j)|ll} + \frac{1}{2}\delta_{jk}P_{i|ll}, \\
S_{ij|k} &= \frac{1}{2}\epsilon_{kl(i}K_{j)l} - \frac{1}{2}\delta_{k(i}Q_{j)}^{(p)} + \frac{1}{2}\delta_{ij}Q_k^{(p)}, \\
S_{ij|kl} &= \delta_{ij}Q_{kl}^{(p)} - (\delta_{i(k}Q_{l)j}^{(p)} + \delta_{j(k}Q_{l)i}^{(p)}) + Q_{ij}^{(p)}\delta_{kl} - \frac{1}{2}\delta_{ij}\delta_{kl}Q_{mm}^{(p)} + \frac{1}{2}\delta_{i(k}\delta_{l)j}Q_{mm}^{(p)},
\end{aligned}$$

where

$$\begin{aligned}
J_{ij} &:= \frac{4}{3}P_{k|l(i}\epsilon_{j)kl}, \\
K_{ij} &:= \frac{4}{3}\epsilon_{kl(i}S_{j)k|l}.
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$$\begin{aligned}
(\partial_t - H)P_{i|jj} &= Q_i^{(p)}, \\
(\partial_t - H)J_{ij} &= -K_{ij}.
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$$(\partial_t - H)P_{i|jj} = Q_i^{(p)}, \qquad (\partial_t - H)J_{ij} = -K_{ij}.$$

Our analysis differs from
[Ashtekar, Bonga, Kesavan, 2015][Chu, 2016]
[Date,Hoque,2016][Hoque,Virmani,2018]

Solving in the quadrupolar truncation

Bounded by the coordinate dimension of the source d

$$\hat{\chi} = 4 \int \frac{d^3x'}{|\vec{x}-\vec{x}'|} \frac{1}{1-\eta^{-1}|\vec{x}-\vec{x}'|} \hat{T}(\eta - |\vec{x}-\vec{x}'|, x'),$$

The solution is expressed in terms of retarded time $\eta_{ret} = \eta - \rho$. We assume that the physical size of the source is smaller than the Hubble scale at retarded time: $a(\eta_{ret})d \ll H^{-1}$. This implies:

$$\frac{d}{\rho} = \frac{a(\eta_{\rm ret})d}{a(\eta_{\rm ret})\rho} \ll \frac{1}{Ha(\eta_{\rm ret})\rho} = 1 - \frac{\eta}{\rho}.$$

Close to $\mathscr{I}^+$, $-\eta/\rho \ll 1$, therefore

$$\frac{d}{\rho} \ll 1, \qquad \frac{d}{-\eta_{\rm ret}} \ll 1.$$

We assume that the source is slowly varying:

$$T_{\mu\nu} > d\,\partial_{\eta_{\rm ret}} T_{\mu\nu} > d^2\,\partial^2_{\eta_{\rm ret}} T_{\mu\nu} > \dots$$

This implies that the quadrupolar radiation is dominant.

Solution in harmonic gauge

- Scalar, vector and tensor mode are obtained in closed form in harmonic gauge

- We keep all monopoles, dipoles and quadrupoles

- Remarkably, the non-local tensorial terms reduce in the quadrupolar truncation to instantaneous terms and terms at the cosmological horizon

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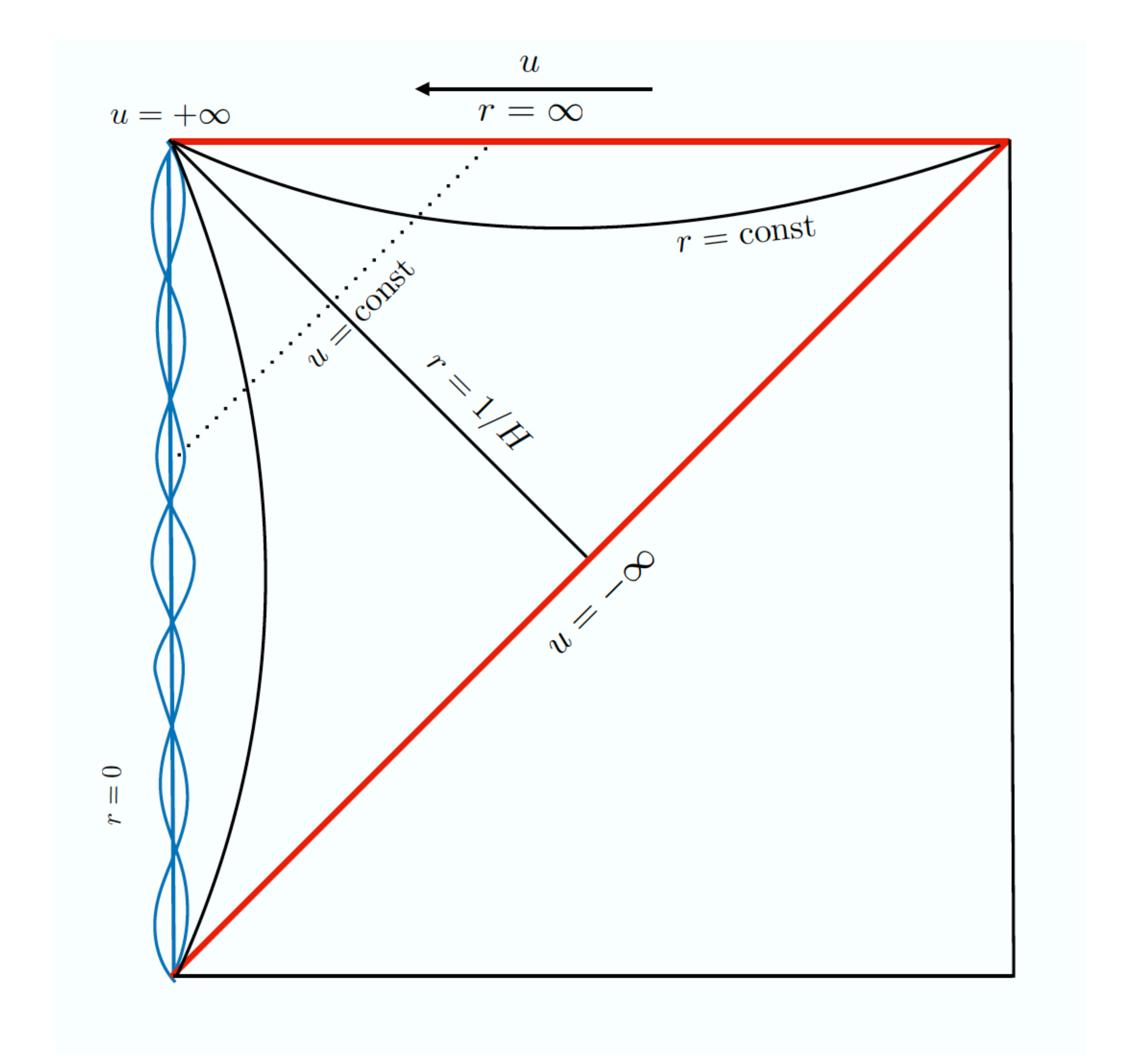
Harmonic gauge still admit residual gauge transformations.
A canonical harmonic gauge exists which is gauge invariant.
The linear solution is expressed in terms of canonical multipole moments $(M_L(u), S_L(u))$.
In the case $Q^{(p)} = 0 = Q_i^{(p)} = Q_{ij}^{(p)} = P_{i|kk}$ the flat limit of the solution is in canonical harmonic gauge.
Otherwise a change of coordinates is required.
The Thorne 1980 metric is recovered in the flat limit and using the simplification with the identification of multipoles

$$Q_L^{(\rho)} \to M_L, \qquad S_{ij} \to \frac{1}{2}\ddot{M}_{ij}, \qquad Q^{(p)} \to \frac{\delta^{ij}}{2}\ddot{M}_{ij}, \qquad P_{i|j} \mapsto \frac{1}{2}\epsilon_{ijk}J_k - \frac{1}{2}\dot{M}_{ij}.$$

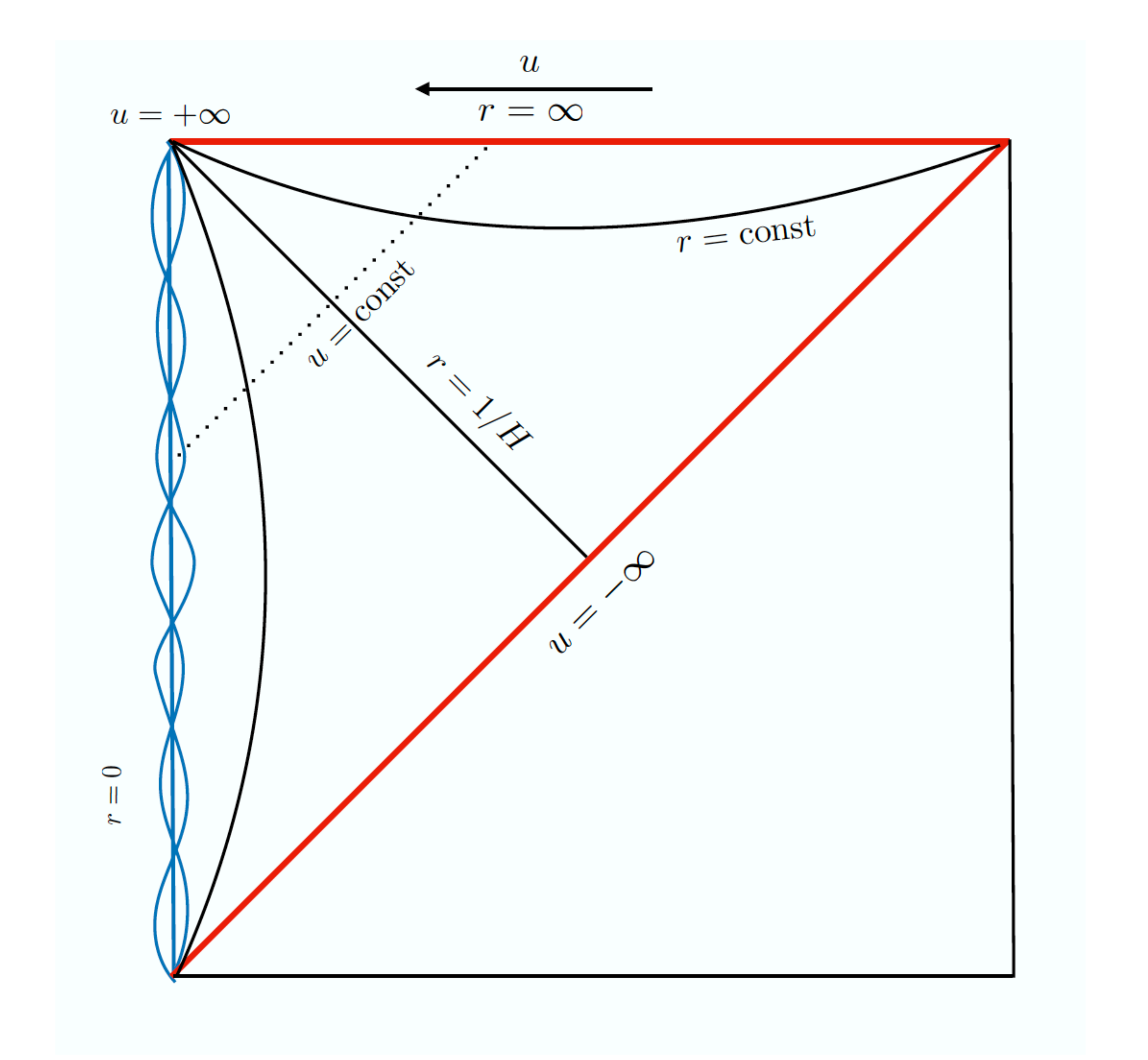
Solution in Bondi gauge



$$H_{\mu\nu}(X) = h_{\mu\nu}(X) + \mathcal{L}_\xi \bar{g}_{\mu\nu}(X), \qquad h_{\mu\nu}(X) := \frac{\partial \bar{x}^\alpha}{\partial X^\mu}\frac{\partial \bar{x}^\beta}{\partial X^\nu} h_{\alpha\beta}(\bar{x}(X)).$$

$$H_{rr} = 0, \qquad H_{rA} = 0, \qquad \partial_r(\mathring{q}^{AB} H_{AB}) = 0.$$

Solution in Λ-BMS gauge



$$H_{\mu\nu}(X) = h_{\mu\nu}(X) + \mathcal{L}_\zeta \bar{g}_{\mu\nu}(X), \qquad h_{\mu\nu}(X) := \frac{\partial \bar{x}^\alpha}{\partial X^\mu}\frac{\partial \bar{x}^\beta}{\partial X^\nu} h_{\alpha\beta}(\bar{x}(X)).$$

$$H_{rr} = 0, \qquad H_{rA} = 0, \qquad \partial_r(\mathring{q}^{AB} H_{AB}) = 0.$$

$$\lim_{r\to\infty}(r^{-2} H_{uu}) = 0, \quad \lim_{r\to\infty}(r^{-2} H_{uA}) = 0.$$

$$\lim_{r\to\infty}(r^{-2}\mathring{q}^{AB} H_{AB}) = 0,$$

Solution in Λ-BMS gauge

- The metric is obtained in closed form. There is no $\log r$. The expansion stops in the $1/r$ expansion.

- We keep all monopoles, dipoles and quadrupoles

- The flat limit $H \mapsto 0$ exactly matches the known linear perturbation at quadrupolar order. [Blanchet, GC, Faye, Oliveri, Seraj, 2020] . The canonical multipole moments are matched as

$$M_\emptyset = Q^{(\rho)}, \qquad M_i = Q_i^{(\rho)}, \qquad M_{ij} = Q_{ij}^{(\rho+p)} - \frac{1}{3}\delta_{ij} Q_{kk}^{(\rho+p)},$$
$$S_i = J_i, \qquad S_{ij} = \frac{3}{4} J_{ij}.$$

Solution in Λ-BMS gauge

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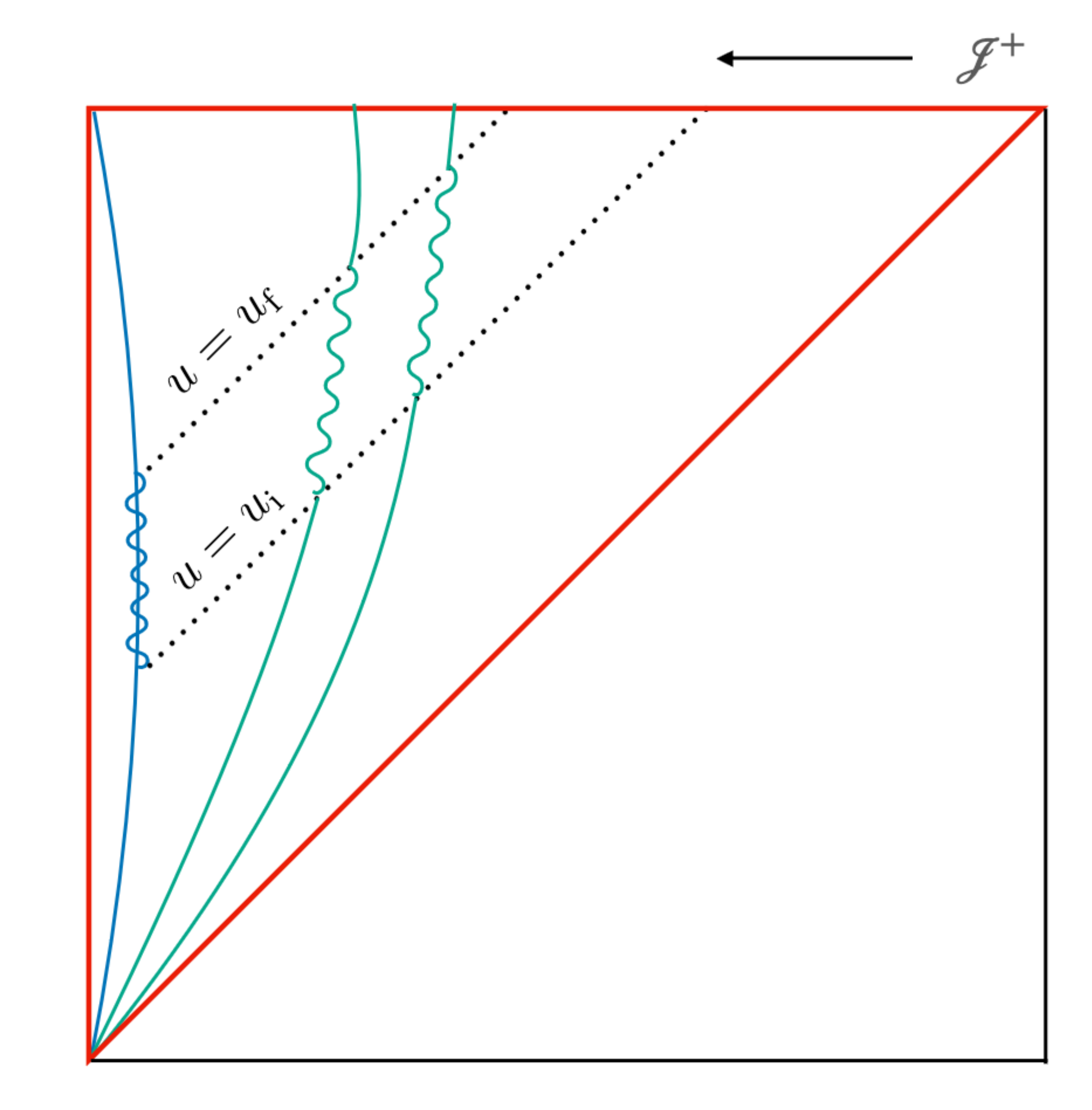
The boundary metric is given by (all quantities are evaluated at $\eta = -H^{-1}e^{-Hu}$)

$$q_{AB} = \mathring{q}_{AB} + 2\mathring{q}_{C\langle A}\mathring{D}_{B\rangle}\xi^C + e^i_{\langle A}e^j_{B\rangle}\left(\partial_u \zeta_{ij} + 2H^2\partial_u Q_{ij}^{(\rho+p)} + 2H^2\epsilon_{ikl}n_k\left(K_{jl} + H\int^u du' K_{jl}(u')\right)\right),$$

where $$\partial_u^2 \zeta_{ij} - 3H^2\zeta_{ij} = -2H^4 Q_{ij}^{(\rho+p)}$$

Summary of the linear analysis



We defined the even parity and odd parity quadrupolar moments of the stress-energy tensor as

$$Q_{ij}^{(\rho+p)}(\eta) \equiv \int d^3x a^3(\eta)(T_{00} + T_{kk})x_i x_j \qquad K_{ij}(\eta) \equiv \frac{4}{3}\int d^3x a^3(\eta)\epsilon_{kl(i}T_{j)k}x_l$$

We evaluate them at $\mathscr{I}^+ : \eta = -H^{-1}e^{-Hu}$. They correspond to a retarded field.

The boundary metric at $\mathscr{I}^+$ of the linear perturbation is given by

$$g_{ab}^{(0)}dx^a dx^b = H^2 du^2 + q_{AB}dx^A dx^B$$

$$q_{AB} = \mathring{q}_{AB} + 2\mathring{q}_{C\langle A}\mathring{D}_{B\rangle}\xi^C + e^i_{\langle A}e^j_{B\rangle}\left(\partial_u\zeta_{ij} + 2H^2\partial_u Q_{ij}^{(\rho+p)} + 2H^2\epsilon_{ikl}n_k(K_{jl} + H\int^u du' K_{jl}(u')\right),$$

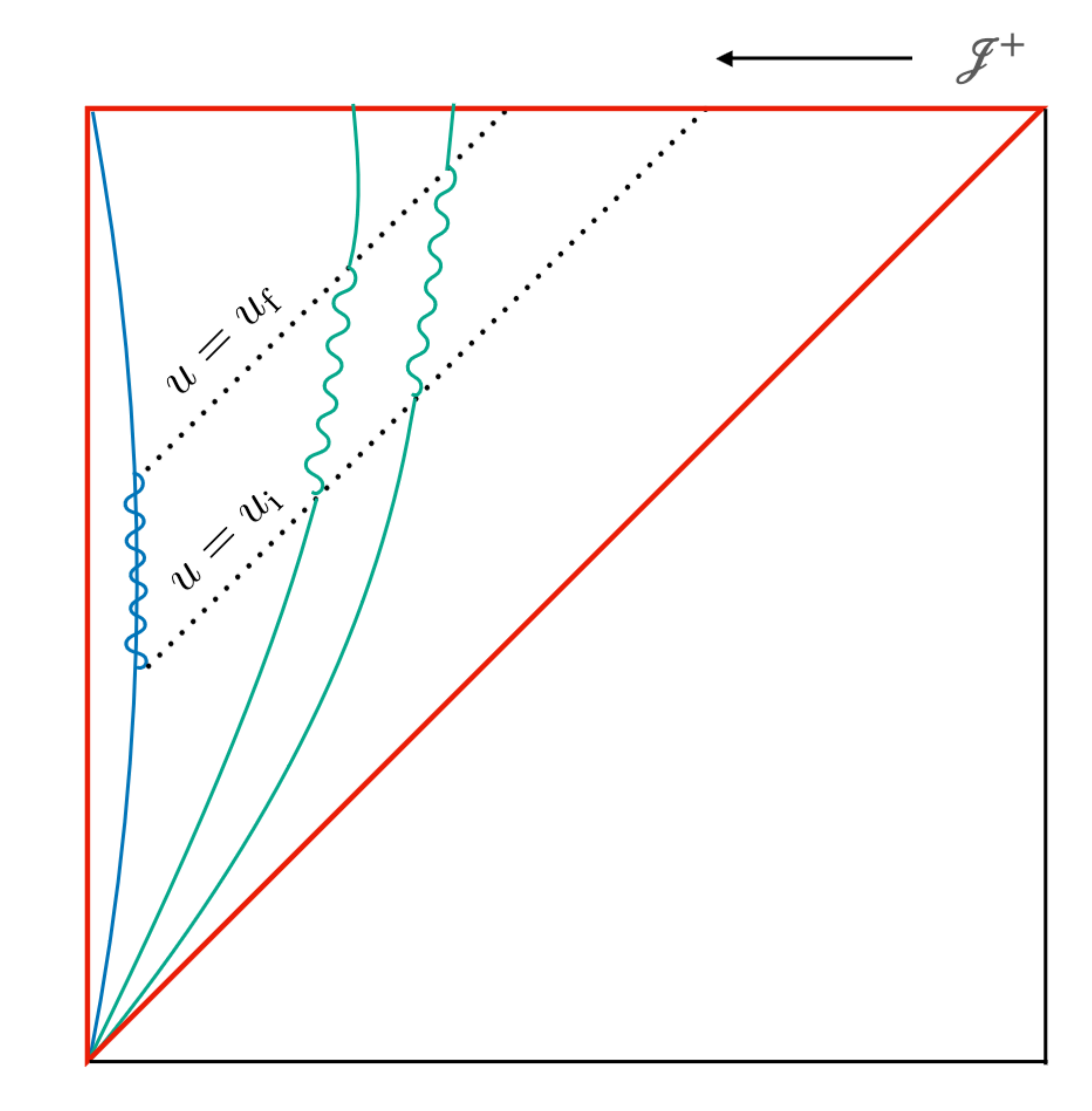
where $\partial_u^2\zeta_{ij} - 3H^2\zeta_{ij} = -2H^4 Q_{ij}^{(\rho+p)}$

This application of the multipolar methods to the linear spin 2 field in de Sitter proves that Dirichlet boundary conditions at $\mathscr{I}^+$ (and therefore conformal symmetry) are fundamentally incompatible with the propagating spin 2 degree of freedom!

This disproves the dS/CFT conjecture [Strominger, 2001]

See also [Ashtekar, Bonga, Kesavan, 2015] [Bunster, Perez, Bonga, 2023]

Memory effects : even sector

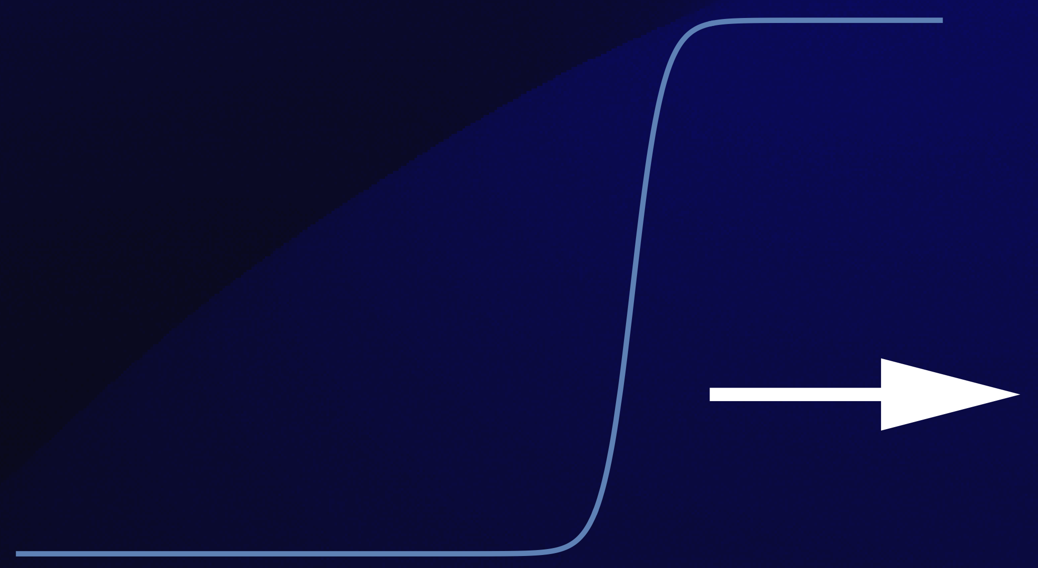


$$q_{AB} = \mathring{q}_{AB} + 2\mathring{q}_{C\langle A}\mathring{D}_{B\rangle}\mathring{\xi}^C + e^i_{\langle A}e^j_{B\rangle}\left(\partial_u\zeta_{ij} + 2H^2\partial_u Q^{(\rho+p)}_{ij} + 2H^2\epsilon_{ikl}n_k\Big(K_{jl} + H\int^u du' K_{jl}(u')\Big)\right),$$

where $\partial_u^2\zeta_{ij} - 3H^2\zeta_{ij} = -2H^4 Q^{(\rho+p)}_{ij}$

In the absence of a detail model for the source, let us assume a step function :

$$Q^{(\rho+p)}_{ij}(u) = Q^{(\rho+p)}_{ij}(u_i) + (Q^{(\rho+p)}_{ij}(u_f) - Q^{(\rho+p)}_{ij}(u_i))\Theta(u)$$



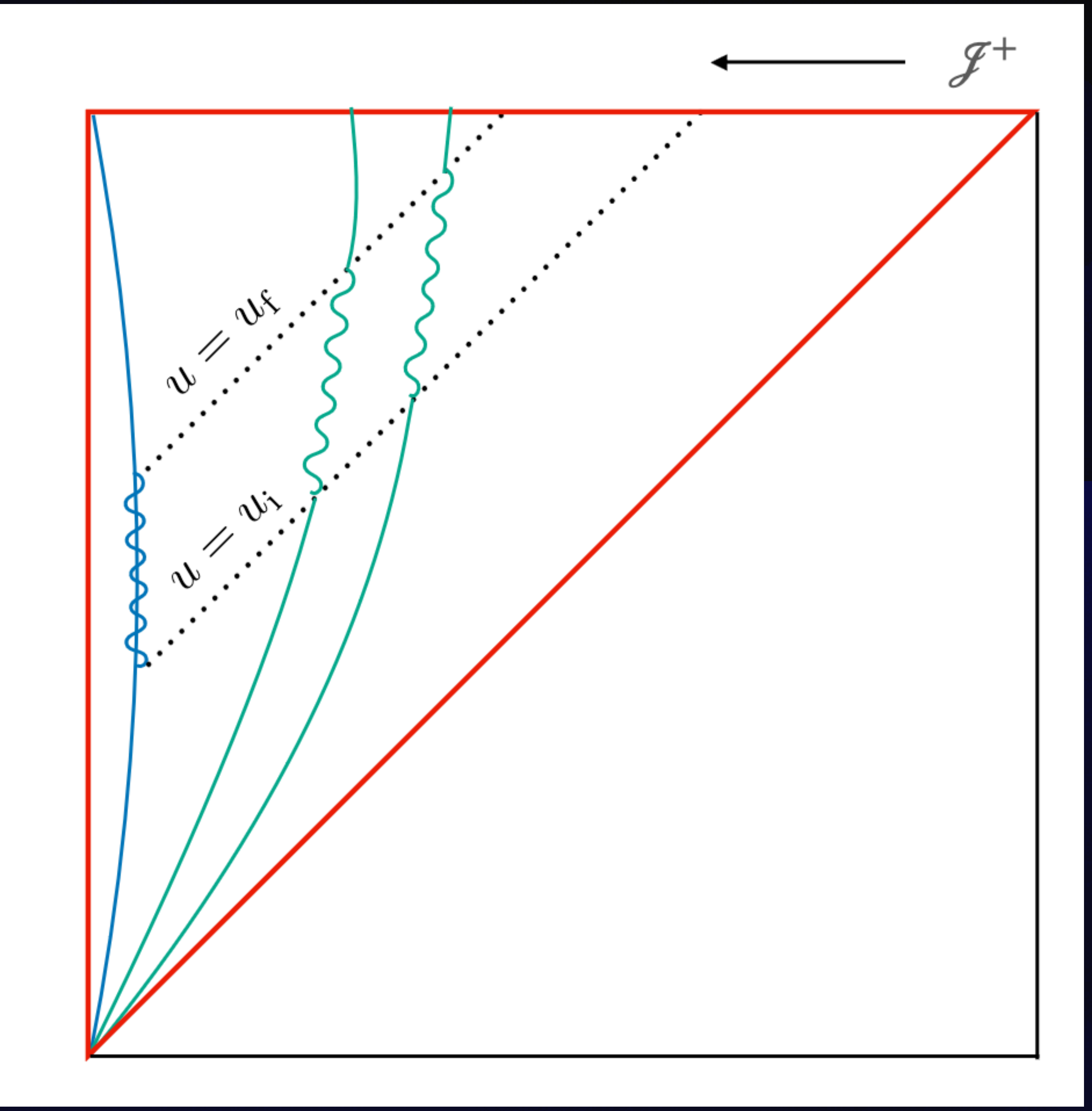
$$\zeta_{ij}(u) = \frac{2H^2}{3}\left(Q^{(\rho+p)}_{ij}(u_\mathrm{i}) + (Q^{(\rho+p)}_{ij}(u_\mathrm{f}) - Q^{(\rho+p)}_{ij}(u_\mathrm{i}))(\cosh(\sqrt{3}Hu) - 1)\Theta(u)\right)$$

$$\partial_u\zeta_{ij}(u) = \frac{2H^3}{\sqrt{3}}(Q^{(\rho+p)}_{ij}(u_\mathrm{f}) - Q^{(\rho+p)}_{ij}(u_\mathrm{i}))\sinh(\sqrt{3}Hu)\Theta(u)$$

Using the Λ-BMS generator $\mathring{\xi}^A = 2\sqrt{3}He^A_i n_j e^{\pm\sqrt{3}Hu}c_{ij}, \quad \mathring{\xi}^u = \pm(\delta_{ij} - 3n_i n_j)c_{ij}e^{\pm\sqrt{3}Hu}$, we can set $q_{AB} = \mathring{q}_{AB}$ at either u_2 or u_1.

The difference between the two non-radiative regions is gauge invariant: this is the memory effect.

Memory effects : odd sector



$$q_{AB} = \mathring{q}_{AB} + 2\mathring{q}_{C\langle A}\mathring{D}_{B\rangle}\xi^C + e^i_{\langle A}e^j_{B\rangle}\left(\partial_u\zeta_{ij} + 2H^2\partial_u Q^{(\rho+p)}_{ij} + 2H^2\epsilon_{ikl}n_k\left(K_{jl} + H\int^u du' K_{jl}(u')\right)\right),$$

where $$\partial_u^2\zeta_{ij} - 3H^2\zeta_{ij} = -2H^4 Q^{(\rho+p)}_{ij}$$

There is no Λ-BMS transition.

$q_{AB} \neq \mathring{q}_{AB}$ at both $u = u_1$ and $u = u_2$.

The presence of an odd parity quadrupole at early times implies that $q_{AB} \neq \mathring{q}_{AB}$ at $\mathscr{I}^+_-$.

Conclusion

- The linear spin 2 field emitted from a source below Hubble scale was solved consistently in the quadrupolar truncation.
- The solution was obtained in closed form both in harmonic gauge and in Bondi gauge.
- It leads to a varying boundary metric which disproves the dS/CFT conjecture
- The varying boundary metric displays a displacement memory effect specific to de Sitter.
- The even parity sector can be understood as a Λ-BMS transition.