Magic zeroes in the black hole response problem and a Love symmetry resolution

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Love numbers in Newtonian gravity A.E.H. Love (1912), Poisson & Will (2014)

Tidal response of spherically symmetric and non-spinning compact body $(L \equiv i_1 \ldots i_\ell,\ x^L \equiv x^{i_1} \ldots x^{i_\ell})$

$$\delta\Phi_{\rm N}(\omega,\vec{x})\Big|_{R_\oplus \leq r \ll L^{☾}} = \sum_{\ell=2}^{\infty} \frac{(\ell-2)!}{\ell!}\left[\underbrace{1}_{\text{“source”}} + \underbrace{k_\ell^{\oplus}(\omega)\left(\frac{R_\oplus}{r}\right)^{2\ell+1}}_{\text{“response”}}\right]\bar{\mathcal{E}}_L^{☾}(\omega)\, x^L$$

Tidal Love numbers $\equiv$ Conservative Tidal Response Coefficients

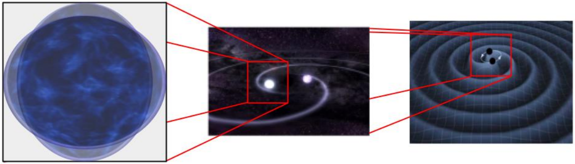
$$k_\ell^{\rm Love}(\omega) = {\rm Re}\left\{k_\ell(\omega)\right\}$$

$$k_\ell^{\rm diss}(\omega) = {\rm Im}\left\{k_\ell(\omega)\right\}$$

. . .

Goldberger et al. (2020), Chia (2020), CDI (2021a), Le Tiec et al. (2020)

Love numbers in GR - Worldline EFT Goldberger&Rothstein (2006), Porto (2006)



$R \sim r_{\text{orbit}} v^2 \ll r_{\text{orbit}} \sim \lambda_{GW} v \ll \lambda_{GW}$

$$S_{\text{EFT}}^{\text{1-body}}[x_{\text{cm}}, h, A, \Phi] = S_{\text{bulk}}[\eta + h, A, \Phi] - M \int d\tau + S_{\text{finite-size}}[x_{\text{cm}}, h, A, \Phi]$$

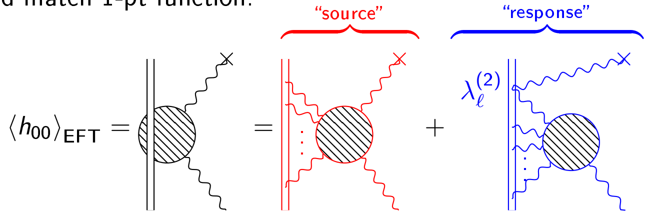
Love numbers in GR $\equiv$ Wilson coefficients in worldline EFT

$$S_{\text{finite-size}} \supset \sum_{s=0}^{2} \frac{\lambda_{\text{el},\ell}^{(s)}}{2\ell!} \int d\tau\, \mathcal{E}_L^{(s)}(x_{\text{cm}}(\tau))\, \mathcal{E}^{(s)\,L}(x_{\text{cm}}(\tau)) + (\mathcal{E} \leftrightarrow \mathcal{B})$$

$$\mathcal{E}_L^{(0)} = \partial_{\langle i_\ell} \ldots \partial_{i_1\rangle} \Phi \;,\quad \mathcal{E}_L^{(1)} = \partial_{\langle i_\ell} \ldots \partial_{i_2} F_{i_1\rangle 0} \;,\quad \mathcal{E}_L^{(2)} = \partial_{\langle i_\ell} \ldots \partial_{i_3} R_{i_2|0|i_1\rangle 0}$$

Newtonian matching Kol & Smolin (2011), Hui et al. (2021), CDI (2021a)

- Put a pure 2^ℓ-pole background with source moments $\bar{\mathcal{E}}_L$ at large distances and match 1-pt function:



$$= \bar{\mathcal{E}}_L x^L \left[\left(1 + a_1 \frac{GM}{r} + \dots \right) + \frac{\lambda_{\text{el},\ell}^{(2)}}{r^{2\ell+1}} \left(b_0 + b_1 \frac{GM}{r} + \dots \right) \right]$$

$$\boxed{\lambda_\ell^{(s)} \propto k_\ell^{(s)\text{Love}} \, (\omega = 0) \; R^{2\ell+1}}$$

- $s = 0$: Scalar response
- $s = 1$: Electric/Magnetic polarization
- $s = 2$: Electric/Magnetic tidal response

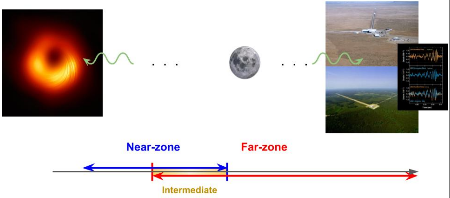
Static scalar Love numbers of Schwarzschild black hole

- Massless scalar field in Schwarzschild black hole background ($r_{\rm h} = r_s = 2GM$, $\Delta(r) = r(r - r_s)$):

$$\mathbb{O}_{\rm full}\,\Phi_{\omega\ell m} = \left[\partial_r\,\Delta\,\partial_r - \frac{r^4}{\Delta}\partial_t^2\right]\Phi_{\omega\ell m} = \ell(\ell+1)\,\Phi_{\omega\ell m}$$

Response as low-energy scattering problem Starobinsky (1965), Chia (2020), CDI (2021a)

- Near-zone region: $\omega\,(r - r_{\rm h}) \ll 1$
- Far-zone region: $\omega r \gg 1$
- Intermediate region: $r_{\rm h} \ll r \ll \frac{1}{\omega}$



$$\mathbb{O}_{\rm full} \simeq \partial_r\,\Delta\,\partial_r - \frac{r_s^4}{\Delta}\partial_t^2 \equiv \mathbb{O}_{\rm NZ} \Rightarrow \boxed{\boxed{k_\ell^{{\rm Love},(0)}(\omega) = 0}} + \mathcal{O}\left(\omega^2 r_s^2\right)$$

The tune of Love

- Totalitarian principle: "*Everything not forbidden is compulsory!*"
- 't Hooft naturalness (1980): "*At any energy scale, a physical parameter is allowed to be small if setting it to zero enhances the symmetry of the system. Otherwise, its natural value is an* $\mathcal{O}(1)$ *number*".

Magic zeroes in the black hole response problem

- For *all* isolated asymptotically flat GR (Kerr-Newman) black holes:

Fang & Lovelace (2005), Damour & Nagar (2009), Binnington & Poisson (2009), Hinderer (2009), Poisson (2015), Le Tiec & Casals (2020), Chia (2020), Le Tiec et. al (2020), CDI (2021a)

$k_{\ell m}^{\text{Love},(s)}(\omega = 0) = 0$ $\Rightarrow$ Fine-tuning or enhanced symmetry? Porto (2016)

Enhanced symmetry in near-zone

Recall near-zone e.o.m. for massless scalar field in Schwarzschild background:

$$\mathbb{O}_{\mathrm{NZ}}\Phi_{\omega\ell m} = \ell\left(\ell+1\right)\Phi_{\omega\ell m}\,, \quad \mathbb{O}_{\mathrm{NZ}} = \partial_r\,\Delta\,\partial_r - \frac{r_s^4}{\Delta}\partial_t^2\,, \quad \Delta = r\left(r-r_s\right)$$

$\mathrm{SL}\left(2,\mathbb{R}\right)$ symmetry of $\mathbb{O}_{\mathrm{NZ}}$: Bertini et al. (2012)

$$L_0 = -\beta\,\partial_t \;\;,\;\; L_{\pm1} = e^{\pm t/\beta}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\beta\,\partial_t\right] \;\;,\;\; \beta = 2r_s$$

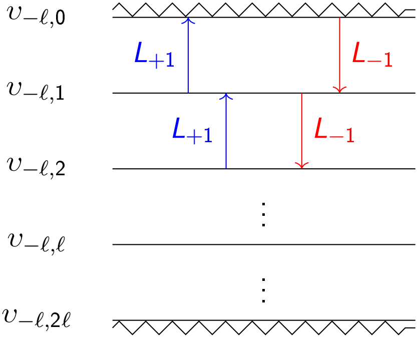
$$\left[L_m, L_n\right] = \left(m-n\right)L_{m+n} \;\;,\;\; \mathcal{C}_2 = L_0^2 - \frac{1}{2}\left(L_{+1}L_{-1} + L_{-1}L_{+1}\right) = \mathbb{O}_{\mathrm{NZ}}$$

- Globally defined and regular at both future and past e.h.'s

$$\boxed{\begin{aligned} L_0\Phi_{\omega\ell m} &= i\beta\omega\Phi_{\omega\ell m} \\ \mathcal{C}_2\Phi_{\omega\ell m} &= \ell\left(\ell+1\right)\Phi_{\omega\ell m}\end{aligned}}$$

Highest-weight banishes Love CDI (2021b, 2022)

Massless scalar field in Schwarzschild background



- Highest-weight vector with fixed Casimir at orbital number $\ell \in \mathbb{N}$:

$$L_0 \upsilon_{-\ell,0} = -\ell \upsilon_{-\ell,0} \ , \quad L_{+1} \upsilon_{-\ell,0} = 0$$

$$\Rightarrow \upsilon_{-\ell,0} = e^{\ell t/\beta} \Delta^{\ell/2}$$

- $\upsilon_{-\ell,0}$: *regular* at e.h., $\omega_{-\ell,0} = i\ell/\beta$. $\Rightarrow \upsilon_{-\ell,n} = \left(L_{-1}\right)^n \upsilon_{-\ell,0}$: *regular* solutions, $\omega_{-\ell,n} = i\left(\ell - n\right)/\beta$.
- $\Phi_{\omega=0,\ell m} \propto \upsilon_{-\ell,\ell}$

$$\left(L_{+1}\right)^{\ell+1} \upsilon_{-\ell,\ell} = 0 \Rightarrow \boxed{\partial_r^{\ell+1} \Phi_{\omega=0,\ell m} = 0 \Rightarrow \text{No } r^{-\ell-1} \text{ term!}}$$

Love symmetry for Kerr-Newman black holes CDI (2021b, 2022)

$$L_0^{(s)} = -\beta\,\partial_t + s$$

$$L_{\pm1}^{(s)} = e^{\pm t/\beta}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\beta\,\partial_t + \frac{a}{\sqrt{\Delta}}\,\partial_\phi - s\,(1\pm1)\left(\sqrt{\Delta}\right)'\right]$$

Teukolsky (1973)

$$\phi_0 = F_{\mu\nu}l^\mu m^\nu\,, \qquad \phi_1 = \tfrac{1}{2}F_{\mu\nu}(l^\mu n^\nu + m^{*\mu}m^\nu)\,, \qquad \phi_2 = F_{\mu\nu}m^{*\mu}n^\nu\,, \tag{1.1}$$

$$\psi_0 = -C_{\alpha\beta\gamma\delta}l^\alpha m^\beta l^\gamma m^\delta\,, \qquad \psi_4 = -C_{\alpha\beta\gamma\delta}n^\alpha m^{*\beta}n^\gamma m^{*\delta}\,, \tag{1.3}$$

$$ds^2 = (1 - 2Mr/\Sigma)dt^2 + (4Mar\sin^2(\theta)/\Sigma)dt\,d\varphi - (\Sigma/\Delta)dr^2 - \Sigma d\theta^2 - \sin^2(\theta)(r^2 + a^2 + 2Ma^2r\sin^2(\theta)/\Sigma)d\varphi^2\,. \tag{4.1}$$

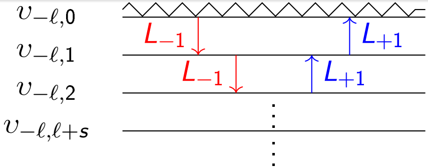
Here M is the mass of the black hole, aM its angular momentum, $\Sigma = r^2 + a^2\cos^2\theta$, and[1] $\Delta = r^2 - 2Mr + a^2$. When $a = 0$, the metric reduces to the Schwarzschild metric, a nonrotating black hole.

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2} - \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi} - 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T\,. \tag{4.7}$$

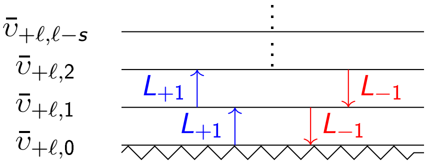
TABLE 1

FIELD QUANTITIES ψ, SPIN-WEIGHT s, AND SOURCE TERMS T FOR EQUATION (4.7)

ψ	s	T
Φ	0	$\Box\Phi = 4\pi T$
χ_0 $\rho^{-1}\chi_1$	$\frac{1}{2}$ $-\frac{1}{2}$	See references in Appendix B
ϕ_0 $\rho^{-2}\phi_2$	1 -1	J_0 (eq. [3.6]) $\rho^{-2}J_2$ (eq. [3.8])
ψ_0^B $\rho^{-4}\psi_4^B$	2 -2	$2T_0$ (eq. [2.13]) $2\rho^{-4}T_4$ (eq. [2.15])



(a) Highest-weight repr. $= \{\upsilon_{-\ell,n}\}$: Regular (Singular) at future (past) e.h.



(b) Lowest-weight repr. $= \{\bar{\upsilon}_{+\ell,n}\}$: Regular (Singular) at past (future) e.h.

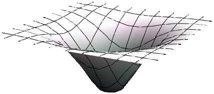
$$R_{s,\omega=0,\ell m}(r) = \left(\frac{r-r_+}{r-r_-}\right)^{im\Omega\beta/2}\Delta^{-s}\sum_{n=0}^{\ell+s} c_{n,\ell m}r^n \Rightarrow$$

No induced multipole moment term

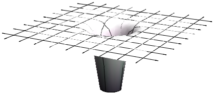
Relation to enhanced NHE isometry CDI (2022)

- Near-horizon geometry of extremal black holes develops an infinite throat and *decouples* from far-horizon geometry.
- Decoupled throat has enhanced isometry from acquired AdS factors.
- RN: $\mathrm{SL}(2,\mathbb{R})_{\mathrm{NHE}} \times SO(3)$
- Kerr: $\mathrm{SL}(2,\mathbb{R})_{\mathrm{NHE}} \times U(1)$

Bardeen & Horowitz (1999)



$Q^2 < M^2$



$Q^2 = M^2$

$$\text{RN:} \quad \lim_{Q^2 \to M^2} \mathrm{SL}(2,\mathbb{R})_{\mathrm{Love}} = \mathrm{SL}(2,\mathbb{R})_{\mathrm{NHE}}$$

$$\text{Kerr:} \quad \lim_{a^2 \to M^2} \left(\mathrm{SL}(2,\mathbb{R})_{\mathrm{BH}} \subset \mathrm{SL}(2,\mathbb{R})_{\mathrm{Love}} \ltimes \hat{U}(1)\right) = \mathrm{SL}(2,\mathbb{R})_{\mathrm{NHE}}$$

Summary

- Love numbers capture the linear conservative "tidal" response of a compact body to external "tidal" fields and can be probed in radiation waveform signals.
- Black holes in $d = 4$ GR have vanishing static Love numbers
  $\rightarrow$ ('t Hooft-)naturalness concerns
- Enhanced $\mathrm{SL}(2,\mathbb{R})$ Love symmetry explains seemingly unnatural properties of black holes Love numbers
  $\rightarrow$ "Highest-weight banishes Love"
- Closely related to enhanced NHE $\mathrm{SL}(2,\mathbb{R})$ isometry.

- In higher dimensions, Love symmetry still exists regardless of details of perturbation and is in accordance with the more intricate vanishing/non-vanishing/running of the black hole static Love numbers.
- For modified GR, Love symmetry in general does not exist and Love numbers have their natural non-zero and RG-flowing values.

What else to do with Love symmetry?

- Re-organization of perturbation theory and QNMs?
- Black p-branes and AdS/CFT? (in progress)
- Full symmetry structure and Kerr/CFT? (in progress)
- "Accidental" symmetry of extremal GR black holes?

  Porfyriadis & Remmen (2021)
- Near-horizon BMS-like algebra and Celestial holography?

  Donnay, Giribet, González, Pino (2016a,2016b)

"Black holes are the hydrogen atom of the 21st century"

't Hooft (2016), EHT (April 10, 2019)

Thank you

$$SL(2,\mathbb{R})_{\text{Love}} \to SL(2,\mathbb{R})_{\text{Love}} \ltimes \hat{U}(1) \supset SL(2,\mathbb{R})_{(\alpha)}$$
$$L_m(\alpha) = L_m^{\text{Love}} + \alpha\, \upsilon_m\, \beta\Omega\, \partial_\phi \quad , \quad \upsilon_m \in \mathcal{V}$$

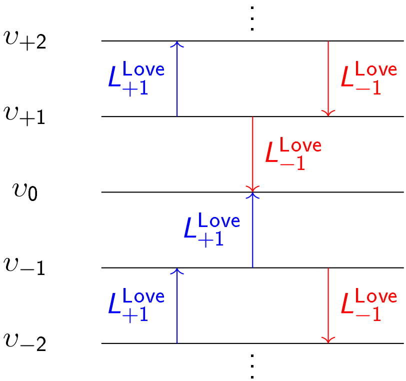
Starobinsky near-zone $SL(2,\mathbb{R}) \times SO(3)$

Starobinsky (1973) $\alpha = 1$

$$L_0 = -\beta\left(\partial_t + \Omega\,\partial_\phi\right)$$
$$L_{\pm 1} = e^{\pm t/\beta}\left[\mp\sqrt{\Delta}\,\partial_r + \left(\sqrt{\Delta}\right)'\beta\left(\partial_t + \Omega\,\partial_\phi\right)\right]$$
$$L_0\Phi_{\omega\ell m} = i\beta\left(\omega - m\Omega\right)\Phi_{\omega\ell m}$$
$$\mathcal{C}_2\Phi_{\omega\ell m} = \ell\left(\ell+1\right)\Phi_{\omega\ell m}$$

Geometrically more interesting!

Hui et al. (2021, 2022)



Love symmetry in higher dimensions CDI (2021b, 2022), CI(2023)

Scalar susceptibility in $d \geq 4$ Schwarzschild Kol & Smolkin (2011), Hui et al. (2021)

$$\hat{\ell} \equiv \frac{\ell}{d-3}$$

$$k_\ell^{(0)} = \begin{cases} c_\ell \neq 0 & \text{if } 2\hat{\ell} \notin \mathbb{N} \\ c_\ell + \beta_\ell \ln \frac{r-r_s}{L} & \text{if } \hat{\ell} \in \mathbb{N} + \frac{1}{2} \\ c_\ell = 0 & \text{if } \hat{\ell} \in \mathbb{N} \end{cases}$$

- Love symmetry still exists in near-zone: Bertini et al. (2012)

$$L_0 \Phi_{\omega\ell m} = i\beta\omega\Phi_{\omega\ell m}$$

$$\mathcal{C}_2 \Phi_{\omega\ell m} = \hat{\ell}(\hat{\ell}+1)\Phi_{\omega\ell m}$$

- Regular static solution $\in$ highest-weight repr. of $SL(2,\mathbb{R})$ iff $\hat{\ell} \in \mathbb{N}$ ✓

EM susceptibilities: $\begin{cases} \text{S-modes} & \checkmark \\ \text{V-modes} & \checkmark \end{cases}$

TLNs: $\begin{cases} \text{T-modes} & \checkmark \\ \text{V-modes (Regge-Wheeler)} & \checkmark \\ \text{S-modes (Zerilli)} & ? \end{cases}$

- Scalar susceptibilities for 5-d rotating (Myers-Perry) black holes ✓

Beyond Einsteinian Love CDI (2022)

- Love symmetry might not be unique to GR, but it certainly does not exist for a generic theory of gravity.

Generic spherically symmetric, non-extremal black hole

$$ds^2 = -f_t(r)\,dt^2 + \frac{dr^2}{f_r(r)} + r^2 d\Omega_2^2$$

- Geometric condition for existence of near-zone $SL(2,\mathbb{R})$ Love symmetry:

$$\frac{f_r(r)}{f_t(r)} = \frac{4r^2 f_t(r) + \left(\frac{\beta_s}{\beta} r_h\right)^2}{\left(r^2 f_t(r)\right)'^2}\ , \quad \beta = \frac{2}{\sqrt{f_t'(r_h)\, f_r'(r_h)}}\ , \ \beta_s = 2r_h$$

- If such geometries are supported, the static Love numbers will vanish due to the highest-weight property.
- Counterexample: $R^3_{\mu\nu\rho\sigma}$ gravity $\rightarrow$ Condition *not* satisfied!
  $\Rightarrow$ No Love symmetry $\rightarrow$ Consistent with $k_\ell^{(0)} = c_\ell + \beta_\ell \ln \frac{r-r_h}{L}$ results

More background multipole moments, e.g. mountains, spin (multi-index notation: $L \equiv i_1 \dots i_\ell$, $x^L \equiv x^{i_1} \dots x^{i_\ell}$)

$$\delta\Phi_{\rm N}(\omega, \vec{x})\Big|_{r \geq R} = \sum_{\ell,\ell'} \frac{(\ell-2)!}{\ell!} \Bigg[\underbrace{\delta_{L,L'}}_{\text{“source”}} + \underbrace{k_{LL'}(\omega)\left(\frac{R}{r}\right)^{2\ell+1}}_{\text{“response”}} \Bigg] \bar{\mathcal{E}}^{L'}(\omega)\, x^L$$

$$k^{\rm Love}_{LL'}(\omega) \equiv k^{\rm cons.}_{LL'}(\omega) \subset k_{LL'}(\omega)$$

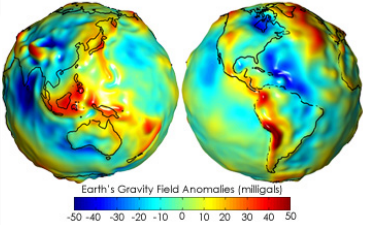
e.g. shape deformation Vs tidal locking

Axisymmetric spinning body with no ℓ-mode mixing

$$k^{\rm Love}_{\ell m}(\omega) = \mathrm{Re}\left\{k_{\ell m}(\omega)\right\}$$

$$k^{\rm diss}_{\ell m}(\omega) = \mathrm{Im}\left\{k_{\ell m}(\omega)\right\}$$

(analogous to $\mathbb{C}$-valued susceptibility in EM)



GRACE data, 2004

Goldberger et al. (2020), Chia (2020), CDI (2021a)

Worldline EFT Goldberger&Rothstein (2006)

$$S_{\text{full}}\left[g,\{x_I\}\right]=S_{\text{EH}}\left[g\right]+S_{\text{int}}[\{x_I\},g^{\text{short}}]$$

- Neutron star made up of $\sim 10^{40}/\text{m}^3$ constituents, with the I'th constituent propagating along a worldline x_I^μ, and $S_{\text{int}}[\{x_I\},g^{\text{short}}]$ capturing the internal dynamics.
- $g_{\mu\nu}=g_{\mu\nu}^{\text{long}}+g_{\mu\nu}^{\text{short}}$; "long"/"short"-distance relative to the size $R\sim 2GM/c^2$ of the body.
- Long-distance dofs: $g_{\mu\nu}^{\text{long}}$, center of mass x_{cm}^μ.
- Short-distance dof: $g_{\mu\nu}^{\text{short}}$, $\delta x_I^\mu=x_I^\mu-x_{\text{cm}}^\mu$.
- Worldline EFT: $e^{iS_{\text{EFT}}\left[x_{\text{cm}},g^{\text{long}}\right]}=\int Dg_{\mu\nu}^{\text{short}}D\delta x_I^\rho e^{iS_{\text{full}}[g,\{x_I\}]}$
- Bottom-up approach:

$$S_{\text{EFT}}\left[x_{\text{cm}},h\right]=S_{\text{EH}}\left[\eta+h\right]-M\int d\tau+S_{\text{finite-size}}\left[x_{\text{cm}},h\right]$$

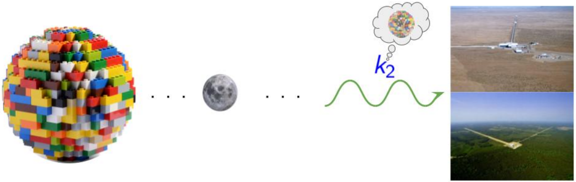
Measuring Love Flanagan & Hinderer (2007), Cardoso et al. (2014), Chatziioannou (2020)

Leading order tidal effects enter at 5PN Flanagan & Hinderer (2007)

$$\Psi^{\rm tidal}_{\rm LO}(f) = -\frac{9}{16}\frac{v^5}{\mu\,(GM)^4}\left[(m_1 + 12m_2)\,\frac{\lambda_1}{m_1} + 1 \leftrightarrow 2\right],$$

$$v = (GM\pi f)^{1/3}\,, \qquad \lambda \equiv \lambda_{\ell=2} = \frac{1}{3}k_{\ell=2}\frac{R^5}{G}$$

Love numbers probe the internal structure of the compact body



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$$v = (GM\pi f)^{1/3}, \quad \lambda \equiv \lambda_{\ell=2} = \tfrac{1}{3} k_{\ell=2}\frac{R^5}{G}$$

Love numbers probe the internal structure of the compact body

- GW170817 NS/NS coalescence

  Abbott et al. (2017)
- Already puts constraints on EoS of involved NSs