Boundaries, dissipation and gravitational charge

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In gravity, every subsystem is an open system

Outline

We are witnessing a shift of perspective (at this conference but also elsewhere) from global aspects of quantum gravity to a more local description of gravitational subsystems.

I will pick three results of the programme thus far.

1. Immirzi parameter, radiative phase space on the lightcone

2. Quantisation of area from deformation of boundary symmetries.

3. Metriplectic geometry for gravitational subsystems

*ww, Gravitational $SL(2,\mathbb{R})$ Algebra on the Light Cone, JHEP **57** (2021), `arXiv:2104.05803`.
*ww, Fock representation of gravitational boundary modes and the discreteness of the area spectrum, Ann. Henri Poincare 18 (2017), 3695, `arXiv:1706.00479`.
*Viktoria Kabel and ww, Metriplectic geometry for gravitational subsystems, (2022), `arXiv:2206.00029`.

Immirzi parameter, boundary symmetries on the lightcone

To understand how gravity couples to boundaries, it is useful to work with differential forms rather than tensors since there is a natural notion of projection onto the boundary, namely the pull-back $\varphi^* : T^*M \to T^*(\partial M)$, which does not require a metric.

Tetrad defines the metric

$$g_{ab} = \eta_{\alpha\beta} e^{\alpha}{}_{a} e^{\beta}{}_{b}.$$

$\mathfrak{so}(1,3)$ connection and covariant derivative

$$\nabla_a V^{\alpha} = \partial_a V^{\alpha} + A^{\alpha}{}_{\beta a} V^{\beta}.$$

The commutator of two covariant derivatives defines the curvature,

$$[\nabla_a, \nabla_b] V^{\alpha} = F^{\alpha}{}_{\beta ab}\,[A] V^{\beta}.$$

There are two scalars that we can form out of the curvature tensor:

$$R[A,e] = F^{\alpha\beta}{}_{ab}[A] e_\alpha{}^a e_b{}^\beta,$$
$$R^*[A,e] = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta ab}[A] e_\mu{}^a e_\nu{}^b \approx 0.$$

Therefore, in the first-order formalism, there are *two coupling constants* at linear order in the curvature,

$$S = \frac{1}{16\pi G}\int_{\mathscr{M}} d^4v \Big[R - \frac{1}{\gamma}R^*\Big] + \text{boundary terms.}$$

G is Newton's constant, γ is the Immirzi parameter.

Back to self-dual variables

Using the isomorphism between spinors and tensors, the action splits into self-dual and anti-selfdual parts

$$S = \frac{1}{16\pi G}\int_{\mathscr{M}} d^4v\Big[R - \frac{1}{\gamma}R^*\Big] =$$
$$= \Big[\frac{\mathrm{i}}{8\pi\gamma G}(\gamma+\mathrm{i})\int_{\mathscr{M}} \Sigma_{AB}\wedge F^{AB}\Big] + \mathrm{cc.}$$

$SL(2,\mathbb{C})$ Spinor indices $A, B, C, \dots$ and $A', B', C', \dots$

Self-dual and anti-self-dual parts, e.g. of Plebański form

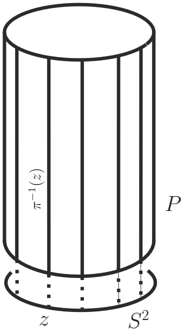
$$e_\alpha \wedge e_\beta =: \Sigma_{\alpha\beta} \equiv \Sigma_{AA'BB'} = -\bar{\epsilon}_{A'B'}\Sigma_{AB} - \epsilon_{A'B'}\bar{\Sigma}_{AB}$$

Field equations

$$\nabla \wedge \Sigma_{AB} = 0, \qquad F_{AB} = \Psi_{ABCD}\Sigma^{CD} = \Psi_{(ABCD)}\Sigma^{CD}.$$

Spacetime region bounded by null surface:

- Compact spacetime region $\mathscr{M}$.
- Bounded by spacelike disks M_0, M_1 and null surface $\mathscr{N}$.
- Null surface boundary $\mathscr{N}$ embedded into abstract bundle (ruled surface) $P(\pi, \mathscr{C}) \simeq \mathbb{R} \times \mathscr{C}$.
- Null generators $\pi^{-1}(z)$.



Metrical structures at the boundary:

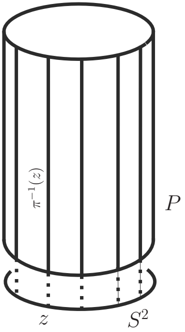
- Signature $(0++)$ metric:
  $\varphi^*_{\mathcal{N}} g_{ab} = q_{ab} = 2m_{(a}\bar{m}_{b)}$.
- Null vectors: $l^a : q_{ab}l^b = 0 \Leftrightarrow \pi_* l^a = 0$.

Abelian symmetries:

- $m_a \longrightarrow \mathrm{e}^{\mathrm{i}\varphi} m_a$
- $l^a \longrightarrow \mathrm{e}^{\lambda} l^a$

Associate spinors:

- Penrose null flag $\ell^A : l^a \simeq \mathrm{i}\,\ell^A \bar{\ell}^{A'}$
- Conjugate spinor-valued two-form
  $\eta_A \in \Omega^2(\mathcal{N} : \mathcal{S}_A)$.
- Area density $\varepsilon = \mathrm{i}\,\eta_A \ell^A \in \Omega^2(\mathcal{N} : \mathbb{R})$
- Abelian symmetries:
  $$\begin{pmatrix} \ell^A \\ \eta_A \end{pmatrix} \longrightarrow \begin{pmatrix} \mathrm{e}^{+\frac{1}{2}(\lambda+\mathrm{i}\varphi)} \ell^A \\ \mathrm{e}^{-\frac{1}{2}(\lambda+\mathrm{i}\varphi)} \eta_A \end{pmatrix}$$



Bulk plus boundary action:

$$S = \frac{\mathrm{i}}{8\pi\gamma G}(\gamma + \mathrm{i})\left[\int_{\mathcal{M}} \Sigma_{AB} \wedge F^{AB} + \int_{\mathcal{N}} \eta_A \wedge \left(D - \tfrac{1}{2}\varkappa\right)\ell^A\right] + \mathrm{cc.}$$

Boundary conditions along $\mathcal{N}$: $\delta[\varkappa_a, l^a, m_a]/_\sim = 0$

- vertical diffeomorphisms $[\varphi^*\varkappa_a, l^a, \varphi^* m_a] \sim [\varkappa_a, \varphi_* l^a, m_a]$
- dilations $[\varkappa_a, l^a, m_a] \sim [\varkappa_a + \nabla_a f, \mathrm{e}^f l^a, m_a]$
- complexified conformal transformations $\lambda = \mu + \mathrm{i}\nu$: $[\varkappa_a, l^a, m_a] \sim \left[\varkappa_a - \frac{1}{\gamma}\nabla_a \nu, \mathrm{e}^{\mu} l^a, \mathrm{e}^{\mu + \mathrm{i}\nu} m_a\right]$
- shifts $[\varkappa_a, l^a, m_a] \sim [\varkappa_a + \bar{\zeta} m_a + \zeta \bar{m}_a, l^a, m_a]$

The equivalence class $g = [\varkappa_a, l^a, m_a]/_\sim$ characterises two degrees of freedom per point.

Covariant pre-symplectic potential for the partial Cauchy surfaces:

$$\Theta_\Sigma = \frac{\mathrm{i}}{8\pi\gamma G}(\gamma+\mathrm{i})\left[-\oint_{\mathscr{C}} \eta_A \mathbb{d}\ell^A + \int_\Sigma \Sigma_{AB}\wedge \mathbb{d}A^{AB}\right] + \mathrm{cc.}$$

Phase space of bulk and boundary degrees of freedom:

$$P_{phys} = (P_{bulk}\times P_{bndry})/\!\!/_{gauge}$$

Poisson brackets at the two-dimensional corner

$$\big\{\pi_A(z), \ell^B(z')\big\}_{\mathscr{C}} = \delta^B_A \delta^{(2)}(z,z').$$

Canonical (spinor-valued) momentum

$$\pi_A = \frac{\mathrm{i}}{8\pi G}\frac{\gamma+\mathrm{i}}{\gamma}\eta_A.$$

First results: Area operator and discrete spectra

- The cross-sectional oriented area is

$$\boxed{\mathrm{Area}[\mathscr{C}] = -8\pi G \frac{\mathrm{i}\gamma}{\gamma + \mathrm{i}} \oint_{\mathscr{C}} d^2x \, \pi_A \ell^A.}$$

- For the area to be real-valued (charge neutral), we have to satisfy the reality conditions,

$$\boxed{K - \gamma L = 0.}$$

- Generators of complexified $U(1)_{\mathbb{C}}$ transformations

$$L = -\frac{1}{2\mathrm{i}} \pi_A \ell^A + \mathrm{cc.} \quad \textit{(generator of U(1) transformations)},$$
$$K = -\frac{1}{2} \pi_A \ell^A + \mathrm{cc.} \quad \textit{(dilatations of the light like direction)}.$$

- Boundary modes: creation and annihilation operators (half densities)

$$a^A = \frac{1}{\sqrt{2}} \Big[\sqrt{d^2\Omega} \, \delta^{AA'} \bar{\ell}_{A'} - \frac{\mathrm{i}}{\sqrt{d^2\Omega}} \pi^A \Big],$$
$$b^A = \frac{1}{\sqrt{2}} \Big[\sqrt{d^2\Omega} \, \ell^A + \frac{\mathrm{i}}{\sqrt{d^2\Omega}} \delta^{AA'} \bar{\pi}_{A'} \Big].$$

Fock quantisation of the area at the boundary

- Boundary Fock vacuum in the continuum

$$\forall z \in \mathscr{C} : a^A(z)\big|\{d^2\Omega, n_\alpha\}, 0\big\rangle = 0,$$
$$b^A(z)\big|\{d^2\Omega, n_\alpha\}, 0\big\rangle = 0.$$

- Boundary operators in terms of harmonic oscillators:

$$\hat{L}(z) = \frac{1}{2}\big[a^\dagger_A(z)a^A(z) - b^\dagger_A(z)b^A(z)\big],$$
$$\hat{K}(z) = \frac{1}{2\mathrm{i}}\big[a_A(z)b^A(z) - \mathrm{hc.}\big],$$
$$\boxed{\big[\hat{K}(z) - \gamma\hat{L}(z)\big]\Psi_{\mathrm{phys}} = 0.}$$

- $\hat{K}$ is a squeeze operator, $\hat{L}$ is difference of number operators.
- Area is quantised on physical states

$$\widehat{\mathrm{Area}_{\boldsymbol{\varepsilon}}}[\mathscr{C}]\Psi_{\mathrm{phys}} = 4\pi\gamma\hbar G/c^3 \oint_{\mathscr{C}} \big[a^\dagger_A a^A - b^\dagger_A b^A\big]\Psi_{\mathrm{phys}}.$$

$SL(2,\mathbb{R})$ variables and radiative modes

Signature $(0{+}{+})$ metric.

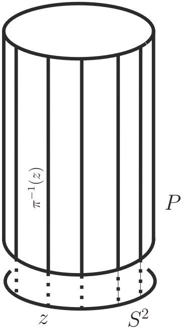
$$q_{ab} = \delta_{ij} e^{i}{}_{a} e^{j}{}_{b}, \qquad i,j = 1,2.$$

Parametrisation of the dyad

$$\boxed{e^{i} = \Omega\, S^{i}{}_{j}\, e^{j}_{(o)}.}$$

Choice of time:

$$\partial^b_U \nabla_b \partial^a_U = -\frac{1}{2}(\Omega^{-2}\frac{\mathrm{d}}{\mathrm{d}U}\Omega^2)\partial^a_U$$



Radiative modes from Holst action [ww2021]

Kinematical phase space for radiation: $\mathscr{P}_{kin} = \mathscr{P}_{abelian} \times T^*SL(2,\mathbb{R})$.

$$\Theta_{\mathcal{N}} = \frac{1}{8\pi G}\int_{\mathcal{N}} d^2v_o \wedge \left[p_K \mathbb{d}\widetilde{K} + \frac{1}{\gamma}\Omega^2\, \mathbb{d}\widetilde{\Phi} + \widetilde{\Pi}^i{}_j \left[S\mathbb{d}S^{-1}\right]^j{}_i\right] + \textit{corner term.}$$

Abelian variables:

$U(1)$ connection: $\widetilde{\Phi}$, area: $\Omega^2\, d^2v_o$, lapse: $\widetilde{K} := \text{đ}U$, expansion: p_K.

Upon imposing 2nd-class constraints: Dirac bracket for radiative modes

$$\left\{S^i{}_m(x), S^j{}_n(y)\right\}^* = -4\pi G\,\Theta(U_x,U_y)\,\delta^{(2)}(\vec{x},\vec{y})\,\Omega^{-1}(x)\,\Omega^{-1}(y)$$
$$\times\left[\mathrm{e}^{-2\,\mathrm{i}\,(\Delta(x)-\Delta(y))}\left[XS(x)\right]^i{}_m\left[\bar{X}S(y)\right]^j{}_n + \text{cc.}\right].$$

Gauge symmetries:

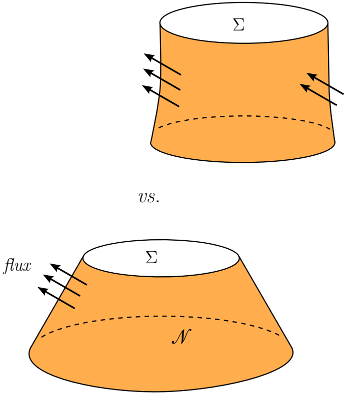
1. $U(1)$ gauge symmetry with $U(1)$ holonomy $h(x) = \mathrm{e}^{-\mathrm{i}\Delta(x)}$
2. vertical diffeomorphisms along null generators

Metriplectic geometry for gravitational subsystems

Subsystems as evolving regions in space

To understand the time evolution of a gravitational subsystem,
two choices must be made.

- Choice of time: A choice must be made for how to extend the boundary of the partial Cauchy surface Σ into a worldtube $\mathscr{N}$.
- A choice must be made how to treat the flux of gravitational radiation across the worldtube of the boundary. Flux drives the time-dependence of the system.
- Metriplectic geometry is a novel algebraic framework to tackle these issues.



N.B.: In spacetime dimensions $d < 4$, there are no gravitational waves, and we can forget about the second issue. The Hamiltonian will be automatically conserved.

In Hamiltonian systems, energy is conserved

Symplectic potential and volume-form on phase space

$$\Theta = p\,\mathrm{d}q, \quad \Omega = \mathrm{d}p \wedge \mathrm{d}q.$$

Hamilton equations

$$\Omega\Big(\delta, \frac{\mathrm{d}}{\mathrm{d}t}\Big) = \delta p\,\dot{q} - \dot{p}\,\delta q = $$
$$= \delta p \frac{\partial H}{\partial p} + \frac{\partial H}{\partial q}\delta q = \delta H.$$

The Hamiltonian is conserved under its own flow

$$\frac{\mathrm{d}}{\mathrm{d}t}H = \Omega\Big(\frac{\mathrm{d}}{\mathrm{d}t}, \frac{\mathrm{d}}{\mathrm{d}t}\Big) = 0.$$

If we insist that there is a Hamiltonian that drives the evolution in a finite region, the standard approach is too restrictive to account for dissipation.

Three possible viewpoints

1. There is no problem: Open systems interact with their environment. There is no Hamiltonian that would measure the gravitational energy in a finite region.

2. Treat the system as explicitly time-dependent.
   - Time dependence induced by the choice of (outer) boundary conditions.
   - Hamiltonian field equations modified (contact geometry).

   $$\frac{\mathrm{d}}{\mathrm{d}t}F_t = \{H, F_t\} + \frac{\partial}{\partial t}F_t.$$

   - By fixing the outgoing flux, radiative data no longer free (highly non-local constraints).
   - Conjecture: Resulting phase space (on which this Hamiltonian operates) is the phase space of edge modes alone. *Seems too restrictive, less useful.*

3. Metriplectic geometry
   - New algebraic approach. New bracket. But many properties of Poisson manifolds lost.
   - Noether charges generate evolution for generic vector fields.
   - Takes into account dissipation.

Metriplectic geometry
work with *Viktoria Kabel*

Metriplectic geometry

Even dimensional manifold $\mathscr{P}$, equipped with a pre-symplectic two-form $\Omega(\cdot,\cdot) \in \Omega^2(\mathscr{P})$ and a signature (p,q,r) metric tensor $G(\cdot,\cdot)$.

A vector field $\mathfrak{X}_F$ is a (right) Hamiltonian vector field of some (gauge invariant) functional $F : \mathscr{P} \to \mathbb{R}$ on $(\mathscr{P}, \Omega, G)$ iff

$$\forall \delta \in T\mathscr{P} : \delta[F] = \Omega(\delta, \mathfrak{X}_F) - G(\delta, \mathfrak{X}_F).$$

The Leibniz bracket between two such functionals is given by

$$(F, G) = \mathfrak{X}_F[G].$$

The metric on phase space encodes dissipation

$$\frac{\mathrm{d}}{\mathrm{d}t} H = (H, H) = -G(\mathfrak{X}_H, \mathfrak{X}_H).$$

*Morrison; Kaufman (1982-); Grmela, GÃ¶ttinger (1997); Guha (2002); Holm, Stanley (2003);...

Metriplectic geometry and extended phase space

Following Freidel, Ciambelli, Leigh, we work on an extended pre-symplectic phase space.

A point on the extended pre-symplectic phase space is labelled by a Einstein metric g_{ab} and choice of coordinate functions x^μ.

Maurer – Cartan form for diffeomorphisms

$$\mathbb{X}^a = \left[\frac{\partial}{\partial x^\mu}\right]^a \mathbb{d}x^\mu.$$

Extended pre-symplectic current

$$\delta[L] \approx \mathrm{d}[\vartheta(\delta)],$$

$$\vartheta_{ext} = \vartheta - \vartheta(\mathscr{L}_{\mathbb{X}}) + \mathbb{X} \lrcorner L = \vartheta - \mathrm{d}q_{\mathbb{X}}.$$

Noether charge and Noether charge aspect

$$Q_{\mathbb{X}} = \oint_{\partial\Sigma} q_{\mathbb{X}} = \int_{\Sigma} \big(\vartheta(\mathscr{L}_{\mathbb{X}}) - \mathbb{X} \lrcorner L\big).$$

*L. Freidel, A canonical bracket for open gravitational system, (2021), arXiv:2111.14747.
*L. Ciambelli, R. Leigh, Pin-Chun Pai, Embeddings and Integrable Charges for Extended Corner Symmetry, Phys. Rev. Lett. **128** (2022), arXiv:2111.13181.

Field dependent vector fields

The coordinate functions $x^\mu : U \subset \mathscr{M} \to \mathbb{R}^4$ are now part of phase space.

Variations of coordinate functions will only contribute a corner term to the extended pre-symlpleictc two-form.

Vector fields that are determined by their component functions $\xi^\mu(x)$ become field dependent vector fields.

$$\xi^a = \xi^\mu(x)\partial_\mu^a,$$

$$\delta[\xi^a] = [\mathbb{X}(\delta), \xi]^a.$$

Metriplectic structure

Extended pre-symplectic structure on the covariant phase space [Freidel; Ciambelli, Leigh]

$$\Omega_{ext}(\delta_1, \delta_2) = \Omega(\delta_1, \delta_2) + Q_{[\mathbb{X}(\delta_1), \mathbb{X}(\delta_2)]} + \oint_{\partial\Sigma} \mathbb{X}(\delta_{[1}) \lrcorner \vartheta(\delta_{2]})$$

Super metric on phase space [Viktoria Kabel, ww]

$$G(\delta_1, \delta_2) = -\oint_{\partial\Sigma} \mathbb{X}(\delta_{(1}) \lrcorner \vartheta(\delta_{2)})$$

Leibniz bracket on extended phase space,

$$\begin{aligned} \delta[F] &= \Omega_{ext}(\delta, \mathfrak{X}_F) - G(\delta, \mathfrak{X}_F), \\ (F, G) &= \mathfrak{X}_F[G]. \end{aligned}$$

On the extended phase space, the Lie derivative $\mathscr{L}_\xi$ is a Hamiltonian vector field with respect to the Leibniz structure.

The corresponding generator is the Noether charge,

$$\delta[Q_\xi] = \Omega_{ext}(\delta, \mathscr{L}_\xi) - G(\delta, \mathscr{L}_\xi).$$

Leibniz bracket captures dissipation

$$(Q_\xi, Q_\xi) = -\oint_{\partial\Sigma} \xi \lrcorner \vartheta(\mathscr{L}_\xi).$$

But violates Jacobi identity and skew-symmetry of Poisson bracket

$$\begin{aligned}(A,(B,C)) + (B,(C,A)) + (C,(A,B)) \neq 0,\\ (A,B) \neq (B,A).\end{aligned}$$

Summary

Summary

We discussed three results:

1. Immirzi parameter mixes $U(1)$ frame rotations and dilations on the null cone. *Provides a geometric explanation for LQG discreteness of geometry.*
2. In gravity, local subsystems are open systems. Characterised the full radiative data for $\gamma \neq 0$ in finite regions.
3. New bracket: Leibniz bracket consists of skew-symmetric-symmetric (symplectic) and symmetric (metric) part. Symmetric part is a corner term that describes dissipation.