BMS algebra in 3d and 4d BF theory

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- Can be understood as a BF theory with additional constraint: Plebaǹski formulation
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Structure of the boundary symmetry algebra?
at **finite distance** and without imposing **anything** on the fields

Gravity as a 3D BF theory

- Theory of first order gravity with Lie algebra ($\mathfrak{su}(2)$) valued

triad 1-form e and connection 1-form ω with curvature $F[\omega] = \mathrm{d}\omega + \frac{1}{2}[\omega \wedge \omega]$

- The Lagrangian reads

$$\boxed{L[e,\omega] \coloneqq 2e \wedge F + \tfrac{\lambda}{3} e \wedge [e \wedge e]}$$

Variation of the Lagrangian?

$$\delta L = \mathrm{d}\Big(2\delta\omega \wedge e\Big) + \delta e \wedge \Big(2F + \lambda[e \wedge e]\Big) + 2\delta\omega \wedge \mathrm{d}_\omega e$$

$$2F + \lambda[e \wedge e] \mathrel{\hat{=}} 0$$

$$\mathrm{d}_\omega e \mathrel{\hat{=}} 0$$

on-shell of torsion equation: $\omega \mathrel{\hat{=}} \Gamma$ $\longrightarrow$ $\boxed{2R \mathrel{\hat{=}} -\lambda[e \wedge e]}$

Gravity as a 3D BF theory

- What about the symmetries?

Lorentz transfo δ^j_α, α any Lie element

$$\delta^j_\alpha e := [e, \alpha] \qquad \text{and} \qquad \delta^j_\alpha \omega := \mathsf{d}_\omega \alpha$$

"Translation" transfo, δ^t_φ, φ any Lie element

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- The underlying algebraic structure is the 6-dimensional algebra

$$[J_i, T_j] = \epsilon_{ij}{}^k T_k \qquad [J_i, J_j] = \epsilon_{ij}{}^k J_k \qquad [T_i, T_j] = -\lambda \epsilon_{ij}{}^k J_k$$

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- Charges via the covariant phase space formalism, we need the symplectic structure

$$\Omega = -2 \int_\Sigma \delta\omega \wedge \delta e$$

Fundamental symmetry algebra of 3D BF

- From the general formula in the covariant phase space formalism, we can easily deduce the Lorentz and translation perhaps-not-integrable charges

$$\delta\!\!\!/\, \mathcal{J}(\alpha) \,\hat{=}\, \oint_S \alpha \, \delta e \quad \text{and} \quad \delta\!\!\!/\, \mathcal{T}(\varphi) \,\hat{=}\, \oint_S \varphi \, \delta\omega$$

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Integrability **without** conditions on the fields

α and φ do not depend on the fields: $\delta\alpha = \delta\varphi = 0$

These conditions give the charges

$$\mathcal{J}(\alpha) \mathrel{\hat{=}} \int_S \alpha e \quad \text{and} \quad \mathcal{T}(\varphi) \mathrel{\hat{=}} \int_S \varphi\omega$$

associated to the 6-dimensional algebra

$$\begin{aligned}
\big\{\mathcal{J}(\alpha), \mathcal{T}(\varphi)\big\} &= \mathcal{T}([\alpha,\varphi]) - 2\oint_S \alpha \mathrm{d}\varphi \\
\big\{\mathcal{J}(\alpha), \mathcal{J}(\beta)\big\} &= \mathcal{J}([\alpha,\beta]) \\
\big\{\mathcal{T}(\varphi), \mathcal{T}(\chi)\big\} &= -\lambda\mathcal{J}([\varphi,\chi])
\end{aligned}$$

About diffeomorphisms?

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$$\delta_\xi^d(e,\omega) \coloneqq \mathcal{L}_\xi(e,\omega)$$

- They can also be understood as field-dependent gauge transformations

$$\delta_\xi^d e = \left(\delta^j_{\xi \lrcorner \omega} + \delta^t_{\xi \lrcorner e}\right) e + \xi \lrcorner \mathrm{d}_\omega e \qquad \delta_\xi^d \omega = \left(\delta^j_{\xi \lrcorner \omega} + \delta^t_{\xi \lrcorner e}\right) \omega + \frac{1}{2} \xi \lrcorner (2Fe - \lambda[e \wedge e])$$

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The diffeo charge is generally not integrable, even with $\delta\xi = 0$

$$\not\delta\mathcal{D}(\xi) \hat{=} \oint_S \delta\left(2(\xi\lrcorner\omega)e\right) - \xi\lrcorner\theta = \not\delta\mathcal{J}(\xi\lrcorner\omega) + \not\delta\mathcal{T}(\xi\lrcorner e)$$

Integrability **without** conditions on the fields

$\xi = (0,0,\xi^\varphi)$ is a tangent vector to S

and the one-dimensional algebra of diffeo is

$$\left\{\mathcal{D}(\xi),\mathcal{D}(\zeta)\right\} = -\mathcal{D}([\xi,\zeta])$$

BMS in 3D BF theory

Let's forget about the Lie derivative. Diffeo (with $\delta\xi = 0$) are "just" particular generators of the sub-algebra of quadratic charges within the envelopping algebra of $(\mathcal{J}, \mathcal{T})$

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- What is the most general quadratic charge one can consider?

$$\not{\delta}\mathcal{H}(\xi) = a\not{\delta}\mathcal{J}(\xi \lrcorner \omega) + b\not{\delta}\mathcal{J}(\xi \lrcorner e) + c\not{\delta}\mathcal{T}(\xi \lrcorner \omega) + d\not{\delta}\mathcal{T}(\xi \lrcorner e)$$

$\rightarrow (a, b, c, d)$ such that $\not{\delta}\mathcal{H}(\xi) = \delta\mathcal{H}(\xi)$ and the corresponding algebra is closed

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$$\boxed{\mathcal{D}^*(\xi) = -\lambda\mathcal{J}(\xi \lrcorner e) + \mathcal{T}(\xi \lrcorner \omega)} \quad \text{with action } \delta^{d^*}_\xi e = \mathcal{L}_\xi \omega\,, \quad \delta^{d^*}_\xi \omega = -\lambda \mathcal{L}_\xi e$$

Their algebra is

$$\begin{aligned} \{\mathcal{D}(\xi), \mathcal{D}(\zeta)\} &= -\mathcal{D}([\xi,\zeta]) \\ \{\mathcal{D}^*(\xi), \mathcal{D}(\zeta)\} &= -\mathcal{D}^*([\xi,\zeta]) \\ \{\mathcal{D}^*(\xi), \mathcal{D}^*(\zeta)\} &= -\lambda\mathcal{D}([\xi,\zeta]) \end{aligned} \quad \xrightarrow{\lambda \to 0} \quad \begin{aligned} \{\mathcal{D}(\xi), \mathcal{D}(\zeta)\} &= -\mathcal{D}([\xi,\zeta]) \\ \{\mathcal{D}^*(\xi), \mathcal{D}(\zeta)\} &= -\mathcal{D}^*([\xi,\zeta]) \\ \{\mathcal{D}^*(\xi), \mathcal{D}^*(\zeta)\} &= 0 \end{aligned}$$

$\mathfrak{witt} \oplus \mathfrak{witt}$ → centreless $\mathfrak{bms}_3$

4D BF theory

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$$L = \langle \mathcal{B} \wedge \mathcal{F} \rangle - \kappa \langle \mathcal{B} \wedge \mathcal{B} \rangle$$

with Lie algebra valued 2-form $\mathcal{B}$ and connection 1-form $\mathcal{A}$

- What about the symmetries?

Lorentz transfo δ^j_α, α any Lie element

$$\delta^j_\alpha \mathcal{B} = [\mathcal{B}, \alpha] \quad \text{and} \quad \delta^j_\alpha \mathcal{A} = \mathrm{d}_\mathcal{A} \alpha$$

Translation transfo, δ^t_φ, φ any Lie valued 1-form

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- $\mathcal{A}$ and $\mathcal{B}$ are forms of different degrees! We cannot "swap" the argument of $(\mathcal{J}, \mathcal{T})$ as in 3d

BMS like structure in 4D BF?

- "(boost+rotation)" decomposition of $g = \mathfrak{so}(4)$, $\mathfrak{so}(3,1)$, $\mathfrak{so}(2,2)$, $\mathfrak{iso}(2,1)$, $\mathfrak{iso}(3)$

$$[J_i, J_j] = \epsilon_{ij}{}^k J_k \ , \qquad [P_i, P_j] = \lambda \epsilon_{ij}{}^k J_k \ , \qquad [J_i, P_j] = \epsilon_{ij}{}^k P_k$$

with pairings $\langle P_i, J_j \rangle = \mu_1 \eta_{ij} \ , \qquad \langle P_i, P_j \rangle = \mu_2 \lambda \eta_{ij} \ , \qquad \langle J_i, J_j \rangle = \mu_2 \eta_{ij}$

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- Fields decomposition $\mathcal{A} = A_1^i J_i + A_2^i P_i$ and $\mathcal{B} = B_1^i J_i + B_2^i P_i$
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$$\not{\delta}\mathcal{J}_1(\alpha_1) = \oint_S \alpha_1 \delta \mathcal{B}_1 \quad \not{\delta}\mathcal{J}_2(\alpha_2) = \oint_S \alpha_2 \delta \mathcal{B}_2 \quad \not{\delta}\mathcal{T}_1(\varphi_1) = \oint_S \varphi_1 \wedge \delta \mathcal{A}_1 \quad \not{\delta}\mathcal{T}_2(\varphi_2) = -\oint_S \varphi_2 \wedge \delta \mathcal{A}_2$$

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- What about diffeos/quadratic charges

$$\begin{aligned}\not{\delta}\mathcal{H}(\xi) =& a\mathcal{J}_1(\xi \lrcorner A_1) + b\mathcal{J}_2(\xi \lrcorner A_2) + e\mathcal{T}_1(\xi \lrcorner B_1) + f\mathcal{T}_2(\xi \lrcorner B_2) \\ &+ c\mathcal{J}_1(\xi \lrcorner A_2) + d\mathcal{J}_2(\xi \lrcorner A_1) + g\mathcal{T}_1(\xi \lrcorner B_2) + h\mathcal{T}_2(\xi \lrcorner B_1)\end{aligned}$$

$\rightarrow (a, b, c, d, e, f, g, h)$ such that $\not{\delta}\mathcal{H}(\xi) = \delta\mathcal{H}(\xi)$ and the corresponding algebra is closed?

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$$\mathcal{D}(\xi) = \mathcal{J}_1(\xi \lrcorner A) + \mathcal{J}_2(\xi \lrcorner C) + \mathcal{T}_1(\xi \lrcorner B) + \mathcal{T}_2(\xi \lrcorner \Sigma)$$
$$\mathcal{D}^*(\xi) = \lambda \mathcal{J}_1(\xi \lrcorner C) + \mathcal{J}_2(\xi \lrcorner A) + \lambda \mathcal{T}_1(\xi \lrcorner \Sigma) + \mathcal{T}_2(\xi \lrcorner B)$$

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Conclusion

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- Obtained without any choice for the boundary and conditions on the fields
- Central charges?

  → in 3d: construction can be understood via a classical Sugawara construction. Central charges can be obtained using a twisted Sugawara construction
  → in 4d: not so clear..
- Additional relations between the choice of gauge group and the algebra of quadratics?
- What about higher-order charges? Relation to W_N algebra (in 3d)?
- 4D gravity: BF theory + constraints (Plebański formulation). Understanding constraints at the level of symmetry algebra and holographic theory reconstruction.

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Thank you for your attention!