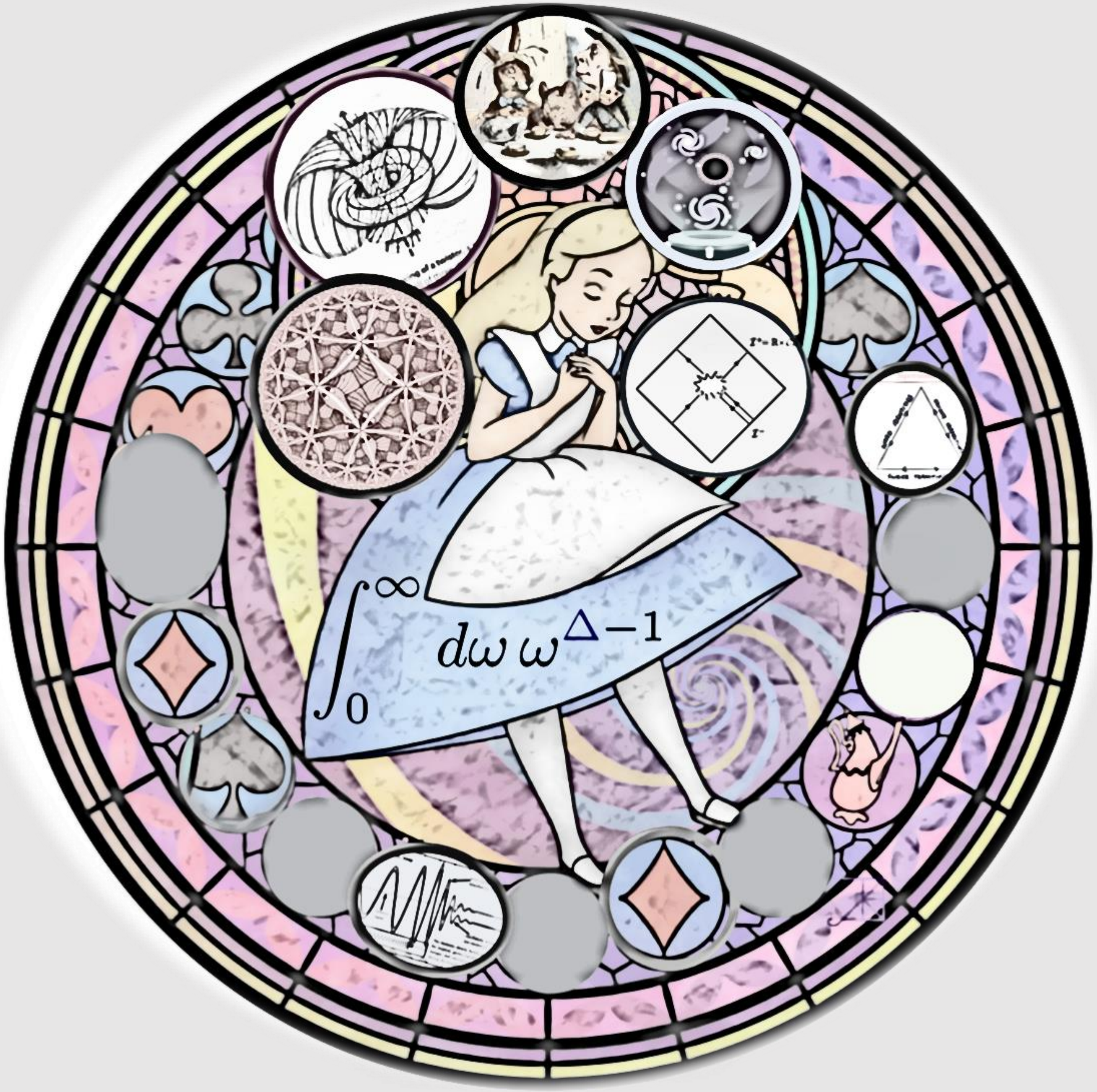


Gravitational S-matrix, celestial CFT & Carrollian holography

Laura DONNAY

UMONS - 2nd Carroll Workshop
12 Sept 2022

SISSA

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes → vanishing cosmological constant

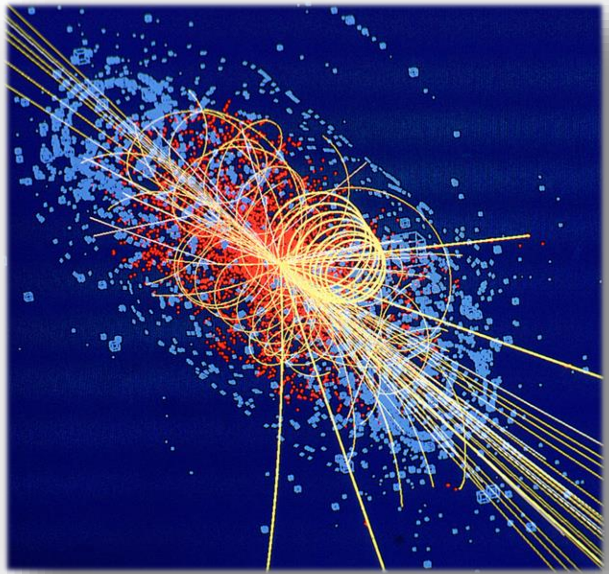
$$\Lambda = 0$$

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes $\longrightarrow$ vanishing cosmological constant

$$\Lambda = 0$$

These spacetimes are relevant



from collider physics …

… to astrophysics
(< cosmological scales)

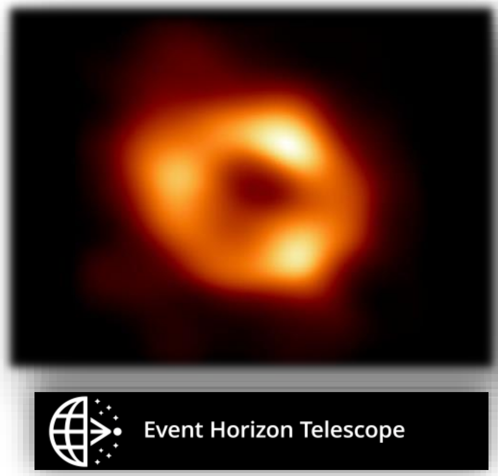
Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes

→ **Black holes**



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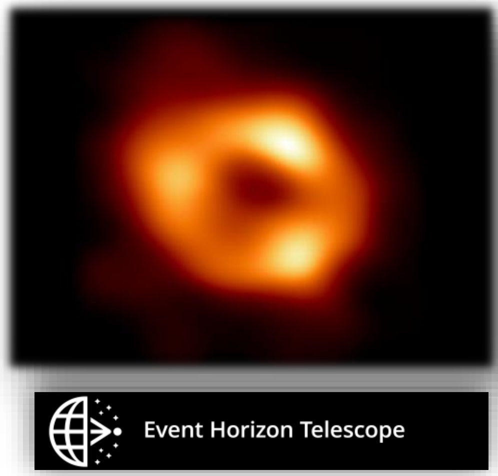
Quantum gravity in 4d asymptotically flat spacetimes

Black holes

Our understanding of quantum properties of black holes goes *hand-in-hand* with the **spectacular advances** of the **holographic** or **AdS/CFT correspondence**.



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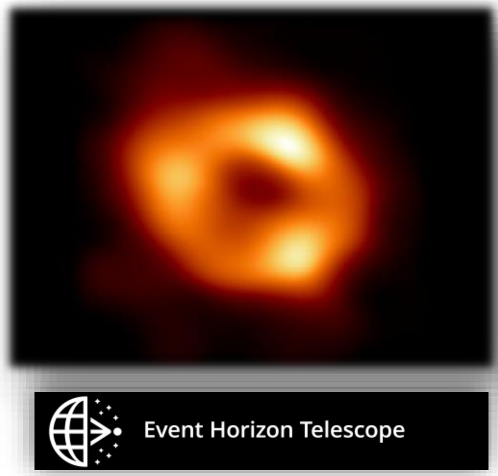
$$\boxed{S_{BH} = \frac{Ac^3}{4G\hbar}}$$

→ 'Primordial holographic relationship'

[Bekenstein][Hawking]



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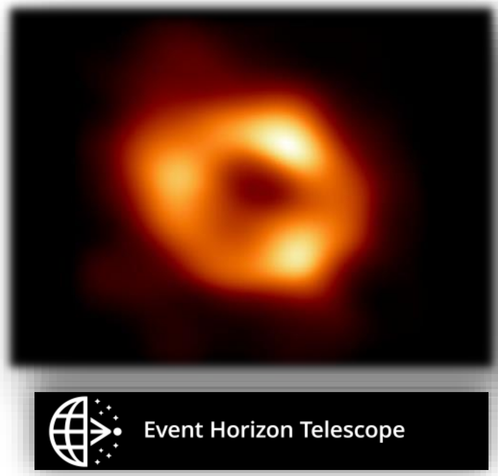
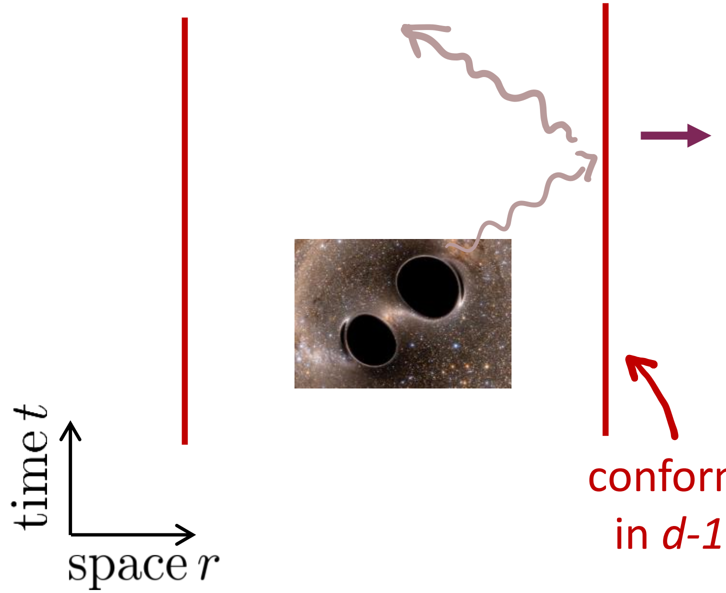


Problem: realistic black holes (e.g. Kerr) do not possess an AdS decoupling region.

→ need to develop a **holographic correspondence** for **flat spacetimes**

Holographic principle

Anti-de Sitter
in *d* dimensions



The holographic principle

> Quantum gravity is *encoded* in a different theory that lives in a *lower-dimensional* spacetime.

['t Hooft '93; Susskind '94; Maldacena '97]

Holographic principle

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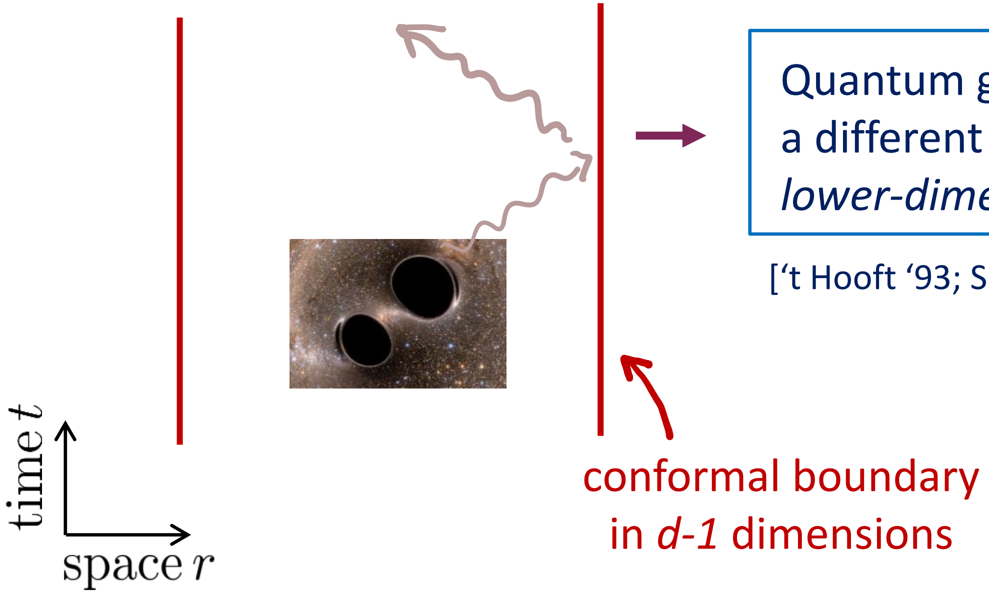

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How general is it?

Go beyond the canonical cases!

Anti-de Sitter	vs	**Flat**
$\Lambda < 0$		$\Lambda = 0$
CFT		

Holographic principle

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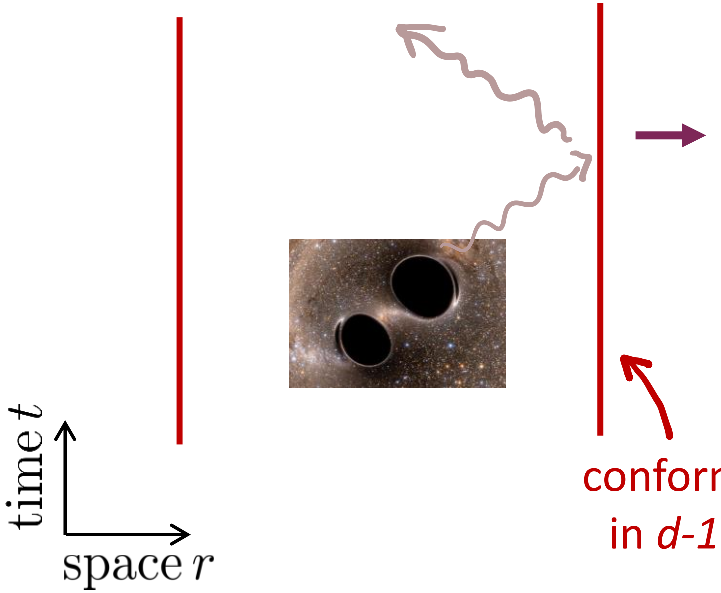

conformal boundary
in *d-1* dimensions

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Anti-de Sitter	vs	Flat
$\Lambda < 0$		$\boxed{\Lambda = 0}$
CFT		**??**
		this talk

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

Flat space holography

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Early attempts:

[Susskind ‘99][Polchinski ‘99][Giddings ’99]
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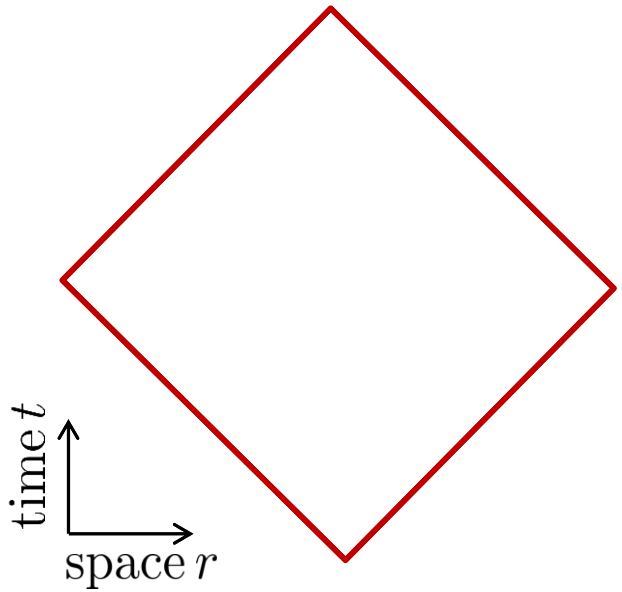
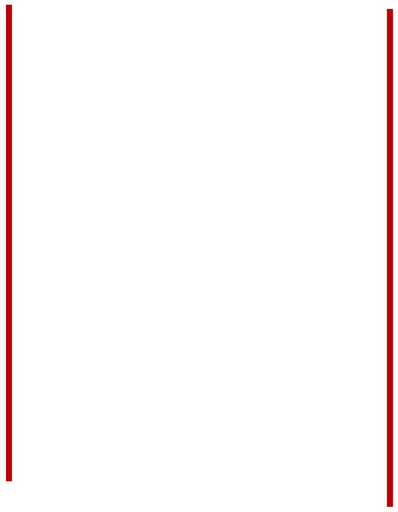
Main obstructions/difficulties:

Minkowski



AdS



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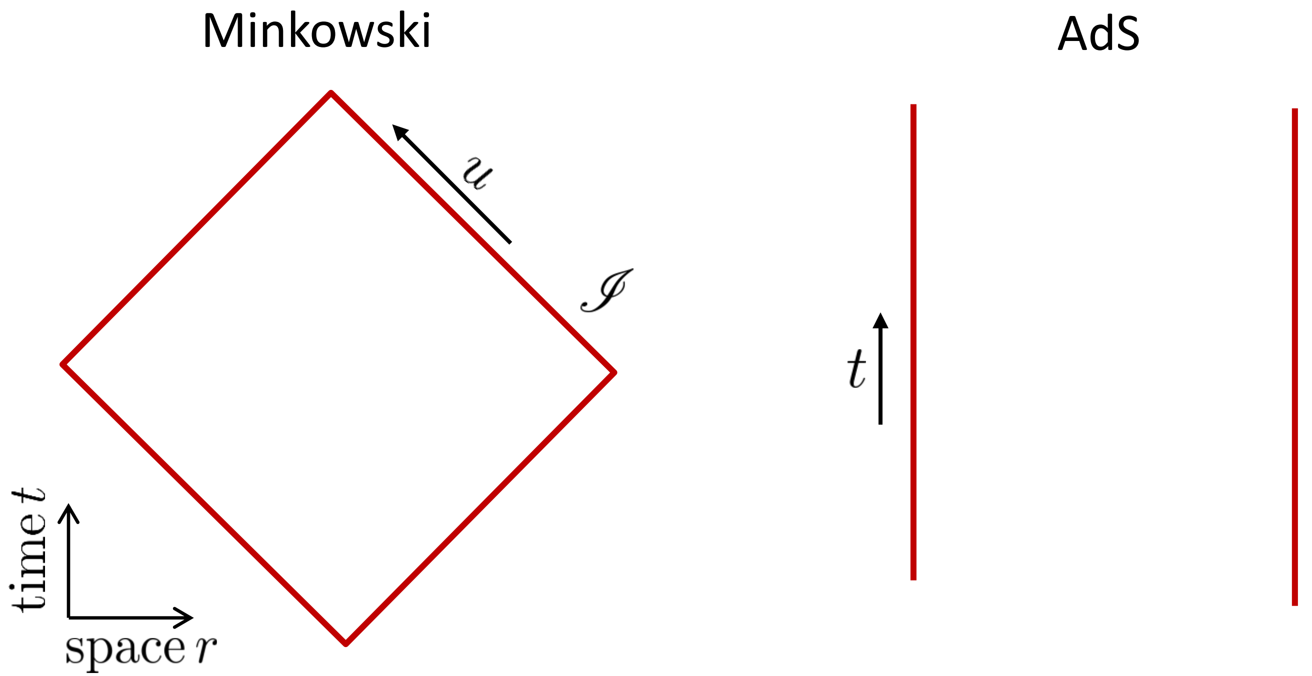
$$u = t - r$$



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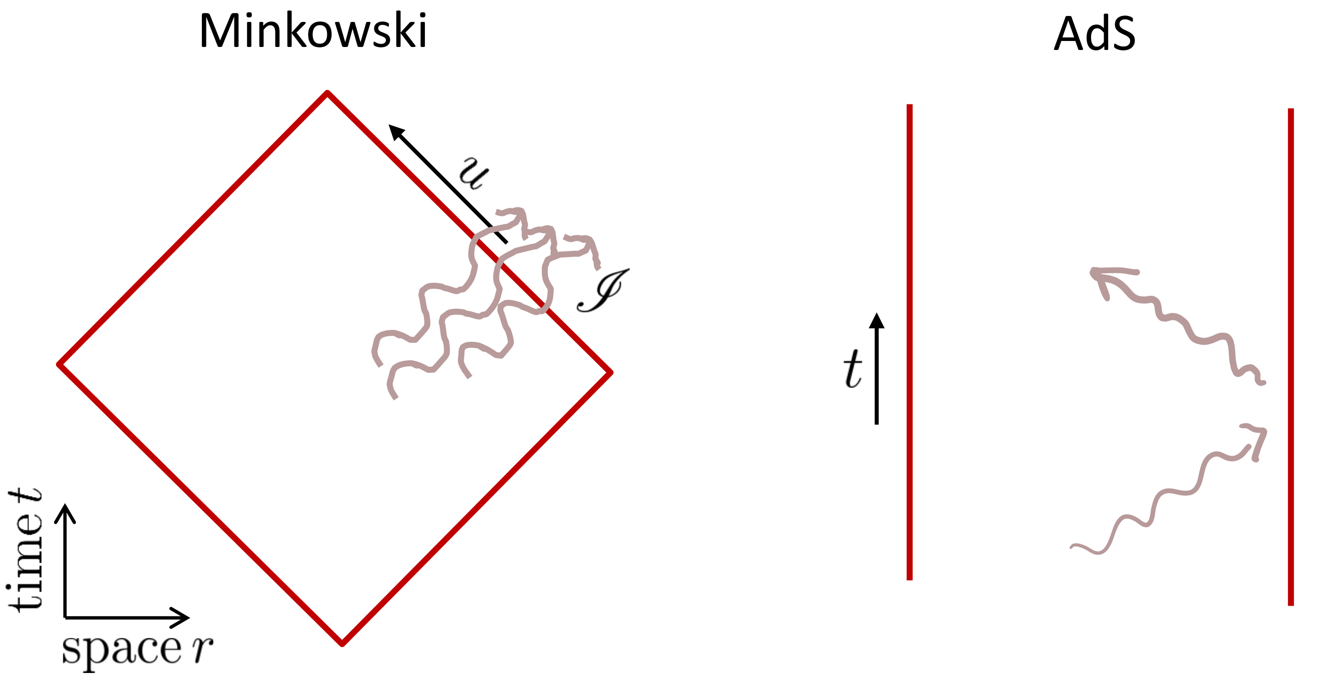

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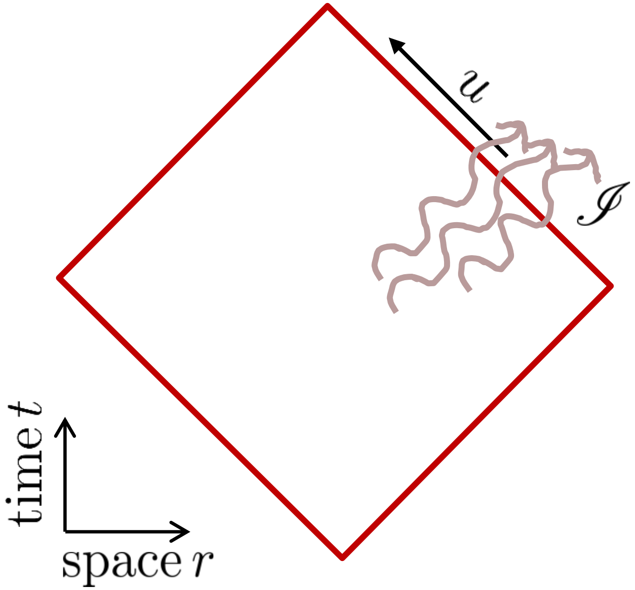

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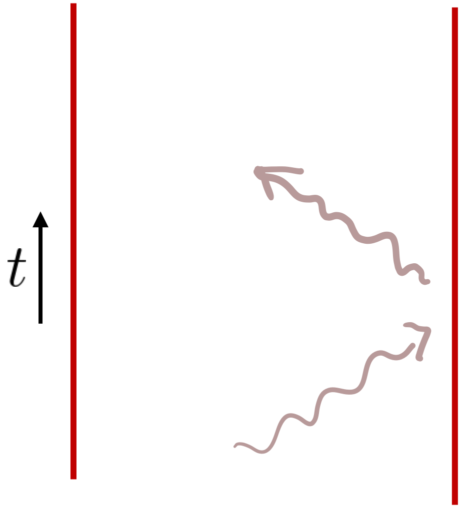

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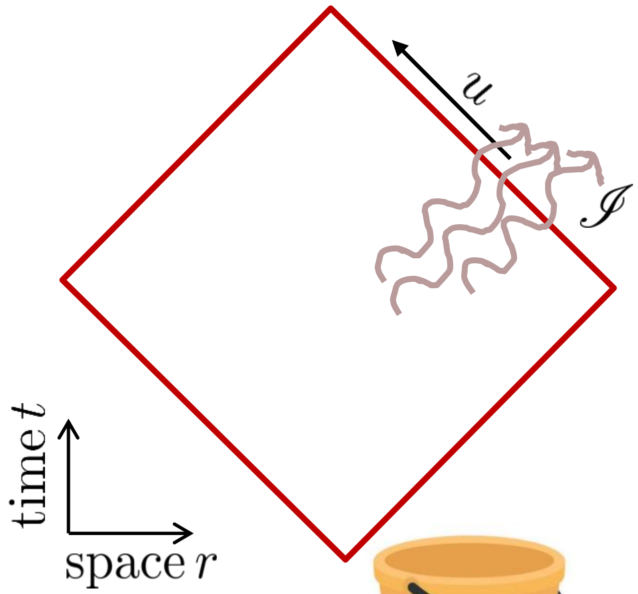

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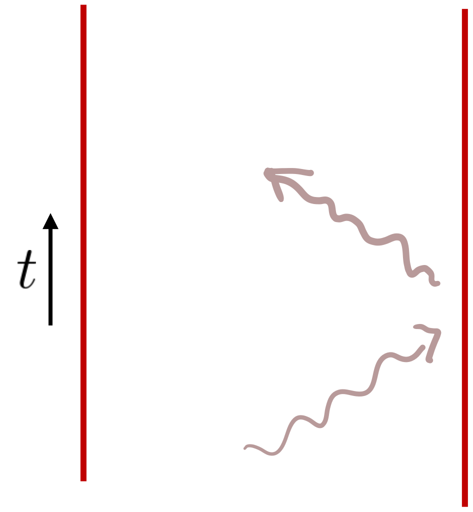

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two natural boundaries/proposals

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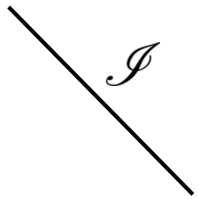

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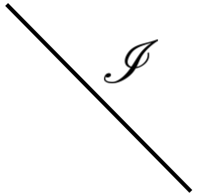

celestial sphere

Euclidean 2-sphere

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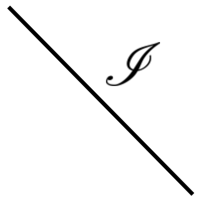

4d bulk/3d holography: ‘Carroll holography’

Dual: 3d ‘BMS field theory’

[Arcioni, Dappiaggi ’03 ’04] [Dappiaggi, Moretti, Pinamonti ’06][Mann, Marolf ’06] [Bagchi, Basu, Kakkar, Melhra ’16] [Bagchi, Melhra, Nandi ’20] [LD, Fiorucci, Herfray, Ruzziconi ’22][Bagchi, Banerjee, Basu, Dutta ‘22][…]

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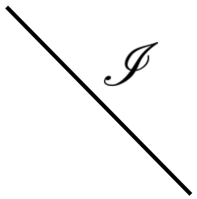

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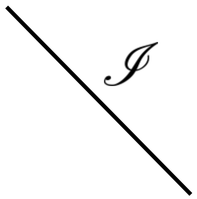

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Features: easier link to AdS/CFT ☺

treatment of fluxes ☹

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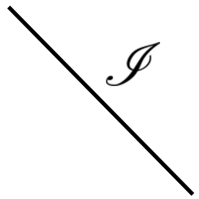

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Features: powerful CFT techniques at hand ☺
role of translations obscured ☹

Goals of this talk

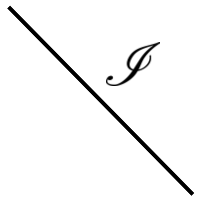
Holographic description of quantum gravity in 4d asymptotically flat spacetimes

two natural boundaries/proposals

3

null infinity

lighlike 3d hypersurface



2

4d bulk/3d holography: ‘Carroll holography’

Dual: 3d ‘BMS field theory’

& more in Romain Ruzziconi’s talk!

Features: easier link to AdS/CFT ☺

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Euclidean 2-sphere

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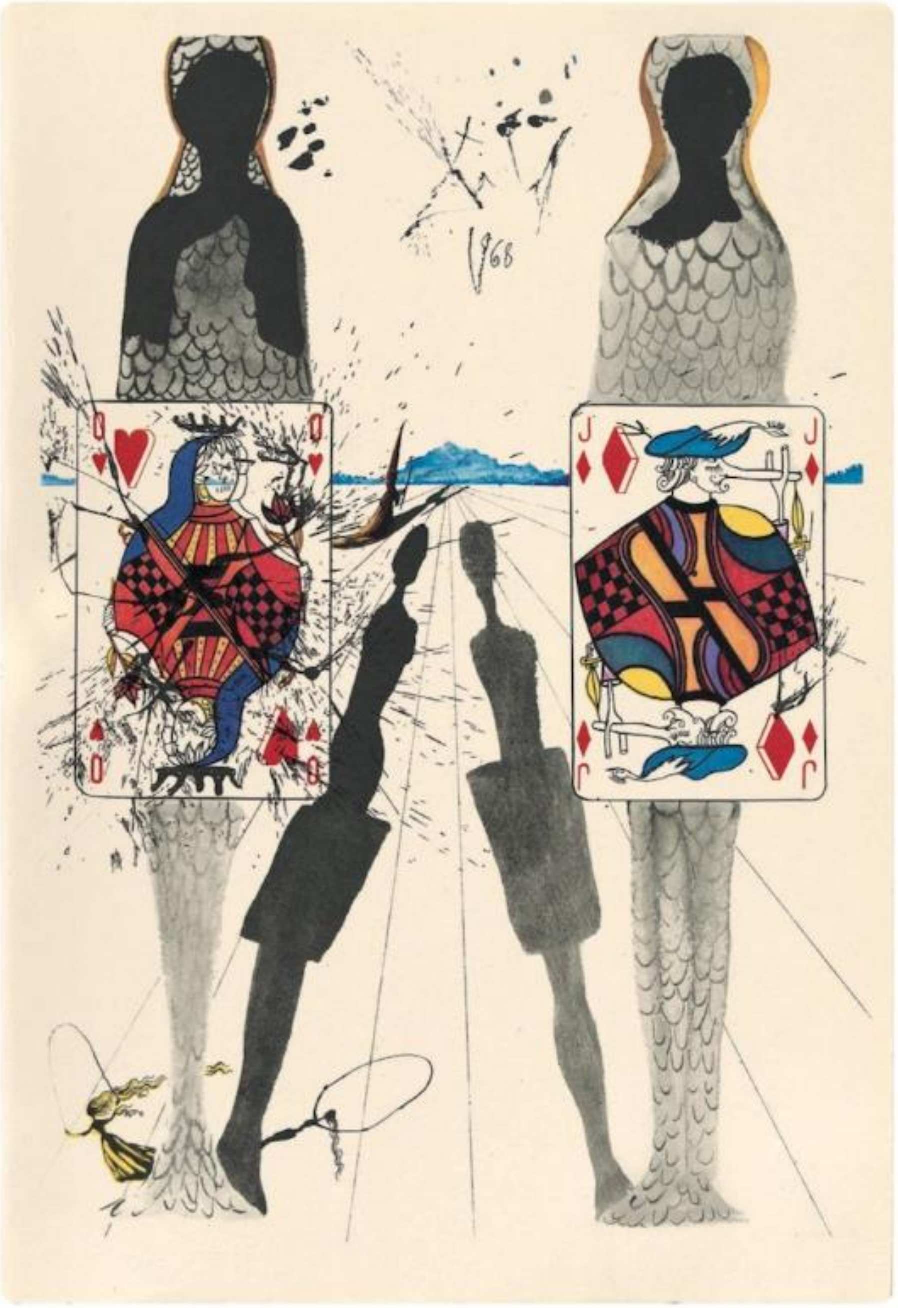
Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

1. BMS & the S-matrix
2. Celestial holography
3. BMS fluxes vs celestial currents
4. Towards Carrollian holography

based on

2108.11969 w/ **Romain RUZZICONI**

2202.04702 PRL (2022) & to appear
w/ **Adrien FIORUCCI, Yannick HERFRAY** & **Romain RUZZICONI**

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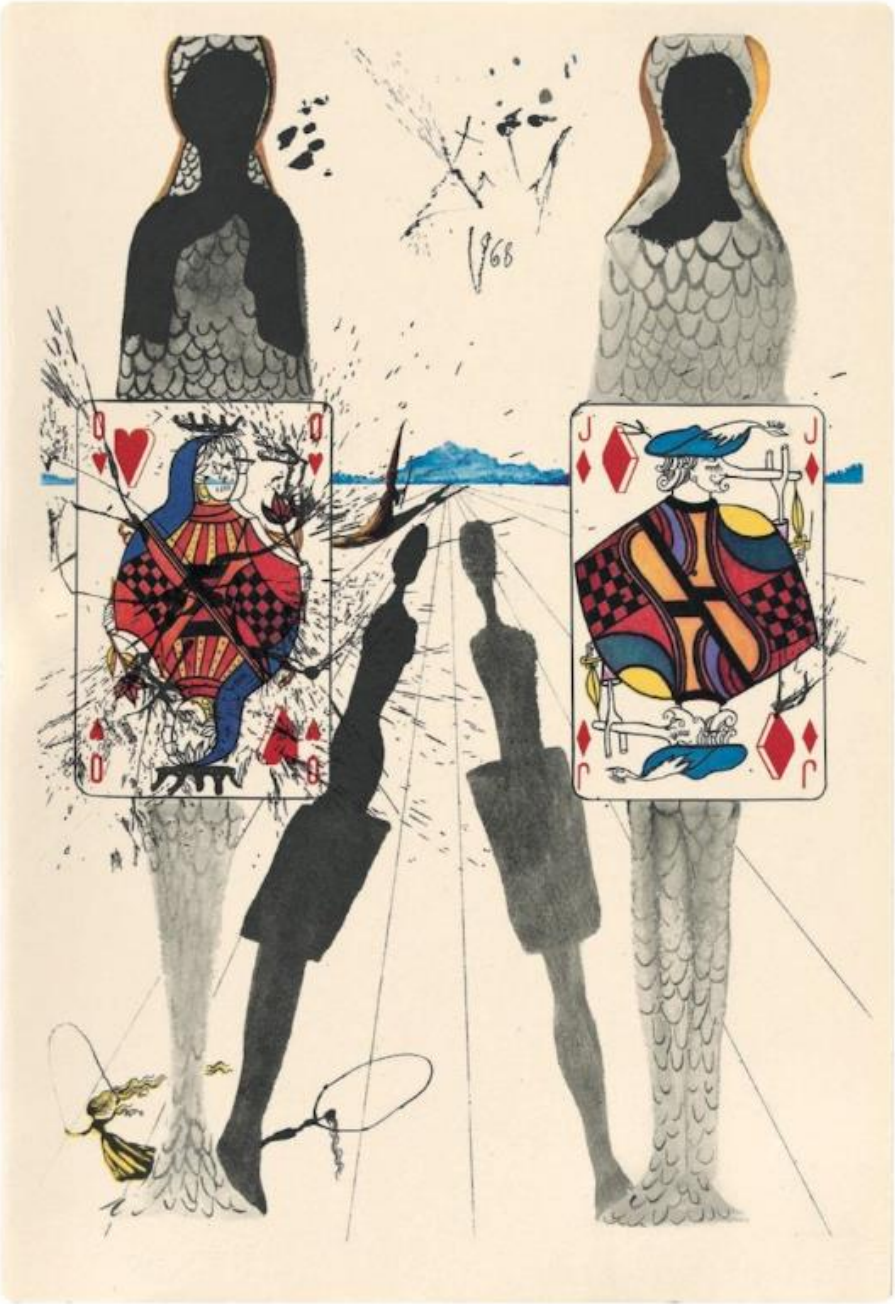

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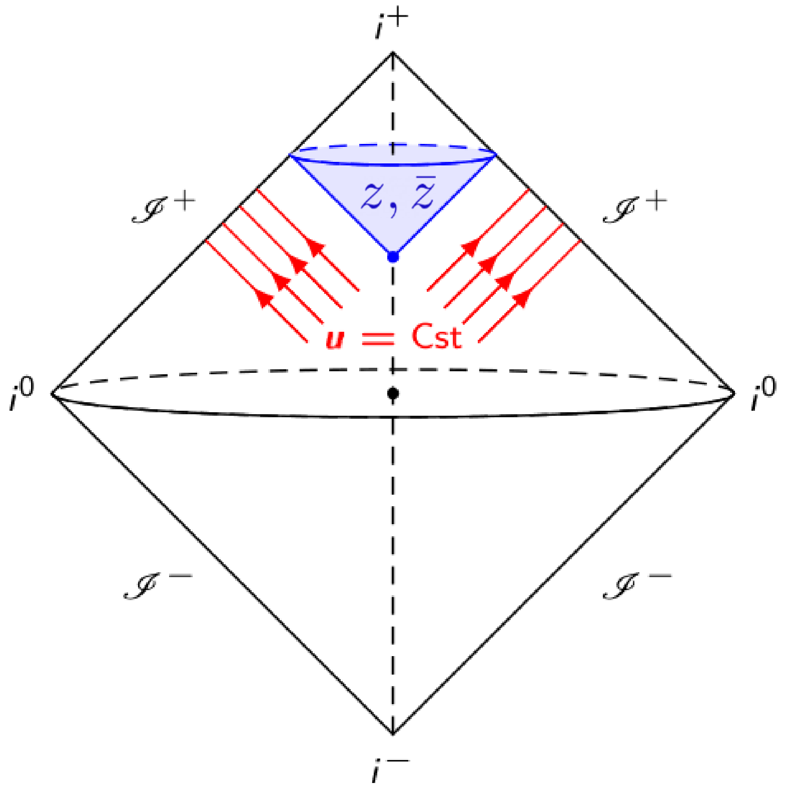
Gravitational solution space

[Bondi, van der Burg, Metzner '62] [Sachs '62]
[Barnich, Troessaert '10]

Asymptotically flat spacetimes in Bondi gauge:

$$r \to \infty \qquad (u, r, x^A)\,, \quad x^A = (z, \bar{z})$$

$$\boxed{\begin{aligned} ds^2 &= -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \\ &\quad + \frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^zC_{zz}\mathrm{d}u\mathrm{d}z \\ &\quad + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots \end{aligned}}$$



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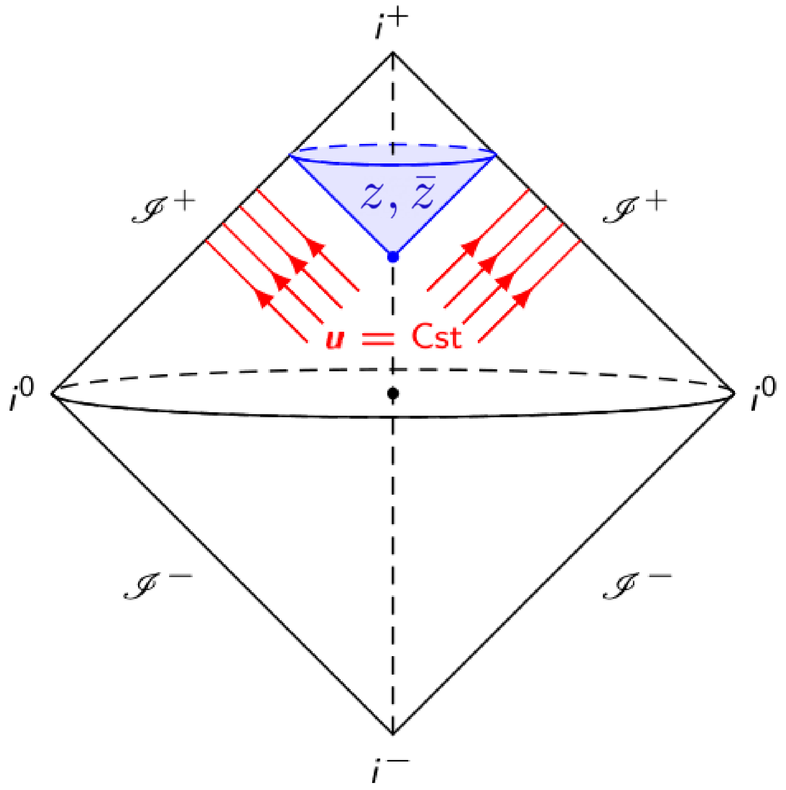

The Bondi **mass** and **angular momentum** aspects satisfy

$$\partial_u M = -\frac{1}{8}N_{AB}N^{AB} + \frac{1}{4}\partial_A\partial_B N^{AB},$$

$$\begin{aligned} \partial_u N_A &= \partial_A M + \frac{1}{16}\partial_A(N_{BC}C^{BC}) - \frac{1}{4}N^{BC}\partial_A C_{BC} \\ &- \frac{1}{4}\partial_B(C^{BC}N_{AC} - N^{BC}C_{AC}) - \frac{1}{4}\partial_B\partial^B\partial^C C_{AC} + \frac{1}{4}\partial_B\partial_A\partial_C C^{BC} \end{aligned}$$

$$N_{AB} \equiv \partial_u C_{AB}$$

Bondi news: encodes **gravitational waves!**

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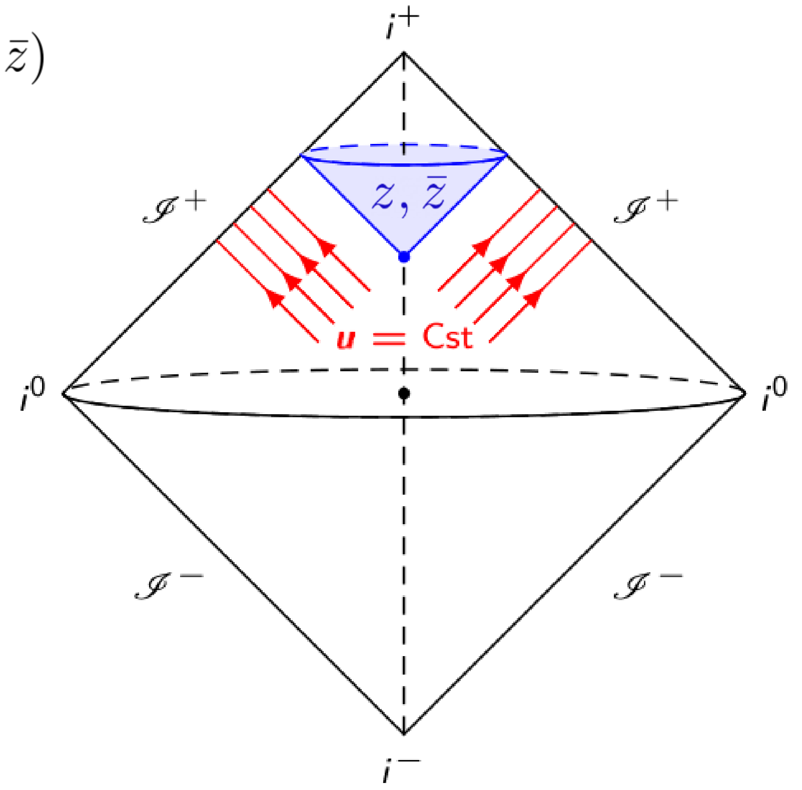


BMS symmetries:

$$\boxed{\begin{array}{l} \text{Poincaré} = 4 \text{ translations} \ltimes 6 \text{ Lorentz} \\ \quad\downarrow \qquad\qquad\qquad \downarrow \qquad\qquad\qquad \downarrow \\ \text{BMS} = \underset{\infty}{\text{supertranslations}} \ltimes \underset{\infty}{\text{superrotations}} \end{array}}$$

$\xi(T) = T(z,\bar z)\partial_u$

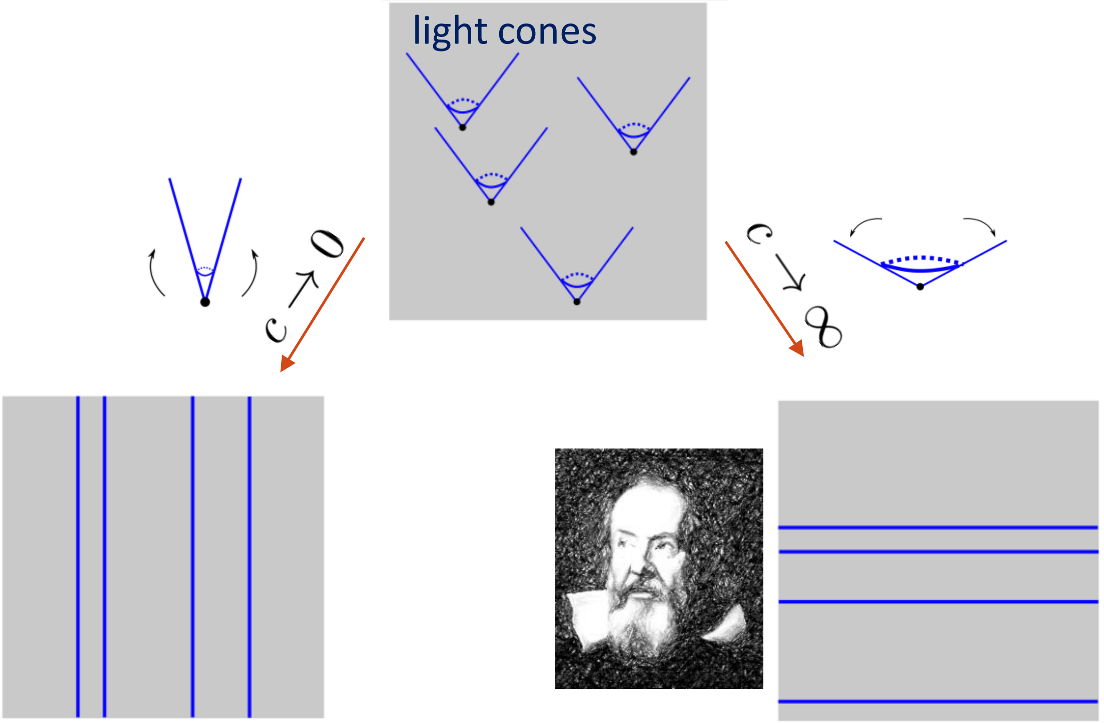
SUPERTRANSLATIONS

$\xi(Y) = Y^z(z)\partial_z + \frac{u}{2}D_zY^z\partial_u + c.c.$

SUPERROTATIONS

[Barnich, Troessaert ’08]

BMS = conformal Carrollian symmetries



[Levy-Leblond ’65] Carrollian spacetime
(space is absolute)

Galilean spacetime
(time is absolute)

[fig. Yannick Herfray]

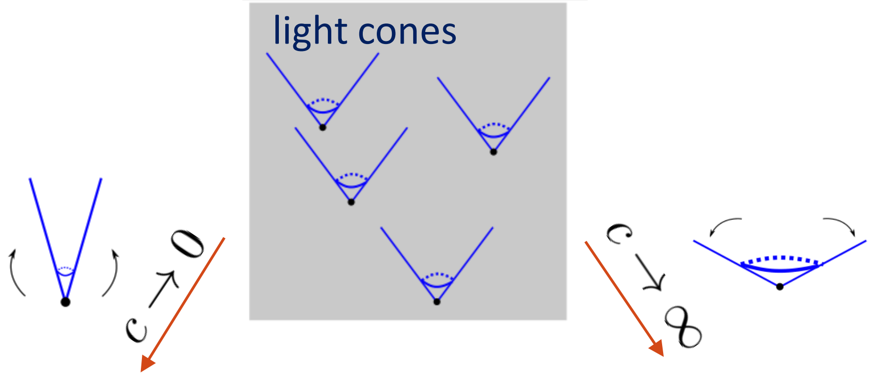
BMS = conformal Carrollian symmetries

- BMS symmetries = **conformal** symmetries of a **Carrollian** structure at null infinity
  [Geroch] [Penrose][Duval, Gibbons, Horvathy] [Hartong] [Ciambelli, Leigh, Marteau, Petropoulos] [Morand] [Herfray]…

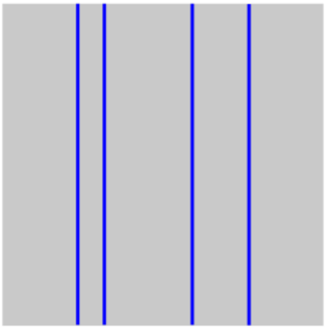
$x^a = (u, z, \bar{z})$

q_{ab} : a degenerate metric $\longrightarrow$ $q_{ab}dx^a dx^b = 0 \times du^2 + 2\gamma_{z\bar{z}}dzd\bar{z}$

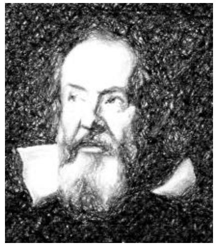
a vector field satisfying $q_{ab}n^b = 0 \rightarrow n = \partial_u$

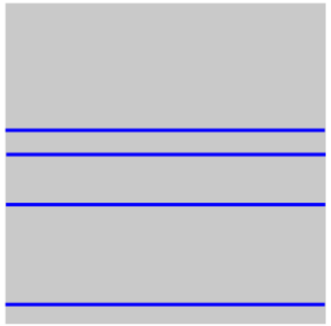


[Levy-Leblond ’65]



Carrollian spacetime
(space is absolute)





Galilean spacetime
(time is absolute)

[fig. Yannick Herfray]

BMS = conformal Carrollian symmetries

- BMS symmetries = **conformal** symmetries of a **Carrollian** structure at null infinity

[Geroch] [Penrose][Duval, Gibbons, Horvathy] [Hartong] [Ciambelli, Leigh, Marteau, Petropoulos] [Morand] [Herfray]…

$x^a = (u, z, \bar{z})$

q_{ab} : a degenerate metric $\longrightarrow$ $q_{ab}dx^a dx^b = 0 \times du^2 + 2\gamma_{z\bar{z}}dzd\bar{z}$

a vector field satisfying $q_{ab}n^b = 0 \rightarrow n = \partial_u$

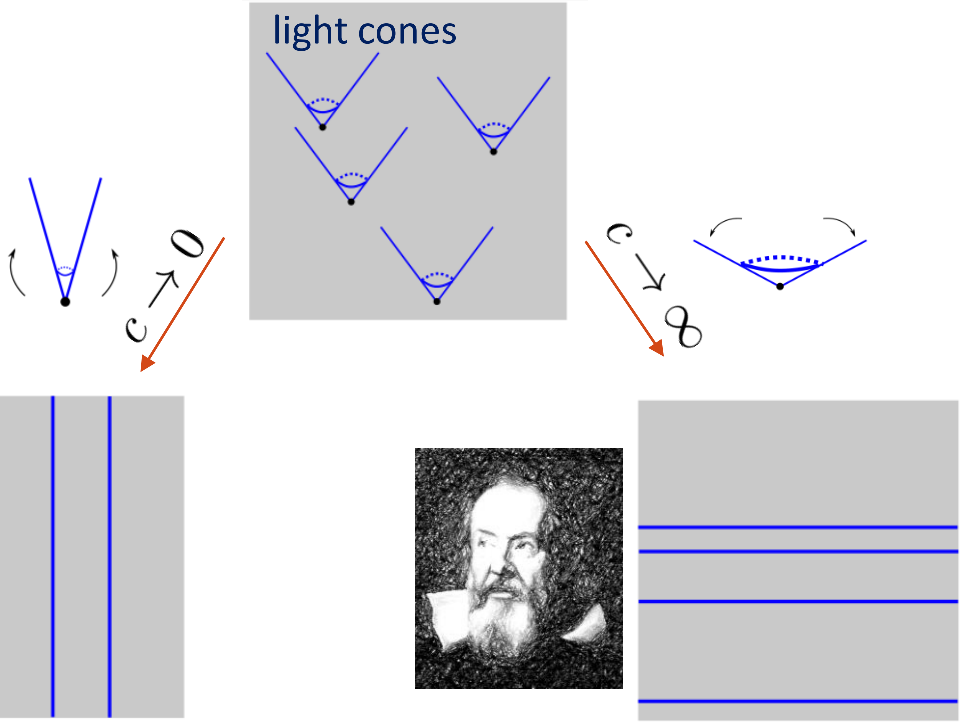
Conformal Carrollian symmetries:

$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab} \qquad \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a$

$\alpha := \frac{1}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})$

$$\boxed{\bar{\xi} = \left[\mathcal{T} + \frac{u}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}}$$

[Levy-Leblond '65]



Carrollian spacetime
(space is absolute)

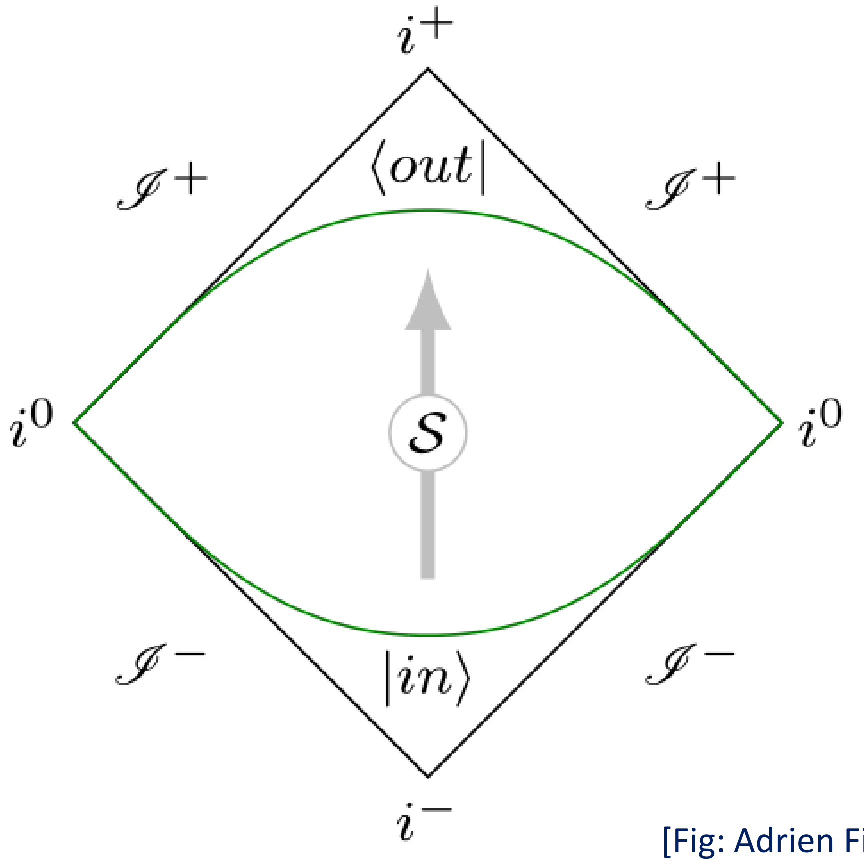
Galilean spacetime
(time is absolute)

[fig. Yannick Herfray]

BMS and the scattering problem

Seminal observation: BMS symmetries constrain the gravitational scattering problem! [Strominger '14]

$\langle out|S|in\rangle$



[Fig: Adrien Fiorucci]

BMS and the scattering problem

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→ 2 key ingredients

BMS and the scattering problem

Seminal observation: BMS symmetries constrain the gravitational scattering problem! [Strominger '14]

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1 Noether charges for BMS symmetries

[Barnich, Troessaert '10]

$$Q_{\mathcal{T}} = \frac{1}{4\pi G}\int d^2z\sqrt{\gamma}\,\mathcal{T}M$$

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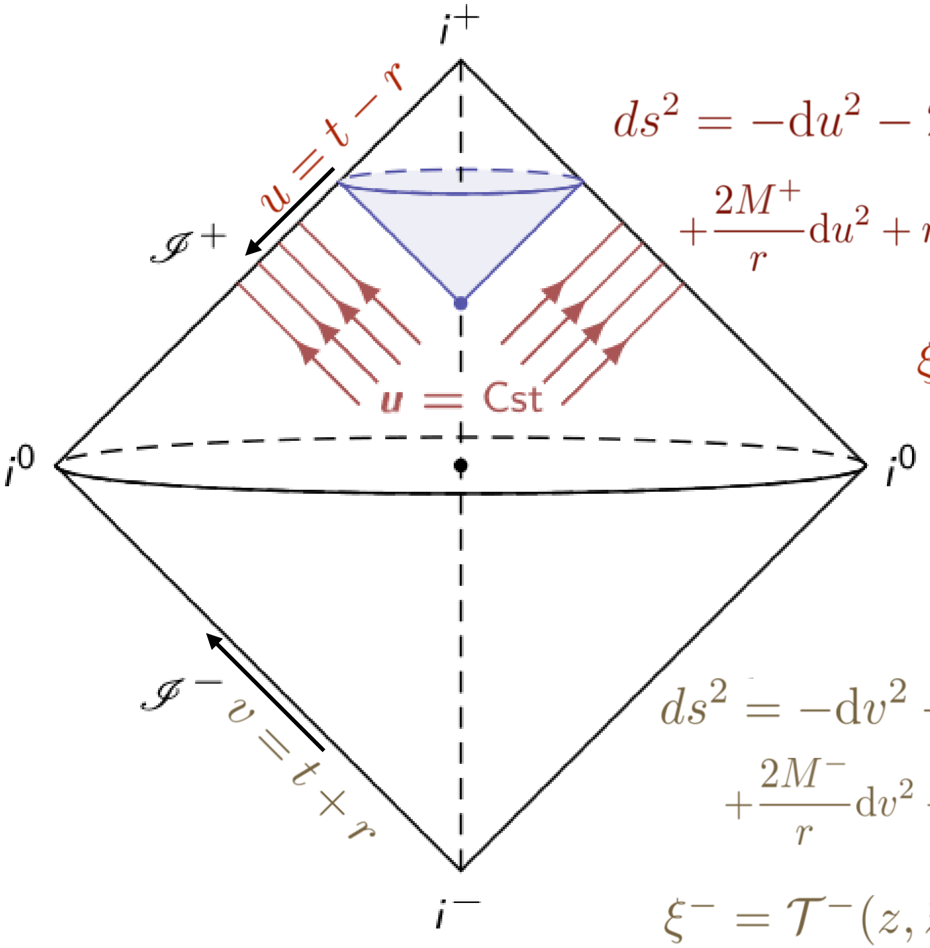
$$Q_{\mathcal{T}} = \frac{1}{4\pi G} \int d^2 z \sqrt{\gamma}\, \mathcal{T} M$$

2. Relating the *past* and the *future*



$$ds^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z}$$
$$+\frac{2M^+}{r}\mathrm{d}u^2 + rC^+_{zz}\mathrm{d}z^2 + D^z C^+_{zz}\mathrm{d}u\mathrm{d}z + \cdots$$

$$\xi^+ = \mathcal{T}^+(z,\bar{z})\partial_u$$

$$ds^2 = -\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z}$$
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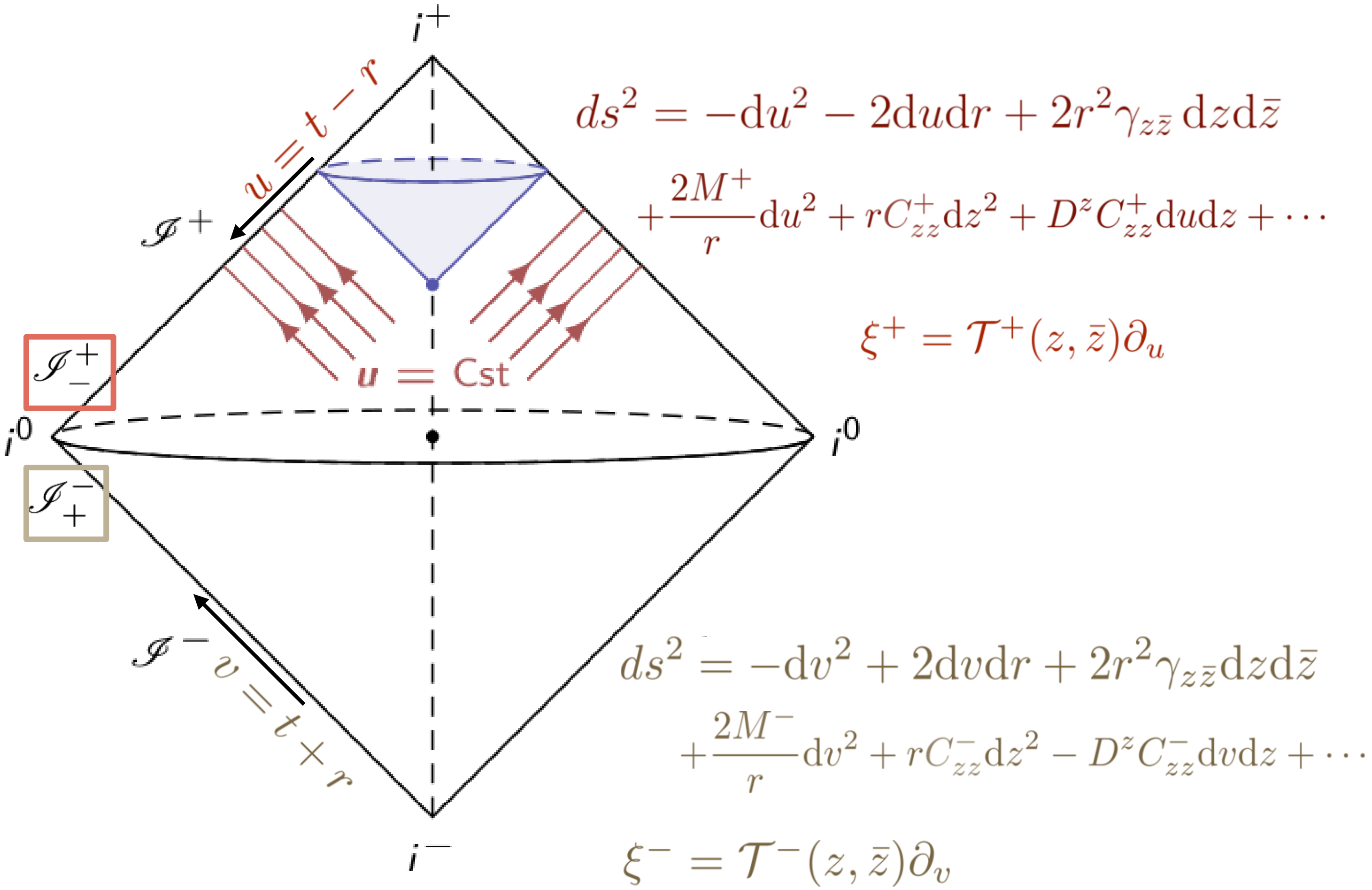

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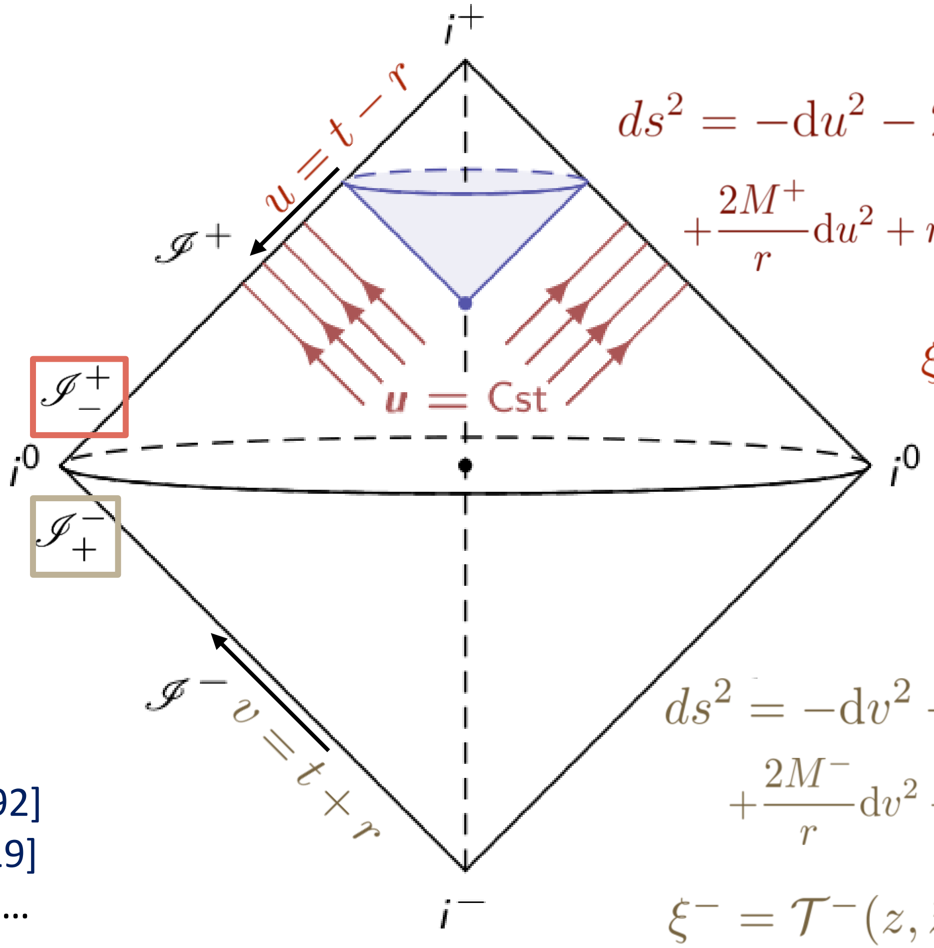
② Relating the *past* and the *future*

Antipodal matching conditions

$$M^-(v,z,\bar z)|_{\mathscr{I}^-_+} = M^+(u,z,\bar z)|_{\mathscr{I}^+_-}$$

$$\mathcal{T}^-(z,\bar z)|_{\mathscr{I}^-_+} = \mathcal{T}^+(z,\bar z)|_{\mathscr{I}^+_-}$$

[Strominger '14]; see also [Herberthson, Ludvigsen '92]
[Troessaert '18][Henneaux, Troessaert '18][Prabhu '19]
[Kroon, Mohamed '21][Capone, Nguyen, Parisini '22]…



$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar z}\,dzd\bar z + \frac{2M^+}{r}du^2 + rC^+_{zz}dz^2 + D^zC^+_{zz}dudz + \cdots$$

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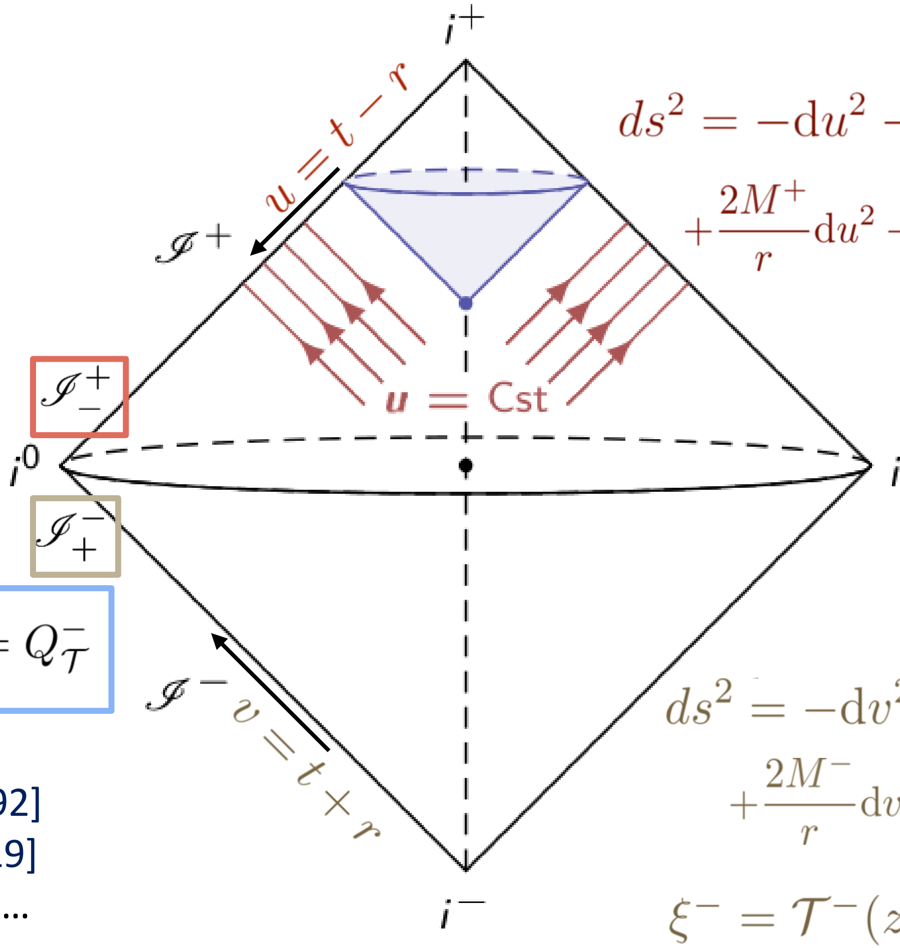

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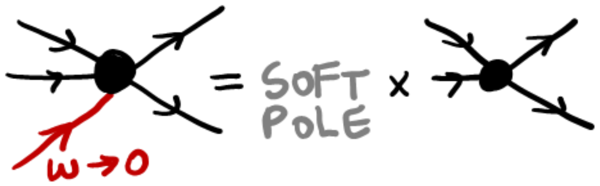
BMS and the scattering problem

Prime example:

The **leading soft graviton theorem** [Weinberg '65]

$$\mathcal{A}_n = \langle out | S | in \rangle$$

+soft particle (energy $\omega \to 0$)



BMS and the scattering problem

Prime example:

The **leading soft graviton theorem** [Weinberg '65]

$$n \text{ hard particles } (p_k) + \text{external graviton } (q)$$

$$\lim_{\omega \to 0} \mathcal{A}_{n+1}(q) = S^{(0)} \mathcal{A}_n + \mathcal{O}(q^0)$$

$$S^{(0)} = \sum_{k=1}^{n} \frac{p_k^\mu p_k^\nu \varepsilon_{\mu\nu}(q)}{p_k \cdot q}$$

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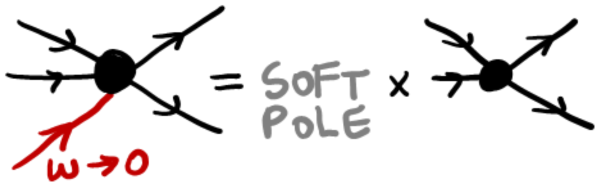

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is nothing but the **Ward identity** associated to **supertranslation** symmetry [He, Lysov, Mitra, Strominger '15]

$$\langle out | Q^+_{\mathcal{T}} \mathcal{S} - \mathcal{S} Q^-_{\mathcal{T}} | in \rangle = 0$$

supertranslation charge

$$Q_{\mathcal{T}} = \frac{1}{4\pi G} \int d^2 z \sqrt{\gamma}\, \mathcal{T} M$$

3 languages for the same IR physics

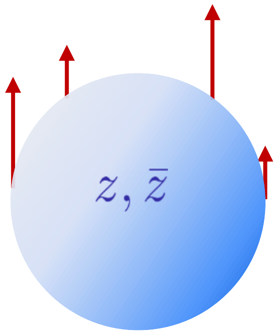
[Strominger '18]

Asymptotic symmetries

General Relativity

supertranslations

[Bondi-Metzner-Sachs '62]



$$\Delta C_{AB} \neq 0$$

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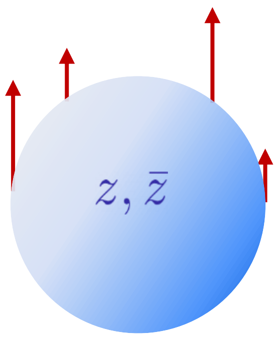

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Soft theorems

Quantum Field Theory

leading soft graviton theorem

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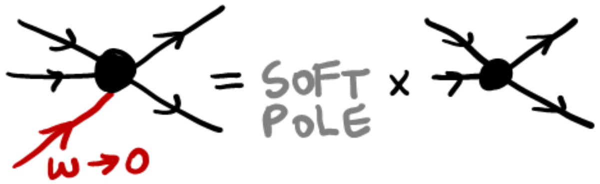

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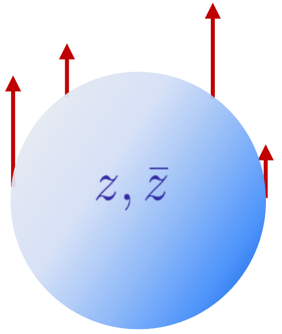

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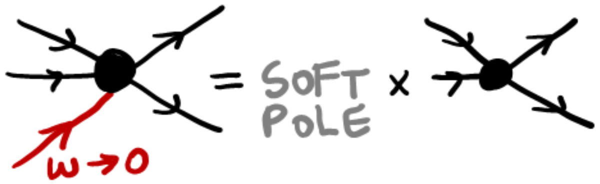
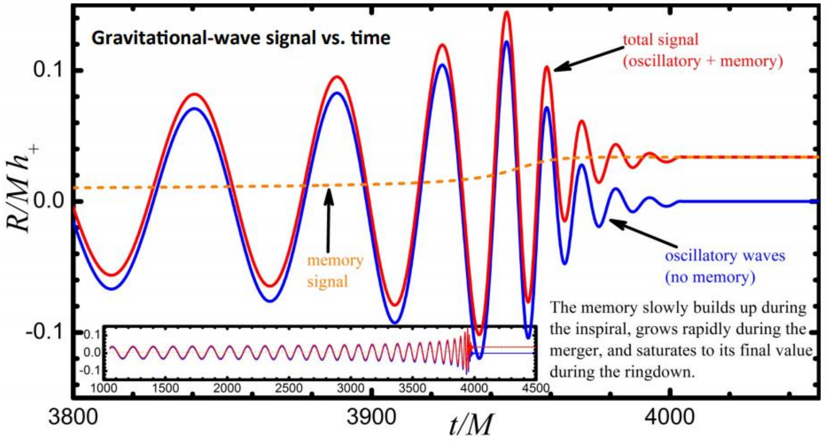


Memory effects

GW observation

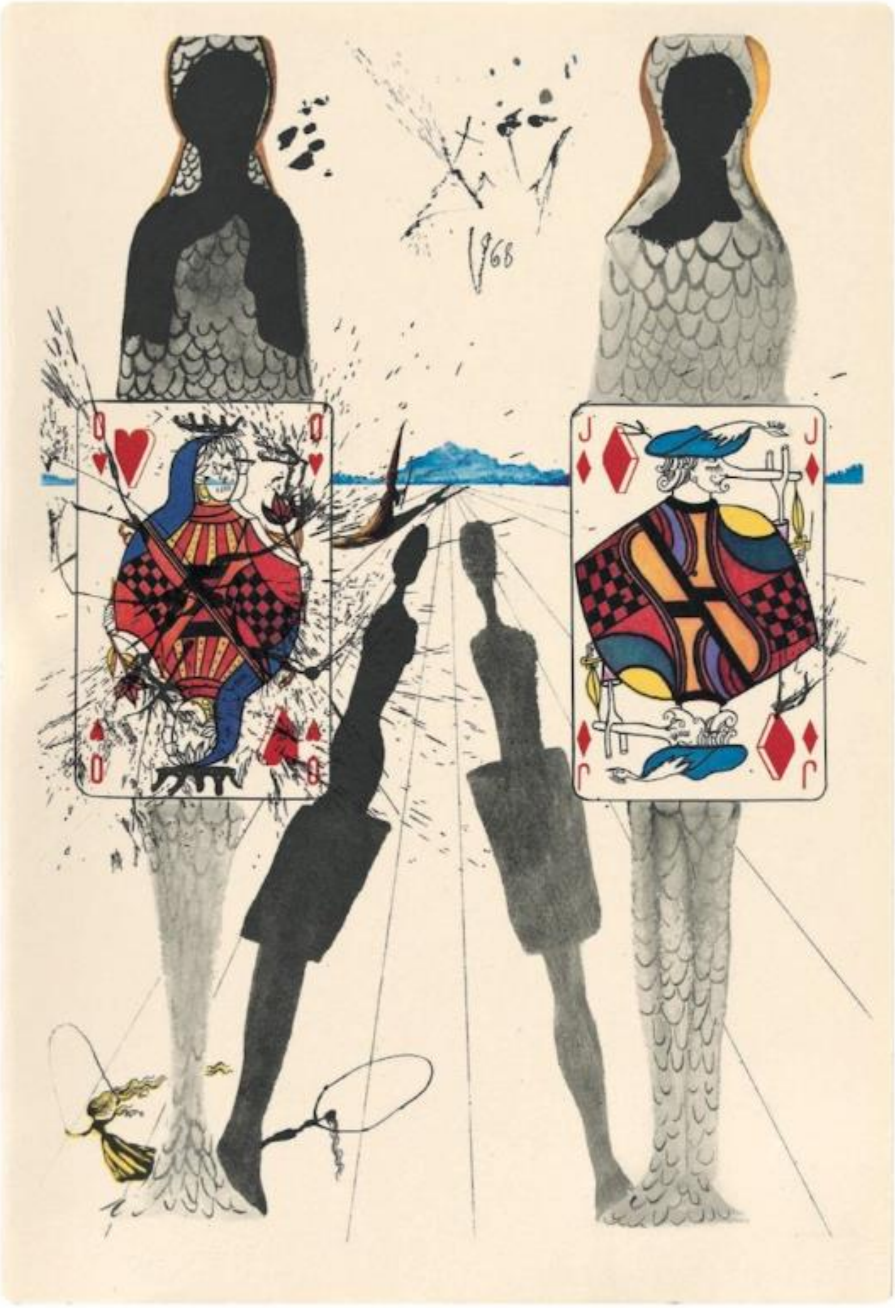
displacement memory

[Zel'dovich, Polnarev, Braginskii, Thorne, Christodoulou] … 70s – 90s



[Favata, '10]

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

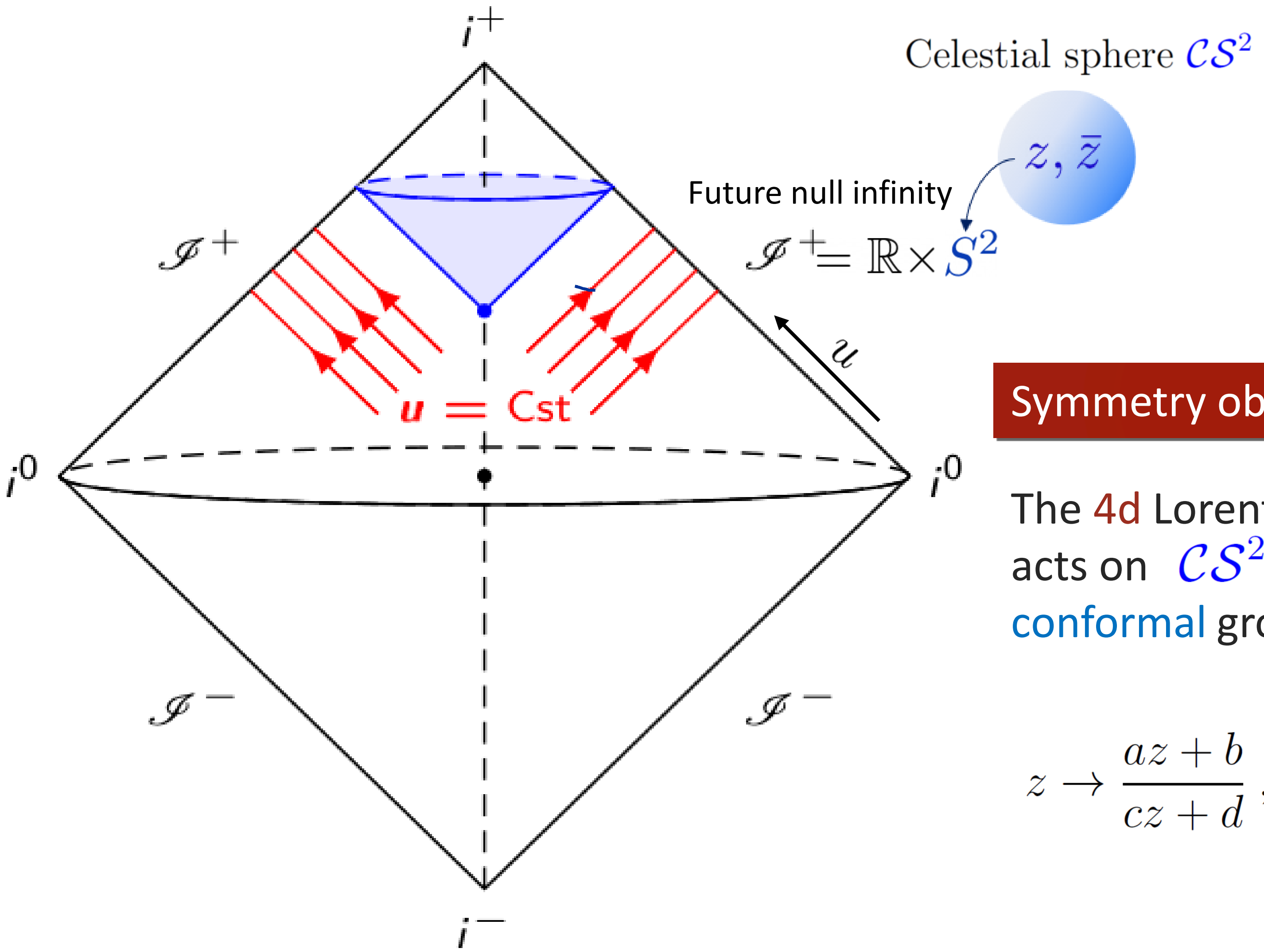
1. BMS & the S-matrix
2. Celestial holography
3. BMS fluxes vs celestial currents
4. Towards Carrollian holography

based on

2108.11969 w/ **Romain RUZZICONI**
2202.04702 PRL (2022) & to appear
w/ **Adrien FIORUCCI, Yannick HERFRAY** & **Romain RUZZICONI**

2205.11477 w/ **Kevin NGUYEN** & **Romain RUZZICONI**

Celestial Holography



Symmetry observation

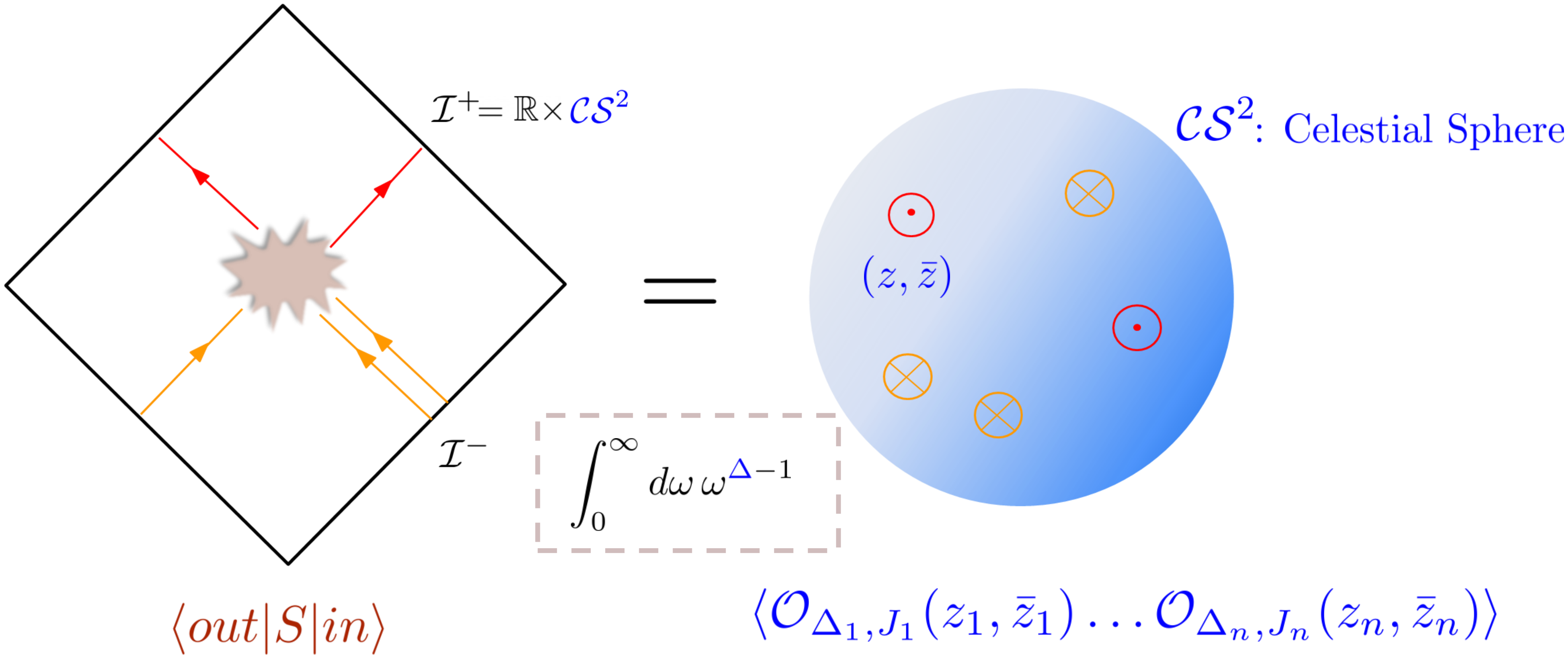
The 4d Lorentz group acts on $\mathcal{CS}^2$ the 2d conformal group:

$$z \to \frac{az+b}{cz+d}\,, \qquad ad - bc = 1$$

$$SL(2, \mathbb{C})$$

Celestial Holography

The 4d spacetime S-matrix is encoded in a 2d 'Celestial Conformal Field Theory'



Basis for celestial holography

Holographic basis:

$$\boxed{\int_0^\infty d\omega\, \omega^{\Delta-1}}$$

Mellin transform

ω: energy

de Boer, Solodukhin
Cheung, de la Fuente, Sundrum
Pasterski, Shao, Strominger

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Plane waves (null momentum $p^\mu = \omega q^\mu(z,\bar{z})$) get mapped to

$$\Psi^{\pm}_{\Delta}(X;z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i p\cdot X}$$

$\Delta = h + \bar{h}$: conformal dimension

$(z,\bar{z})$: a point on $\mathcal{CS}^2$

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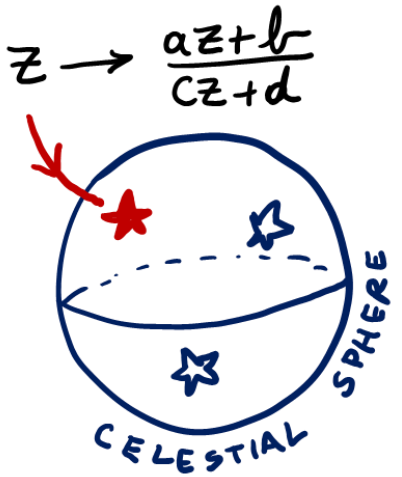
'conformal primary wavefunctions' which transform under $SL(2,\mathbb{C})$

$$\Psi_{h,\bar{h}}(z,\bar{z}) \to (cz+d)^{2h}(\bar{c}\bar{z}+\bar{d})^{2\bar{h}}\Psi_{h,\bar{h}}(z,\bar{z})$$



as primaries of weights

$$(h,\bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

2d spin J

Basis for celestial holography

- **Celestial operators** are defined as [LD, Pasterski, Puhm '20]

$$\boxed{\mathcal{O}_\Delta(z,\bar{z}) = (\Phi(X), \Psi_\Delta(X;z,\bar{z}))}$$

bulk operator

$(\cdot,\cdot)$: Klein-Gordon inner product pushed at $\mathscr{I}$

X : a point in the bulk

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Recall:

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X : a point in the bulk

$p^\mu = \omega q^\mu(z,\bar{z})$

NB: for simplicity, I consider here only scalar operators (and some labels are sometimes omitted)

Basis for celestial holography

- Celestial operators are defined as [LD, Pasterski, Puhm '20]

$$\boxed{\mathcal{O}_\Delta(z,\bar z) = (\Phi(X), \Psi_\Delta(X; z, \bar z))} \sim a_\Delta(z,\bar z) \equiv \int_0^\infty d\omega \omega^{\Delta-1} a(\omega, z, \bar z)$$

bulk operator

conformal primary wavefunction

$(\cdot,\cdot)$: Klein-Gordon inner product pushed at $\mathscr{I}$

Recall:

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Basis for celestial holography - let's repeat

- Momentum basis

$$\Phi(X) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[a(p)e^{ip\cdot X} + a(p)^\dagger e^{-ip\cdot X}\right]$$

$$p^\mu = \omega q^\mu(\vec{w})$$

Basis for celestial holography - let's repeat

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$$p^\mu = \omega q^\mu(\vec{w})$$

$$= \int \frac{d^2\vec{w}}{2(2\pi)^3} \omega d\omega \left[a(\omega,\vec{w})e^{i\omega q\cdot X} + a(\omega,\vec{w})^\dagger e^{-i\omega q\cdot X}\right]$$

Basis for celestial holography - let's repeat

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- Celestial basis

$$\Phi(X) = \int \frac{d^2\vec{w}}{2(2\pi)^3} \overbrace{\int_{c-i\infty}^{c+i\infty} \frac{d\Delta}{i2\pi}}^{\text{inverse Mellin}} \left[a_{2-\Delta}(\vec{w})\Psi^+_\Delta(X;\vec{w}) + a_{2-\Delta}(\vec{w})^\dagger \Psi^-_\Delta(X;\vec{w})\right]$$

Basis for celestial holography - let's repeat

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- Celestial basis

inverse Mellin

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$$\Psi^\pm_\Delta(X;\vec{w}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i\omega q\cdot X}$$

Mellin of plane waves

$$a_\Delta(\vec{w}) = \int_0^\infty d\omega\, \omega^{\Delta-1} a(\omega,\vec{w})$$

Mellin of ladder operators

Basis for celestial holography - let's repeat

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Mellin of plane waves

Mellin of ladder operators

Ladder operators

$$a(p) = (\Phi(X), e^{ip\cdot X})$$

Celestial operators

$$a_\Delta(\vec{w}) = (\Phi(X), \Psi^+_\Delta(X;\vec{w}))$$

$$\equiv \mathcal{O}_{\Delta,J=0}(\vec{w})$$

Celestial currents

$$\int_0^\infty d\omega\, \omega^{\Delta-1}$$

$$\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})$$

$$(h,\bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

weights of the celestial operators

Celestial currents are obtained by taking 'conformally soft' limits $\Delta \to \mathbb{Z}$

[LD, Puhm, Strominger '18]

- dual notion to energetically soft limit $\omega \to 0$ -

QED $(J=1)$**:**

$\Delta \to 1$

- U(1) Kac-Moody current

$$J(z) = \mathcal{O}_{\Delta=1,J=1} \; : (1,0)$$

$$\langle J(z)\mathcal{O}_1 \cdots \mathcal{O}_n\rangle = \sum_{k=1}^{n} \frac{1}{(z-z_k)} \langle \mathcal{O}_1 \cdots \mathcal{O}_n\rangle$$

Celestial version of Weinberg's soft photon theorem!

Celestial currents

$$\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})$$

$$(h,\bar{h}) = \frac{1}{2}(\Delta + J, \Delta - J)$$

weights of the celestial operators

Celestial currents are obtained by taking 'conformally soft' limits $\Delta \to \mathbb{Z}$

[LD, Puhm, Strominger '18]

- dual notion to energetically soft limit $\omega \to 0$ -

Gravity $(J=2)$**:**

$\Delta \to 1$

- Supertranslation current

$$P(z,\bar{z}) = \partial_{\bar{z}} \mathcal{O}_{\Delta=1,J=2}$$

$$(\tfrac{3}{2}, -\tfrac{1}{2}+1) = (\tfrac{3}{2}, \tfrac{1}{2})$$

$$P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)} \mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$$

Celestial version of Weinberg's (leading) soft graviton theorem!

[Strominger '14][He, Lysov, Mitra, Strominger '15]
[LD, Puhm, Strominger '18][Stieberger, Taylor '19]

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Gravity $(J=2)$**:**

- 2d stress tensor $T(z) : (2,0)$!!

$$\boxed{T(z)\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{h}{(z-w)^2}\mathcal{O}_{h,\bar{h}}(w,\bar{w}) + \frac{\partial \mathcal{O}_{h,\bar{h}}(w,\bar{w})}{z-w}}$$

[LD, Puhm, Strominger][Stieberger, Taylor]
[Kapec, Mitra, Raclariu, Strominger][Cheung, de la Fuente, Sundrum][Fotopoulos, Stieberger,Taylor]

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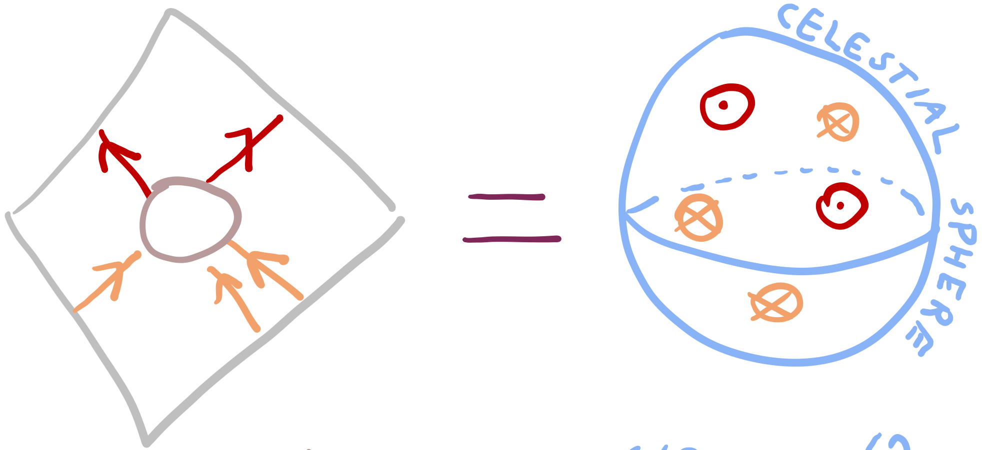
This promotes celestial operators to full **Virasoro** primaries on the celestial sphere!

[LD, Puhm, Strominger][Stieberger, Taylor]
[Kapec, Mitra, Raclariu, Strominger][Cheung, de la Fuente, Sundrum][Fotopoulos, Stieberger,Taylor]

Summary: celestial holography

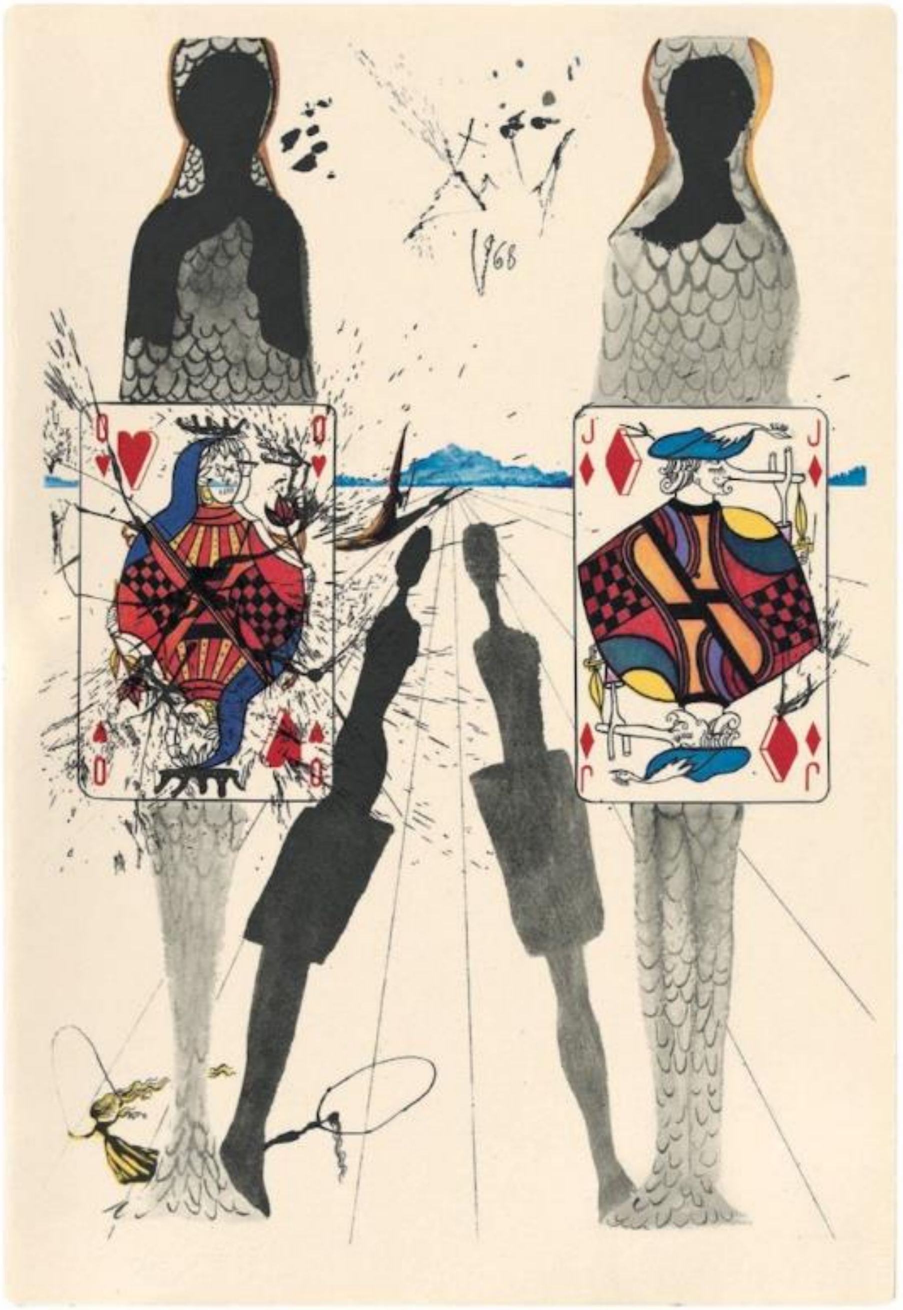


$\langle out|S|in \rangle$ = $\langle \mathcal{O}_{\Delta_1,J_1} \cdots \mathcal{O}_{\Delta_m,J_m} \rangle_{CCFT_2}$

$$\int_0^\infty d\omega\, \omega^{\Delta-1}$$

The soft sector of scattering is captures by celestial currents $\Delta \to \mathbb{Z}$

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

1. BMS & the S-matrix
2. Celestial holography
3. BMS fluxes vs celestial currents
4. Towards Carrollian holography

based on

2108.11969 w/ **Romain RUZZICONI**

2202.04702 PRL (2022) & to appear
w/ **Adrien FIORUCCI, Yannick HERFRAY** & **Romain RUZZICONI**

2205.11477 w/ **Kevin NGUYEN** & **Romain RUZZICONI**

Question

Which objects from the gravitational phase space

$$
\begin{aligned}
ds^2 &= -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \\
&\quad + \frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^z C_{zz}\mathrm{d}u\mathrm{d}z \\
&\quad + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots
\end{aligned}
$$

transform as conformal primaries under the action of extended BMS symmetries?

$$
\mathcal{O}_{h,\bar{h}}(z,\bar{z}) \to \left(\frac{\partial z}{\partial z'}\right)^h \left(\frac{\partial \bar{z}}{\partial \bar{z}'}\right)^{\bar{h}} \mathcal{O}_{h,\bar{h}}(z,\bar{z})
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How are they related to the celestial CFT currents?
What is their algebra?

$$P(z,\bar{z}) : (\tfrac{3}{2}, \tfrac{1}{2})$$
$$T(z) : (2,0)$$

BMS extended symmetries

BMS (supertanslations + superrotations) symmetries act as

$$\delta_{(f,Y)} C_{AB} = [f\partial_u + \mathcal{L}_Y - \frac{1}{2} D_C Y^C] C_{AB} - 2 D_A D_B f + \mathring{q}_{AB} D_C D^C f,$$

$$\delta_{(f,Y)} N_{AB} = [f\partial_u + \mathcal{L}_Y] N_{AB} - (D_A D_B D_C Y^C - \frac{1}{2} \mathring{q}_{AB} D_C D^C D_D Y^D),$$

$$\begin{aligned}\delta_{(f,Y)} M &= [f\partial_u + \mathcal{L}_Y + \frac{3}{2} D_C Y^C] M \\ &\quad + \frac{1}{8} D_C D_B D_A Y^A C^{BC} + \frac{1}{4} N^{AB} D_A D_B f + \frac{1}{2} D_A f D_B N^{AB},\end{aligned}$$

$$\begin{aligned}\delta_{(f,Y)} N_A &= [f\partial_u + \mathcal{L}_Y + D_C Y^C] N_A + 3 M D_A f - \frac{3}{16} D_A f N_{BC} C^{BC} \\ &\quad - \frac{1}{32} D_A D_B Y^B C_{CD} C^{CD} + \frac{1}{4}(2 D^B f + D^B D_C D^C f) C_{AB} \\ &\quad - \frac{3}{4} D_B f (D^B D^C C_{AC} - D_A D_C C^{BC}) + \frac{3}{8} D_A (D_C D_B f C^{BC}) \\ &\quad + \frac{1}{2}(D_A D_B f - \frac{1}{2} D_C D^C f \mathring{q}_{AB}) D_C C^{BC} + \frac{1}{2} D_B f N^{BC} C_{AC}.\end{aligned}$$

$$\xi^u = \mathcal{T} + \frac{u}{2}(\partial\mathcal{Y} + \bar\partial\bar{\mathcal{Y}}) \equiv f$$

$$\xi^z = \mathcal{Y} + \mathcal{O}(r^{-1}), \qquad \xi^{\bar z} = \bar{\mathcal{Y}} + \mathcal{O}(r^{-1}),$$

$$\xi^r = -\frac{r}{2}(\partial\mathcal{Y} + \bar\partial\bar{\mathcal{Y}}) + \mathcal{O}(r^0),$$

BMS symmetry generators

(nb: from now on, I will work with the flat 2d metric, for simplicity)

BMS charges and fluxes

BMS charges and fluxes

- At each cut $\{u = \text{constant}\}$ of $\mathscr{I}^+$, the prescription for BMS charges is

[Barnich, Troessaert '11] [He, Lysov, Mitra, Strominger '14] [Kapec, Lysov, Pasterski, Strominger '14]
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$$\boxed{\begin{aligned}
&Q_\xi = \frac{1}{8\pi G}\int_{\mathcal{S}} d^2z\,[2\mathcal{T}\mathcal{M} + \mathcal{Y}\bar{\mathcal{N}} + \bar{\mathcal{Y}}\mathcal{N}]\,,\\
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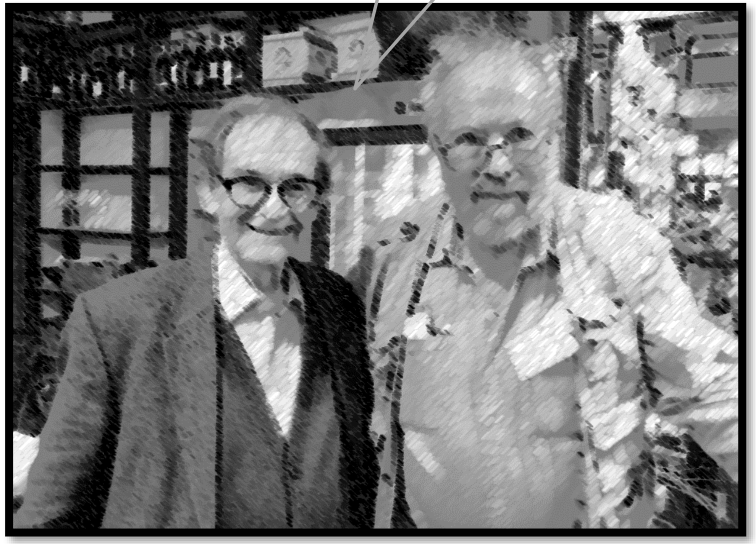
$$\mathcal{M} = -\frac{1}{2}(\Psi_2^0 + \bar{\Psi}_2^0)$$

$$\mathcal{N} = -\Psi_1^0 + u\bar{\partial}\Psi_2^0$$



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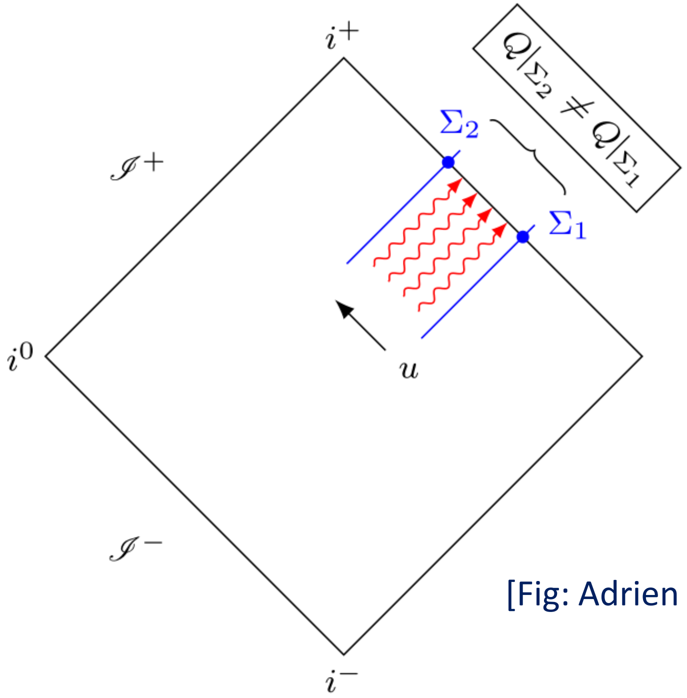

[Fig: Adrien Fiorucci]

- Outgoing radiation $\longrightarrow$ BMS charges are not conserved but satisfy flux balance laws

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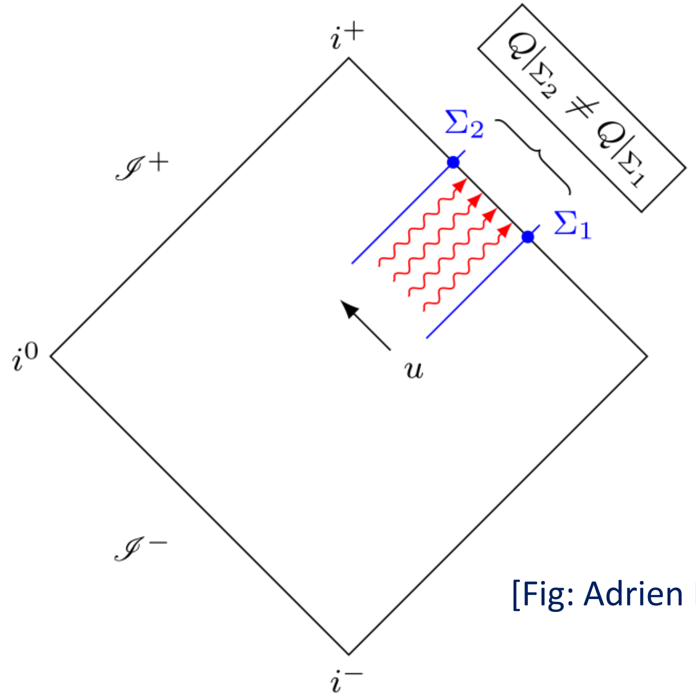

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- The integrated BMS fluxes are

$$F_\xi = \int_{-\infty}^{+\infty} du\,\partial_u Q_\xi = \int_{\mathcal{S}} d^2 z\,[\mathcal{T}\mathcal{P} + \mathcal{Y}\bar{\mathcal{J}} + \bar{\mathcal{Y}}\mathcal{J}]$$

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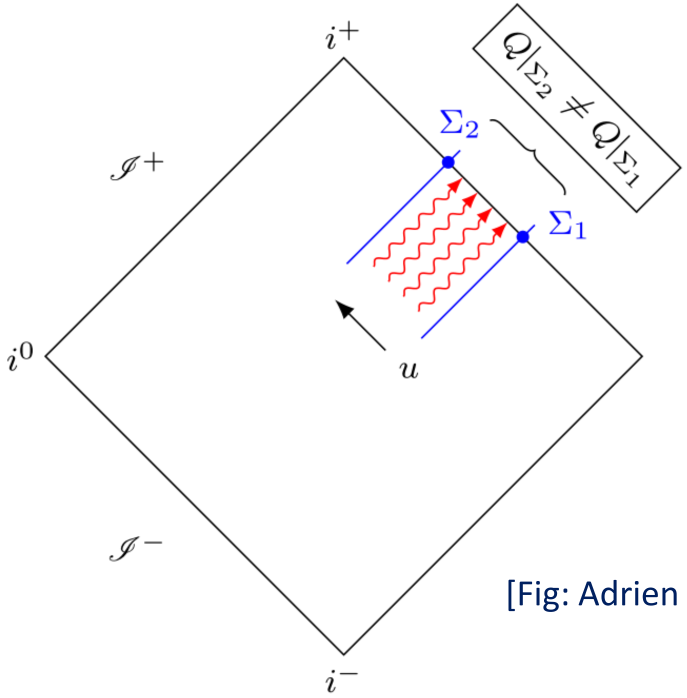

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one can check indeed that

$$\delta_{(\mathcal{T},\mathcal{Y},\bar{\mathcal{Y}})}\mathcal{P} = \Big[\mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + \frac{3}{2}\partial\mathcal{Y} + \frac{3}{2}\bar{\partial}\bar{\mathcal{Y}}\Big]\mathcal{P}$$: $\left(\frac{3}{2}, \frac{3}{2}\right)$ primary

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1. a refined analysis of the radiative phase space with superrotations

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$$\boxed{\mathcal{J} = \frac{1}{8\pi G}\int_{-\infty}^{+\infty} du\,\partial_u \mathcal{N}} \qquad (2,1)$$

- How are they related to the celestial CFT currents? What is their algebra?

The answer to these questions requires

1. a refined analysis of the radiative phase space with superrotations
2. a crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries

Extended radiative phase space

1. Refined analysis of the radiative phase space with superrotations

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  [Compère, Long '16][Compère, Fiorucci, Ruzziconi '18]

$$\widetilde{N}_{zz}(u,x) \equiv N_{zz}(u,x) - N^{vac}_{zz}(x)$$

tracefree part of the Geroch tensor

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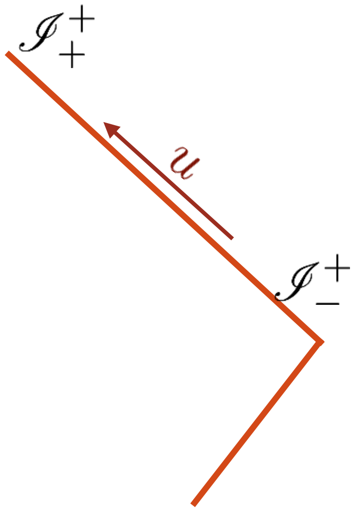

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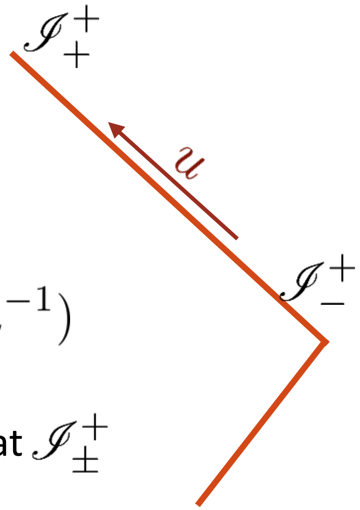

- fall-offs as $u \to \pm\infty$: $\quad N_{zz} = N^{vac}_{zz} + o(u^{-2})$

$$C_{zz} = (u + C_\pm)N^{vac}_{zz} - 2\partial^2 C_\pm + o(u^{-1})$$

value of the supertranslation field at $\mathscr{I}^+_\pm$

Extended radiative phase space

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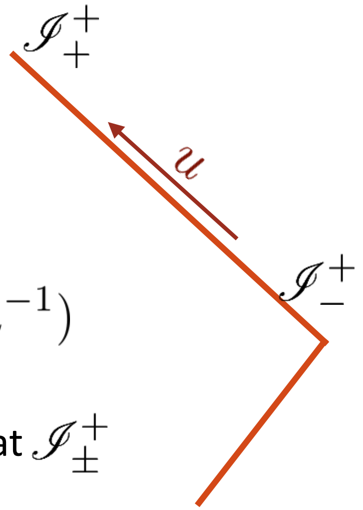

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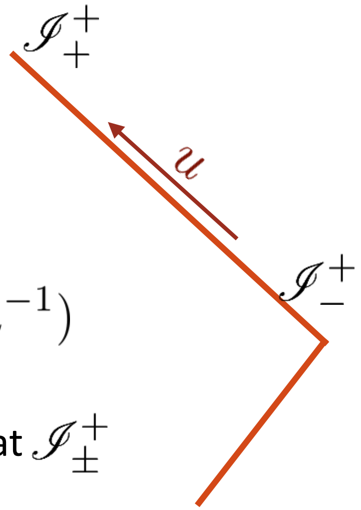

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  value of the supertranslation field at $\mathscr{I}^+_\pm$

- we also define (please bear with me) [Compère, Fiorucci, Ruzziconi '18][Campiglia, Laddha '21][LD, Nguyen, Ruzziconi '22]

$$\widetilde{C}_{zz} \equiv C_{zz} - uN^{vac}_{zz} - C^{(0)}_{zz}$$
$$C^{(0)}_{zz} \equiv -\mathscr{D}^2(C_+ + C_-)$$

$\mathscr{D}$: "superrotation covariant derivative"
[Campiglia, Peraza '20][LD, Ruzziconi '21][Barnich, Ruzziconi '21]

Soft fluxes and celestial currents

2 Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries

[LD, Ruzziconi '21]

Soft fluxes and celestial currents

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leading soft graviton operator

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- Remember: 'supertranslation current'

$$\Delta \to 1 \qquad P(z,\bar{z}) : \left(\tfrac{3}{2}, \tfrac{1}{2}\right) \qquad P(z,\bar{z})\mathcal{O}_{h,\bar{h}}(w,\bar{w}) \sim \frac{1}{(z-w)}\mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(w,\bar{w})$$

$$\boxed{\mathcal{P}^{soft}(z,\bar{z}) = \bar{\mathcal{D}}\, P(z,\bar{z}) + \mathcal{D}\, \bar{P}(z,\bar{z})}$$

Soft fluxes and celestial currents

2 Crucial split between 'hard' and 'soft' pieces of the flux such that *both* transform separately as Virasoro primaries [LD, Ruzziconi '21]

$$(2,1) \quad \bar{\mathcal{J}}^{hard} = \frac{1}{16\pi G}\int du \left[\frac{3}{2}\tilde{C}_{zz}\partial\tilde{N}_{\bar{z}\bar{z}} + \frac{1}{2}\tilde{N}_{\bar{z}\bar{z}}\partial\tilde{C}_{zz} + \frac{u}{2}\partial(\tilde{N}_{zz}\tilde{N}_{\bar{z}\bar{z}})\right]$$

$$\bar{\mathcal{J}}^{soft} = \frac{1}{16\pi G}\left[-\mathscr{D}^3\mathcal{N}^{(1)}_{\bar{z}\bar{z}} + \frac{3}{2}C^{(0)}_{zz}\mathscr{D}\mathcal{N}^{(0)}_{\bar{z}\bar{z}} + \frac{1}{2}\mathcal{N}^{(0)}_{\bar{z}\bar{z}}\mathscr{D}C^{(0)}_{zz}\right]$$

$$\mathcal{N}^{(0)}_{\bar{z}\bar{z}} = \int du\,\tilde{N}_{\bar{z}\bar{z}} \quad \leftarrow$$ leading soft graviton operator $\left(\frac{3}{2}, -\frac{1}{2}\right)$

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- This leads to a **corrected** celestial stress tensor!!

$$T(z) = \frac{i}{8\pi G}\int_S d^2w\, \frac{1}{z-w}\left(-\mathscr{D}^3\mathcal{N}^{(1)}_{\bar{w}\bar{w}} + \frac{3}{2}C^{(0)}_{ww}\mathscr{D}\mathcal{N}^{(0)}_{\bar{w}\bar{w}} + \frac{1}{2}\mathcal{N}^{(0)}_{\bar{w}\bar{w}}\mathscr{D}C^{(0)}_{ww}\right)$$

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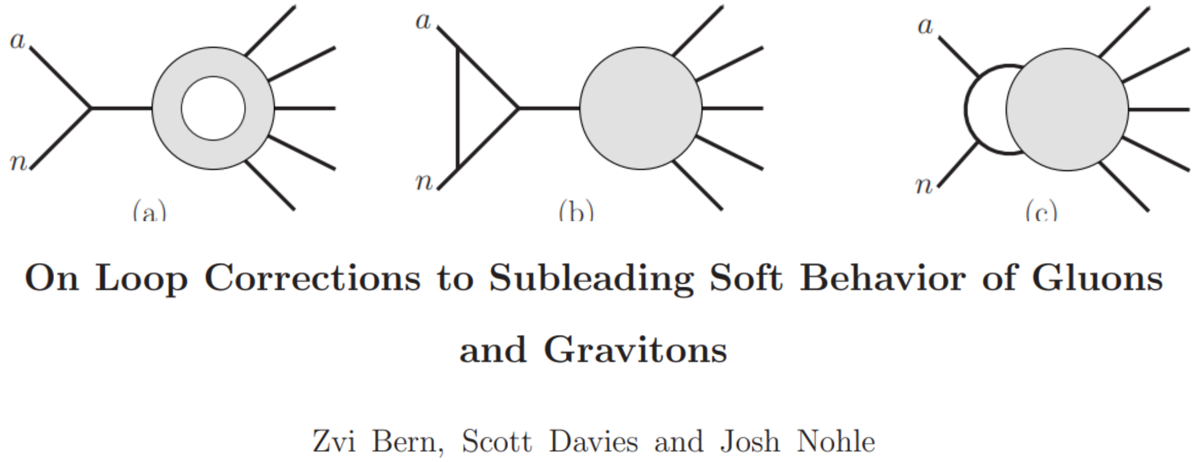
- Remarkably, the new quadratic pieces exactly account for the one-loop corrections of Cachazo-Strominger subleading soft graviton theorem!

  [LD, Nguyen, Ruzziconi '22][Pasterski '22]



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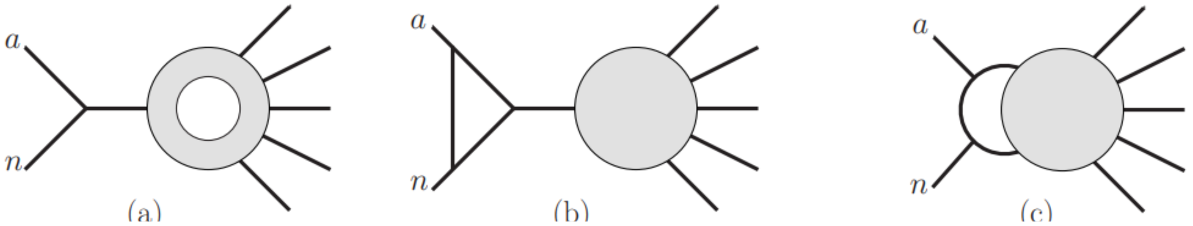


On Loop Corrections to Subleading Soft Behavior of Gluons and Gravitons

Zvi Bern, Scott Davies and Josh Nohle

Since it is one-loop exact, this shows that superrotations are genuine symmetries of the gravitational S-matrix beyond semiclassical level.

Celestial CFT OPE from BMS flux algebra

- Finally, one can deduce the following OPE for the celestial CFT [LD, Ruzziconi '21]

$$P(z,\bar z)P(w,\bar w) \sim 0$$

$$P(z,\bar z)\bar P(w,\bar w) \sim 0$$

$$T(z)P(w,\bar w) \sim \frac{1}{(z-w)}\partial_w P(w,\bar w) + \frac{3/2}{(z-w)^2}P(w,\bar w)$$

$$\bar T(\bar z)P(w,\bar w) \sim \frac{1}{(\bar z-\bar w)}\partial_{\bar w} P(w,\bar w) + \frac{1/2}{(\bar z-\bar w)^2}P(w,\bar w)$$

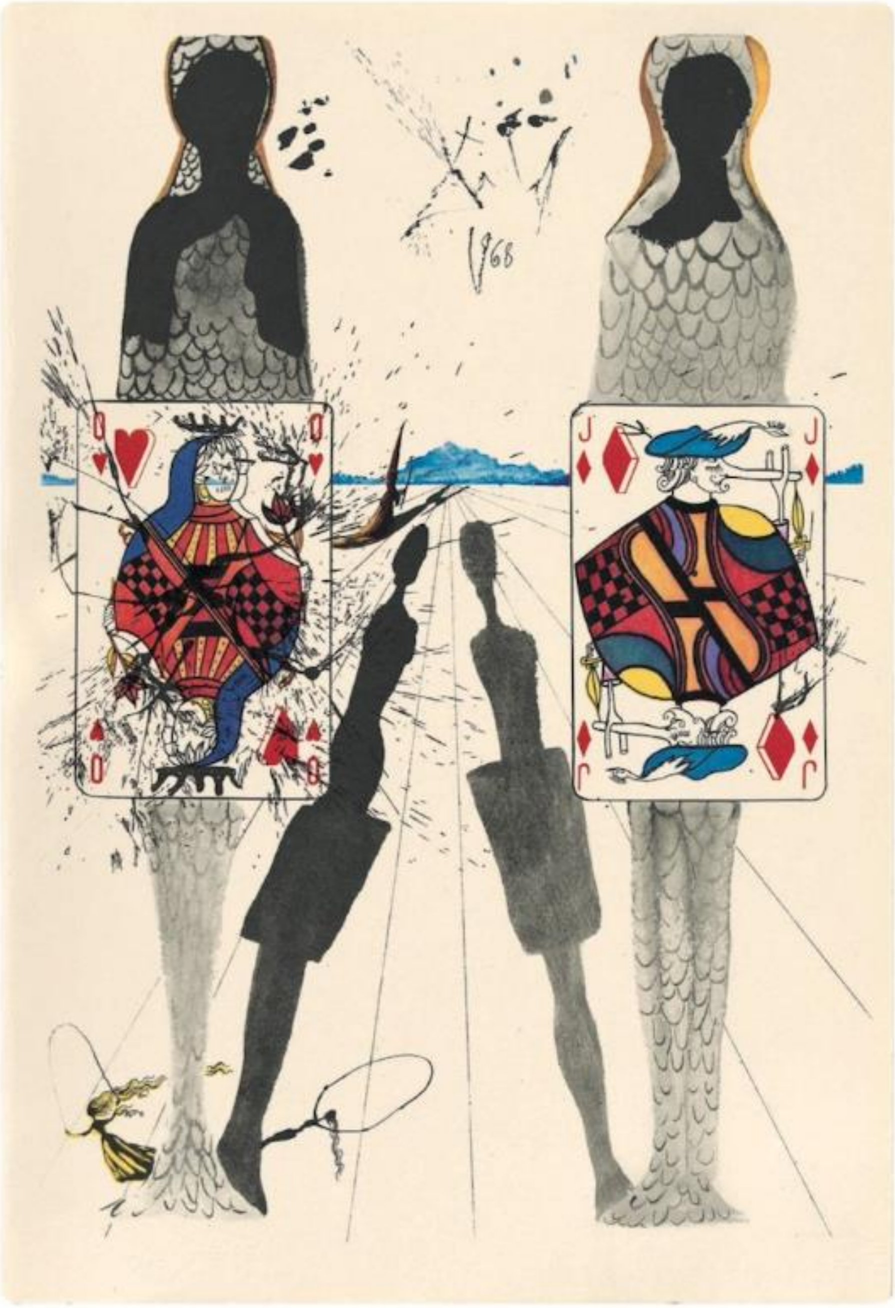
$$P(z,\bar z)T(w) \sim \frac{1/2}{(z-w)}\partial_w P(w,\bar z) + \frac{3/2}{(z-w)^2}P(w,\bar z),$$

$$\boxed{T(z)T(w) \sim \frac{1}{(z-w)}\partial_w T(w) + \frac{2}{(z-w)^2}T(w)} \rightarrow \boxed{c=0}$$

$$\bar T(\bar z)T(w) \sim 0$$

see also [Fotopoulos, Stieberger Taylor, Zhu '19]

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

1. BMS & the S-matrix
2. Celestial holography
3. BMS fluxes vs celestial currents
4. Towards Carrollian holography

based on

2108.11969 w/ **Romain RUZZICONI**

2202.04702 PRL (2022) & to appear
w/ **Adrien FIORUCCI, Yannick HERFRAY** & **Romain RUZZICONI**

2205.11477 w/ **Kevin NGUYEN** & **Romain RUZZICONI**

3 bases for the scattering problem

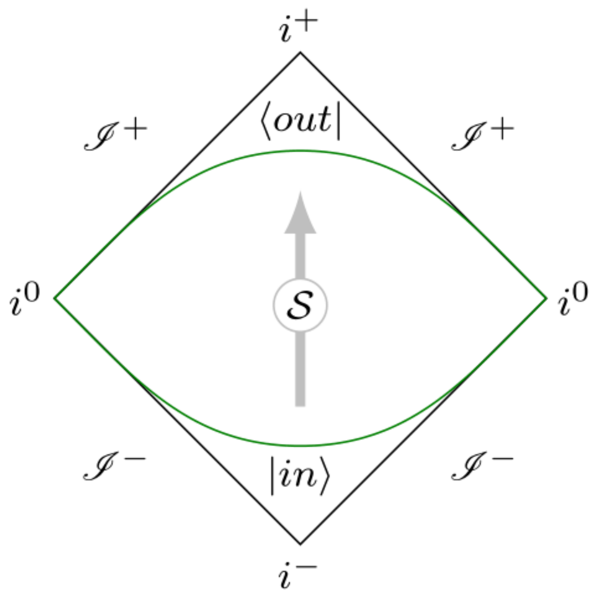
Momentum basis $\mathcal{A}_N = \langle out|\mathcal{S}|in\rangle_{\text{momentum}}$

i.e. the usual formulation of the scattering amplitudes

$$|\omega, z, \bar{z}, \pm s\rangle = a_{\pm}^{(s)}(\omega, z, \bar{z})^{\dagger}\,|0\rangle$$

$$|in\rangle = |\omega_1, z_1, \bar{z}_1, \pm s_1\rangle \otimes \cdots \otimes |\omega_n, z_n, \bar{z}_n, \pm s_n\rangle$$

$$\langle out| = \langle \omega_{n+1}, z_{n+1}, \bar{z}_{n+1}, \pm s_{n+1}| \otimes \cdots \otimes \langle \omega_N, z_N, \bar{z}_N, \pm s_N|$$



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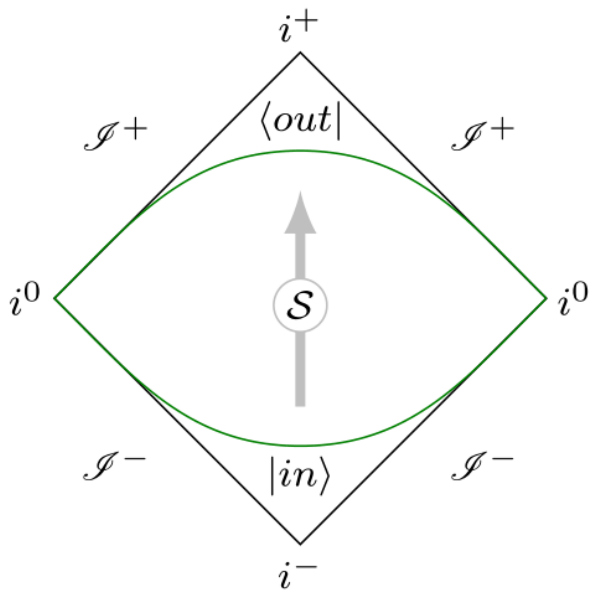
Celestial basis $$\mathcal{M}_N = \langle out|\mathcal{S}|in\rangle_{\text{boost}}$$

used in celestial holography, obtained via Mellin transforms

$$|\Delta, z, \bar{z}, \pm s\rangle = a^{(s)}_{\Delta,\pm}(z,\bar{z})^\dagger\,|0\rangle = \int_0^{+\infty} d\omega\,\omega^{\Delta-1}\,|\omega, z, \bar{z}, \pm s\rangle$$

$$\boxed{\mathcal{M}_N = \int_0^{+\infty} d\omega_1\,\omega_1^{\Delta_1-1}\cdots\int_0^{+\infty} d\omega_N\,\omega_N^{\Delta_N-1}\mathcal{A}_N}$$

loads of these celestial amplitudes have been explicitly computed recently



3 bases for the scattering problem $u \leftrightarrow \omega \leftrightarrow \Delta$

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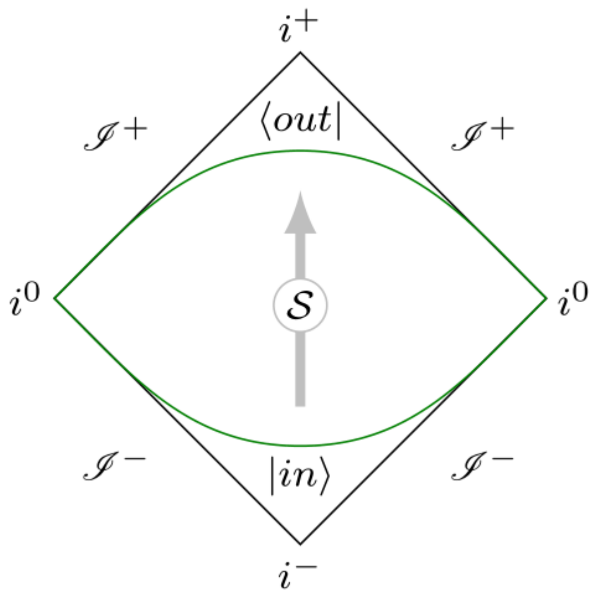


Position space basis $\mathcal{C}_N = \langle out|\mathcal{S}|in\rangle_{\text{position}}$

obtained via Fourier transforms from momentum basis

$$\mathcal{C}_N = \int_0^{+\infty} d\omega_1\,e^{-i\omega_1 u_1}\dots\int_0^{+\infty} d\omega_N\,e^{i\omega_N u_N}\mathcal{A}_N$$

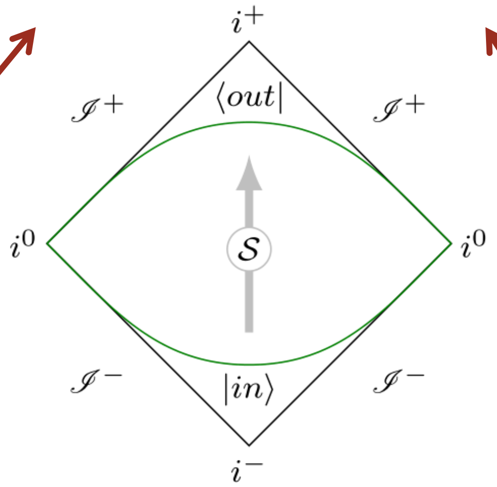
3 bases for the scattering problem $u \leftrightarrow \omega \leftrightarrow \Delta$

Momentum basis

$$\mathcal{A}_N = \langle out|\mathcal{S}|in\rangle_{\text{momentum}}$$

$$\int_0^{+\infty} d\omega\, \omega^{\Delta-1}$$

Mellin



Fourier

$$\int_0^{+\infty} d\omega\, e^{-i\omega u}$$

Celestial basis

$$\mathcal{M}_N = \langle out|\mathcal{S}|in\rangle_{\text{boost}}$$

Position space basis

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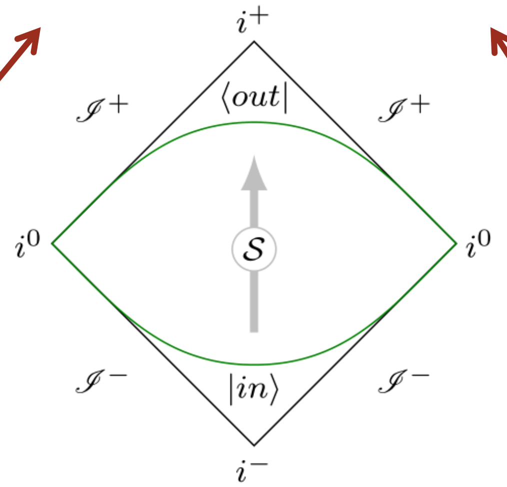

Fourier

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Position space basis

$$\mathcal{C}_N = \langle out|\mathcal{S}|in\rangle_{\text{position}}$$

Mellin + Fourier

$$\int_{-\infty}^{+\infty} du\, u^{-\Delta}$$

see also ‘extrapolate dictionary’ [Pasterski, Puhm, Trevisani ’21]

Towards Carrollian holography…

Celestial basis ⟷ **Position space basis**

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Mellin + Fourier

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The S-matrix has an intrinsic holographic flavor.

In celestial holography, scattering elements -written in a boost eigenstate basis- are interpreted as correlation functions of a 'celestial CFT'.

Towards Carrollian holography…

Celestial basis $\longleftrightarrow$ **Position space basis**

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Can we interpret S-matrix elements as correlation functions of a conformal Carrollian field theory?

[LD, Fiorucci, Herfray, Ruzziconi ’22]

Towards Carrollian holography…

Carrollian holography

S-matrix elements as correlation functions of a conformal Carrollian field theory

Dictionnary between celestial operators and conformal Carrollian fields (defined as the boundary value at null infinity of bulk field operators)

$$\mathcal{O}_{\Delta,J}(z,\bar{z}) \propto \int_{-\infty}^{+\infty} du\, u^{-\Delta}\, \Phi_{(k,\bar{k})}(u,z,\bar{z})$$

$$k = \frac{1}{2}(1+J)\,, \qquad \bar{k} = \frac{1}{2}(1-J)$$

[LD, Fiorucci, Herfray, Ruzziconi ’22]

See Romain Ruzziconi’s talk!

Towards Carrollian holography…

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Proof that the sourced conformal Carrollian Ward identities reproduce the celestial Ward identities

See Romain Ruzziconi's talk!

Carrollian holography

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A different dictionary was proposed in [Bagchi, Banerjee, Basu & Dutta ‘22] using a “modified Mellin transform” [see Arjun & Sudipta’s talk]

$$\tilde{\mathcal{M}}\left(\{u_i, z_i, \bar{z}_i, h_i, \bar{h}_i, \epsilon_i\}\right) = \prod_{i=1}^{n} \int_0^{\infty} d\omega_i \omega_i^{\Delta_i - 1} e^{-i\epsilon_i \omega_i u_i} S\left(\{\epsilon_i \omega_i, z_i, \bar{z}_i, \sigma_i\}\right) = \prod_i \langle \phi^{\epsilon_i}_{h_i,\bar{h}_i}(u_i, z_i, \bar{z}_i)\rangle$$

Towards Carrollian holography…

Carrollian holography

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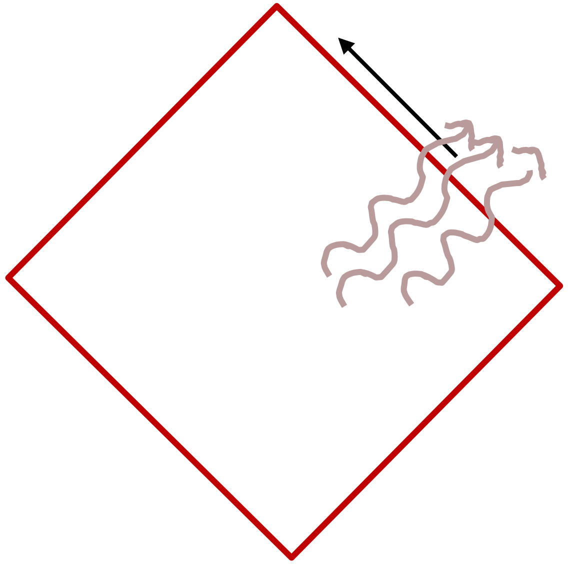
while our Carrollian correlators depend only on the time – not on the celestial weights –
(cf. the map of scattering bases)

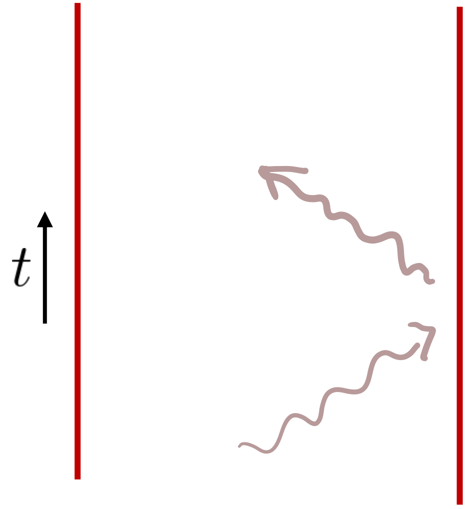
Conclusion and outlook

Flat space holography

Very different from holography in Anti-de Sitter (AdS acts like a box)!

Flat holography forces us to deal with leaks of radiation through the boundary.





Infinitely many **symmetry constraints** beyond conformal invariance.

e.g. constraints coming from supertranslation symmetry have no analog in usual holography.

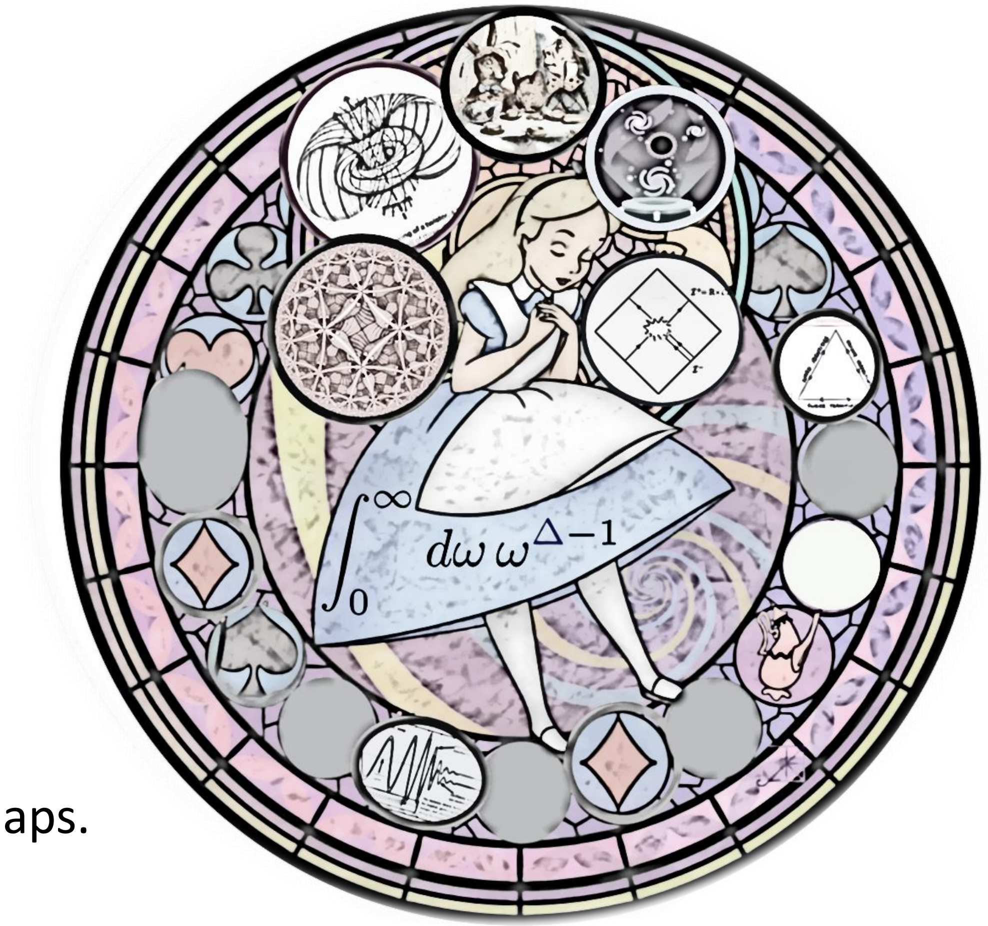
$$P(z)\mathcal{O}_\Delta(w,\bar{w}) \sim \frac{1}{z-w}\mathcal{O}_{\Delta+1}(w,\bar{w})$$

Two roads: celestial vs Carrollian

Some of the outstanding challenges ahead

what is a Celestial CFT? what is a conformal Carrollian CFT? Beyond kinematics? Top-down constructions?

full tower of currents $\Delta \in \mathbb{Z}$
link with AdS/CFT, dS/CFT
building representations
log corrections
bootstrapping CCFT
higher dimensions
massive particles
relationship to string theory
adding black holes
…



Many things remain to be understood !

We have to keep building up the celestial and Carrollian maps.
Let's see where it leads us.

Thank you for listening!