Holographic Lorentz and Carroll Frames

Arnaud DELFANTE

Service de Physique de l'Univers, Champs et Gravitation

Université de Mons

September 14th, 2022 – 2^{nd} Carroll Workshop

Based on [A. Campoleoni, L. Ciambelli, AD, C. Marteau, P. M. Petropoulos, R. Ruzziconi (2208.07575)]

Outline

I. **Plan and Motivations**

II. Covariant Bondi gauge in AdS and holographic frames

III. Flat limit and boundary Carroll frames

IV. Summary

I. Plan and Motivations

- Study of the classical phase space of 3D asymptotically AdS gravity:

  Select the allowed metric fluctuations at infinity [Brown-Henneaux '86]

- No requirement to fix any particular gauge but it is often convenient

  For example: Fefferman–Graham, Bondi gauge

- In this talk: covariant Bondi gauge, allow for a smooth flat-space limit

  Originally from fluid/gravity correspondence (see K. Siampos' talk)

  Study holographically Lorentz and Carroll-boost anomalies

Outline

I. Plan and Motivations

II. Covariant Bondi gauge in AdS and holographic frames

III. Flat limit and boundary Carroll frames

IV. Summary

II. Covariant Bondi gauge in AdS

- Key idea: relax the AdS Bondi gauge → dependence on the boundary dyad

$$\mathrm{d}s^2_{\mathrm{AdS}} = \frac{2}{k^2}\,\mathrm{u}\,(\mathrm{d}r + r\,\mathrm{A}) + r^2 g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \frac{8\pi\mathcal{G}}{k^4}\,\mathrm{u}\,(\varepsilon\,\mathrm{u} + \chi *\mathrm{u})$$

- Boundary metric and Cartan frame:

$$g_{\mu\nu} = \frac{1}{k^2}\left(-u_\mu u_\nu + *u_\mu *u_\nu\right)$$

Weyl connection: [Loganayagam '08]

$$\mathrm{A} = \frac{1}{k^2}\left(\Theta^* *\mathrm{u} - \Theta\,\mathrm{u}\right), \qquad \Theta = \nabla_\mu u^\mu, \qquad \Theta^* = \nabla_\mu *u^\mu$$

- Energy-momentum tensor: [Brown-York '93]

$$\mathrm{T} = \mathrm{T}(\varepsilon, \chi) \;: \qquad \nabla_\mu T^{\mu\nu} = 0\,, \qquad T^\mu{}_\mu = \frac{R}{16\pi\mathcal{G}k}$$

II. Covariant Bondi gauge in AdS: residual symmetries

- Asymptotic Killing vectors: [Ciambelli-Marteau-Petropoulos-Ruzziconi '20]

$$v = \left(\xi^\mu - \frac{1}{k^2 r}\,\eta *u^\mu\right)\partial_\mu + \left(r\,\sigma + \frac{1}{k^2}\left(*u^\nu\,\partial_\nu\eta + \Theta^*\eta\right) + \frac{4\pi\mathcal{G}}{k^2 r}\,\chi\,\eta\right)\partial_r$$

$\hookrightarrow$ bdy diffeomorphisms $\xi^\mu(x)$, Weyl rescalings $\sigma(x)$ and Lorentz boosts $\eta(x)$

$$\delta_{(\xi,\sigma,\eta)}\mathrm{u} = \mathcal{L}_\xi\mathrm{u} + \sigma\,\mathrm{u} + \eta\,*\mathrm{u}\,, \quad \delta_{(\xi,\sigma,\eta)}*\mathrm{u} = \mathcal{L}_\xi\,*\mathrm{u} + \sigma\,*\mathrm{u} + \eta\,\mathrm{u}$$

where

$$\delta_{(\xi,\sigma,\eta)}g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu} + 2\,\sigma\,g_{\mu\nu}$$

and

$$\begin{pmatrix}\mathrm{u}'\\ *\mathrm{u}'\end{pmatrix} = \begin{pmatrix}\cosh\eta & \sinh\eta\\ \sinh\eta & \cosh\eta\end{pmatrix}\begin{pmatrix}\mathrm{u}\\ *\mathrm{u}\end{pmatrix}$$

- Question: What are the asymptotic symmetries?

II. Covariant Bondi gauge in AdS: symplectic structure

- Einstein–Hilbert presymplectic potential: [Iyer-Wald '94]

$$\Theta_{\text{EH}}[G;\delta G] = \frac{\sqrt{-G}}{32\pi\mathcal{G}}\left[\nabla^N \delta G_{PN}\, G^{PM} - \nabla^M \delta G_{PN}\, G^{PN}\right] \epsilon_{MQS}\, \mathrm{d}x^Q \wedge \mathrm{d}x^S$$

Radial divergences: need for renormalization

$$\Theta^{(r)}_{\text{EH}}[G;\delta G] = r^2\,\Theta_{(2)} + r\,\Theta_{(1)} + \Theta_{(0)} + \mathcal{O}\left(r^{-1}\right)$$

Ambiguous definition:

$$\Theta_{\text{EH}}[G;\delta G] \to \Theta_{\text{EH}}[G;\delta G] + \delta Z[G] - \mathrm{d}\,Y[G;\delta G]$$

- Choices of prescription:
  - i. same results as obtained in FFG [de Haro-Solodukhin-Skenderis (2000)]
  - ii. presymplectic potential that remains finite in the flat-space limit

II. Covariant Bondi gauge in AdS: symplectic structure

- Einstein–Hilbert presymplectic potential: [Iyer-Wald '94]

$$\Theta_{\mathrm{EH}}[G;\delta G] = \frac{\sqrt{-G}}{32\pi\mathcal{G}} \left[\nabla^N \delta G_{PN}\, G^{PM} - \nabla^M \delta G_{PN}\, G^{PN}\right] \epsilon_{MQS}\, \mathrm{d}x^Q \wedge \mathrm{d}x^S$$

  Radial divergences: need for renormalization

$$\Theta^{(r)}_{\mathrm{EH}}[G;\delta G] = r^2\, \Theta_{(2)} + r\, \Theta_{(1)} + \Theta_{(0)} + \mathcal{O}\left(r^{-1}\right)$$

  Ambiguous definition:

$$\Theta_{\mathrm{EH}}[G;\delta G] \to \Theta_{\mathrm{EH}}[G;\delta G] + \delta Z[G] - \mathrm{d}Y[G;\delta G]$$

- Choices of prescription:

  i. ~~same results as obtained in FFG~~ [de Haro-Solodukhin-Skenderis (2000)]

  ii. <u>presymplectic potential that remains finite in the flat-space limit</u>

II. Covariant Bondi gauge in AdS: surface charges

- Conformal gauge: conformally flat bdy metric $(x^{\pm} = \phi \pm k\, u)$

$$\mathrm{d}s^2 = \mathrm{e}^{2\varphi}\, \mathrm{d}x^+ \mathrm{d}x^-$$

Parametrization of the Cartan frame: $(\varphi = \varphi(x^+, x^-),\, \zeta = \zeta(x^+, x^-))$

$$\mathrm{u} = -\frac{k}{2}\, \mathrm{e}^{\varphi} \left(\mathrm{e}^{\zeta}\, \mathrm{d}x^+ - \mathrm{e}^{-\zeta}\, \mathrm{d}x^- \right), \qquad *\mathrm{u} = \frac{k}{2}\, \mathrm{e}^{\varphi} \left(\mathrm{e}^{\zeta}\, \mathrm{d}x^+ + \mathrm{e}^{-\zeta}\, \mathrm{d}x^- \right)$$

- Charges associated with the Weyl–Lorentz symmetries: $(\delta_v \varphi = \varpi,\, \delta_v \zeta = h)$

$$Q_{(\varpi, h)} = \frac{1}{4\pi \mathcal{G} k} \int_0^{2\pi} \mathrm{d}\phi \left(h\, (\partial_- - \partial_+)\, \zeta \right)$$

$\hookrightarrow$ integrable and non-conserved: Lorentz is anomalous, Weyl is pure gauge

- Anomaly in the Lorentz symmetry in the dual theory $(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$

$$\delta_{(\xi,\sigma,\eta)} S_{\mathrm{L}} = \int \left(\eta \, \frac{F}{8\pi\mathcal{G}}\right) \mathsf{Vol}_{\partial\mathcal{M}}$$

$\hookrightarrow$ flat limit: yes

- If we choose the first prescription $\rightarrow$ anomaly in the Weyl symmetry in the dual theory [Alessio-Barnich-Ciambelli-Mao-Ruzziconi '20]

$$\delta_{(\xi,\sigma,\eta)} S_{\mathrm{W}} = \int \left(\sigma \, \frac{R}{8\pi\mathcal{G}}\right) \mathsf{Vol}_{\partial\mathcal{M}}$$

$\hookrightarrow$ flat limit: no

- Displacement of the anomaly: two different representatives in the same cohomology class $\rightarrow$ BRST formulation [Ciambelli-Leigh-Jia (to appear)]

Outline

III. Flat limit and boundary Carroll frames

- **Key idea:** null gauges in AdS admit a proper flat limit

  $\hookrightarrow$ in the flat limit: timelike AdS bdy $\to$ null manifold

  $\hookrightarrow$ bdy metric $\to$ degenerate $\Rightarrow$ Carrollian geometry

- **Bulk metric:** [Campoleoni-Ciambelli-Marteau-Petropoulos-Siampos '19]

$$\mathrm{d}s^2_{\text{Flat}} = \lim_{k\to 0} \mathrm{d}s^2_{\text{AdS}} = 2\,\mu\,(\mathrm{d}r + r\mathcal{A}) + r^2\,\mu^*\,\mu^* + 8\pi\mathcal{G}\,\mu\,(\epsilon\,\mu + \alpha\,\mu^*)$$

$\hookrightarrow$ small-k behavior for the line element quantities:

$$\mu = \lim_{k\to 0}\frac{\mathrm{u}}{k^2}\,, \quad \mu^* = \lim_{k\to 0}\frac{*\mathrm{u}}{k}\,, \quad \upsilon = \lim_{k\to 0} u\,, \quad \upsilon_* = \lim_{k\to 0}\frac{*u}{k}\,,$$

$$\alpha = \lim_{k\to 0}\frac{\chi}{k}\,, \quad \epsilon = \lim_{k\to 0}\varepsilon\,, \quad \mathcal{A} = \lim_{k\to 0}\mathrm{A} = \mu^*\,\theta^* - \mu\,\theta$$

III. Flat limit: residual symmetries

- Asymptotic Killing vectors: [Ciambelli-Marteau-Petropoulos-Ruzziconi '20]

$$v = \left(\xi^\mu - \frac{1}{r}\,\lambda\,\upsilon_*^\mu\right)\partial_\mu + \left(r\,\sigma + \upsilon_*^\nu\partial_\nu\lambda + \theta^*\lambda + \frac{4\pi\mathcal{G}}{r}\,\alpha\,\lambda\right)\partial_r$$

$\hookrightarrow$ bdy diffeomorphisms $\xi^\mu(x)$, Weyl rescalings $\sigma(x)$ and Carroll boosts $\lambda(x)$

$$\lambda(x) = \lim_{k\to 0}\frac{\eta(x)}{k}$$

- Residual symmetries: transformations of the bdy Carroll dyad

$$\delta_{(\xi,\sigma,\lambda)}\mu = \mathcal{L}_\xi\mu + \sigma\,\mu + \lambda\,\mu^*, \qquad \delta_{(\xi,\sigma,\lambda)}\mu^* = \mathcal{L}_\xi\mu^* + \sigma\,\mu^*$$

- Next step: study the symplectic structure

$\hookrightarrow$ only the 2nd AdS prescription remains finite in the flat-space limit

III. Flat limit: surface charges and anomalies

- Conformal gauge: parametrization of the Carrollian dyad $(\beta = \lim_{k\to 0} \frac{\zeta}{k})$

$$\mu = -e^{\varphi}\left(du + \beta\, d\phi\right), \quad \mu^* = e^{\varphi}\, d\phi$$

Charges associated with the Weyl–boost symmetries: $(\delta_v \varphi = \varpi,\ \delta_v \beta = \tilde{h})$

$$Q_{(\varpi,\tilde{h})} = \frac{1}{4\pi\mathcal{G}} \int_0^{2\pi} d\phi \left(\partial_u \tilde{h}\, \beta\right)$$

$\hookrightarrow$ integrable, non-conserved: Carroll boost is anomalous, Weyl is pure gauge

- Anomalies: $(\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu)$

$$\delta_{(\xi,\sigma,\lambda)} S_{\mathrm{C}} = \int \left(\lambda \frac{\mathcal{F}}{8\pi\mathcal{G}}\right) \mathrm{vol}_{\partial\mathcal{M}}$$

$\hookrightarrow$ new holographic prediction, calling for further investigation

Outline

IV. Summary

Main goal:

- Explore the charges of 3D (AdS or flat) gravity in covariant Bondi gauge

  $\hookrightarrow$ bdy diffeomorphisms, Weyl rescalings and local frame boosts

Results:

- Divergences in the symplectic structure
- Renormalization via ambiguities
- Surface charges and anomalies
- New holographic Carrollian prediction

Future possibilities:

- Relate to asymptotic corner group [Donnelly-Freidel '16, Freidel-Geiller-Pranzetti '20, Ciambelli-Leigh-Pai '21]
- Connect to the celestial holography proposal [Strominger '17, Pasterski-Pate-Raclariu '21, Donnay-Fiorucci-Herfray-Ruzziconi '22]
- Extension to higher dimensions [Petkou-Petropoulos-Betancour-Siampos '22]

IV. Summary

Main goal:

- Explore the charges of 3D (AdS or flat) gravity in covariant Bondi gauge

  $\hookrightarrow$ bdy diffeomorphisms, ~~Weyl rescalings~~ and local frame boosts

Results:

- Divergences in the symplectic structure
- Renormalization via ambiguities
- Surface charges and anomalies
- New holographic Carrollian prediction

Future possibilities:

- Relate to asymptotic corner group [Donnelly-Freidel '16, Freidel-Geiller-Pranzetti '20, Ciambelli-Leigh-Pai '21]
- Connect to the celestial holography proposal [Strominger '17, Pasterski-Pate-Raclariu '21, Donnay-Fiorucci-Herfray-Ruzziconi '22]
- Extension to higher dimensions [Petkou-Petropoulos-Betancour-Siampos '22]

IV. Summary

"I'm Late", Alice in Wonderland, White Rabbit, by Sir John Tenniel

Thank you for listening!

References I

J. D. Brown and M. Henneaux, "Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity," Commun. Math. Phys. **104** (1986), 207-226

I. Papadimitriou and K. Skenderis, "Thermodynamics of asymptotically locally AdS spacetimes," JHEP **08** (2005), 004

L. Ciambelli and R. G. Leigh, "Weyl Connections and their Role in Holography," Phys. Rev. D **101** (2020) no.8, 086020

F. Alessio, G. Barnich, L. Ciambelli, P. Mao and R. Ruzziconi, "Weyl charges in asymptotically locally AdS_3 spacetimes," Phys. Rev. D **103** (2021) no.4, 046003

L. Ciambelli, C. Marteau, A. C. Petkou, P. M. Petropoulos and K. Siampos, "Flat holography and Carrollian fluids," JHEP **07** (2018), 165

A. Campoleoni, L. Ciambelli, A. Delfante, C. Marteau, P. M. Petropoulos and R. Ruzziconi, "Holographic Lorentz and Carroll Frames," [arXiv:2208.07575 [hep-th]].

References II

- A. Campoleoni, L. Ciambelli, C. Marteau, P. M. Petropoulos and K. Siampos, "Two-dimensional fluids and their holographic duals," Nucl. Phys. B **946** (2019), 114692
- R. Loganayagam, "Entropy Current in Conformal Hydrodynamics," JHEP **05** (2008), 087
- L. Ciambelli, C. Marteau, P. M. Petropoulos and R. Ruzziconi, "Gauges in Three-Dimensional Gravity and Holographic Fluids," JHEP **11** (2020), 092
- J. Lee and R. M. Wald, "Local symmetries and constraints," J. Math. Phys. **31** (1990), 725-743
- V. Iyer and R. M. Wald, "Some properties of Noether charge and a proposal for dynamical black hole entropy," Phys. Rev. D **50** (1994), 846-864
- S. de Haro, S. N. Solodukhin and K. Skenderis, "Holographic reconstruction of space-time and renormalization in the AdS / CFT correspondence," Commun. Math. Phys. **217** (2001), 595-622

References III

- K. Skenderis, "Lecture notes on holographic renormalization," Class. Quant. Grav. **19** (2002), 5849-5876
- R. A. Bertlmann, "Anomalies in quantum field theory,"
- R. Loganayagam, "Entropy Current in Conformal Hydrodynamics," JHEP **05** (2008), 087
- V. E. Hubeny, S. Minwalla and M. Rangamani, "The fluid/gravity correspondence," [arXiv:1107.5780 [hep-th]].
- G. Barnich, A. Gomberoff and H. A. Gonzalez, "The Flat limit of three dimensional asymptotically anti-de Sitter spacetimes," Phys. Rev. D **86** (2012), 024020
- C. Duval, G. W. Gibbons and P. A. Horvathy, "Conformal Carroll groups and BMS symmetry," Class. Quant. Grav. **31** (2014), 092001
- C. Duval, G. W. Gibbons and P. A. Horvathy, "Conformal Carroll groups," J. Phys. A **47** (2014) no.33, 335204

References IV

- W. Donnelly and L. Freidel, "Local subsystems in gauge theory and gravity," JHEP **09** (2016), 102
- L. Freidel, M. Geiller and D. Pranzetti, "Edge modes of gravity. Part I. Corner potentials and charges," JHEP **11** (2020), 026
- L. Ciambelli, R. G. Leigh and P. C. Pai, "Embeddings and Integrable Charges for Extended Corner Symmetry," Phys. Rev. Lett. **128** (2022), 171302
- A. Strominger, "Lectures on the Infrared Structure of Gravity and Gauge Theory," [arXiv:1703.05448 [hep-th]].
- S. Pasterski, M. Pate and A. M. Raclariu, "Celestial Holography," [arXiv:2111.11392 [hep-th]].
- L. Donnay, A. Fiorucci, Y. Herfray and R. Ruzziconi, "Carrollian Perspective on Celestial Holography," Phys. Rev. Lett. **129** (2022) no.7, 071602

References V

A. C. Petkou, P. M. Petropoulos, D. R. Betancour and K. Siampos, "Relativistic Fluids, Hydrodynamic Frames and their Galilean versus Carrollian Avatars," [arXiv:2205.09142 [hep-th]].